Structural Engineering and Mechanics, *Vol. 56, No. 6 (2015) 1003-1020*
DOI: http://dx.doi.org/10.12989/sem.2015.56.6.1003

Free vibration analysis of edge cracked symmetric functionally graded sandwich beams

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(Received May 1, 2015, Revised August 7, 2015, Accepted November 24, 2015)

Abstract. In this study, free vibration analysis of an edge cracked multilayered symmetric sandwich beams made of functionally graded materials are investigated. Modelling of the cracked structure is based on the linear elastic fracture mechanics theory. Material properties of the functionally graded beams change in the thickness direction according to the power and exponential laws. To represent functionally graded symmetric sandwich beams more realistic, fifty layered beam is considered. Composition of each layer is different although each layer is isotropic and homogeneous. The considered problem is carried out within the Timoshenko first order shear deformation beam theory by using finite element method. A MATLAB code developed to calculate natural frequencies for clamped and simply supported conditions. The obtained results are compared with published studies and excellent agreement is observed. In the study, the effects of crack location, depth of the crack, power law index and slenderness ratio on the natural frequencies are investigated.

Keywords: functionally graded materials; cracked beam; free vibration; FEM

1. Introduction

Functionally graded materials (FGMs) idea was first announced in 1984 by a group of scientists during the design of thermal barrier (Demir *et al*. 2013). Due to continuous transition of material properties through the thickness is obtained by changing constituents of the materials, the problem such as delaminating occurred between laminated composite surfaces is not encountered. Generally, a FGM is prepared from a mixture of a metal and a ceramic in such a way that the ceramic can resist high temperature in thermal surroundings and protects the metal from corrosion, while the metal provides toughness and strength. As a consequence, FGMs have been effectively used in many structures of aerospace, civil and automotive industry. Beam elements are widely employed in the structural design. However, existence of a crack in beam affects its stiffness and changes its vibration characteristics. Therefore, the existence of a crack in structures must be taken in consideration at the design stage.

There are several studies about free vibration behavior of homogeneous cracked beams in the literature (Kisa *et al*. 1998, Kisa and Brandon 2000, Kisa 2004). On the other hand, there are many

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

studies investigated dynamic and free vibration characteristics of FGM beams. For instance, Alshorbagy *et al*. (2011) investigated the dynamic characteristics of functionally graded beam. In this study, material variation is assumed in axially or transversally through the thickness based on the power law. The theory of the work is derived by using the principle of virtual work under the assumptions of the Euler–Bernoulli beam theory. Simsek *et al*. (2012) presented linear dynamic investigation of an axially functionally graded (AFG) beam with simply supported boundary conditions owing to a moving harmonic load. Dynamic responses of AFG beam are obtained by employing Newmark method. In this study, velocity of the moving load, material distribution and excitation frequency parameters are explored. Giunta *et al*. (2011) used several axiomatic refined theories for free vibration analysis of FG beams. Distribution of the materials is assumed to vary continuously on the transverse direction according to power law dispersion in terms of the mixture ratio of the material components. Shahba *et al*. (2011) investigated the free vibration and stability analysis of axially functionally graded tapered Timoshenko beams by using finite element approach. Thai and Vo (2012) developed several higher-order shear deformation beam theories to analyze the free vibration and bending of functionally graded beams. Hamilton’s principle is utilized to drive the equations of motion. The influences of shear deformation and power law index on the bending and free vibration responses of FG beams are studied. Li (2008) introduced a new methodology to analyses the static and dynamic behaviors of FG beams by incorporating shear deformation and rotary inertia. Li *et al*. (2013) presented a novel finite element method to solve the free vibration and static problems of axially and transversally FG beams with variable cross section profile. Pradhan and Chakraverty (2013) investigated free vibration analysis of FGM beams for different boundary conditions. The study is based on the classical and first order shear deformation beam theories. Elishakoff and Candan (2001) resented free vibration of non-uniform beams whose elastic modulus and material density varying axially. The analysis is carried out for different boundary conditions. Aydogdu and Taskin (2007) studied the free vibration behavior of simply supported FG beam whose elastic modulus varies in the transverse direction according to exponential and power law. Su and Banerjee (2015) investigated free vibration of FG Timoshenko beams by dynamic stiffness method. In this study, changing of the material properties is assumed in the thickness direction.

At the same time, there are many studies related to the cracked FGM beams. For example, Wei *et al*. (2012) proposed an analytical solution for the free vibration of cracked FGM beams with axial loading, shear deformation and rotary inertia effects. The crack is simulated by a rotational spring element. Yang and Chen (2008) examined free vibration and elastic buckling of FGM beams containing open edge cracks theoretically by means of Bernoulli–Euler beam theory and the rotational spring model. Aydin (2013) investigated the free vibration of FGM beams with multiple open edge cracks. The analyses are based on Euler-Bernoulli beam and massless rotational springs joined two intact parts of the beam. Variation of the material is an exponential through the beam thickness direction. Yan and Yang (2011) presented an analytical study on the forced flexural vibration of FG beams via open edge cracks under an axial compressive force and a concentrated transverse load moving along the longitudinal direction. The crack is represented by a rotational spring whose sectional flexibility is calculated through fracture mechanics. Kitipornchai *et al*. (2009) studied the nonlinear vibration of FGM beams containing an open edge crack. Timoshenko beam theory and von Karman geometric nonlinearity are used in this paper. A massless elastic rotational spring is used to model the cracked section. Yan *et al*. (2011) worked the parametric instability of FGM beams with an open edge crack exposed to an axial pulsating excitation which is a combination of a harmonic excitation force and a static compressive force. Timoshenko beam theory and linear rotational spring model are used in the analysis. Matbuly *et al*. (2009) investigated the free vibration analysis of an elastically supported cracked beam by differential quadrature method. Yan *et al*. (2011) analyzed the dynamic analyses of FGM beams with an open edge crack resting on an elastic foundation exposed to a transverse load moving at a constant speed. Timoshenko beam theory is used to consider the transverse shear deformation. Modelling of the cracked section is symbolized as an assembly of two sub-beams linked through a linear rotational spring. Ferezqi *et al*. (2010) introduced an analytical method to investigate the free vibrations of cracked Timoshenko beams prepared of FGMs. Akbas (2013) studied a geometrically nonlinear static analysis of edge cracked Timoshenko beams made of FGM. The material property is changed in the transverse direction according to exponential distributions. Modelling of the cracked beam is represented as an assembly of two sub beams linked through a massless elastic rotational spring. Akbaş (2013) presented free vibration behavior of an edge cracked FG cantilever beam. Hamilton's principle is used to drive differential equations of the motion. Investigation of the problem is based on the Euler-Bernoulli beam theory by finite element method. A massless elastic rotational spring element used to represent the crack in the beam. Wattanasakulpong *et al*. (2013) improved a third order shear deformation theory to formulate a governing equation for predicting free vibration of layered FG beams. The Ritz method is employed to solve the governing equation for various types of boundary conditions. As it is seen from the literature there are several approach and modelling techniques to determine the dynamic and free vibration characteristics of FGM beams. Although, there are some studies relating to prediction of natural frequencies of the layered FG beams, estimations of the natural frequencies of the edge cracked layered symmetric sandwich beams made of FGM by using the effective mass density and Young's modulus using mixture rules approach is not considered in the literature.

In this study, free vibration analysis of an edge cracked multilayered symmetric sandwich beams made of FGMs are investigated. The considered problem is carried out within the Timoshenko first order shear deformation beam theory by using finite element method. It is assumed that the material properties of the FG sandwich beam follow power and exponential distributions through the beam thickness. As a consequence, the originality of this work is to estimate the natural frequencies of the edge cracked FGM beams by using the effective mass density and Young's modulus using mixture rules and laminate theory. Hence, natural frequencies are computed by using obtained effective material properties for edge cracked cantilever and simply supported boundary conditions. The results obtained from the study are compared with the available results in the literature. Excellent agreement is observed. In the study, the effects of crack location, depth of the crack, power law index and slenderness ratio on the natural frequencies are also investigated.

2. Modelling and calculation of the effective material properties

The considered symmetric FGM sandwich beam is shown in Fig. 1. Here; L, b, d and N represent length, thickness, width and number of layers of the beam respectively. To represent symmetric FGM sandwich beam more realistic, fifty layered beam is considered. Each layer of the beam is composed by a mixture of alumina phases (Al_2O_3) and aluminum (Al). Layers are prepared symmetrically to the neutral axis of the beam as shown in Fig. 1.

The neutral plane of the beam is assumed to be pure aluminum while the upper and lower surfaces of the beam are Al_2O_3. FGM is arranged between the neutral surface and upper and lower

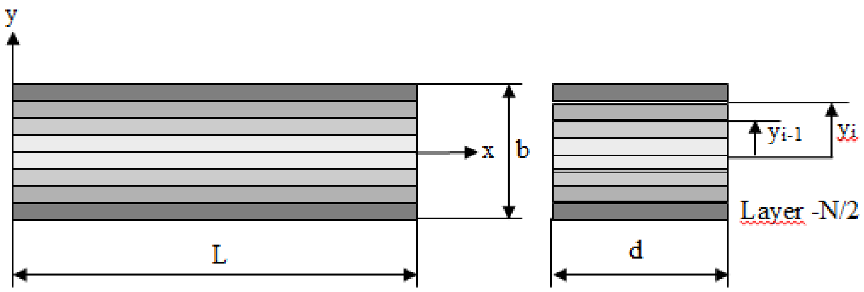


Fig. 1 Symmetric FGM sandwich beam

surfaces. The mixture ratio is determined according to the polynomial or an exponential function. The mixture is varied continuously from the upper and lower surfaces to the neutral surface of the beam.

The Young's modulus for each layer of the upper half of the beam are calculated from exponential and power laws respectively by the following equations (Demir *et al*. 2013)

$$E(y) = E_c e^{(-\delta(1-2y))}, \delta = \frac{1}{2}\ln\left(\frac{E_c}{E_m}\right) \tag{1}$$

$$E(y) = (E_c - E_m)\left(y + \frac{1}{2}\right)^n + E_m \tag{2}$$

where E_c, E_m, y and n are Young's modulus of the ceramic (Al_2O_3), metal (Al), the coordinate axis and power law index, respectively. To calculate each layer density of the upper half of the beam, the equations given for Young's modulus are adopted for density by the following form

$$\rho(y) = \rho_c e^{(-\delta(1-2y))}, \delta = \frac{1}{2}\ln\left(\frac{\rho_c}{\rho_m}\right) \tag{3}$$

$$E(y) = (E_c - E_m)\left(y + \frac{1}{2}\right)^n + E_m \tag{4}$$

Where ρ_c, ρ_m represent the mass density of the ceramic and metal. The variable y in the above equations is defined as

$$y = -\frac{1}{2}, -\frac{1}{2} + \frac{1}{\eta}, -\frac{1}{2} + \frac{2}{\eta}, \ldots, \frac{1}{2}$$

where η is equal to $(N/2)$-1.

To obtain the mass density and the Young's modulus throughout the thickness of the beam, the effective Young's modulus and mass density are calculated by the laminate theory. The effective Young's modulus and density are given by the following equations (Gibson 1994)

$$E_{ef} = \frac{8}{b^3}\sum_{i=1}^{N/2}\left(E_y\right)_i \left(y_i^3 - y_{i-1}^3\right) \tag{5}$$

$$\rho_{ef} = \frac{8}{b^3}\sum_{i=1}^{N/2}\left(\rho_y\right)_i\left(y_i^3 - y_{i-1}^3\right) \tag{6}$$

3. Stiffness and mass matrices for the Timoshenko beam element

Derivation of the stiffness and mass matrices for a beam element having two nodes with 2 degrees of freedoms (d.o.f.s) $\{v, \theta\}$ at each node are given by Petyt (1990)as

$$[K_1] = \frac{EI_z}{2k^3(1+3\beta)}\begin{bmatrix} 3 & 3k & -3 & 3k \\ 3k & (4+3\beta)k^2 & -3k & (2-3\beta)k^2 \\ -3 & -3k & 3 & -3k \\ 3k & (2-3\beta)k^2 & -3k & (4+3\beta)k^2 \end{bmatrix} \tag{7}$$

and

$$[M_1] = \frac{\rho Ak}{210(1+3\beta)^2}\begin{bmatrix} m_1 & m_2 & m_3 & m_4 \\ m_2 & m_5 & -m_4 & m_6 \\ m_3 & -m_4 & m_1 & -m_2 \\ m_4 & m_6 & -m_2 & m_5 \end{bmatrix} + \frac{\rho I_z}{30k(1+3\beta)^2}\begin{bmatrix} m_7 & m_8 & -m_7 & m_8 \\ m_8 & m_9 & -m_8 & m_{10} \\ -m_7 & -m_8 & m_7 & -m_8 \\ m_8 & m_{10} & -m_8 & m_9 \end{bmatrix} \tag{8}$$

where $k = L/2$, E is Young's modulus, A is cross section area of the beam and I_z is the section moment of inertia with respect to the z axis,

$$\beta = \frac{EI_z}{\kappa GAk^2} \tag{9}$$

Where κ is the shear correction factor, G is the shear modulus and

$$\begin{aligned} &m_1 = 156 + 882\beta + 1260\beta^2, m_2 = \left(44 + 231\beta + 315\beta^2\right)k, \\ &m_3 = 54 + 378\beta + 630\beta^2, m_4 = \left(-26 - 189\beta - 315\beta^2\right)k, \\ &m_5 = \left(16 + 84\beta + 126\beta^2\right)k^2, m_6 = \left(-12 - 84\beta - 126\beta^2\right)k^2, \\ &m_7 = 18, m_8 = (3 - 45\beta)k, \\ &m_9 = \left(8 + 30\beta + 180\beta^2\right)k^2, m_{10} = \left(-2 - 30\beta + 90\beta^2\right)k^2 \end{aligned} \tag{10}$$

The stiffness and mass matrices for a beam element having two nodes with 1 d.o.f.s $\{u\}$ local axial displacement in the x direction at each node are given by Petyt (1990) as

$$[K_2] = \frac{EA}{k}\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad [M_2] = \rho Ak\begin{bmatrix} \frac{3}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \tag{11}$$

In the analyses, a beam element having two nodes with 3 d.o.f.s $d=\{u, v, \theta\}$ at each node is used. Therefore, the obtained stiffness and mass matrices above are adopted to get the resulting stiffness and mass matrices with 3 d.o.f.s at each node. The resulting stiffness and mass matrices for a beam element are

$$K_{el} = \begin{bmatrix} K_2 11 & & & K_2 12 & & \\ & K_1 11 & K_1 12 & & K_1 13 & K_1 14 \\ & K_1 21 & K_1 22 & & K_1 23 & K_1 24 \\ K_2 21 & & & K_2 22 & & \\ & K_1 31 & K_1 32 & & K_1 33 & K_1 34 \\ & K_1 41 & K_1 42 & & K_1 43 & K_1 44 \end{bmatrix}_{(6x6)} \tag{12}$$

and

$$M_{el} = \begin{bmatrix} M_2 11 & & & M_2 12 & & \\ & M_1 11 & M_1 12 & & M_1 13 & M_1 14 \\ & M_1 21 & M_1 22 & & M_1 23 & M_1 24 \\ M_2 21 & & & M_2 22 & & \\ & M_1 31 & M_1 32 & & M_1 33 & M_1 34 \\ & M_1 41 & M_1 42 & & M_1 43 & M_1 44 \end{bmatrix}_{(6x6)} \tag{13}$$

4. The stiffness matrix for the crack

In this study, the cracked node as a cracked element of zero mass and zero length is considered to represent the crack (Kisa *et al*. 1998). The strain energy induced from the crack leads to flexibility coefficients. Derivations of the flexibility coefficients functions of the crack shape and stress intensity factors are given in detail by (Kisa and Brandon 2000). The compliance coefficients matrix is constructed by the flexibility coefficients according to the d.o.f.s $d=\{u, v, \theta\}$ as

$$C = \begin{bmatrix} c_{11} & 0 & c_{13} \\ 0 & c_{22} & 0 \\ c_{31} & 0 & c_{33} \end{bmatrix}_{(3x3)} \tag{14}$$

The inverse of the compliance matrix C^{-1} is the stiffness matrix of the nodal point. Finally, the stiffness matrix of the cracked nodal element is written as

$$K_{cr} = \begin{bmatrix} [C]^{-1} & -[C]^{-1} \\ -[C]^{-1} & [C]^{-1} \end{bmatrix}_{(6x6)} \tag{15}$$

For free vibration of an intact (Cunedioglu and Beylergil 2014) and cracked beam (Akbaş

2013), the eigenvalue problem is defined by the following equations, respectively

$$\left([K]-\omega^2[M]\right)\{\bar{d}\}=0 \tag{16}$$

$$\left(\left([K]+[K_{cr}]\right)-\omega^2[M]\right)\{\bar{d}\}=0 \tag{17}$$

where [*K*], [*M*], ω and { $\bar{d}$ } are the global stiffness matrix, the global mass matrix, the natural angular frequency in radians per second and the mode shape, respectively.

5. Verification of the cracked beam

In order to validate the accuracy of the suggested approach and the finite element MATLAB code developed by author, an example taken from the literature (Akbaş 2013) is analyzed and then numerical results are compared with existing results in the literature. For this aim, the following cantilever isotropic homogeneous with an edge cracked Timoshenko beam shown in Fig. 2 is considered to calculate the first natural frequencies for different crack location (L_1/L) for L=0.2 m, b=0.0078 m, a/b=0.2, E=216 GPa, G=3/8E, κ=5/6, ρ=7850 kg/m^3, υ=0.28 and width d=0.025 m.

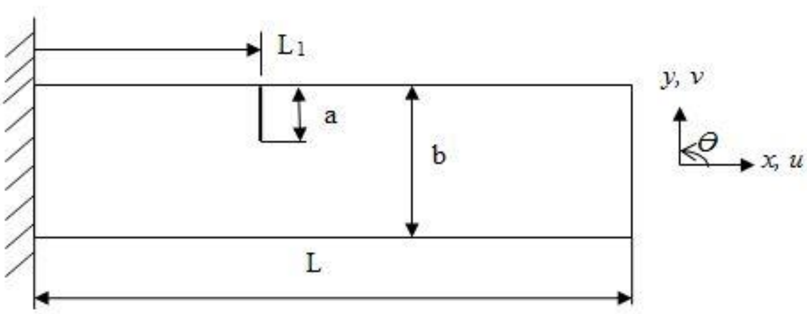


Fig. 2 Geometry of the cantilever isotropic homogeneous beam with an edge crack

100 finite elements are used in the analyses and the obtained results are given in the Table 1.

Table 1 First natural frequency values of the cantilever isotropic homogeneous beam with an edge crack

L_1/L	0.2	0.4	0.6	Intact Beam
Kisa *et al*. (1998)	1020.0137	1030.095	1035.284	1037.0189
Ke *et al*. (2009)	1020.098	1029.853	1034.932	1037.0106
Akbaş (2013)	1021.6	1031.2	1036.2	1037.09
Present	1020.046	1030.004	1035.196	1036.932

As seen from the Table 1, the present results are very close to the reference values.

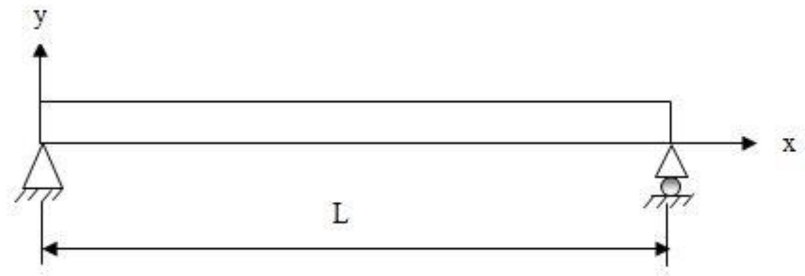


Fig. 3 Simply supported FG symmetric sandwich beam

Table 2 The material properties of the constituents of the sandwich beam

Material	*E* (GPa)	*ρ* (kg/m3)	*υ*
Al	70	2700	0.3
Al2O3	380	3950	0.3

Table 3 Convergence analysis of the simply supported FG symmetric sandwich beam

Function type	Nat. freq. (Hz)	40 elements	50 elements	60 elements	70 elements	80 elements	90 elements	100 elements
Exponential	1st mode	486.426	486.423	486.423	486.422	486.422	486.422	486.422
	2nd mode	1939.516	1939.479	1939.463	1939.454	1939.450	1939.447	1939.445
	3rd mode	4341.146	4340.935	4340.836	4340.784	4340.755	4340.737	4340.726
	4th mode	7662.503	7661.716	7661.340	7661.139	7661.021	7660.948	7660.899
Polynomial (*n*=10)	1st mode	404.674	404.673	404.672	404.671	404.671	404.671	404.671
	2nd mode	1613.550	1613.520	1613.506	1613.499	1613.495	1613.493	1613.492
	3rd mode	3611.550	3611.374	3611.292	3611.249	3611.225	3611.210	3611.200
	4th mode	6374.702	6374.047	6373.734	6373.567	6373.469	6373.408	6373.368

6. Verification of the FGM beam

The FG symmetric sandwich beam is characterized by fifty layered elements. The rotary inertia and shear deformation effects are taken into account to calculate the natural frequencies of a simply supported beam. For convergence and validation, an example taken from the literature as shown in Fig. 3 is analyzed and then numerical results are compared with published results. The geometric dimensions of the beam are taken as length (L=200 mm), thickness (b=5 mm) and width (d=20 mm).

The material properties of the constituents of the sandwich beam are given as in Table 2 (Demir *et al*. 2013). Poisson's ratio is taken constant.

Convergence analysis is carried out for exponential and polynomial (n=10) functions to show the accuracy of the results. The results of the analysis are given in Table 3. Due to the fact that the number of elements used satisfies the convergence, only 100 elements are enough in the finite element model.

Table 4 The first four natural frequencies (Hz) of the symmetric FG sandwich Timoshenko beam

Function Type			Mode1	Mode2	Mode3	Mode4
Exponential		Demir *et al*. (2013)	486.52	1939.6	4341.48	7663.15
		Present	486.422	1939.445	4340.726	7660.899
Polynomial	n=0.1	Demir *et al*. (2013)	550.91	2196.89	4917.36	8679.65
		Present	550.944	2196.707	4916.511	8677.095
	n=0.5	Demir *et al*. (2013)	534.98	2133.31	4775.06	8428.46
		Present	534.999	2133.135	4774.228	8425.982
	n=1	Demir *et al*. (2013)	518.2	2065.88	4624.09	8162
		Present	518.086	2065.695	4623.289	8159.592
	n=5	Demir *et al*. (2013)	443.06	1766.61	3954.27	6979.69
		Present	443.038	1766.469	3953.582	6977.634
	n=10	Demir *et al*. (2013)	404.67	1613.63	3611.82	6375.24
		Present	404.671	1613.492	3611.200	6373.368

The obtained validation results from the finite element MATLAB code developed by author and published results are listed in Table 4. It is seen from the Table 4 that a good agreement is observed.

7. Statement of the problem

The analyses are carried out for the cracked cantilever and simply supported symmetric FG sandwich Timoshenko beams. The beams are modeled by fifty layered beam elements to represent the FG beams. Also, the geometric dimensions stated before in the verification of the FG beam are taken in the analyses. In addition, finite element models are generated by 100 elements and it provides converge. Variations of the elastic modulus and mass density through the thickness are realized by power and exponential laws. The material model for n=0 represents the isotropic homogeneous beam. For that reason, verification of the cracked beam procedure is valid for the FGM beam.

In the study, the effects of crack location, depth of the crack, power law index, slenderness ratio and different material distribution on the natural frequencies are investigated.

8. Results and discussion

Consider that the edge cracked cantilever isotropic homogeneous beam indicated in Fig. 2 is made of symmetric FG sandwich Timoshenko beam. The analyses are carried out according to the stated assumption. The effects of crack depth ratio (a/b) on the first four mode natural frequency values for crack location (L_1/L=0.2) and different power index (n) are demonstrated in Fig. 4.

As shown in Fig. 4, the natural frequency values decreases with increasing the ratio (a/b) and (n). But, the second mode natural frequency values are slightly affected from the ratio (a/b). The effects of crack location (L_1/L) on the first four mode natural frequency values for (n=0.1) and

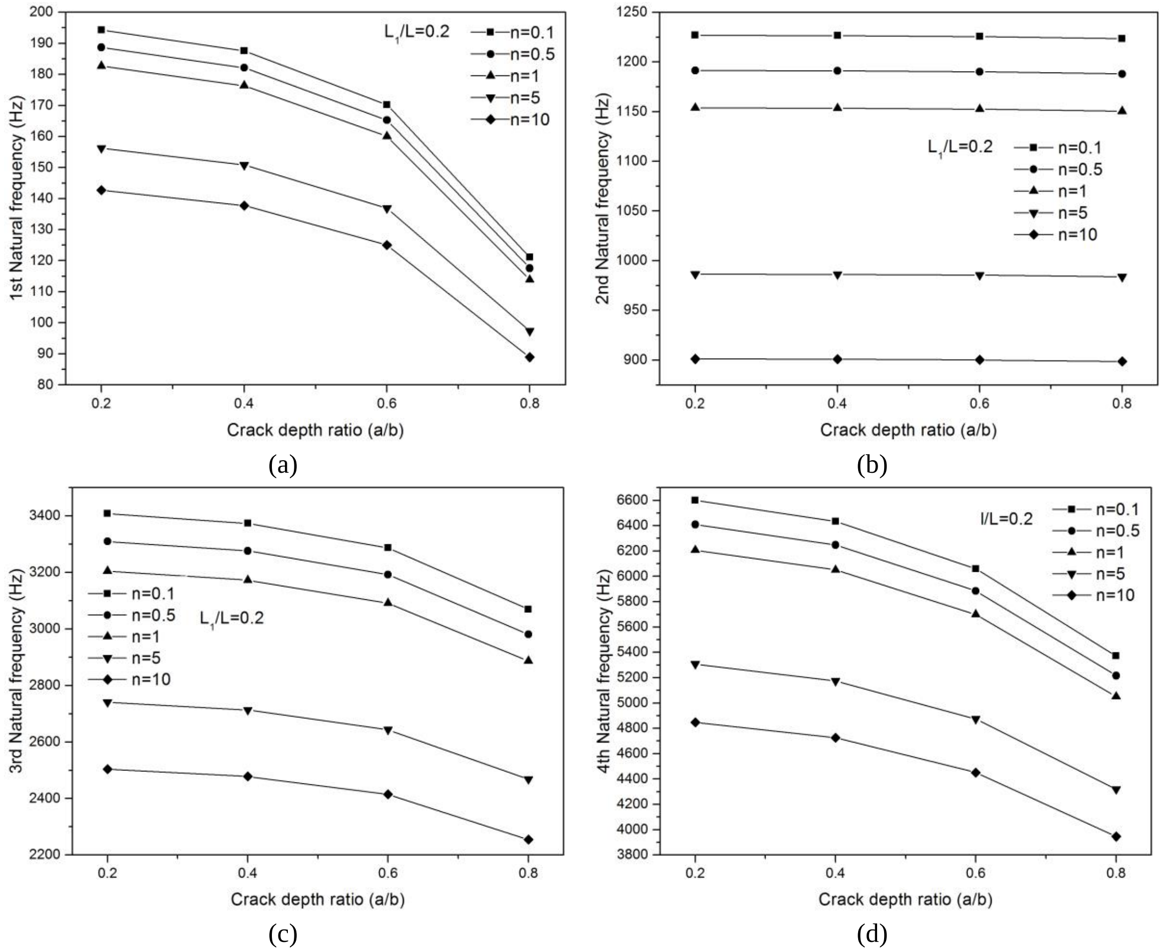


Fig. 4 The effects of the crack depth ratio (*a/b*) on the first four mode natural frequency values for crack location (L_1/L=0.2) and different power index (*n*)

different ratio (*a/b*) are shown in Fig. 5.

As shown in Fig. 5(a), the minimum first mode natural frequency values are observed when the (L_1/L) is the nearest to the clamped edge of the beam. However, when the location (L_1/L) approaches to the free end of the beam, the first mode values increase. The minimum 2nd mode and the maximum 4th mode natural frequency values are observed at the (L_1/L=0.6) as shown in Figs. 5 (b) and (d) respectively. The 3rd mode natural frequency values decreases when the location (L_1/L) approaches to the free end of the beam as shown in Fig. 5(c). On the other hand, the natural frequency values decreases with increasing the ratio (*a/b*) for all modes. It is known that the natural frequency is a function of the elasticity modulus and the mass density of the structure. Increasing the (*n*) parameter is resulted in decrease in values of the elasticity modulus (*E*), mass density (*ρ*) and the ratio (*E*/*ρ*). Because of these reasons, the natural frequencies decrease by increasing the (*n*) parameter. Nevertheless, the effects of (*n*) on the first four mode natural frequency values for crack location (L_1/L = 0.2), intact beam and different ratio (*a/b*) are demonstrated in Fig. 6.

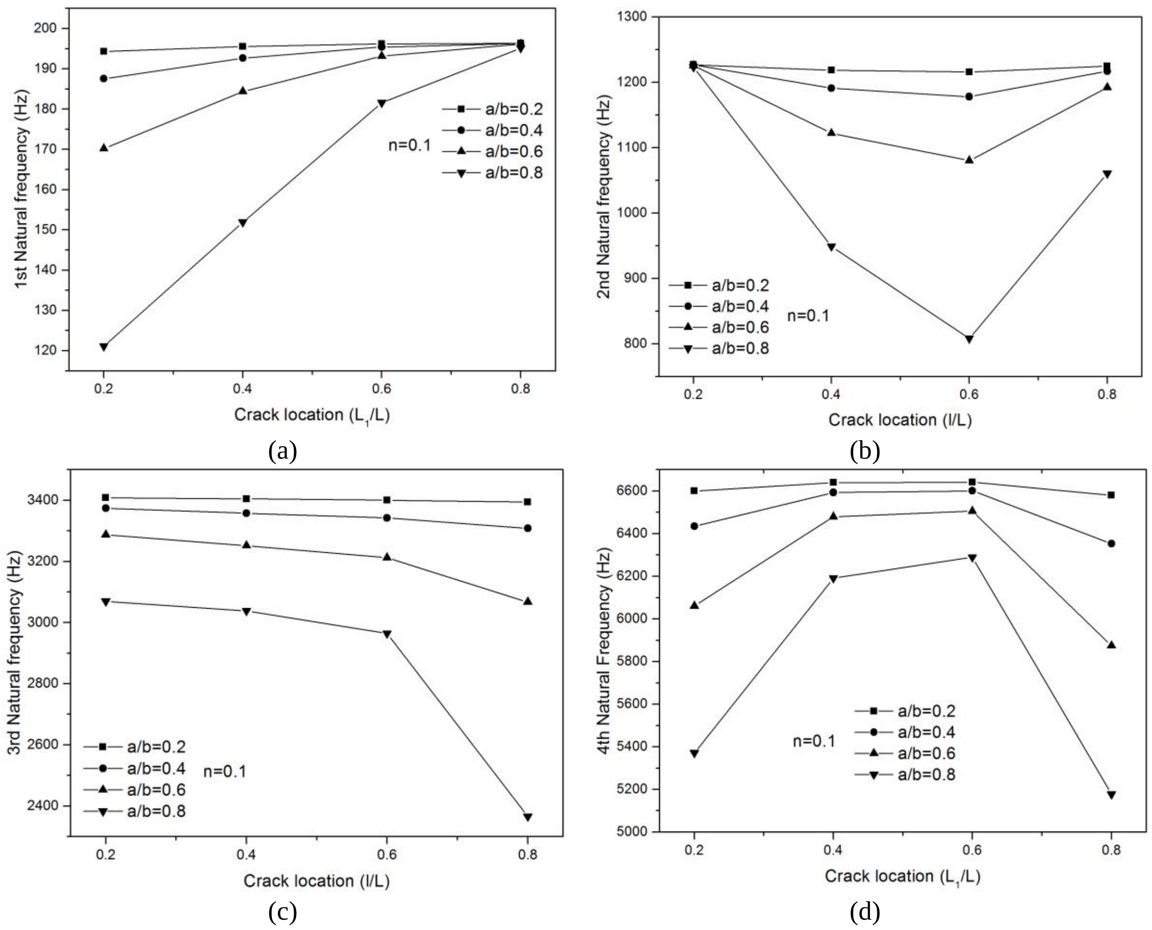


Fig. 5 The effects of the crack location (L_1/L) on the first four mode natural frequency values for (n=0.1) and different ratio (a/b)

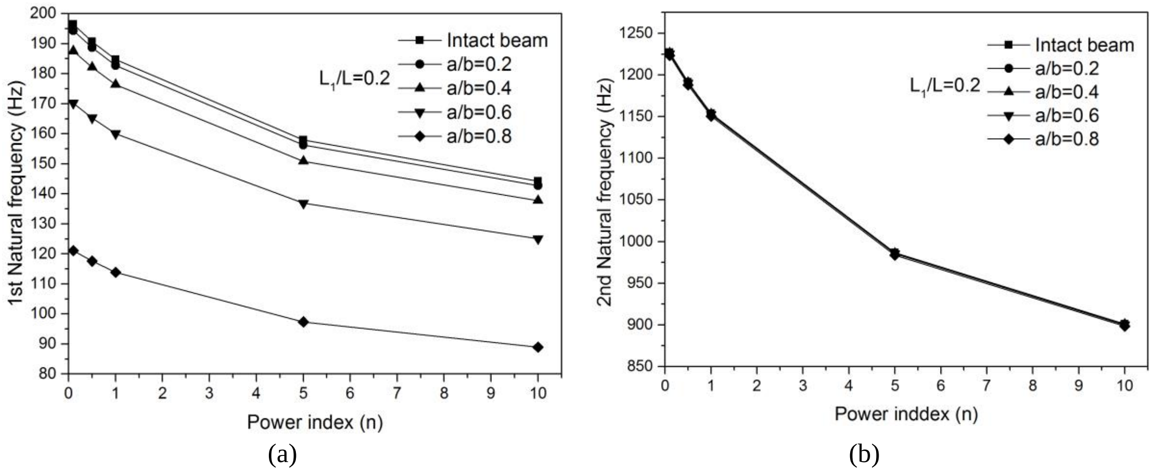
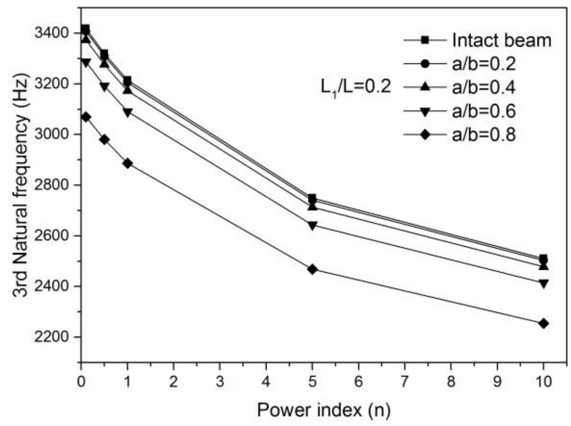
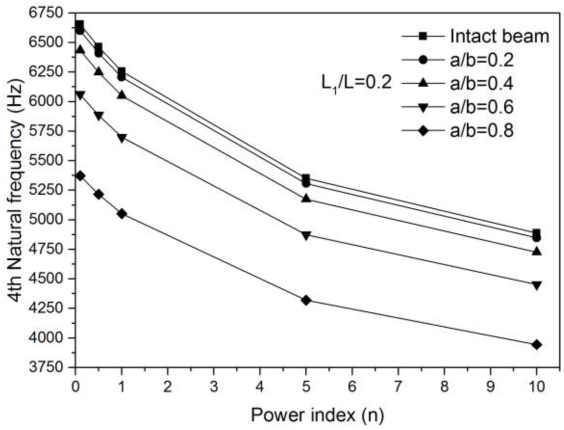


Fig. 6 The effects power index *(n)* on the first four mode natural frequency values for crack location (L_1/L=0.2), intact beam and different ratio (a/b)

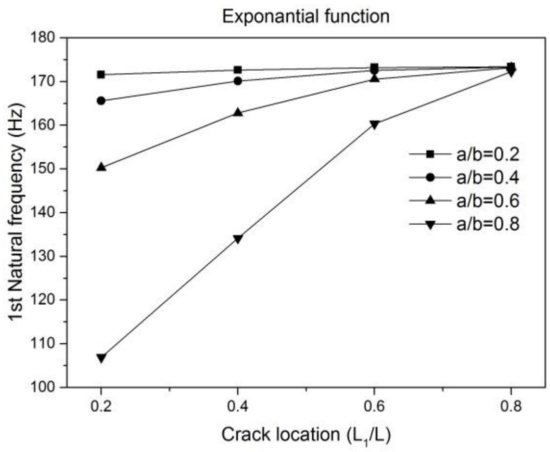


(c)

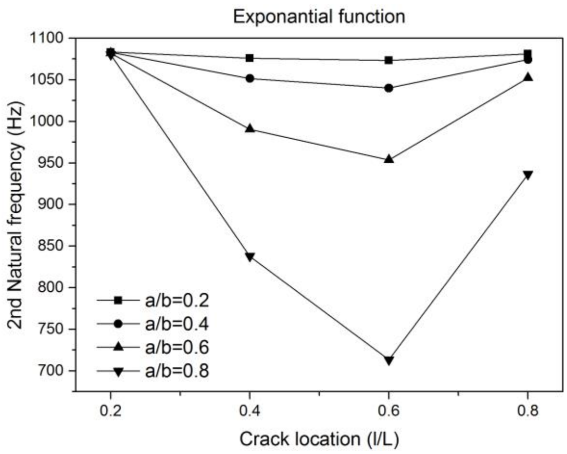


(d)

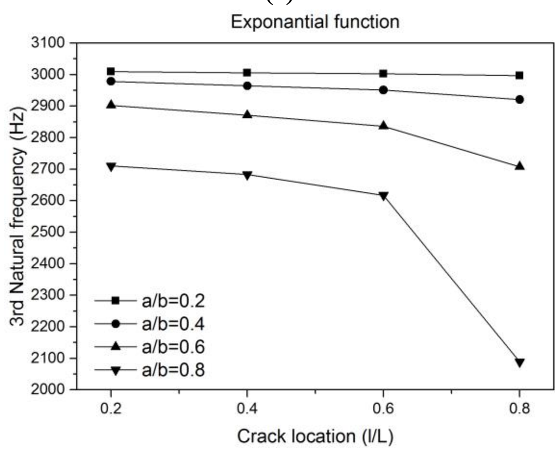
Fig. 6 Continued



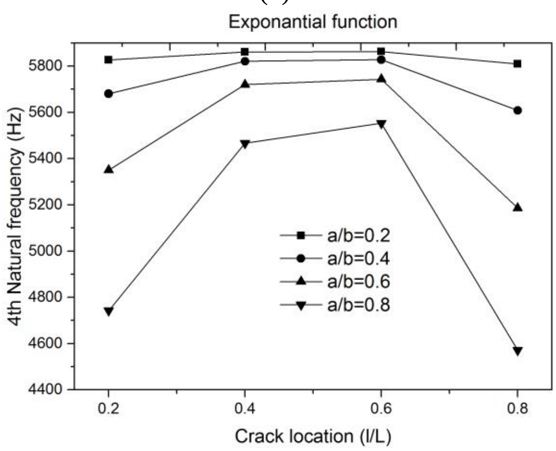
(a)



(b)



(c)



(d)

Fig. 7 The effects of the crack location (L_1/L) on the first four mode natural frequency values for different ratio (*a/b*)

As shown in Fig. 6, the natural frequency values of the intact and cracked beam decreases with increasing the parameter (*n*) and the ratio (*a/b*). However, the 2nd mode natural frequency values are almost not affected by the ratio (*a/b*). In addition, the analyses were carried out for exponential function. The effects of crack location (L_1/L) on the first four mode natural frequency values for different crack ratio (*a/b*) are shown in Fig. 7 and the same trends of Fig. 5 are observed.

Consider that the edge cracked cantilever isotropic homogeneous beam shown in Fig. 2 is made of symmetric FG sandwich Timoshenko beam and the boundary conditions are modified to the simply supported condition. In that case, if the same analyses are performed for the simply supported condition, some observations can be obtained. For instance, the effects of crack depth ratio (*a/b*) on the first four mode natural frequency values for crack location (L_1/L=0.2) and different power index (*n*) are represented in Fig 8.

As shown in Fig. 8, the natural frequency values decrease with increasing the ratio (*a/b*) and (*n*). The effects of crack location (L_1/L) on the first four mode natural frequency values for (*n*=0.1) and different ratio (*a/b*) are shown in Fig. 9. As shown in Figs. 9 (a) and (d), the minimum natural

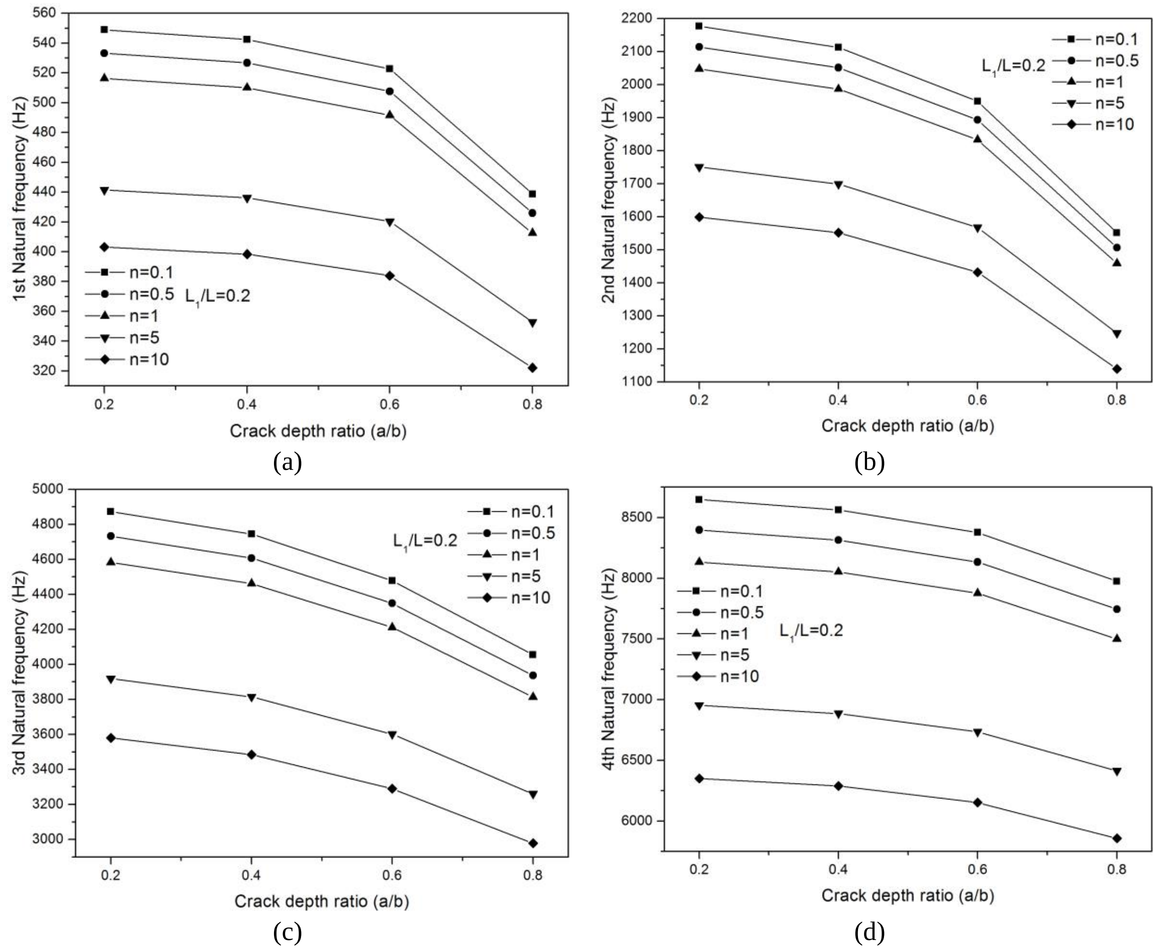


Fig. 8 The effects of the crack depth ratio (*a/b*) on the first four mode natural frequency values for crack location (L_1/L=0.2) and different power index (*n*)

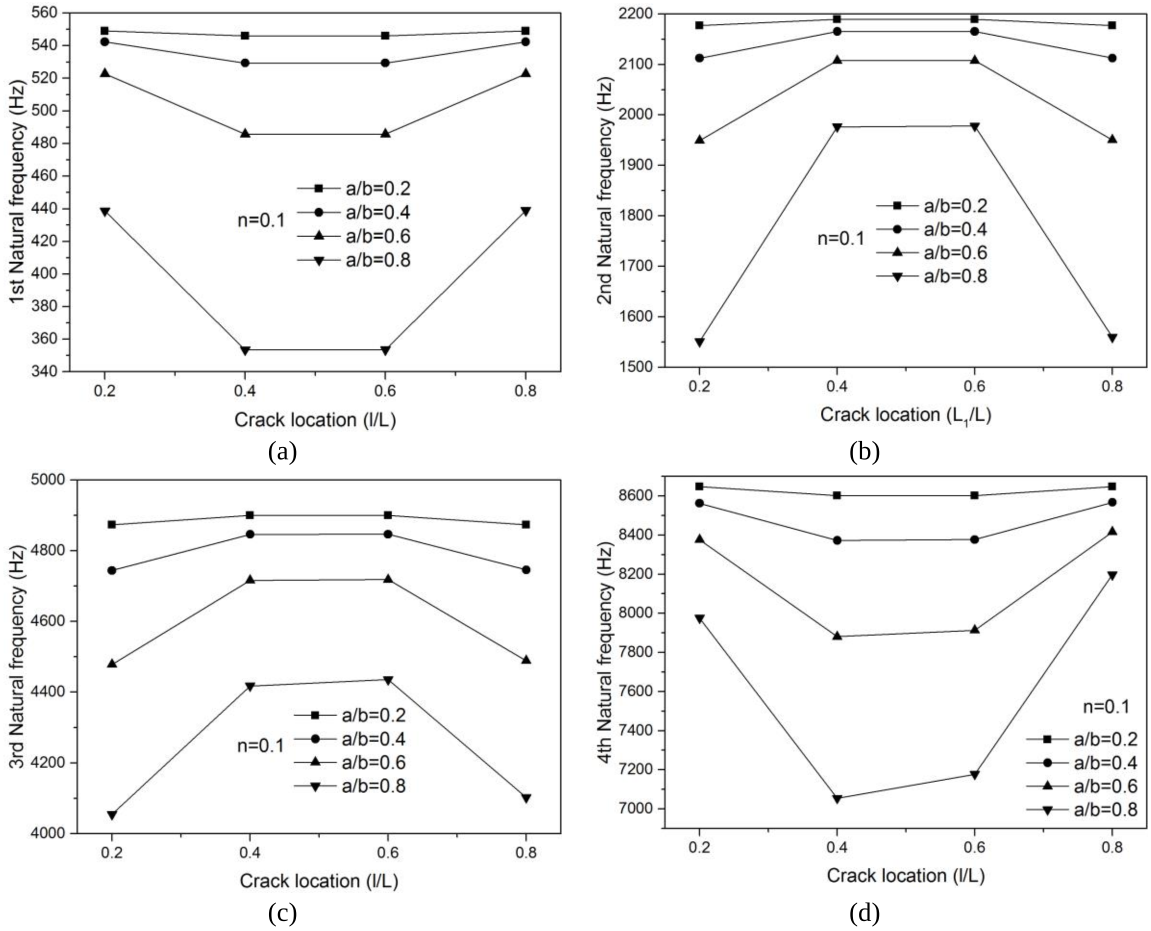


Fig. 9 The effects of the crack location (L_1/L) on the first four mode natural frequency values for (n=0.1) and different ratio (a/b)

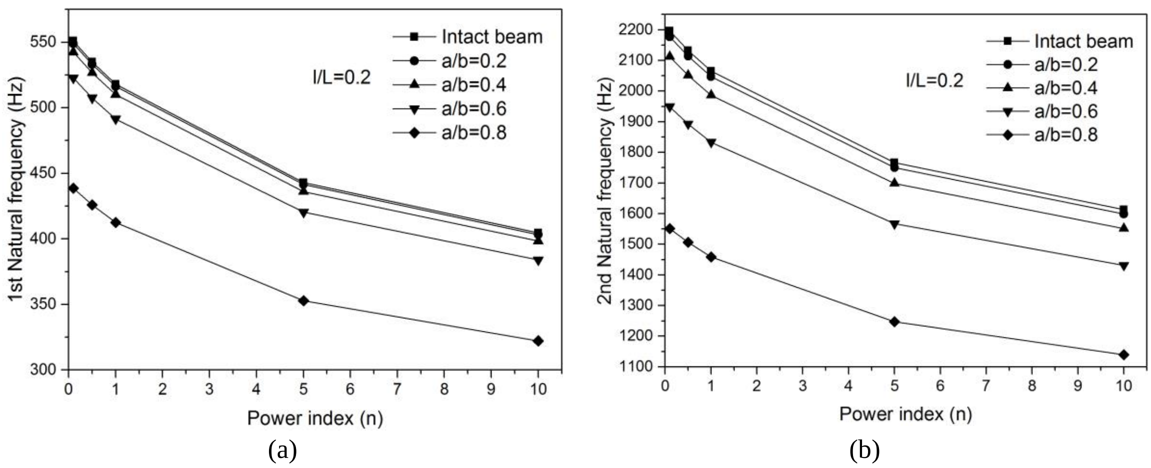
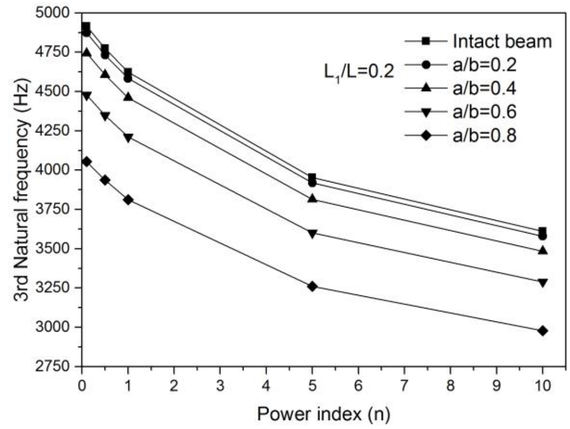
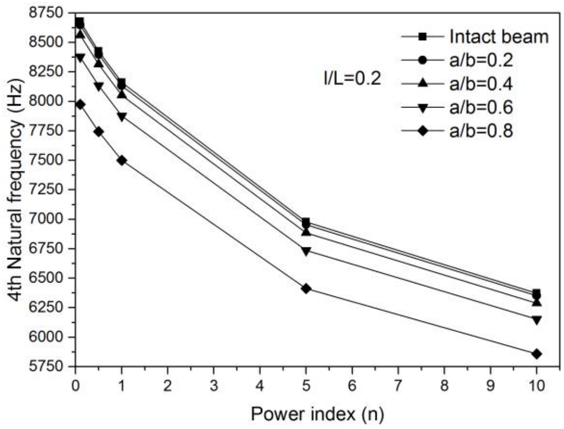


Fig. 10 The effects of power index (n) on the first four mode natural frequency values for crack location (L_1/L=0.2), intact beam and different ratio (a/b)

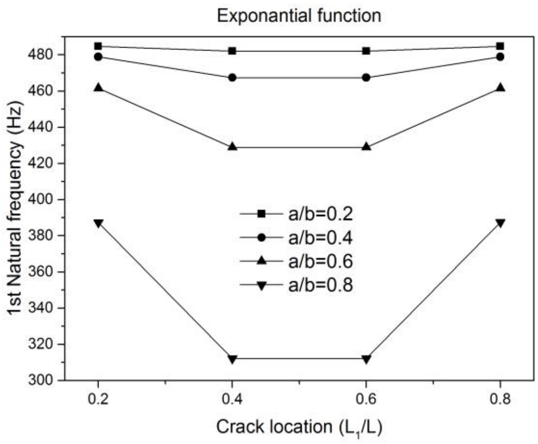


(c)

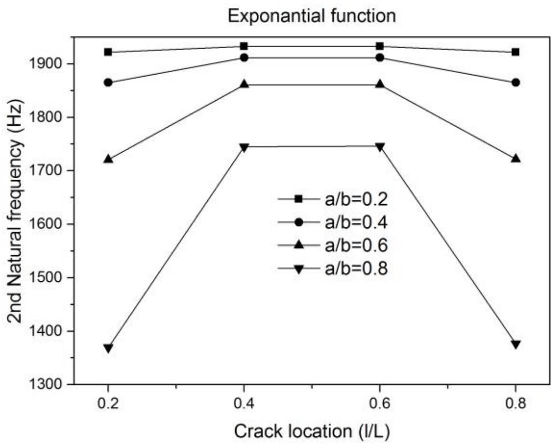


(d)

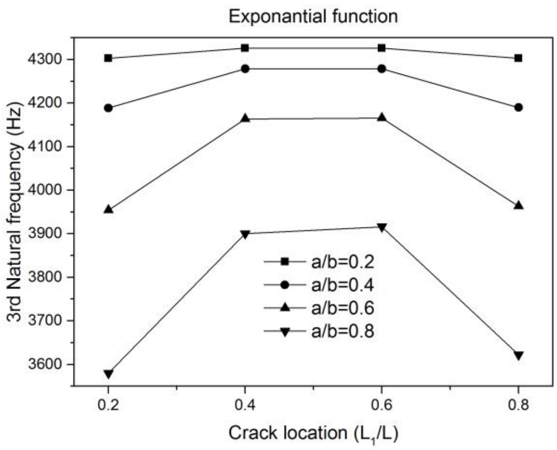
Fig. 10 Continued



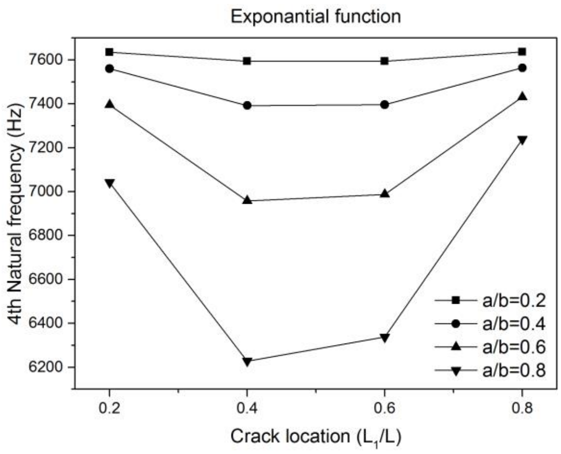
(a)



(b)



(c)



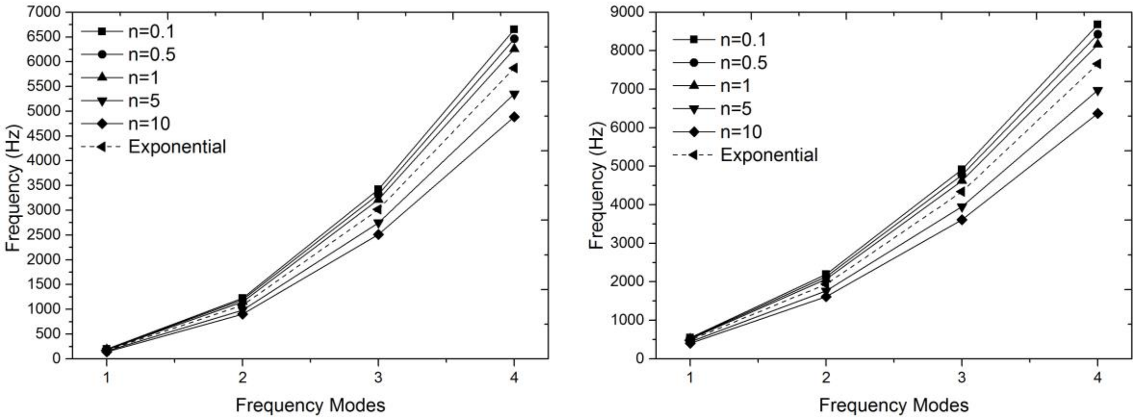
(d)

Fig. 11 The effects of the crack location (L_1/L) on the first four mode natural frequency values for different ratio *(a/b)* frequency values are observed at the location (L_1/L=0.4) then the natural frequency values start to increase while the (L_1/L) is close to the right end of the beam. However, the minimum 2nd and the maximum 3rd mode natural frequency values are observed at the (L_1/L=0.2) and (L_1/L=0.6) as shown in Figs. 9 (b)-(c) respectively. Also, the natural frequency values decreases with increasing the ratio (a/b) for all modes.

Nonetheless, the effects of (n) on the first four mode natural frequency values for crack location (L_1/L=0.2), intact beam and different ratio (a/b) are demonstrated in Fig. 10. As shown in Fig. 10, the natural frequency values of the intact and cracked beam decreases with increasing ratio (a/b) and the parameter (n).

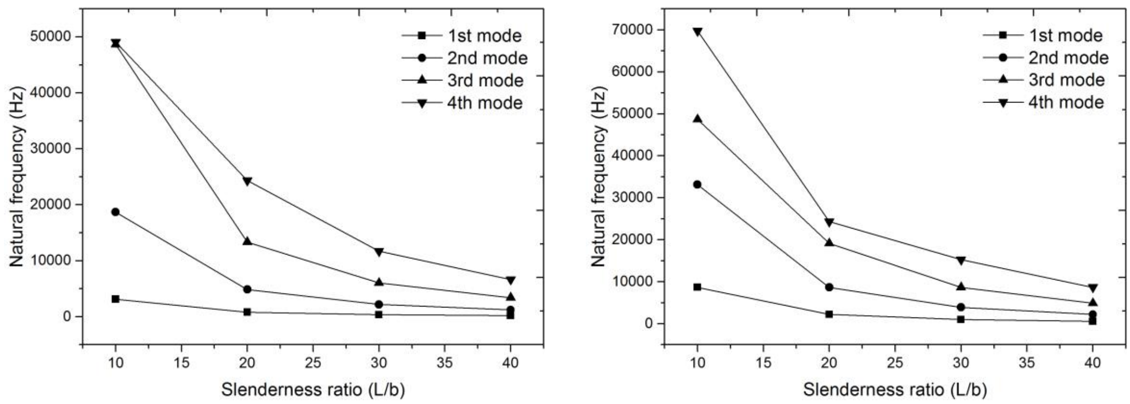
On the other hand, the analyses are performed for exponential function. The effects of crack location (L_1/L) on the first four mode natural frequency values for different crack ratio (a/b) are shown in Fig. 11 and the same trends are observed when Fig. 11 is compared with Fig. 9.

The effects of the different material distributions on the first four mode natural frequencies for



(a) Cantilever boundry condition (b) Simply supported boundry condition

Fig. 12 The effects of the different material distributions on the first four mode natural frequencies for clamped and simply supported boundary conditions



(a) Cantilever boundry condition (b) Simply supported boundry condition

Fig. 13 The effects of the slenderness ratio on the first four mode natural frequencies for clamped and simply supported boundary conditions clamped and simply supported boundary conditions are given in Fig. 12. It is observed from the Fig. 12 that the natural frequency values of the exponential function are less than for (n=0.1, 0.5, 1).

On the other hand, the effects of the slenderness ratio on the first four mode natural frequencies for clamped and simply supported boundary conditions are given in Fig. 13. It is seen from the Fig. 13 that the natural frequency values decrease by increasing the slenderness ratio.

9. Conclusions

In this study, free vibration behavior of the clamped and simply supported symmetric FG sandwich Timoshenko beams is investigated and some conclusions can be drawn as follows;

• The effective mass density and Young's modulus obtained by mixture rules and laminate theory can be used to estimate the natural frequency of the FGM beams.

• The crack ratio (a/b) and the crack location (L_1/L) are affected by the natural frequency values significantly. But, the 2nd mode natural frequency of the clamped beam is almost not affected from the existence of the crack ratio (a/b).

• The desired natural frequency of the structure can be obtained by adjusting volume fraction of the constituent of the FGM.

References

Akbas, S.D. (2013), "Geometrically nonlinear static analysis of edge cracked Timoshenko beams composed of functionally graded material", *Math. Prob. Eng.*, Article ID 871815, 14.

Akbaş, Ş.D. (2013), "Free vibration characteristics of edge cracked functionally graded beams by using finite element method", *IJETT*, **4**(10), 4590-4597.

Alshorbagy, A.E., Eltaher, M.A. and Mahmoud, F.F. (2011), "Free vibration characteristics of a functionally graded beam by finite element method", *Appl. Math. Model.*, **35**(1), 412-425.

Aydin, K. (2013), "Free vibration of functionally graded beams with arbitrary number of surface cracks", *Eur. J. Mech. A-Solid.*, **42**, 112-124.

Aydogdu, M. and Taskin, V. (2007), "Free vibration analysis of functionally graded beams with simply supported edges", *Mater. Des.*, **28**(5), 1651-1656.

Cunedioglu, Y. and Beylergil, B. (2014), "Free vibration analysis of laminated composite beam under room and high temperatures", *Struct. Eng. Mech.*, **51**(1), 111-130.

Demir, E., Callioglu, H. and Sayer, M. (2013), "Free vibration of symmetric FG sandwich Timoshenko beam with simply supported edges", *Indian J. Eng. Mater. S*, **20**(6), 515-521.

Elishakoff, I. and Candan, S. (2001), "Apparently first closed-form solution for vibrating: inhomogeneous beams", *Int. J. Solid. Struct.*, **38**(19), 3411-3441.

Ferezqi, H.Z., Tahani, M. and Toussi, H.E. (2010), "Analytical approach to free vibrations of cracked Timoshenko beams made of functionally graded materials", *Mech. Adv. Mater. Struct.*, **17**(5), 353-365.

Gibson, R.F. (1994), *Principles of Composite Materials*, First Edition, McGraw-Hill, New York.

Giunta, G., Crisafulli, D., Belouettar, S. and Carrera, E. (2011), "Hierarchical theories for the free vibration analysis of functionally graded beams", *Compos. Struct.*, **94**(1), 68-74.

Ke, L.L., Yang, J., Kitipornchai, S. and Xiang, Y. (2009), "Flexural vibration and elastic buckling of a cracked Timoshenko beam made of functionally graded materials", *Mech. Adv. Mater. Struct.*, **16**(6), 488-502.

Kisa, M. (2004), "Free vibration analysis of a cantilever composite beam with multiple cracks", *Compos. Sci. Technol.*, **64**(9), 1391-1402.

Kisa, M. and Brandon, J. (2000), “The effects of closure of cracks on the dynamics of a cracked cantilever beam”, *J. Sound. Vib.*, **238**(1), 1-18.

Kisa, M., Brandon, J. and Topcu, M. (1998), “Free vibration analysis of cracked beams by a combination of finite elements and component mode synthesis methods”, *Comput. Struct.*, **67**(4), 215-223.

Kitipornchai, S., Ke, L.L., Yang, J. and Xiang, Y. (2009), “Nonlinear vibration of edge cracked functionally graded Timoshenko beams”, *J. Sound. Vib.*, **324**(3-5), 962-982.

Li, S., Hu, J.J., Zhai, C.H. and Xie, L.L. (2013), “A unified method for modeling of axially and/or transversally functionally graded beams with variable cross-section profile”, *Mech. Bas. Des. Struc.*, **41**(2), 168-188.

Li, X.F. (2008), “A unified approach for analyzing static and dynamic behaviors of functionally graded Timoshenko and Euler-Bernoulli beams”, *J. Sound Vib.*, **318**(4-5), 1210-1229.

Matbuly, M.S., Ragb, O. and Nassar, M. (2009), “Natural frequencies of a functionally graded cracked beam using the differential quadrature method”, *Appl. Math. Comput.*, **215**(6), 2307-2316.

Petyt, M. (1990), *Introduction toFfinite Element Vibration Analysis*, First Edition, Cambridge University Press, Cambridge.

Pradhan, K.K. and Chakraverty, S. (2013), “Free vibration of Euler and Timoshenko functionally graded beams by Rayleigh-Ritz method”, *Compos. Part B-Eng*., **51**, 175-184.

Shahba, A., Attarnejad, R., Marvi, M.T. and Hajilar, S. (2011), “Free vibration and stability analysis of axially functionally graded tapered Timoshenko beams with classical and non-classical boundary conditions”, *Compos. Part B-Eng.*, **42**(4), 801-808.

Simsek, M., Kocaturk, T. and Akbas, S.D. (2012), “Dynamic behavior of an axially functionally graded beam under action of a moving harmonic load”, *Compos. Struct.*, **94**(8), 2358-2364.

Su, H. and Banerjee, J.R. (2015), “Development of dynamic stiffness method for free vibration of functionally graded Timoshenko beams”, *Comput. Struct*,. **147**, 107-116.

Thai, H.T. and Vo, T.P. (2012), “Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories”, *Int. J. Mech. Sci.*, **62**(1), 57-66.

Wattanasakulpong, N., Prusty, G.B. and Kelly, D.W. (2013), “Free and forced vibration analysis using improved third-order shear deformation theory for functionally graded plates under high temperature loading”, *J. Sandw. Struct. Mater.*, **15**(5), 583-606.

Wei, D., Liu, Y.H. and Xiang, Z.H. (2012), “An analytical method for free vibration analysis of functionally graded beams with edge cracks”, *J. Sound Vib.*, **331**(7), 1686-1700.

Yan, T., Kitipornchai, S. and Yang, J. (2011), “Parametric instability of functionally graded beams with an open edge crack under axial pulsating excitation”, *Compos. Struct.*, **93**(7), 1801-1808.

Yan, T., Kitipornchai, S., Yang, J. and He, X.Q. (2011), “Dynamic behaviour of edge-cracked shear deformable functionally graded beams on an elastic foundation under a moving load”, *Compos. Struct.*, **93**(11), 2992-3001.

Yan, T. and Yang, J. (2011), “Forced vibration of edge-cracked functionally graded beams due to a transverse moving load”, *Procedia Eng.*, **14**, 3293-3300.

Yang, J. and Chen, Y. (2008), “Free vibration and buckling analyses of functionally graded beams with edge cracks”, *Compos. Struct.*, **83**(1), 48-60.

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