Structural Engineering and Mechanics, *Vol. 56, No. 6 (2015) 1063-1093*
DOI: http://dx.doi.org/10.12989/sem.2015.56.6.1063

New indices of structural robustness and structural fragility

João André[*1], Robert Beale[2a] and António M. Baptista[1b]

[1]Department of Structures, National Laboratory for Civil Engineering, Av. Brasil, 101, 1700-066 Lisbon, Portugal
[2]Department of Mechanical Engineering and Mathematical Sciences, Oxford Brookes University, Wheatley Campus, OX33 1HX Oxford, UK

(Received August 6, 2015, Revised November 20, 2015, Accepted December 2, 2015)

Abstract. Structural robustness has become an important design variable. However, based on the existing definitions of structural robustness it is often difficult to analyse and evaluate structural robustness, and sometimes not efficient since they mix structural robustness with several other structural variables. This paper concerns the development of a new structural robustness definition, and structural robustness and structural fragility indices. The basis for the development of the new indices is the analysis of the damage energy of structural systems for a given hazard scenario and involves a criterion to define an "unavoidable collapse" state. Illustrative examples are given detailing the steps and calculations needed to obtain values for both the structural robustness and the structural fragility indices. Finally, this paper presents the main advantages of the newly proposed definition and indices for the structural risk analysis over existing traditional methods.

Keywords: structural robustness; structural fragility; structural collapse; energy analysis; falsework

1. Introduction

The design of engineering structures can essentially be defined as a continuous process of making difficult engineering decisions based on the available knowledge under severe constraints imposed by society and nature.

In the traditional approach, engineers resort to structural design codes to make decisions. These documents are developed specifically to address areas where significant past experience exists and where critical societal risks are not involved. Thereby, codes are established for the purpose of providing a general, simple, safe and economically efficient basis for the design of ordinary structures under normal loading, operational and environmental conditions.

However, problems do exist. The present design codes are based on semi-probabilistic limit states design. In general, the Limit State Design (LSD) methodology was calibrated to provide an appropriate reliability only at the individual element level. Therefore, as resistance safety checks are merely considered at a local level (single or element's cross section) and the global resistance is not directly

∗Corresponding author, Ph.D., E-mail: jandre@lnec.pt
[a]Ph.D., E-mail: rgbeale@brookes.ac.uk
[b]Ph.D., E-mail: ambaptista@lnec.pt

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

accounted for, the design efficiency and the global target reliability may not be achieved in practice. As highlighted by Starossek (2006), the safety of any structure depends not only on the safety of all the elements against local failure but also of the system response to local failure. The implied assumption that the adequate resistance of the structure is guaranteed by the resistance of its elements is generally not valid, as shown by Starossek and Wolff (2005). Additionally, structural robustness requirements specified in codes are not linked with a quantifiable reliability level of the whole structural system, since target reliabilities specified in codes address only single elements.

Therefore, the present basis for design does not assure optimal design in terms of resources allocation and risk acceptance. The traditional standards-based approach is becoming increasingly inadequate to handle the allocation of limited resources for structures design, operation, repair or improvement, in a climate of growing public scrutiny.

In this paper, new structural robustness and fragility indices are presented and the latter forms the basis of the risk analysis framework. The paper starts with a new structural robustness definition. Afterwards, a brief bibliographic review of past studies on structural robustness is presented. Finally, the formulation of new structural robustness and fragility indices are detailed with illustrative application examples. The advantages over existing methods are demonstrated.

2. Definitions

2.1 Structural robustness

Robustness is defined in ISO 2394 (ISO 1998) as the "*ability of a structure not to be damaged by events like fire, explosions, impact or consequences of human errors, to an extent disproportionate to the original cause*". In this definition robustness can be defined as a parameter indicating the sensitivity of a structure to disproportionate collapse, i.e., a distinct disproportion between the triggering, spatially limited failure and the resulting widespread collapse, due to a local failure (Starossek 2009). In other words, the robustness of a system is a measure of the ability of a system to restrict the failures to those damaged elements directly involved with a given local hazard scenario.

Structural robustness was only defined and introduced as a design objective, even if just qualitatively, following the partial collapse that occurred at Ronan Point in the UK in 1968. As usual, following a peak of inflated focus the research attention given to robustness attenuated, until the Oklahoma City bomb attack on the Alfred P. Murrah Federal Building took place in 1995, and furthermore after the 2001 terrorist attacks to the Twin Towers in New York, turned the spotlights again towards robustness.

According to Faber (2009) "*despite many significant theoretical, methodical and technological advances over the recent years, structural robustness is still an issue of controversy and poses difficulties in regard to interpretation as well as regulation*".

A general design method for structural robustness, or structural integrity, is not yet explicitly specified in the existing design codes, although general design guidance is now given to assure appropriate strength and ductility of the structural connections between members. An exception to this general observation are the rules that most recent codes provide regarding the structural analysis and structural requirements for accidental load cases, such as failure of a member (typically a column) due to an explosion or vehicle (or ship) collision.

Why is structural robustness important? The answer to this question is given in by Starossek (2009): "*Progressive collapse is arguably the most dramatic and feared form of failure in structural* *engineering. It usually occurs unexpectedly and causes high losses*".

In the present paper, a broader definition of structural robustness is introduced: robustness is a measure of the predisposition of a structural system to loss of global equilibrium and global stability, as a result of a failure scenario, e.g., a failure of one or more elements of the structure, for a given hazard event. It is applicable to all design situations and not only those unforeseen, accidental, or concerning local failures (difficult to define and select).

It is the evaluation of the "what if" scenario, which is absent from present codes or standards. This omission can lead to unsafe (damage intolerant) structures, since a non-robust structure can crumble in a progressive and disproportionate collapse fashion if submitted to (i) a failure scenario under normal operation conditions, (ii) an accidental load case, with low probability of occurrence, (iii) an unexpected load case, or (iv) a load case unaccounted for.

2.2 Structural fragility

The structural fragility of a system is an expression of the system's structural performance, typically in terms of damage extension, for a given hazard event. Traditionally, fragility of a structure or element may be expressed by the conditional probability of failure for a given hazard event. Structural fragility is a system characteristic, independent of the probability of occurrence of the hazard event.

3. A new framework for structural robustness and structural fragility analysis

3.1 Structural robustness analysis

Structural robustness is a measure of the predisposition of a structural system to progressive and disproportionate collapse. It is an essential tool to design damage tolerant structures because citing Todinov (2007) *"maximising the reliability of a system does not necessarily guarantee smaller losses from failures"*.

Using the new definition, structural robustness can be determined for any given deterministic combination of loads, or loads with a given conditional probability, which cause a failure in the structural system, irrespective of the system context and exposure. If the uncertainty of the resistance properties of the structure, and consequently of the system's response, is accounted for, then robustness is not defined by a single value but by a probabilistic distribution of possible values.

Using the new definition, it is clear that structural robustness and reliability are two different concepts, although related.

Reliability of a system is associated with the probability of structural failure, which depends on not only the definition of structural failure (local or global level), the resistance properties of the system but also on the likelihood of occurrence of the hazard scenario.

In general, reliability analysis consists in a comparison between a value (p) of an applied action, P, defined within a particular hazard scenario, H, with a given probabilistic distribution, and the ultimate resistance of the structural system expressed as the value (p_{max}) of the action P when structure failure state is attained. The probability of structural failure can be determined based on the differences between corresponding p values and p_{max} values. What happens between the first structural failure state and the complete structural failure state, and also between p and p_{max}, is not assessed.

In structural robustness analysis, the focus is not in assessing the probability of structural failure but in what happens after the first structural failure state, measuring the predisposition for failure (damage) propagation within the system. In addition, the probabilistic distribution of a particular hazard scenario, *H*, used in reliability analysis and in robustness analysis is in general different, see section 3.1.3.

However related these two concepts are different. A system can have a very high reliability but if it is governed by the reliability of very few elements, there is always a load scenario for which the system's structural robustness is very low. The opposite is also true, a system can have a very high structural robustness but if all the elements have the same ultimate resistance there is always a hazard scenario for which the system's reliability is very low.

According to the new definition, structural robustness is considered to be a structural property, not dependent of possible human or economical risks associated with a failure or collapse. Structural robustness is a measure of damages and not of consequences, contrary to what is suggested by Baker *et al.* (2008), since the latter concept has a broader scope as it encompasses structural consequences (damages) but also social and economical consequences, for example.

As robustness is understood to be a structural property, it cannot be controlled by external measures such as: (i) reduction of the structure exposure to hazard events or (ii) introduction of external elements to minimise the effects of those hazard scenarios on the structure. However, these types of measures would increase the reliability of the structure and decrease the risk of disproportionate collapse.

In conformity with the new definition, it is possible to have a system with a very high resistance but with a very low structural robustness, and vice-versa. Increasing the resistance of all elements of a structure, for instance by choosing materials with a higher tensile strength, although a sufficient condition to increase the system's resistance (and reliability for the same exposure), is not a sufficient condition for increasing the system's structural robustness. This is clearly demonstrated by comparing two steel structures, *A* and *B* for example, with the same geometry at global and local levels but where *A* was built with steel of a higher grade than *B*, but the steel used in structure *A* is so brittle, that the increase in resistance does not make up for the loss in deformation capacity. Therefore, the robustness of *A* is smaller than of *B*.

Additionally, increasing material strength, or member resistance, is not always the most cost-effective approach to increase structural robustness. In some structures, it may even be counterproductive. For instance, the robustness may decrease by increasing the resistance of joints between different parts of a structure as the collapse might propagate to other initially undamaged (or even unloaded) areas. Finally, resistance is not a suitable property to measure robustness since it must always have to be expressed in terms of the local behaviour of the structure, which might differ greatly within the structure.

The main advantages of the proposed new definition of structural robustness in relation to the existing definitions are:

- Structural robustness, structural resistance, reliability and risk can now be considered to be four different concepts. The existing structural robustness definitions mixed these four concepts which made the analysis, interpretation and evaluation of the former variables difficult tasks. Furthermore, by coupling in the same definition of structural robustness up to four different concepts the benefits of determining robustness was not clear. The present definition makes structural robustness a property than can be measured independently of the system's resistance, reliability and risk. Structural robustness can for the first time be considered an independent requirement for the structural performance of civil engineering infrastructures. Together with the structural resistance and reliability they become powerful tools that can, and should, be used in the risk management of civil engineering infrastructures;
- The second advantage of the new definition, is that for the first time, progressive and

disproportionate collapse analysis is clearly defined as a requirement not only for unforeseen and accidental situations affecting localised areas of a given structure, but also for normal service conditions covering for instance design cases where the permanent load is the dominant action.

Structural robustness is a function of resistance variables, *R*, of the structural system (Knoll and Vogel 2009) and also a function of the hazard scenario, *H*: loads, imposed displacements, etc.

3.1.1 Traditional methods for analysis of robustness of structures

Robustness has been present directly or indirectly in several structural codes throughout the last thirty years. However, to date there is not one document that specifies a general purpose design method for determining robustness in a consistent and effective manner. Until now, such additional considerations have only been made in individual cases, e.g., for government buildings, and mostly at the engineer's own discretion; mandatory and specific procedures for general structures do not exist (Starossek and Wolff 2005). Moreover, there is an almost complete absence in existing codes and guidance documents regarding rules, design requirements and methods or procedures to evaluate the robustness of temporary structures, such as bridge falsework systems.

However, it is noteworthy to summarise how the most advanced structural codes treat robustness and what are the current design requirements for robustness.

Starting from the first structural codes adopting limit state design theory, a structural insensitivity requirement was incorporated to avoid progressive collapse scenarios, i.e., the structure would not collapse if subjected to a limited damage (CEB 1993). Thus robustness was treated qualitatively and indirectly by specifying standard prescriptive detailing rules for members; linked with an undesirable failure mode. No rules for design and verification were specified.

More recently, EN 1990 (BSI 2002) establishes "*robustness (structural integrity)*" as a way to achieve required levels of reliability relating to structural resistance and serviceability. EN 1991-1-7 (BSI 2006) defines robustness as "*the ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent disproportionate to the original cause*".

Depending on the consequence class of the structure, see EN 1990 (BSI 2002), different types of design rules must be followed. For consequence class CC3 (the most stringent class) a complete risk assessment may be required. Typical structures that fall in this class are grandstands, public buildings where consequences of failure are high. Specific guidance for buildings is given in Annex A of EN 1991-1-7 (BSI 2006). An additional review of robustness related rules present in the structural Eurocodes is given by Narasimhan and Faber (2009).

In the USA, the following national codes for the design of buildings were prepared, defining requirements, rules and verification procedures to achieve collapse-resistant buildings in the event of abnormal loading: the NIST "*Best Practices for Reducing the Potential for Progressive Collapse in Buildings*" report (Ellingwood et al 2007) and the US Defence Department United Facilities Criteria (USDOD 2010).

The first document defines progressive collapse as "*the spread of local damage, from an initiating event, from element to element resulting, eventually, in the collapse of an entire structure or a disproportionately large part of it; also known as disproportionate collapse*".

Therefore, structural robustness analysis is yet not fully implemented in the existing structural codes, except for the scenario of an accidental action.

3.1.2 Advanced methods for analysis of robustness of structures

There are however, methods available to analyse structural robustness, some of them developed quite recently. All of these methods are based on structural robustness definitions different than the one introduced in the present paper, and all of them suggest different approaches to measure structural robustness.

As observed by Starossek and Haberland (2008) existing methodologies can either (i) be based on structural behaviour or be based on structural attributes; (ii) assume an initial local damage or be based on the identification of a collapse sequence. Finally, it is also possible to distinguish between deterministic approaches and probabilistic approaches.

A common trend in structural robustness evaluation is to define a structural robustness measure, often in the form of a structural robustness index. The first question that needs an answering is why is it useful to measure quantitatively the structural robustness of a system? One could simply compare resistances or load *vs*. displacement curves associated with different failure scenarios of the same structure. The answer to this question is found in the definition of robustness presented previously. To assess structural robustness, i.e., its global stability reserve, complex non-linear analyses must be performed. It is thus a requisite that the maximum information should be extracted in order to obtain knowledge return for the additional computational and analysis effort. Therefore, the results obtained should be informative and easy to apply, e.g., should allow a straightforward comparison between different structures. All the latter favour the development and use of a simple measure of structural robustness.

The first structural robustness indices to be developed were deterministic relating for instance the resistance of the damaged structure to a given load case with the corresponding resistance of the undamaged structure. More recently, probabilistic-based and risk-based indices have been developed. Whilst the former usually compares the probability of failure of the damaged structure with the probability of failure of the undamaged structure, the latter is more complex as it weights the indirect risks with the total (direct and indirect) risks.

Deterministic approaches

A number of different deterministic approaches to analyse robustness of a structural system have been proposed, based either on the load carrying capacity, or on the energy dissipation or on the extent of damages. Let us start by reviewing energy dissipation based approaches.

Smith (2006) presented an analytical procedure coupled with finite element simulations to analyse the progressive collapse of building structures based on the parallelism of progressive collapse with the theory of unstable fast fracture in fracture mechanics. The idea behind the method is: "*if the energy released by loss of a damaged member is greater than the energy absorbed by the destroyed member and other damaged members, then progressive collapse will occur*" (Smith 2006).

Smith determined the energy required to destroy sufficient structural members to develop an unstable mechanism (which he named damage energy) and using a minimisation process, consisting basically in continuously deleting damaged elements from the mesh, coupled with a sorting procedure, he identified the sequence of damage events that required the least amount of damage energy. Smith then used this minimum damage energy as a measure of the structural system robustness.

However, this measure has some limitations. Besides being based on an absolute value, which is inconvenient when comparing different structures, it assumes that higher damage energy necessarily translates to higher structural robustness. This is not always the case. For example, it is possible to have two similar structures, *A* and *B*, subject to the same loading conditions but with different yield loads, $p_{y,A} < p_{y,B}$, where *A* is more resistant than *B* ($p_{u,A} > p_{u,B}$) and *B* has more damage energy than *A* ($W_{d,B} > W_{d,A}$), because the materials of *B* have a larger deformation capacity, for instance, see Fig. 1.

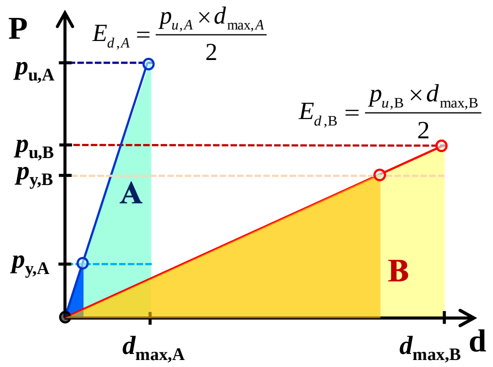


Fig. 1 Example of the limitations of Smith's definition of robustness

The damage energy dissipated by structure *B* is larger than the one dissipated by structure *A*, but the relative energy reserve after yield load of structure *A* is larger than of structure *B*. Therefore, *A* must be more robust than *B*, which contradicts Smith definition. Increasing the damage energy is not a sufficient condition to achieve higher structural robustness. Smith approach lacks a comparative term.

Alternative energy-based measures of robustness were suggested by Starossek and Haberland (2009), Fang (2007). However, as the authors recognise the measures have some important drawbacks.

Another possible deterministic robustness index could relate the resistance of the damaged structure with the resistance of the undamaged structure. However, this index has evident weaknesses.

Probabilistic-based approaches

In terms of probabilistic-based approaches, two different methodologies exist: one focusing in the probability of failure and the other on the risk of the structure.

The first measures based on the former methodology were the redundancy indices proposed by Frangopol and Curley (1987), Fu and Frangopol (1990)

$$RI = \frac{\mathrm{P}_f\left(\text{damaged}\right) - \mathrm{P}_f\left(\text{intact}\right)}{\mathrm{P}_f\left(\text{intact}\right)} \tag{1}$$

$$\beta_R = \frac{\beta_{\text{intact}}}{\beta_{\text{intact}} - \beta_{\text{damaged}}} \tag{2}$$

where P_f represents the probability of failure and β represents the reliability index.

Finally, in 2008, a new structural robustness index was presented based on a complete risk analysis where the consequences are divided into direct and indirect risks (R_{Dir} and R_{Ind}, respectively) and the measure is given by the ratio between the direct risks with the total risk (sum of the direct and indirect risks) (Baker *et al.* 2008)

$$I_{Rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \tag{3}$$

3.1.3 A new measure of structural robustness

From the analysis of the existing deterministic and probabilistic measures it could be concluded that two types of analyses are used: (i) one where the failure modes of the structure are analysed explicitly and in a rigorous way, see Smith (2006) and the other (ii) where a particular fictitious damage is assumed to have occurred to the structure. Using the latter methodology one can only obtain an indication of the sensitivity of the structure to a local failure, although a possible unconservative estimate since in reality when an element fails several other elements could have suffered severe deformations (close to failure) and the configuration of the global system could have changed also. Therefore, important second-order effects are neglected and possible secondary load paths and resistance models are not taken into account, which can introduce a significant bias to the proposed simplified measures of structural robustness.

From the information presented previously, it can be concluded that the reviewed structural robustness evaluation strategies are not consistent with the new structural robustness definition presented in this paper. In some indices, mainly in the newly developed risk-based index, see Eq. (3), it is considered that robustness of a structural system depends not only on the structural characteristics of the structure but also on the variation of the loads and the exposure of the structure (probability of occurrence of the loads). In the risk-based index, robustness is also linked with the consequences (economic, social, etc.) of the collapse. Therefore, an alternative structural robustness measure is proposed.

The idea behind developing a new robustness index stems from analysing the strengths and weaknesses of the existing indices. Most of them require the separate evaluation of direct and indirect damages (or consequences). This step is often very difficult and depends on the validity of the adopted assumptions, which are frequently subjective. Damages are a continuum and it is very difficult to break apart direct and indirect damages. The proposed structural robustness index relates to structural damages rather than to risk or reliability, making structural robustness, risk and reliability three different variables that can be evaluated independently. Together, these three variables can be used to support and improve rational decision-making in civil engineering.

The basis for the development of the new structural robustness index is the analysis of the structural behaviour in terms of energy balance. There are plenty of advantages of energy-based measures over resistance-based or reliability-based robustness measures. Energy-based measures concern the global behaviour of the structure, which removes the need for subjective selection of the parameter which robustness depends on with all the possible loss of objectivity, expressiveness and generality that comes with it.

In a closed system, the principle of conservation of energy defines that the change in the total energy (E_{Total}) between a system and its surroundings is constant, i.e., energy is not created or destroyed but can be transformed

$$\Delta E_{Total} = \Delta P + \Delta K = \Delta E + \Delta W + \Delta K = \text{constant} \tag{4}$$

where:

ΔP represents the variation of the potential energy of the system;

ΔK represents the variation of the kinetic energy of the system;

ΔE represents the variation of the internal energy of the system: internal strain energy of the system plus the variation of the other sources of energy such as electrical energy, chemical energy and energy dissipated by friction, creep, etc.;

ΔW represents the variation of the work done by external actions on the system.

The internal strain energy, E_S, is the total potential energy contained in the system. In structural mechanics, the internal potential energy may be expressed by

$$E_S = E_{El} + E_{Pl} \tag{5}$$

where:

E_{El} represents the elastic (recoverable) strain (potential) energy of the structure;

E_{Pl} represents the plastic (dissipated) strain (potential) energy of the structure.

By using energy based structural robustness criterion it is possible to get a deeper insight into the behaviour of the structure. For example, analysing energies it is possible to distinguish between recoverable (e.g., elastic strain energy) and unrecoverable energies (e.g., viscous energy and plastic strain energy). The presence of unrecoverable energy makes the system non-conservative because some of the energy is dissipated and after unloading the system does not recover the initial state (path dependent problem). These energies can be used as an evidence of the existence of damages in specific dissipative elements of the system.

Several structural robustness indices were developed and tested. In the end, an index which is capable of overcoming the limitations was finally developed as the mathematical expression of

$$\mathrm{I_R}\left(\mathrm{A}_L \mid H\right) = \frac{\text{Damages up to unavoidable collapse state for hazard } h}{\text{Damages up to collapse state for hazard } h} \tag{6}$$

The general expression is given by

$$\mathrm{I_R}\left(\mathrm{A}_L \mid H\right) = \frac{D_{uc} - D_{\text{1st failure}}}{D_c - D_{\text{1st failure}}} \quad \text{with} \begin{cases} 0 \le \mathrm{I_R} \le 1 \\ D_c - D_{\text{1st failure}} = 0 \Rightarrow \mathrm{I_R} = 1 \end{cases} \tag{7}$$

where (see Fig. 2):

A_L represents the leading action;

$H = \{h_1, h_2, ..., h, ..., h_n\}$ is a set of hazard scenarios: {base conditions} + {impact on column 1, impact on column 2, ..., impact on column n}, a set of different actions or a combination of different actions, for example;

$D_{1st\ failure}$ represents the damage energy of the structure when the "first failure" state takes place for the hazard scenario considered;

D_{uc} represents the damage energy corresponding to the state where collapse is unavoidable for the hazard scenario considered;

D_c represents the damage energy corresponding to the collapse state for the hazard scenario considered.

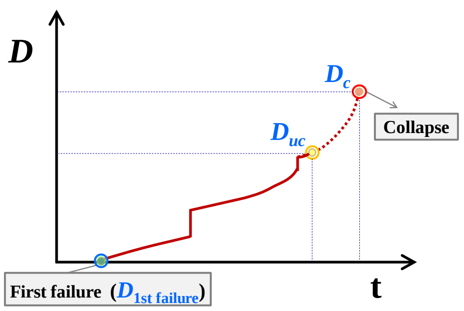


Fig. 2 Illustration of the structural robustness index notation

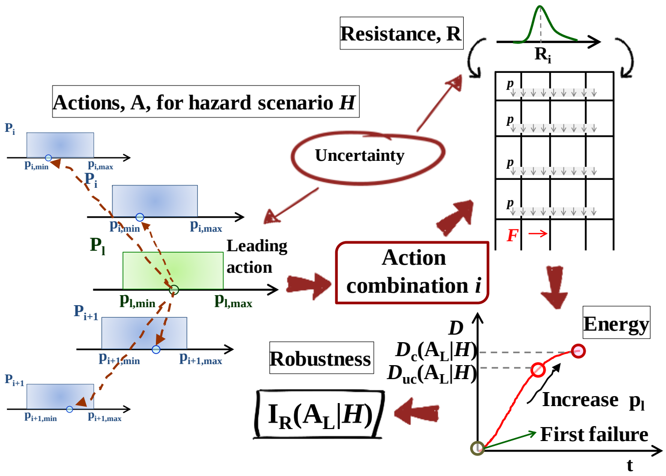


Fig. 3 Procedure to determine the structural robustness index

The selected criteria for monitoring the damages and the collapse of a structure is the system's damage energy (D) evolution, because it gives a good estimate of the capability of the system to redistribute forces by alternative load paths and resistance mechanisms.

The damage energy is given by the sum of the plastic strain energy (non-decreasing function) with the internal energy released in each failure (stepped function). Therefore, the damage energy is a non-decreasing function. Failure is any state where there is a release of internal energy: it can be the formation of a crack, failure of a joint, failure of a cross-section, for example. Care should be paid not to consider the plastic strain energy of a failed element twice: in the plastic strain energy and in the internal energy released in the failure.

It is assumed that the damage energy is zero if there are no plastic strains within the structure and that no failures have occurred. Therefore, a system whose collapse is solely triggered by elastic instabilities lacking post-buckling resistance or by loss of overall stability of foundations, without prior failures, has a robustness index equal to zero, because no system damage energy is needed to attain the "unavoidable collapse" state. On the other hand, a system where all elements are brittle, i.e., which cannot deform plastically, and in which the collapse trigger mechanism involves the failure of all elements, has a robustness index equal to one, because the entire damage energy available in the system has been used to attain collapse.

The above expression is slightly different than the one presented in an earlier work (André *et al.* 2013), in that the strain energy was replaced by the damaged energy. This is due to the fact that only the damage energy is relevant for structural robustness.

Also, structural robustness is no longer a function of the system's energy for a given action value, p. This is needed to disassociate structural robustness from the applied action value. As stated earlier, structural robustness and reliability are two different parameters. Structural robustness is only a function of the damage initiation mechanism and of its structural consequences.

In order to calculate the structural robustness index a three-step procedure must be followed, see Fig. 3.

First step: define the *nominal* loading conditions (hazard scenario), i.e., the sequence of action (loads) application is rationally chosen and the initial values of the various actions, material properties, system imperfections, etc. are generated, corresponding to values obtained from the corresponding probability density functions (actions are modelled with uniform probability density distributions and resistance variables can also be modelled with uniform probability density distributions but preferably with more informative distributions). Correlations could be considered, for example: complete or incomplete correlation between action values-incomplete correlation means that values of different actions can be correlated for only a range of values of one of the actions; for example seismic actions and traffic actions.

Second step: while holding everything constant ("*ceteris paribus*"), a leading action that can cause the structure to collapse (if it has not already occurred during the first step) is selected and increased until the collapse is attained. Alternatively, several actions can be defined as leading actions and increased simultaneously if it is considered appropriate (if actions are correlated for example). The aim should be to obtain a realistic safety assessment of the structure and therefore of the most likely damage propagation within the structure. In the example illustrated in Fig. 4(a) it may be necessary to evaluate the robustness index for increasing values of the action p and not of F, while in Fig. 4(b) it may be possible to select either actions. The value of D_{uc} is determined.

Third step: the structural robustness index is determined from Eq. (7) based on the adopted limit state which defines the first failure state (the value of $D_{1st\ \mathrm{failure}}$ is determined).

The value of D_c can be determined in any of the three steps, depending on the method used.

This index is also flexible since the inputs can change, for example: the "first failure" state can be replaced by another criterion, possibly related to a particular element failure, and the "unavoidable collapse" and "collapse" states can also be changed to represent a maximum limit of acceptable damage, D_{max}, for instance. In the latter case $D_{max} \leq D_{uc}$.

The value of the structural robustness index is a function of the hazard scenario, H, (i.e., action values) because in general D_{uc} and $D_{1st\ failure}$ depend on the loading conditions.

A value of the structural robustness index equal to one means that the structure is completely optimised in terms of structural robustness, for the hazard scenario considered. In the contrary, a value of the structural robustness index equal to zero may indicate that the structure completely lacks optimisation in terms of structural robustness, for the hazard scenario considered.

The value of D_{uc} can be estimated from the theory formulated by Dusenberry and Hamburger (2006). As explained by these authors, following a structural failure the system's energy balance can be expressed *"through comparison of the release of potential energy that occurs as the structure falls, the strain energy that accumulates as the structure deforms, and the kinetic energy associated with the moving mass. (...) For all conditions when the kinetic energy is positive, portions of the*

(a) Case 1 (b) Case 2

Fig. 4 Possible different selections of leading actions to evaluate structural robustness

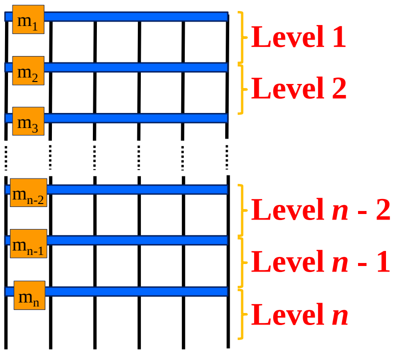


Fig. 5 Example of a framed structural system

structure are in motion, and collapse has not been arrested. When kinetic energy attains a value of zero strain energy accumulated by the structure equals the change in potential energy the moving portion of the structure is at rest, and collapse potentially has been averted. If strain energy never equals the change in potential energy, the mass remains in motion and collapse is inevitable" (Dusenberry and Hamburger 2006).

Applying this theory to a framed structural system, see Fig. 5 for example, such as a bridge falsework, the collapse of a lower level of elements due to the failure of an upper level of elements can only be arrested if and if only (Bažant and Verdure 2007)

$$W(t) < E_S(t) \tag{8}$$

where:

$W(t)$ represents the value of the work done at time t by external actions on the lower level elements, including the potential energy associated with the kinetic energy of the moving mass of the upper level elements.

$E_S(t)$ represents the value of the internal energy of the lower level non-failed elements at time t.

For such a system, the progressive collapse criterion given by Eq. (9) can be expressed by

$$W(t) < E_{S,\max} \tag{9}$$

where $E_{S,\max}$, represents the maximum internal energy dissipated by all the non-failed elements of the lower level. $E_{S,\max}$ can be estimated analytically, numerically (Smith 2006, Bažant *et al.* 2008, Bažant and Zhou 2002), or experimentally (Korol and Sivakumaran 2014) for each element and summed over all the elements of the level.

In each level of a typical framed structural system, the energy can potentially be dissipated by the floor elements (slabs and beams), the columns and the joints between these structural elements. The contribution of non-structural elements can also be considered. This characteristic structural layout leads to two different mechanisms that control the value of $E_{S,\max}$. One concerns the column elements of the level – their collapse leads to the level collapse. The other relates to the elements of the level floor – if all beam-to-column joints fail it is very likely that the entire floor collapses, for example.

In general, it is necessary to consider in the analysis all of the above-mentioned mechanisms. With respect to the energy dissipation by the column elements, for a steel frame and assuming a three plastic hinge dissipative mechanism for the columns of each level, $E_{S,\max}$ of level j can be estimated by (Bažant *et al.* 2008)

$$E_{S,\max}^{j} = \sum_{i=1}^{N_{columns}^{j}} \left[2 \times \int_{0}^{\theta_{e,pl}} M_{e,i}^{j}(\theta) d\theta + \int_{0}^{\theta_{m,pl}} M_{m,i}^{j}(\theta) d\theta \right] \tag{10}$$

where:

θ represent the rotation at the column i extremities and middle of column i, respectively;

$\theta_{e,pl}$ and $\theta_{m,pl}$ represent the maximum rotation capacity at the column i extremities and middle of column i, respectively, with $\theta_m = 2\times\theta_e$;

$M_{e,i}$ and $M_{m,i}$ represent the bending moments at the extremities and middle of column i;

$N^{j}_{columns}$ represents the number of columns of level j that have not failed.

It is assumed that failures will only occur due to excessive bending rotations. Furthermore, it is assumed that outside the plastic hinges length, the elements behave elastically. However, this plastic hinge model neglects any contribution of the axial and shear deformation energy capacity to the internal energy and therefore may underestimate the actual dissipated energy (Korol and Sivakumaran 2014).

In order to improve the accuracy of the $E_{S,\max}$ value, the values of the maximum bending moments at the plastic hinges, $M_{e,i}$ and $M_{m,i}$ in Eq. (10), can be obtained using exact analytical expressions, taking into account the interaction between the bending moment and axial forces, the material constitutive model and the maximum deformation capacity of the element's material. For steel CHS (circular hollow sections) under monotonic loading, see Baptista and Muzeau (2001). For cyclic loading the values of $M_{e,i}$ and $M_{m,i}$ may be reduced due to damage accumulation. If local buckling occurs, the maximum deformation and bending moment capacities are lower than their maximum plastic values and can be estimated by the method developed by Gardner (2008), with some restrictions. Additionally, the maximum rotations at the plastic hinges, $\theta_{m,pl}$ and $\theta_{e,pl}$ in Eq. (10), can be determined by the following procedure:

Under the Bernoulli hypothesis, the extensions, e, and deformations, ε, at any cross-section of a linear element are given by André (2014)

$$\text{Extension: } e(z) = e_N - \theta \times z \quad , \quad e_N = e(z=0) \tag{11}$$

$$\text{Deformation: } \varepsilon(z) = \varepsilon_N - \chi \times z \quad , \quad \varepsilon_N = \varepsilon(z=0) \tag{12}$$

where:

z represents the coordinate along an axis perpendicular to the longitudinal axis of the element with origin at the cross-section geometric centre;

ε_N and e_N represent the deformation and extension due to the axial force, respectively;

χ and θ represent the curvature and rotation at a given cross-section, respectively.

Considering a linear function for the deformations, ε, along the longitudinal development (x coordinate) of the element - this simplification potentially overestimates the value of $E_{S,\max}$:

Functions of ε over the plastic hinge length

Hinge at column middle:

$$\varepsilon(z = z_{\max}, x)_m \approx \begin{cases} \varepsilon_y + \dfrac{2\times(\varepsilon_u - \varepsilon_y)}{l_{pl,m}} x & \text{for} \quad \dfrac{L}{2} - \dfrac{1}{2}\times l_{pl,m} \le x \le \dfrac{L}{2} \\ \varepsilon_u - \dfrac{2\times(\varepsilon_u - \varepsilon_y)}{l_{pl,m}} x & \text{for} \quad \dfrac{L}{2} \le x \le \dfrac{L}{2} + \dfrac{1}{2}\times l_{pl,m} \end{cases} \tag{13}$$

Hinge at column extremities:

$$\varepsilon\left(z=z_{\max},x\right)_e \approx \begin{cases} \text{Column top: } \varepsilon_y + \dfrac{\left(\varepsilon_e-\varepsilon_y\right)}{l_{pl,\mathrm{e}}}x \quad \mathit{for} \quad L-l_{pl,\mathrm{e}} \le x \le L \\ \text{Column bottom: } \varepsilon_e - \dfrac{\left(\varepsilon_e-\varepsilon_y\right)}{l_{pl,\mathrm{e}}}x \quad \mathit{for} \quad 0 \le x \le l_{pl,\mathrm{e}} \end{cases} \tag{14}$$

where:

ε_u and ε_y represent the ultimate and yield deformations of the element's material, respectively. For cyclic loading the values of ε_u and ε_y may be reduced due to damage accumulation;

ε_e represents the maximum deformation of the element's material at column extremities hinges;

L represents the column length;

l_{pl} represent the plastic hinge length. For steel CHS a conservative estimate of l_{pl} was determined, based on the results of numerical simulations, equal to

$$l_{pl} \approx d_{ext} \rightarrow \begin{cases} \text{Hinge at column middle: } l_{pl,m} = d_{ext} \\ \text{Hinge at column extrimities: } l_{pl,e} = l_{pl,m} \end{cases} \tag{15}$$

where d_{ext} is the external diameter of the cross-section. The value of l_{pl} of each plastic hinge can be obtained more accurately directly from the numerical model results.

Knowing that the extensions are a function of the deformations

$$e\left(z,x\right)=\int_l \varepsilon\left(z,x\right)dx \tag{16}$$

Introducing Eqs. (12)-(16) in Eq. (11) gives

$$\theta_{m,pl} = \left[\frac{\left(\varepsilon_u+\varepsilon_y\right)}{2} \times l_{pl} - e_N\right] \Bigg/ \left(\frac{d_{ext}}{2}\right) \tag{17}$$

with

$$\theta_{\mathrm{e},pl} = \frac{\theta_{m,pl}}{2} \tag{18}$$

The value of ε_e can be obtained by using Eqs. (11)-(18).

Having determined an estimate of $E_{S,\max}$, D_{uc} can be estimated by the following procedure

Energy demand at level h, E_D^h:

$$E_D^h\left(t\right)=K_D^h\left(t\right)+E_S^h\left(t\right) \tag{19}$$

where K_D^h represents the kinetic energy at level h transferred by the levels above level h.

Considering levels as rigid blocks, K_D^j is determined by:

$$\text{If: } E_D^{j-1}\left(t\right) < E_{S,\max}^{j-1} \Rightarrow K_D^j\left(t\right) = \sum_{i=1}^{j-1} K^i\left(t\right) \text{ Else: } K_D^j\left(t\right) \equiv \sum_{i=1}^{j-1} m_i \times g \times h^{ij} \tag{20}$$

where:

m_i represents the moving mass of level i;
g represents the gravitational acceleration;
h^{ij} represents the vertical distance between levels i and j;
Finally, an estimate of D_{uc} is obtained by:

$$E_D^{n_{level}}\left(t^{n_{level}}\right) = E_{S,\max}^{n_{level}} \Rightarrow \begin{cases} t_{uc} = t^{n_{level}} \\ D_{uc} = D\left(t_{uc}\right) \end{cases} \quad , \quad n_{level} \text{ represents the bottom level index} \tag{21}$$

In order to further improve the accuracy of the procedure, to capture local effects due to gravity action, the values of the leading action should only be increased when a static equilibrium between the applied loads and the internal forces has been reached. A suitable failure search and detection algorithm should also be developed and included inside the numerical protocol in order to update the values of E_D and E_S of each level, by for example considering the effect of the failed beam-to-column connections in the columns' energy demand and in the column's length, but also that the failed column elements (attained either by joint failure or plastic failure) do not enter in the calculation of E_S.

If the analysis additionally considers the dissipative energy capacity of the floor elements, a similar model as the one described above can be used for the beam elements, whereas for the slab a specific model has to be developed for each type of structural solution.

In all cases, it is assumed that the upper level of elements will fall onto a lower level of elements. If not, the upper level of elements will free fall until reaching the ground. In this case, the kinetic energy of the upper level of elements should not be considered when evaluating the resistance capacity of the lower level of elements. In addition, care should be paid in choosing the leading action, since the entire mass where the selected leading action is applied may be free falling and thus might not cause the complete collapse of the system.

The denominator of Eq. (7), D_c, is also difficult to determine since it involves the calculation of the potential sum of damage energy and ideally an iterative search for the maximum potential sum of damage energy. Alternatively, D_c may be considered to represent the sum of damage energy for the collapse mode aimed during structural design, but consequently robustness index values higher than one may be obtained. The value of D_c can be estimated numerically, for example by the sum of damage energy up to collapse. Nevertheless, this method poses considerably difficult numerical challenges. An alternative method, simpler but also approximate, is to numerically, or analytically, estimate the damage energy value for each individual element and sum over all the elements of the system.

The ratio presented in Eq. (7) may have a potential limitation. For example, for a structure which resistance is controlled by very few elements (only one element in the extreme case), in general in this case $I_R \approx 0$ ($D_c >> D_{uc}$) but when $D_c \approx D_{uc}$, which could happen when the sum of available damage energy of the remaining elements is exceptionally low when compared with the sum of available damage energy of the controlling elements, then $I_R \approx 1$ which may not be the expected result. In these cases, Eq. (7) could be modified by introducing a parameter, α, that accounts for the relation between the total number of failed elements needed to attain the "unavoidable collapse" state and the total number of elements present in the system

$$\mathrm{I_R}\left(\mathrm{A}_L \mid H\right) = \alpha \times \frac{D_{uc} - D_{\text{1st failure}}}{D_c - D_{\text{1st failure}}} \text{ with } \begin{cases} \alpha = \dfrac{n_{uc}}{n_{total}} \\ 0 \le \mathrm{I_R} \le 1 \\ D_c - D_{1st\ failure} = 0 \Rightarrow \mathrm{I_R} = 1 \end{cases} \tag{22}$$

Fig. 6 Example of bridge falsework Cuplok® systems

where:

n_{uc} represents the number of failed elements for the “unavoidable collapse” state, i.e., when $D = D_{uc}$;

n_{total} represents the total number of elements present in the system.

In a framed system, as a plausible simplification, n_{uc} can be given by the number of element failures (including joints) and n_{total} can be given by the total number of elements present in the system.

By using advanced finite element analysis programs it is also possible to follow the damage (failure) path throughout the system as the loading increases, for instance by using flag variables in the numerical model which are activated if a given damage criterion is met. This information can be used to modify the value of the robustness index by giving more emphasis to the existence of damages in selected critical areas or critical elements of the system. This can be easily done by introducing penalty weight factors ($\zeta < 1$) into the calculation of D_{uc}, specifically applied to the damage energy of those critical areas or critical elements, see Eq. (23).

However, this type of differentiation should preferably be done only in the vulnerability analysis where the costs associated to damages are calculated to avoid introducing risk related parameters or subjectivity into the determination of structural robustness.

$$\mathrm{I_R}\left(\mathrm{A}_L \mid H\right) = \frac{\sum_{i}^{j}\left(\zeta_i \times D_{uc,\mathrm{i}}\right) - D_{\text{1st failure}}}{D_c - D_{\text{1st failure}}} \quad \text{with} \begin{cases} 0 \le \mathrm{I_R} \le 1 \\ D_c - D_{1st\ failure} = 0 \Rightarrow \mathrm{I_R} = 1 \end{cases} \tag{23}$$

The proposed index can also be used to calculate the residual robustness of a system against follow-up hazards after a selected hazard event has taken place, see André (2014).

The robustness index presented does not exhibit the same limitations as the existing indices. Thus, it is thought that the new structural robustness index fulfils all conditions listed by Starossek and Haberland (2008) and can be used as a measure of the robustness of a structural system.

As an illustration example, a bridge falsework Cuplok® system, see Fig. 6, will be analysed.

Bridge falsework systems are typically low robust structures since they are an assemblage of similar and slender steel linear elements prone to instability phenomena, and weak loose connections, where the critical design load is often present at its maximum value, uniformly distributed over the entire, or a significant part, of the structure. Additionally, since the elements of these systems are linear elements they do not possess alternate resistance models like the slabs of

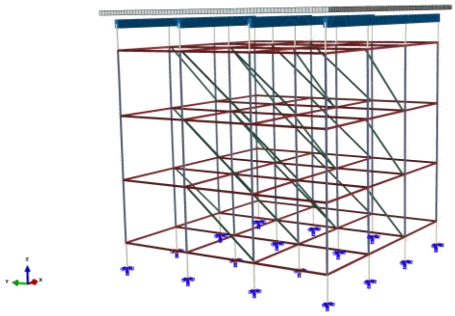
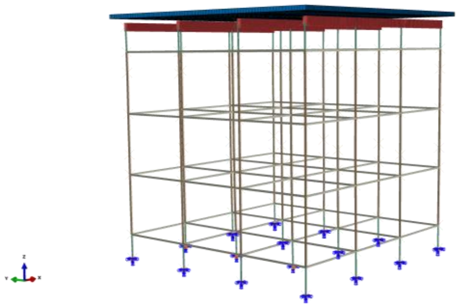
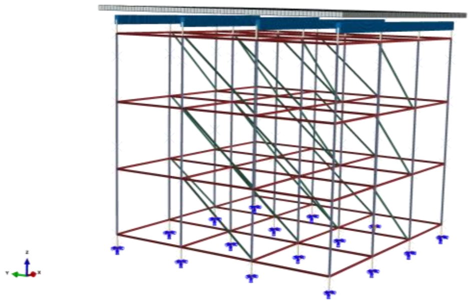


Fig. 7 Overview of Model A2



(a) Overview of Model A4



(b) Overview of Model A2m

Fig. 8 Overview of other models

bridge decks, and thus complete failures of elements are more easily reached in bridge falsework systems than complete failures of bridge elements.

Additionally, factors such as lack of competence in design, absence of rigorous quality control supervision, reuse of damaged elements, etc. have large impact on the structural behaviour of bridge falsework systems, which contribute to the existence of high levels of uncertainty associated with these temporary structures, see André *et al.* (2013, 2012), Beale (2014).

Therefore, it is extremely important to evaluate the robustness of these temporary structures since (i) their margin of safety given an irreversible failure is usually much lower than the one achieved in permanent structures, (ii) the failure of one element may lead to the progressive and disproportionate collapse of the entire, or the majority, of the structure, and (iii) their exposure to critical hazard scenarios is also much larger than the one of permanent structures.

As demonstrative examples, the Model A2 tested in the University of Sydney, see Chandrangsu and Rasmussen (2011), will be considered. The cross-section geometrical characteristics as well as the material properties of the various elements which are part of the falsework system are given in Chandrangsu and Rasmussen (2011). The finite element model properties are detailed in André, Beale and Baptista (2015a). The formwork was explicitly modelled, with an equivalent thickness equal to 100 mm, and the joint characteristics considered were taken as the average values of the joint tests results reported in André *et al.* (2015b). The value of the top and bottom jacks' extension lengths was equal to 600 mm. The initial geometrical imperfections were taken as the values measured *in situ* during the full-scale test performed at University of Sydney, see Chandrangsu and Rasmussen (2011). Fig. 7 illustrates the numerical representation of Model A2.

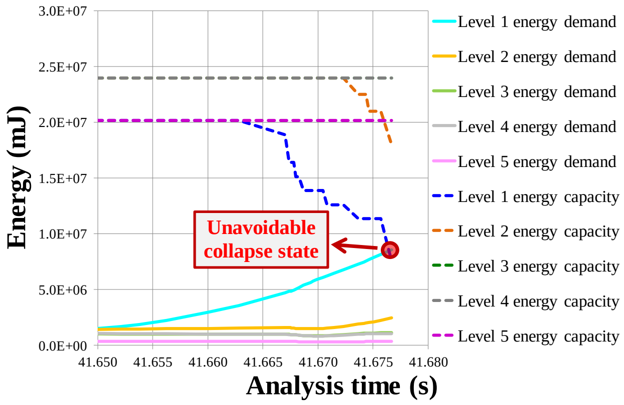


Fig. 9 Application of the procedure used to detect the “unavoidable collapse” state of Model A2

Additionally to Model A2, two variations of this model will also be used: (i) Model A4, which is the braceless version of Model A2, and Model A2m which is equal to Model A2 except the value of the top and bottom jacks extension lengths is equal to 300 mm. Fig. 8 illustrates the numerical representation of models A4 and A2m. The only action considered in this example, besides the materials’ self-weight, was the weight of the concrete slab. This action was selected as the leading action, applied uniformly to the formwork elements and increased monotonically over a step period of 100 s (implicit dynamic quasi-static analysis).

Taking Model A2 as the reference model, Fig. 9 shows how the procedure developed to detect the “unavoidable collapse” state is applied.

For each one of the five levels of Model A2, see Figs. 5 and 7, the values of the energy deformation demand (E_D) and energy deformation capacity (E_S) were determined for increasing values of the leading action. As the load value increased, so did the internal forces and material strains in the various elements of the structure. Eventually, an element(s) breaks or looses substantially its stiffness. Table 1 presents the various element failures of Model A2. It took a total of 13 element failures for the collapse of Model A2 to become unavoidable, the majority of which located at level 1 (*i.e.* at the top level), more specifically due to excessive rotation of the forkhead plates making them loose their rigidity and resistance capacity, and consequently of the attached jack elements.

Column failure was considered when three plastic hinges (plastic deformations) formed at sections of the column spaced by at least 15% of the column length from each other.

Each element failure means that there are fewer elements available to contribute to the resistance of the structure and consequently the energy deformation capacity (E_S) decreases. On the other hand, energy deformation demand (E_D) increases. When the energy deformation demand at one level equals the energy deformation capacity of that level, collapse of that level is unavoidable. The collapse of one level implies that the potential energy stored at the levels above will be converted, mainly, into kinetic energy. The motion of this moving mass will only be stopped if and if only the energy deformation capacity of the levels below is greater than the kinetic energy of the moving mass, see Eq. (9).

Considering Model A2, the collapse of level 1 means that the weight of the formwork and of the poured concrete will move at least from their respective height at time of collapse of level 1 to the height of level 2. Therefore, a mass equal to the applied pressure times the formwork area is put in motion by gravity as it drops from height of level 1 to the height of level 2. The kinetic

Table 1 Element failures of Model A2

Element type	Time (s)	Level
Forkhead joint	42,6670	1
Forkhead joint	42,6674	1
Forkhead joint	42,6674	1
Column	42,6680	1
Forkhead joint	42,6687	1
Column	42,6708	1
Column	42,6737	2
Column	42,6746	2
Column	42,6767	1
Column	42,6767	1
Column	42,6767	1
Column	42,6767	2
Column	42,6767	2

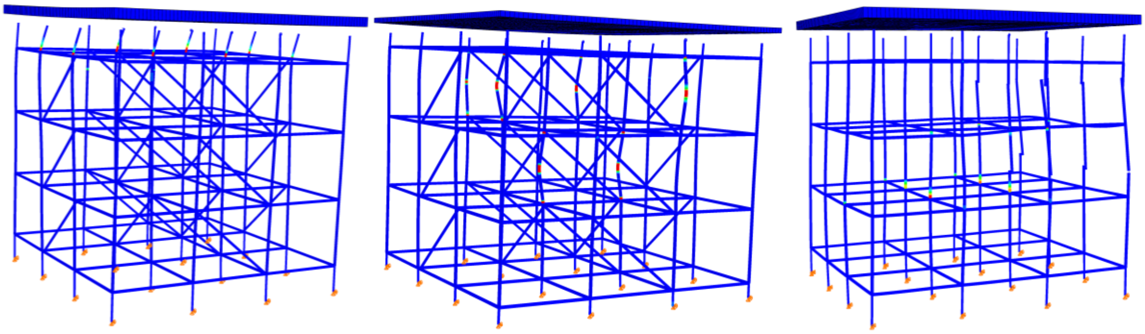
energy associated with this movement equals $8{,}42\times10^{8}$ mJ for the model considered. The estimated maximum energy deformation capacity of level 2, determined according to Eq. (10), is $2{,}40\times10^{7}$ mJ (at the time of collapse of level 1). Therefore, level 2 will also collapse, and the same happens for levels 3, 4 and 5. In conclusion, the collapse of level 1 induces the global collapse of the entire structure, and therefore it corresponds to the "unavoidable collapse" state of Model A2.

Robustness index of Model A2 is given by Eq. (7). From the analysis results, the value of D_{uc} is equal to $9{,}15\times10^{6}$ mJ. The value of D_c is equal to $1{,}12\times10^{8}$ mJ. The latter value was determined using Eq. (24). Therefore, as a simplification, the contribution of the beam (ledger) and brace elements was not considered. However, as the height of bridge falsework structures is typically higher than the maximum length of a single column (standard) element, as is the case of the example models, spigot joints will exist in some levels and these joints have a much lower maximum damage energy capacity than a column plastic hinge. This overestimation of the columns damage energy at least partially compensates the abovementioned simplification, since the maximum damage energy capacity of one ledger element is lower than of one spigotless column element. This occurs because the maximum damage energy capacity of the ledger-to-standard joints is not large. Also, Eq. (24) is a function of the internal energy, which is larger than the damage energy. As a result, for the example models considered the value of D_c given by Eq. (24) represents a reasonable estimate of the true value.

$$D_c = \sum_{i=1}^{N_{levels}} E^{i}_{S,\max} \tag{24}$$

Consequently, a robustness index equal to 0,082 is obtained. If Eq. (22) is used instead then a value equal to 0,006 is obtained. The number of elements in the model was considered equal to the number of columns at each level, plus the number of ledger-to-standard joints, plus the number of brace-to-beam joints (if applicable), plus the number of forkhead and baseplate joints. For Model A2, the number of elements is 200 (the number of columns represents 40% of this number).

Applying the same procedure to models A4 and A2m, the following robustness indices were obtained: 0,129 (0,012) and 0,164 (0,013), respectively. The values in brackets were obtained



(a) Model A2 (c) Model A2m (a) Model A4

Fig. 10 Deformed shape and plastic strain distribution at "unavoidable collapse" state*

*High plastic strain are shown in red, small in green and zero in blue

using Eq. (22).

The maximum resistance to concrete pressures applied to the formwork of Model A2, A4 and A2m is equal to 0,03909 N/mm^2, 0,01401 N/mm^2 and 0,04776 N/mm^2, respectively.

It can be observed that Model A2 has the smallest robustness index value of the three models considered. This is justified because the critical elements to the collapse resistance of this model were the forkhead plates and top jacks. The jacks have an energy deformation capacity lower than the standard elements since their cross-section dimensions are smaller. Therefore, damage concentrated in few elements which have a lower energy deformation capacity than the rest of the elements of the structure, see Fig. 10(a).

Decreasing the jacks extension length allowed damages to concentrate at the columns (of levels 2 and 3), which can dissipate more energy and thus can endure more damages, see Fig. 10(b). It also provided extra resistance to Model A2m. Removing the brace elements makes the structure to exhibit large sway displacements and concentrates damages at the columns of level 3, see Fig. 10(c). However, because the spigot joints, which do not have a large energy deformation capacity, are also severely deformed, they end up limiting the collapse resistance of A4 model, in terms of robustness and resistance.

No matter what equation is used to calculate structural robustness, all values are low, meaning that the systems have a small structural robustness against uniformly applied actions to the formwork. Also, damages up to "unavoidable collapse" state are concentrated in a very small number of elements.

As a closing remark to this application example, it must be pointed out that the applied procedure thus provide an accurate (numerical wise) value of the system's maximum resistance and of the robustness index. If the value of the leading action was set to just a fraction lower than the maximum action value that the structure is capable of resist, collapse would not have occurred. Fig. 11 shows the energy demand and energy capacity evolution of the five levels of Model A2 loaded with concrete pressures applied to the formwork up to 0,03908 N/mm^2, *i.e.* 99,97% of the maximum action value. As it can be seen, for this action value the damages that occur are not sufficient to bring the structure to the ground. This occurs because the kinetic energy of the mass of the Model A2 is very small in this case, but becomes very large when loaded up to the maximum action value, see Figs. 12-13.

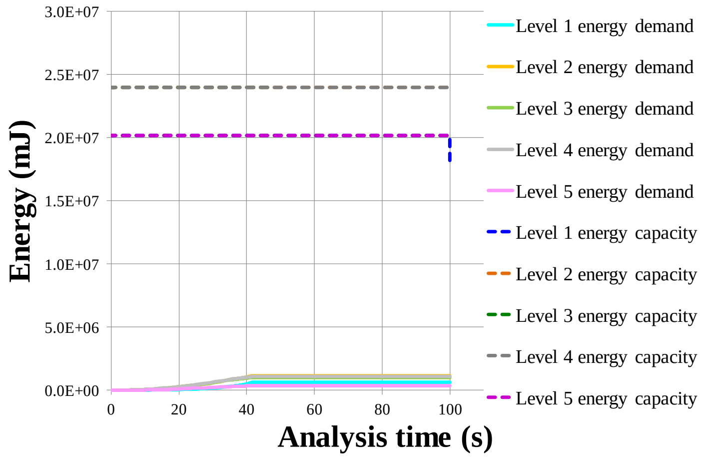


Fig. 11 Energy demand and energy capacity for Model A2 loaded just a fraction less than the maximum load, i.e., the "unavoidable collapse" state

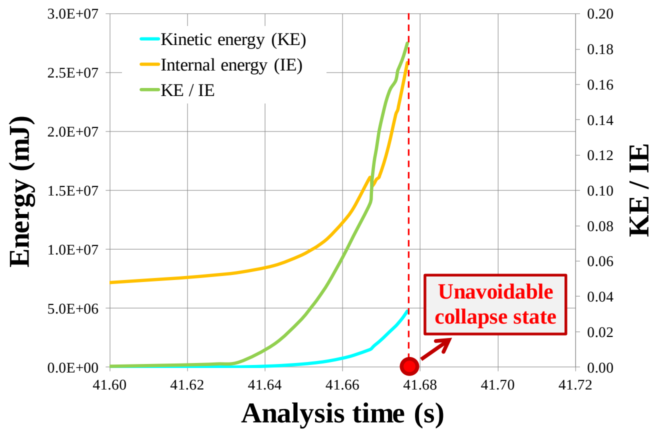


Fig. 12 Kinetic energy and internal energy for Model A2 loaded until "unavoidable collapse" state

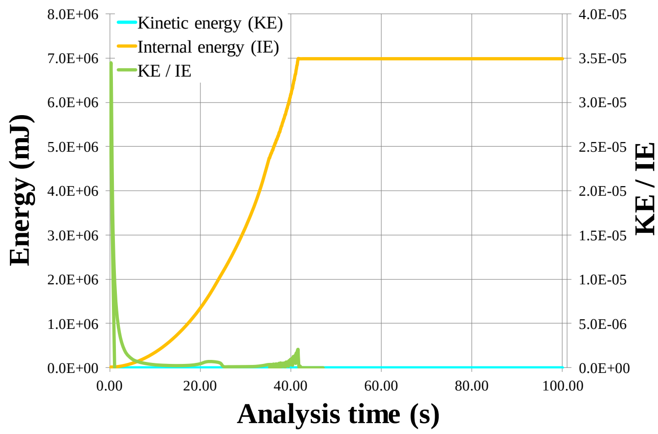


Fig. 13 Kinetic energy and internal energy for Model A2 loaded just a fraction less than the maximum load, i.e., the "unavoidable collapse" state

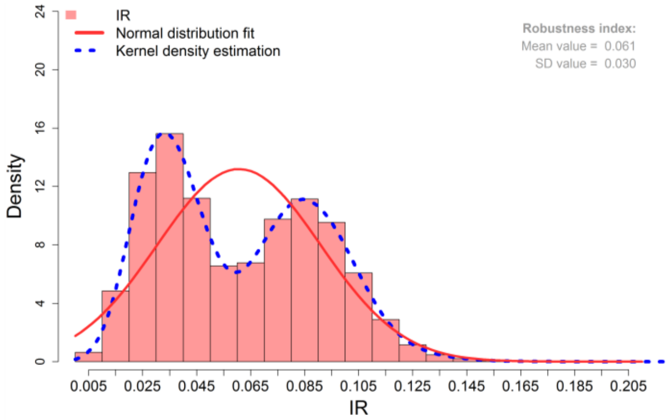


Fig. 14 Illustrative example of robustness index probability density function (pdf)

In the analyses, the kinetic energy value was controlled and if it increased between consecutive increments, using a minimum reference time increment, the leading action value was not increased in order to obtain near static solutions, if possible.

Structural robustness is a function of the hazard scenario, *H*, in particular of the actions values, A, which have an impact on the initial damage mechanism (e.g., an explosion of magnitude *a*) and damage propagation, and of the resistance variables of the structural system, *R*. Resistance variables are random variables and structural robustness directly depends on the resistance variables of the system. Therefore, structural robustness is a random variable, function of resistance variables and action variables.

Determining analytical expressions for the functions relating actions with resistance is quite difficult. Therefore, simulation schemes, like Monte Carlo or other, are a viable alternative solution to determine the structural robustness probability density function (pdf), see Fig. 14 for example.

3.2 Structural fragility analysis

Structural robustness is a measure of the predisposition of a structural system to progressive and disproportionate collapse. Therefore, it is not the best parameter to evaluate when the objective is to assess the system's resistance against the applied actions. The development of such a measure is of great benefit, and even more, if this measure could relate directly to the damage extension within the system for a given action combination. A structural fragility index, F_R, which is capable of addressing adequately these objectives, was developed as the mathematical expression of

$$\mathrm{F_R}\left(\mathrm{A}_R, \mathrm{A}_L \mid H\right) = \frac{\text{Damages up to equilibrium state for action } \mathrm{A}_R \text{ for hazard } h}{\text{Damages up to collapse state for hazard } h} \tag{25}$$

The general expression is given by

$$\mathrm{F_R}\left(\mathrm{A}_R, \mathrm{A}_L \mid H\right) = \frac{D_p - D_{\text{1st failure}}}{D_c - D_{\text{1st failure}}} \text{ with } \begin{cases} 0 \le \mathrm{F_R} \le 1 \\ D_c - D_{\text{1st failure}} = 0 \Rightarrow \mathrm{F_R} = 1 \\ A_L \ge A_{L,uc} \Rightarrow \mathrm{F_R} = 1 \end{cases} \tag{26}$$

where (see Fig. 15):

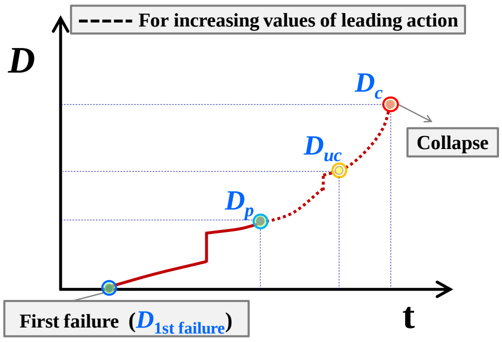


Fig. 15 Illustration of the fragility index notation

A_R represents the reference action;

A_L represents the leading action, which can be different from the reference action. $A_{L,uc}$ represents the value associated with D_{uc};

$H = \{h_1, h_2, ..., h, ..., h_n\}$ is a set of hazard scenarios: {base conditions} + {impact on column 1, impact on column 2, ..., impact on column n}, a set of different actions or a combination of different actions, for example;

D_p represents the value of the damage energy of the structure when the new static equilibrium state is reached for value p of the reference action within the considered hazard scenario. Even though in the remainder of the paper the latter definition is used, for certain analyses cases, for example when seismic actions are involved (or other non-monotonic actions), to attain an improved interpretation, D_p can alternatively represent the damage energy associated with: (i) a given analysis time value, (ii) a displacement (absolute or relative) value, (iii) a force value, (iv) or a value of other variable considered appropriate. The value of D_p can also be given by the damage energy at the end of the application of the reference action;

$D_{1st\ failure}$ represents the damage energy of the structure when the "first failure" state takes place for the hazard scenario considered;

D_{uc} represents the damage energy corresponding to the state where collapse is unavoidable for the hazard scenario considered;

D_c represents the damage energy corresponding to the collapse state for the hazard scenario considered.

The above expression is slightly different from Eq. (7) used to calculate the robustness index. Since structural fragility relates to system's damage energy reserve capacity under the applied actions, in the numerator $D_{uc} - D_{1st\ failure}$ is replaced by $D_p - D_{1st\ failure}$. In the denominator, D_c is kept to obtain a relative damage measure with respect to the maximum available damage energy. Therefore, structural robustness and structural fragility are two closely related structural parameters.

In order to calculate the structural fragility index a three-step procedure must be followed, similar to the one detailed for the structural robustness index, see Fig. 16. The main differences are:

First step: Equal to structural robustness index. Additionally, the reference action must be also chosen and the value of D_p determined. The system performance should preferably be sensitive to the selected reference action values.

Second step: Equal to structural robustness index.

Third step: The fragility index is determined from Eq. (26) based on the adopted limit state, which defines the first failure state (the value of $D_{1st\ failure}$ is determined).

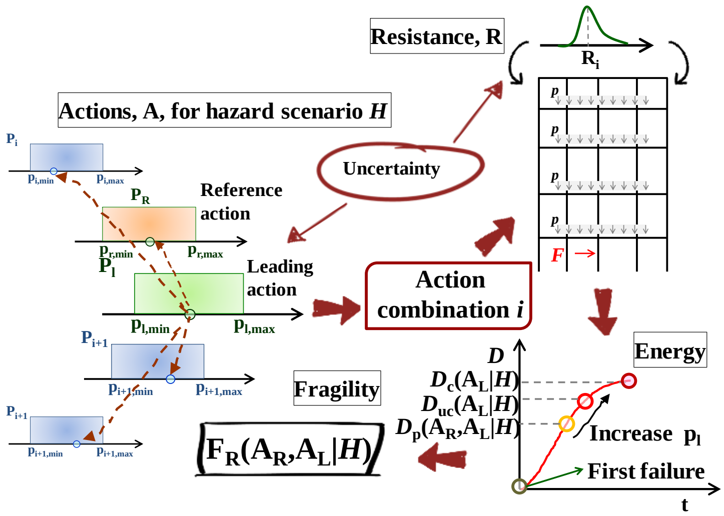


Fig. 16 Procedure to determine the fragility index

For damage energy values higher than D_{uc}, the fragility index is equal to one; for values below $D_{1st\ failure}$, the fragility index is equal to zero.

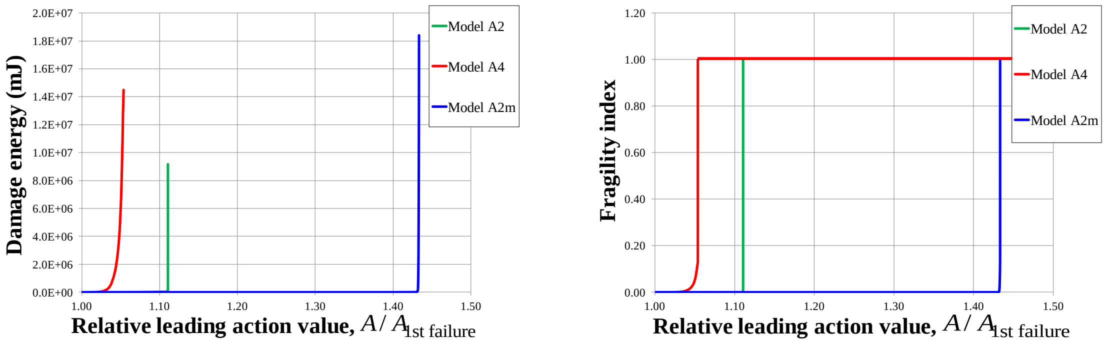
This index is also flexible since the inputs can change, for example: the "first failure" state can be replaced by another criterion, possibly related to a particular element failure, and the "unavoidable collapse" state can also be changed to represent a maximum limit of acceptable damage, for instance.

This flexibility is important. For example, in structural systems where there is a large discrepancy between the damage energy of great part of the elements (e.g., they are very brittle and weak and thus have a very low damage energy) and the remaining few (e.g., they are very resistant and ductile and thus have a very high damage energy), a hazard scenario may occur where only the majority of these weak elements fail. Since the sum of their damage energy is only a fraction of the sum of the damage energy of the strong elements, the fragility index will still be close to zero despite the bulk of the elements have failed. In these cases, it is possible to define a maximum limit of acceptable damage, namely the sum of the damage energy of the weak elements, and assign it to D_{uc}. An alternative, is to include in Eq. (26) the parameter α used in the robustness index calculation, see Eq. (22).

Another extreme case is where there are very few controlling elements. In these cases, there is no need to adapt the parameters of the fragility index since D_{uc} is almost only defined by the damage energy of these controlling elements.

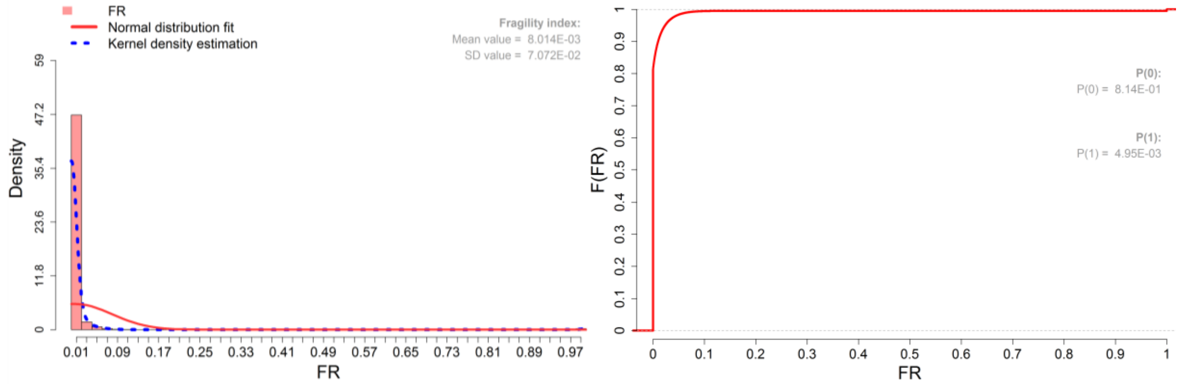
The fragility index also shares many of other features of the robustness index that were already presented in section 3.1.3. An additional remark should be made about the analysis of structural fragility using the proposed index. It is possible that the same increment of the action value (A: load, displacement, rotation, temperature, etc.) causes different increments in structures *A* and *B* fragility index, despite having the same yield and ultimate energy values. This translates to structural fragility (damage accumulation) sensitivity to action values, which may be important when performing risk analysis.

The same bridge falsework models presented in section 3.1.3 will be used as an example to evaluate structural fragility. Fig. 17 illustrates the damage energy and fragility index sensitivity to action values, for all three models (A2, A4 and A2m).



(a) Damage energy (b) Fragility index

Fig. 17 Sensitivity to action values for Models A2, A4 and A2m



(a) Fragility index pdf (b) Fragility index cdf

Fig. 18 Illustrative example of fragility index pdf and cdf

It was already shown that for the structural configurations and external actions considered, robustness index values were small which means that collapse is disproportionate. From Fig. 17, it is possible to observe that damage progression (after first failure) in Model A2m is slower than in Model A4 (where collapse occurs almost after first failure) and than in Model A2, so collapse in the latter two cases is abrupt after first damage. Additionally, in both A2 and A2m models, damages propagate slowly until the "unavoidable collapse" state is attained (after which increase exponentially), whereas in Model A4 damages propagate more gradually for increasing action values, despite having a lower structural resistance after first damage than the braced systems. This means that in Model A4 it may be easier, although more critical, to identify a potential collapse than in models A2 and A2m. Therefore, costs that stem from structural damages propagate differently depending on the structural configuration and external actions considered, see also André (2014), so if a damage detection system is used it should be developed having this in mind.

It may be concluded that adopting small values for the jacks' extension length is one strategy to increase both the resistance and robustness of the bridge falsework Model A2, and to decrease its structural fragility, which has beneficial implications in terms of structural risks and economic risks.

Structural fragility is also a random variable, function of resistance variables and action variables. The probabilistic description of fragility follows closely the one described for structural

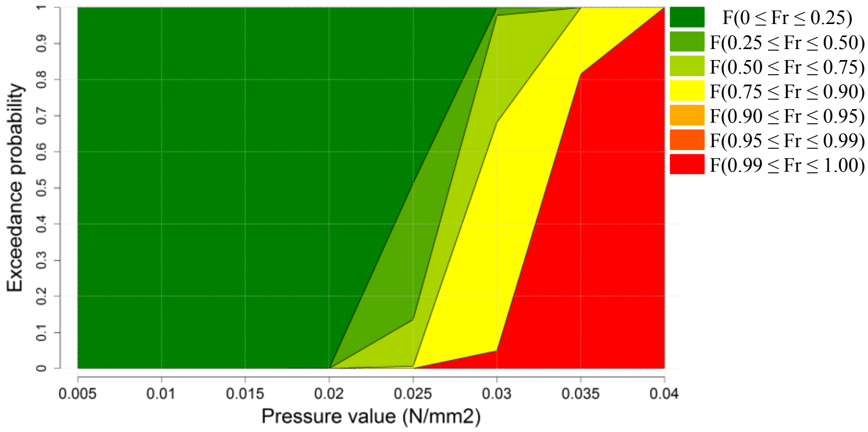


Fig. 19 Example of a representation of fragility curves

robustness. From the cdf of fragility, see Fig. 18 for an illustrative example, a graphic representation of fragility curves can be obtained, which could express simply fragility as a function of action values, or the probability of non-exceedance of fragility values as a function of the actions values, see Fig. 19, for example.

3.3 Vulnerability analysis

Vulnerability, in terms of costs of consequences, is related with structural fragility by a cost function κ(C), which translates levels of structural damages to costs of consequences. An example of a cost function is detailed in André (2014).

3.4 Risk measures

Risk is generally expressed in terms of the probability of structural collapse times the cost of the consequences given the collapse. Additionally, in the classical approach, risk can also be expressed by a probability of failure. However, these definitions are quite limited since they do not account for the various damage states that might occur (damage is a continuous function) but that do not directly imply the global collapse of the structure. Therefore, valuable information is lost that could be used during the risk informed decision-making process. For instance, two structural systems *A* and *B* can have the same probability of failure but the damage evolution in *A* can be quite different than in *B*.

In the suggested framework, if actions and resistance variables are simulated by their real probability distributions and not by uniform probability distributions, structural fragility becomes an expression of the damage extension (D) of the structural system, a measure of the system's structural risk and damage tolerance, and vulnerability becomes a measure of risk that can be used in a Cost-Benefit analysis (CBA). With this approach it is possible to analyse how risk changes with structural robustness or with other risk control measures thus contributing to a better decision-making process.

If the consequences are only assessed in terms of structural damages, then risk is expressed by

$$RISK = f\left(\mathrm{D}\right) \tag{27}$$

and its expected value is given by

$$E(RISK) = \int_D \left[f(\mathrm{D}) \times \mathrm{D} \right] d\mathrm{D} \tag{28}$$

where D represents the structural damages and f(D) is the pdf of the structural damages.

Structural vulnerability may be analysed if the consequences are assessed in terms of costs of all consequences

$$RISK = f(\mathrm{C}) \tag{29}$$

and its expected value is given by

$$E(RISK) = \int_C \left[f(\mathrm{C}) \times \mathrm{C} \right] d\mathrm{C} \tag{30}$$

where C represents the costs related to the consequences and f(C) is the pdf of the costs.

Using this framework, it is possible to perform comparative analysis between alternative scenarios. Fig. 20 illustrates the results of such an analysis, expressed in terms of the cdf of the relative net value between two alternative scenarios for bridge falsework structures, see André (2014), André *et al.* (2015c) for details.

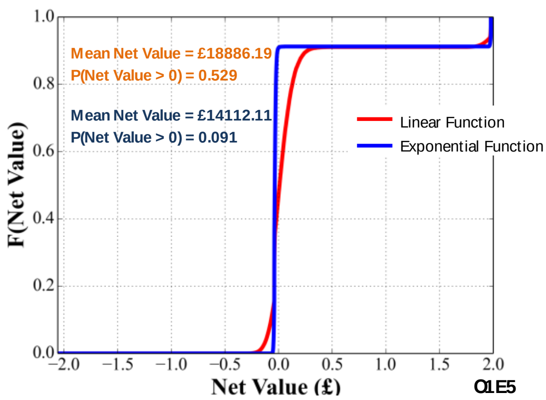


Fig. 20 Cdf of relative Net Value

The potential benefits of using the suggested fragility index over the traditional risk measures, i.e., probability of failure × total cost, can be readily observed. In the traditional risk framework, only one damage state is usually analysed, typically structural collapse. This corresponds to a single value of cost of consequences. With the new proposed methodology, several damage states are already included in the fragility index calculation and therefore it is possible to obtain with no added effort additional and important information for a wide range of probable damage states that if not accounted for in the decision-making process could lead to inefficient solutions.

Of course, more intricate, complex cost functions can be used depending on the problem. For instance, flag variables can be included in the numerical model so to indicate that a given criterion (structural, e.g., nature of damage, or other such as type of operation, e.g., number of persons in the affected area) in a certain critical location of the system has been met. Different cost functions can be attributed to each criteria and location, and the overall cost is determined by the sum of all these particular functions. In the limit, a different cost function can be used for each element.

Multiple failure criteria can be used simultaneously, e.g., failure is attained when the first criterion is met. As in general, there is not a univocal (single) relationship (function) between the damage costs and the damages intensity (fragility) for every failure criterion a slight change must be considered in deriving the probabilistic models for vulnerability. Instead of determining the probabilistic models for vulnerability based on the probabilistic model for fragility, the vulnerability must be determined for each combination of input values, for which the function between the damage costs and the fragility is known. Having a large sample of vulnerability values, obtained from a surrogate model for example, it is possible to estimate the probabilistic model for vulnerability.

In addition, with the new definition of fragility, different possible definitions for failure can be used. If the objective is to analyse a structural system until a damage state other than the complete collapse, for instance to control the rotation of a particular joint, then, as was already mentioned, it is just necessary to assign a fragility index equal to one to that target damage state.

Furthermore, in the existing probability of failure based stochastic design methods it is not straightforward to analyse the sensitivity of the system's probability of failure to a change in the input variables, as well as to perform uncertainty propagation analysis. Consider a structural system to which an acceptable probability of failure was determined using certain input probabilistic models and model parameters. What would happen to the system's probability of failure if these initial hypotheses change? Also, uncertainty may be unevenly distributed across all possible damage states.

In the majority of cases it is not possible to know with appropriate confidence the types of probabilistic models of the input variables and of the distributions parameters, or the degree of dependence/independence between input variables. Knowing that many engineering problems are governed by the extreme values of the input variables, the analyst choices play a crucial role in the follow-up assessment of the results and in the decision-making process. Therefore, uncertainty propagation needs to be considered in the analysis.

The new fragility index gives a direct insight to the consequences of changing the initial hypotheses and if coupled with simulation methods it can also easily incorporate directly the influences of different uncertainties sources.

It is important to emphasise that, in principle, risk in structural engineering can be controlled without structural robustness. This can be readily seen by analysing how risk of consequence *X* is determined

$$RISK(X) = \mathrm{P}(X \mid \mathrm{CL}) \times \mathrm{P}(CL \mid F \cap H) \times \mathrm{P}(F \mid H) \times \mathrm{P}(H) \tag{31}$$

where:

H represents the hazard event;

P(H) represents the probability of occurrence of the hazard event;

P(F|H) represents the conditional probability of occurrence of a local failure given *H*;

P(CL|F∩H) represents the conditional probability of occurrence of a collapse (CL) given *H* and *F*.

Risk can be controlled:

• At the source, i.e., the hazard event, by diminishing its probability of occurrence, *P*(*H*), by eliminating the hazard source or by reducing the hazard source, e.g., by better control of the application of concrete casting loads, reducing the dynamic load effects and their variability, or by specifying maximum working wind velocities for the assembly and operation phases;

• By diminishing the severity of the hazard, *P(F|H)*: externally by reducing the magnitude of the loads effects, adopting protective barriers outside the structure for example, or internally (structurally) by increasing the structure's resistance and/or reducing the resistance variability of each element of the system (especially in the lower-tail region of the probabilistic distributions). It is also possible to use passive isolation techniques such as base isolation of the structure;

• By managing the consequences of the hazard applying protective (reactive) measures: (i) structurally by increasing the resistance (reliability) and/or the robustness of the structure, i.e., by modifying $P(CL|F \cap H)$, or (ii) by changing the context (e.g., by moving valuable goods, people to safer areas or by installing alarm systems and defining efficient exit routes), i.e., by modifying $P(X|CL)$.

The first two measures are preventive (proactive) measures and will increase the system's structural reliability. It is possible to choose a combination of both measures.

It is also necessary to recognise that material properties, geometrical characteristics and actions values vary with time. This fact implies that the behaviour, resistance, reliability, robustness and risk of a structural system changes with time. Therefore, it is important that risk management includes prediction of the risk measures over time: a time variant problem. Here, it is beneficial to refine and to update the models with information, new and more accurate, acquired over time.

4. Conclusions

This paper started by giving a new definition of structural robustness. The main advantages of the newly proposed definition of robustness in relation to the existing definitions are:

• Structural robustness, structural resistance, reliability and risk can now be considered to be four different concepts. The existing structural robustness definitions mixed these four concepts, which made the analysis, interpretation and evaluation of the former variables difficult tasks. Furthermore, by coupling in the same definition of structural robustness up to four different concepts the benefits of determining robustness was not clear. The present definition makes structural robustness a property than can be measured independently of the system's resistance, reliability and risk. Structural robustness can for the first time be considered an independent requirement for the structural performance of civil engineering infrastructures. Together with the structural resistance and reliability they become powerful tools that can, and should, be used in the risk management of civil engineering infrastructures;

• The second advantage of the new definition, is that for the first time, progressive and disproportionate collapse analysis is clearly defined as a requirement not only for unforeseen and accidental situations affecting localised areas of a given structure, but also for normal service conditions covering for instance design cases where the permanent load is the dominant action.

After, new structural robustness and structural fragility indices were presented and formed the basis of a risk management framework. This new methodology is applicable, in principle, to all structures. This paper gave a complete insight to the methods and procedures to be used to determine structural robustness, structural fragility and vulnerability.

An explicit expression to determine the robustness index was given based on the concept of damage energy and representative illustration examples were presented using bridge falsework case studies.

The fragility index was developed as a tool to assess the system's structural damages for a given action combination. Using this measure, it is possible to perform progressive collapse analysis and also evaluate the sensitivity of damage accumulation to action values, which may be important when performing risk analysis.

In general, traditional structural risk analyses focus on probability of failure. These analyses are quite limited since they do not account for the various damage states that might occur (damage is a continuous function) but that do not directly imply the global collapse of the structure. Therefore, valuable information is lost that could be used during the risk informed decision-making process potentially leading to inefficient solutions. For instance, two structural systems *A* and *B* can have the same probability of failure but the damage evolution in *A* can be quite different than in *B*.

As a conclusion, the newly developed structural robustness index can be used as a design option to reduce the structural risk and the newly developed structural fragility index is an analysis tool that should be used to assess the structural risk.

Acknowledgments

The authors acknowledge financial support from Fundação para a Tecnologia e a Ciência through the PhD scholarship SFRH / BD / 70993 / 2010 given to the first author.

References

André, J., Beale, R. and Baptista, A. (2012), "A survey of failures of bridge falsework systems since 1970", *Proceedings of the ICE - Forensic Engineering*, **65**(4), 161-172.

André, J., Beale, R. and Baptista, A. (2013), "Recent advances and existing challenges in the design of bridge falsework systems", *Civil Eng. Environ. Syst.*, **30**(2), 130-145.

André, J., Beale, R. and Baptista, A. (2015a), "Finite element models of bridge falsework systems", *Struct. Eng. Mech*. (under Review)

André, J., Beale, R. and Baptista, A. (2015b), "Experimental analysis of bridge falsework Cuplok joints", *Steel Compos. Struct*. (under Review)

André, J., Beale, R. and Baptista, A. (2015c), "Risk analysis of bridge falsework Cuplok systems", *Struct. Eng. Mech*. (under Review)

André, J. (2014), "Determination of the main parameters affecting the performance of bridge falsework systems", PhD Thesis, Oxford Brookes University, Oxford, UK.

Baker, J., Schubert, M. and Faber, M. (2008), "On the assessment of robustness", *Struct. Saf.*, **30**(3), 253-267.

Baptista, A. and Muzeau, J. (2001), "Analytical formulation of the design of circular hollow sections in the elastoplastic regime", *Proceedings of III Portuguese National Conference of Steel and Composite Construction*, Aveiro, Portugal, December. (in Portuguese)

Bažant, Z., Le, J., Greening, F. and Benson, D. (2008), "What did and did not cause collapse of World Trade Center Twin Towers in New York?", *J. Eng. Mech.*, **134**(10), 892-906.

Bažant, Z. and Verdure, M. (2007), "Mechanics of progressive collapse: Learning from World Trade Center and building demolitions", *J. Eng. Mech.*, **133**(3), 308-319.

Bažant, Z. and Zhou, Y. (2002), "Why did the World Trade Center collapse? - simple analysis", *J. Eng. Mech.*, **128**(1), 2-6.

Beale, R. (2014), "Scaffold research - a review", *J. Constr. Steel Res.*, **98**, 188-200.

BS EN 1990 (2002), *Eurocode 0: Basis of Structural Design*, UK.

BS EN 1991-1-7 (2006), *Eurocode 1: Actions on Structures - Part 1-7: General Actions - Accidental Actions*, UK.

Chandrangsu, T. and Rasmussen, K. (2011), "Structural modelling of support scaffold systems", *J. Constr. Steel Res.*, **67**(5), 866-875.

CEB (1993), *CEB-FIP Model Code 90*, Thomas Telford, London, UK.

Dusenberry, D. and Hamburger, R. (2006), "Practical means for energy-based analyses of disproportionate collapse potential", *J. Perform. Construct. Facil.*, **20**(4), 336-348.

Ellingwood, B., Smilowitz, R., Dusenberry, D., Duthinh, D., Lew, H. and Carino, N. (2007), *Best Practices*

for Reducing the Potential for Progressive Collapse in Buildings, National Institute of Standards and Technology (NIST), USA.
Faber, M. (2009), *Risk and Safety in Engineering Course Lecture Notes*, Swiss Federal Institute of Technology Zurich (ETHZ), Switzerland.
Fang, Z. (2007), "Energy-based approach to structural robustness", *J. Southwest Jiaotong Univ.*, **15**(4), 319-234.
Frangopol, D. and Curley, J. (1987), "Effects of damage and redundancy on structural reliability", *J. Struct. Eng.*, **113**(7), 1533-1549.
Fu, G. and Frangopol, D. (1990), "Balancing weight, system reliability and redundancy in a multiobjective optimization framework", *Struct. Saf.*, **7**(2-4), 165-175.
Gardner, L. (2008), "The continuous strength method", *Proceedings of the ICE - Structures and Buildings*, **161**(3), 127-133.
ISO 2394 (2002), *General Principles on Reliability for Structures*, Switzerland.
Knoll, F. and Vogel, T. (2009), *Design for Robustness*, International Association for Bridge and Structural Engineering (IABSE), Zurich, Switzerland.
Korol, R. and Sivakumaran, K. (2014), "Reassessing the plastic hinge model for energy dissipation of axially loaded columns", *J. Struct.*, **2014**(1), 1-7.
Narasimhan, H. and Faber, M. (2009), "Categorisation and assessment of robustness related provisions in European standards", *Proceedings of the Joint Workshop of COST Actions TU0601 and E55*, Ljubljana, Slovenia, September.
Smith, J. (2006), "Structural robustness analysis and the fast fracture analogy", *Struct. Eng. Int. (SEI)*, **16**(2), 18-123.
Starossek, U. and Haberland, M. (2008), "Measures of structural robustness - Requirements & applications", *Proceedings of the ASCE SEI 2008 Structures Congress – Crossing Borders*, Vancouver, Canada, April.
Starossek, U. and Haberland, M. (2009), "Evaluating measures of structural robustness", *Proceedings of the Structures Congress 2009: Don't Mess with Structural Engineers - Expanding Our Role*, Austin, Texas, May.
Starossek, U. and Wolff, M. (2005), "Progressive collapse: design strategies", *Proceedings of IABSE Symposium, Structures and Extreme Events*, Lisbon, Portugal, September.
Starossek, U. (2006), "Progressive collapse of structures: Nomenclature and procedures", *Struct. Eng. Int. (SEI)*, **16**(2), 113-117.
Starossek, U. (2009), *Progressive Collapse of Structures*, Thomas Telford, London, UK.
Todinov, M. (2007), *Risk-Based Reliability Analysis and Generic Principles for Risk Reduction*, Elsevier, Amsterdam, Netherlands.
USDOD (2010), *Unified Facilities Criteria (UFC), Design of Buildings to Resist Progressive Collapse*, United States of America Defence Department, USA.

PL