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Disturbance due to internal heat source in thermoelastic solid using dual phase lag model

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Abstract. The dual-phase lag heat transfer model is employed to study the problem of isotropic generalized thermoelastic medium with internal heat source. The normal mode analysis is used to obtain the exact expressions for displacement components, force stress and temperature distribution. The variations of the considered variables through the horizontal distance are illustrated graphically. The results are discussed and depicted graphically.

Keywords: dual-phase-lag model; thermoelasticity; temperature distribution; normal-mode

1. Introduction

Thermoelasticity theories which involve finite speed of thermal signals (second sound) have created much interset during the last three decades. The conventional coupled dynamic thermoelasticity theory (CTE) based on the mixed parabolic-hyperbolic governing equations of (Biot 1956, Chadwick 1960) predicts an infinite speed of propagation of thermoelastic disturbance. To remove the paradox of infinite speed for propagation of thermoelastic disturbance, several generalized thermoelasticity theories have been developed, which involve hyperbolic governing equations. Among these generalized theories, the extended thermoelasticity theory (ETE) proposed by Lord and Shulman (1967) involving one relaxation time (called single-phase-lag-model) and the temperature-rate-dependent theory of thermoelasticity (TRDTE) proposed by Green and Lindsay (1972) involving two relaxation times are two important models of generalized theory of thermoelasticity. Experimental studies (Kaminski 1990, Mitra *et al*. 1995, Tzou 1995a, b) indicate that the relaxation times can be of relevance in the cases involving a rapidly propagating crack tip, a localized moving heat source with high intensity, shock wave propagation, laser technique etc. Because of the experimental evidence in support of finiteness of heat propagation speed, the generalized thermoelasticity theories are considered to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes at short intervals like those occurring in laser units and energy channels. For a review of the relevant literature, see (Chandrasekharaiah 1986, Ignaczak 1989).

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Green and Naghdi (1977, 1992, 1993) formulated three different models of thermoelasticity among which, in one of these models, there is no dissipation of thermoelastic energy. This model is referred to as the G-N model of thermoelasticity without energy dissipation (TEWOED). Problems concerning generalized thermoelasticity theories and G-N theory have been studied by many authors (Roy Choudhuri and Debnath 1983, Roy Choudhuri 1984, 1985, 1987, Dhaliwal and Rokne 1988, 1989, RoyChoudhuri 1990, Chandrasekharaiah and Murthy 1993, Chandrasekharaiah and Srinath 1996, RoyChoudhuri and Banerjee 2004, RoyChoudhuri and Bandyopadhyay 2005, RoyChoudhuri and Dutta 2005). Tzou (1995a, b), Ozisik and Tzou (1994) have developed a new model called dual-phase-lag model for heat transport mechanism in which Fourier's law is replaced by an approximation to a modification of Fourier's law with two different time translations for the heat flux and the temperature gradient. According to this model, classical Fourier's law $\vec{q} = -K\,\vec{\nabla}T$ has been generalized as $\vec{q}(P, T+\tau_q) = -K\,\vec{\nabla}T(P, t+\tau_\theta)$, where the temperature gradient $\vec{\nabla}T$ at a point P of the material at time $t+\tau_\theta$ corresponds to the heat flux vector $\vec{q}$ at the same point at the time $t+\tau_q$. Here K is the thermal conductivity of the material. The delay time τ_θ is interpreted as that caused by the microstructural interactions (small-scale heat transport mechanisms occurring in microscale) and is called the phase-lag-of the temperature gradient. The other delay time is τ_q interpreted as the relaxation time due to the fast transient effects of thermal inertia(small scale effects of heat transport in time) and is called the phase-lag of the heat flux. If $\tau_q=\tau$ and $\tau_\theta=0$, Tzou (1995a, b) refers to the model as the single phase-lag model. The case τ_θ τ_q ($\neq 0$) corresponds to the dual phase-lag model of the constitutive equation connecting the heat flux vector and the temperature gradient. The case $\tau_q=\tau_\theta$ ($\neq 0$) becomes identical with the classical Fourier's law. Further for materials with τ_q τ_θ, the heat flux vector is the result of a temperature gradient and for materials with τ_θ τ_q, the temperature gradient is the result of a heat flux vector. For a review of the relevant literature, see (Chandrasekharaiah (1998)). A hyperbolic thermoelastic model was developed in this same reference, taking into account the phase-lag of both temperature gradient and heat flux vector and also the second order term in τ_q in Taylor's expansion of the heat flux vector and the first order term in τ_θ in Taylor's expansion of the temperature gradient in the generalization of classical Fourier's law. It may be pointed out that ETE was formulated by taking into account the thermal relaxation time, which is in fact the phase-lag of the heat flux vector (single-phase-lag model). Chakravorty and Chakravorty (1998) discussed the transient disturbances in a relaxing thermoelastic half space due to moving stable internal heat source. Kumar and Devi (2008) studied thermomechanical interactions in porous generalized thermoelastic material permeated with heat source. Lotfy (2010) have studied the transient disturbance in a half-space under generalized magneto-thermoelasticity with a stable internal heat source. Lotfy (2011) discussed the transient thermo-elastic disturbances in a visco-elastic semi-space due to moving internal heat source. Othman (2011) studied the generalized thermoelastic problem with temperature-dependent elastic moduli and internal heat sources. Kothari and Mukhopadhyay (2013) presented some theorems in the linear theory of thermoelasticity with dual-phase lags for an anisotropic medium. El-Karamany and Ezzat (2014) proved uniqueness and reciprocal theorems for dual-phase-lag thermoelasticity theory without using Laplace transforms. Banerjee (2015) studied the potential of dual phase microstructures of extra low carbon steel.

In the present paper, the effect of dual phase-lag is studied on isotropic generalized thermoelastic medium with internal heat source. The normal mode method is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically.

2. Formulation of the problem and fundamental equations

An isotropic homogeneous thermally conducting elastic solid**,** at uniform absolute temperature T_0, in the undistributed state is considered.

We consider a fixed rectangular cartesian coordinate system (x,y,t) having origin on the surface y=0 and negative y-axis pointing normally into the medium, which is thus represented by y=0.

The field equations and constitutive relations for a homogeneous, generalized thermoelastic solid in the absence of incremental body forces and heat sources are given by

$$t_{ij} = \lambda \mathrm{div}(\vec{u})\delta_{ij} + \mu(u_{i,j} + u_{j,i}) - \vartheta T\delta_{ij} \tag{1}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

$$(\lambda + \mu)\mathrm{grad}(\mathrm{div}(\vec{u})) + \mu\nabla^2\vec{u} - \vartheta \mathrm{grad}(T) = \rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{3}$$

Then the heat conduction equation in the context of dual phase lag thermoelasticity proposed by Tzou in this case takes the form

$$K\,(1+\tau_\theta\frac{\partial}{\partial t})\nabla^2 T = \rho C_\theta(1+\tau_\theta\frac{\partial}{\partial t})\frac{\partial T}{\partial t} + \vartheta T_0(1+\tau_q\frac{\partial}{\partial t})\frac{\partial e}{\partial t} - (1+\tau_q\frac{\partial}{\partial t})Q \tag{4}$$

In the above equations, u_i are the components of displacement, e_{ij} is the strain tensor, t_{ij} is the stress tensor, λ and μ are Lame's elastic constants, T is the absolute temperature. T_0 is the reference temperature, ρ is the mass density, C_e is the specific heat at constant strain. α_t is the coefficient of linear thermal expansion of the material, τ_q and τ_θ are the phase-lag of the temperature gradient and of the heat flux respectively, often referred to as the delay times. e is the dilatation.

In addition, $\Delta=\mathrm{div}\,\vec{u}$ and $v=(3\lambda+2\mu)\alpha_t$.

3. Solution of the problem

If we restrict our analysis parallel to xy plane and $\partial/\partial z\equiv 0$, the displacement components have the following form

$u_x=u(x,y,t)$, $v_y=v(x,y,t)$, $w_z=0$(5)

From Eqs. (2) and (5), we obtain the strain components

$$e_{xx} = \frac{\partial u}{\partial x}, e_{yy} = \frac{\partial v}{\partial y}, e_{xy} = \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$$

$$e_{zz} = e_{xz} = e_{zx} = 0, e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{6}$$

To facilitate the solution, following dimensionless quantities are introduced

$$\{x', y'\} = \frac{\omega}{c_1}\{x, y\}, \{u', v'\} = \frac{\rho c_1 \omega}{\vartheta T_0}\{u, v\}, T' = \frac{T}{T_0}, t'_{ij} = \frac{\sigma_{ij}}{\vartheta T_0}$$

$$t' = \omega \quad t, \tau'_0 = \omega \quad \tau_0, \tau'_0 = \omega \quad Q'_0 = \frac{1}{\lambda\omega} Q_0 \tag{7}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \omega = \frac{\rho C_e c_1^2}{K}.$$

Eq. (3), with the help of Eqs. (1) and (5)-(7) may be recast into the dimensionless form after suppressing the primes as

$$\frac{\partial^2 u}{\partial x^2} + \xi_{12}\frac{\partial^2 u}{\partial y^2} + \xi_{11}\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^{2'}} \tag{8}$$

$$\frac{\partial^2 v}{\partial y^2} + \xi_{11}\frac{\partial^2 u}{\partial x \partial y} + \xi_{12}\frac{\partial^2 \upsilon}{\partial x^2} - \frac{\partial T}{\partial y} = \frac{\partial^2 v}{\partial t^{2'}} \tag{9}$$

The heat conduction equation is given by

$$\nabla^2 T = \left\{ \frac{(1 + \tau_q \frac{\partial}{\partial t})}{(1 + \tau_\theta \frac{\partial}{\partial t})} \right\} \left\{ \frac{\partial T}{\partial t} + \xi_{21}\frac{\partial e}{\partial t} - \xi_{22} Q \right\} \tag{10}$$

Using the expression relating displacement components $u(x,y,t)$, $v(x,y,t)$ to the scalar potential functions $\varphi(x,y,t)$ and $\psi(x,y,t)$

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y}, v = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x'} \tag{11}$$

in Eqs. (8)-(10), we obtain

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial t^2}\right)\varphi - T = 0 \tag{12}$$

$$\left\{ \xi_{12}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{\partial^2}{\partial t^2} \right\}\psi = 0 \tag{13}$$

$$\nabla^2 T = \left\{ \frac{(1 + \tau_q \frac{\partial}{\partial t})}{(1 + \tau_\theta \frac{\partial}{\partial t})} \right\} \left\{ \frac{\partial T}{\partial t} + \xi_{21}\frac{\partial}{\partial t}\nabla^2\varphi - \xi_{22} Q \right\} \tag{14}$$

where

$$\xi_{11} = \frac{\lambda + \mu}{\rho c_1^2}, \xi_{12} = \frac{\mu}{\rho c_1^2}, \xi_{13} = \frac{\lambda}{\rho c_1^2}, \xi_{21} = \frac{\vartheta^2 T_0}{\rho K \omega}, \xi_{22} = \frac{\lambda c_1^2}{K T_0 \omega}.$$

4. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$\left[\varphi, \varphi, T, t_{ij}\right](x, y, t) = \left[\overline{\varphi}, \overline{\psi}, \overline{T}, \overline{\sigma}_{ij}\right](y) e^{(\omega t + iax)} \tag{15}$$

$$Q = Q_0 e^{(\omega t + iax)}, \overline{Q} = Q_0 \tag{16}$$

where [$\overline{\varphi}, \overline{\psi}, \overline{T}, \overline{\sigma}_{ij}$] are the magnitude of the functions, ω is the complex time constant and a is the wave number in x-direction and Q_0 is the magnitude of stable internal heat source.

Using (15)-(16), in Eqs. (12)-(14) we obtain

$$[D^2 - a^2 - \omega^2]\overline{\varphi} - \overline{T} = 0 \tag{17}$$

$$\xi_{24}[D^2 - a^2]\overline{\varphi} - [D^2 - \xi_{23}]\overline{T} = \xi_{22}\varepsilon Q_0 \tag{18}$$

$$[\xi_{12}(D^2 - a^2) - \omega^2]\overline{\psi} = 0 \tag{19}$$

where

$$\xi_{23} = (a^2 + \varepsilon\omega), \xi_{24} = (\xi_{21}\varepsilon\omega), \varepsilon = \frac{(1 + \tau_q \omega)}{(1 + \tau_\theta \omega)}.$$

Eliminating $\overline{T}$ from Eqs. (17)-(18), we obtain

$$[D^4 - \lambda_1 D^2 + \lambda_2](\overline{\varphi}(y)) = -\varepsilon \xi_{22} Q_0 \tag{20}$$

where, $D = \dfrac{d}{dy}$,

$\lambda_1 = (a^2 + \omega^2 + \xi_{23} + \xi_{24}), \lambda_2 = (a^2 + \omega^2)\xi_{23} + a^2 \xi_{24}$.

The solution of Eq. (20) is given by

$$\overline{\phi}(y) = \sum_{j=1}^{2} S_j(a, \omega) e^{-k_j y} + \sum_{j=1}^{2} R_j(a, \omega) e^{k_j y} - l_1 \tag{21}$$

In a similar way, we get

$$\overline{T}(y)=\sum_{j=1}^{2}a_jS_j(a,\omega)e^{-k_jy}+\sum_{j=1}^{2}a_jR_j(a,\omega)e^{k_jy}+l_2 \tag{22}$$

The solution of Eq. (19) is given by

$$\overline{\psi}(y)=S_3(a,\omega)e^{-k_3y}+R_3(a,\omega)e^{k_3y} \tag{23}$$

where

$$l_1=\frac{\varepsilon\xi_{22}Q_0}{\lambda_2},l_2=l_1(a^2+\omega^2),k_3^2=\frac{\xi_{12}a^2+\omega^2}{\lambda_2},a_j=(k_j^2-a^2-\omega^2),\ j=1,2 \tag{24}$$

and $S_j(a,\omega)$, $R_j(a,\omega)$ are some parameters depending on a and ω. k_j^2 (j=1,2) are the roots of the characteristic Eq. (20).

5. Applications

The boundary conditions at the interface y=0 subjected to an arbitrary normal force P_1 are

$$\text{(i) } t_{22}(x,0^+,t)-t_{22}(x,0^-,t)=-P_1e^{(\omega t+iax)},\ \text{(ii) } t_{21}(x,0^+,t)-t_{21}(x,0^-,t)=0$$

$$\text{(iii) } u(x,0^+,t)=u(x,0^-,t),\ \text{(iv) } v(x,0^+,t)=v(x,0^-,t)$$

$$\text{(v) } T(x,0^+,t)=T(x,0^-,t),\ \text{(vi) } \frac{\partial T}{\partial y}(x,0^+,t)=\frac{\partial T}{\partial y}(x,0^-,t) \tag{25}$$

where P_1 is the magnitude of mechanical force. Using Eqs. (1) and (7) on the non-dimensional boundary conditions and then using (21)-(23), we get the expressions of displacement, force stress and temperature distributions for isotropic generalized thermoelastic medium as

$$u=\{\sum_{j=1}^{2}iaS_j(a,\omega)e^{-k_jy}+\sum_{j=1}^{2}iaR_j(a,\omega)e^{k_jy}+k_3S_3e^{-k_3y}-k_3R_3e^{k_3y}\}e^{(\omega t+iax)}-ial_1 \tag{26}$$

$$v=\{-\sum_{j=1}^{2}k_jS_j(a,\omega)e^{-k_jy}+\sum_{j=1}^{2}k_jR_j(a,\omega)e^{k_jy}+iaS_3e^{-k_3y}+iaR_3e^{k_3y}\}e^{(\omega t+iax)} \tag{27}$$

$$t_{22}=\{\sum_{j=1}^{2}b_jS_j(a,\omega)e^{-k_jy}+\sum_{j=1}^{2}b_jR_j(a,\omega)e^{k_jy}+N_1S_3e^{-k_3y}-N_1R_3e^{k_3y}\}e^{(\omega t+iax)}+N_2 \tag{28}$$

$$t_{21} = \{\sum_{j=1}^{2} c_j S_j(a,\omega) e^{-k_j y} - \sum_{j=1}^{2} c_j R_j(a,\omega) e^{k_j y} + N_3 S_3 e^{-k_3 y} + N_3 R_3 e^{k_3 y}\} e^{(\omega t + iax)} \tag{29}$$

$$u = \{\sum_{j=1}^{2} a_j^{\,*} S_j(a,\omega) e^{-k_j y} + \sum_{j=1}^{2} a_j^{\,*} R_j(a,\omega) e^{k_j y}\} e^{(\omega t + iax)} + l_2 \tag{30}$$

where

$$b_j = (k_j^2 - a^2 \xi_{13} - a_j), c_j = -2iak_j \xi_{12}, j = 1,2, N_1 = iak_3(\xi_{13} - 1)$$

$$N_2 = (a^2 \xi_{13} l_1 + l_2), N_3 = -(a^2 + k_3^2)\ .$$

Invoking the boundary conditions (25) at the surface y=0, we obtain a system of six equations, and applying the inverse of matrix method, we obtain the values of six constants S_j and R_j, j=1,2,3, as

$$S_1 = \frac{\Delta_1}{\Delta}, S_2 = \frac{\Delta_2}{\Delta}, S_3 = \frac{\Delta_3}{\Delta}, R_1 = \frac{\Delta_4}{\Delta}, R_2 = \frac{\Delta_5}{\Delta}, R_3 = \frac{\Delta_6}{\Delta}$$

where Δ, Δ_i, i=1,2,3,...,6 are defined in appendix A.

6. Numerical results

For computational work, to illustrate the analytical procedure presented earlier, we consider now a numerical example. The results depict the variations of displacements, force stress and temperature distribution. For this purpose, sand stone is considered as the thermoelastic material body for which we use the physical constants as follows:

ρ=2.30×10^3 Kg/m^3, α_0=0.4×10^{-5}/℃, λ=μ=0.8×10^{10} N/m^2,

K=2.512 J/m sec ℃, C_E=0.9629×10^3 J/Kg ℃, T_0=23 ℃.

The computations are carried out in the range 0≤x≤10 and on the surface y=1.0. The numerical values for normal displacement u_2, normal force stress t_{22} and temperature distribution T are shown in Figs. 1-6 for mechanical force with $\omega=\omega_0+i\zeta$, ω_0=2.3, a=2.1 for an

(a) Isotropic generalized thermoelastic medium with internal heat source (τ_q=0.1 and τ_θ=0.05) by solid line i.e., Dual phase lag (DPL) model.

(b) Isotropic generalized thermoelastic medium with internal heat source (τ_q=0.1 and τ_θ=0) by dashed line i.e., Single phase lag (SPL) model.

(c) Isotropic generalized thermoelastic medium with internal heat source ($\tau_q=\tau_\theta$=0.1) by solid line with centered symbol (*) i.e., Classical Fourier's law (CFL).

(d) Isotropic generalized thermoelastic medium with internal heat source (τ_q=0.05 and τ_θ=0.1) by solid line i.e., Dual phase lag (DPL) model.

(e) Isotropic generalized thermoelastic medium with internal heat source (τ_q=0.1 and τ_θ=0) by dashed line. i.e., Single phase lag (SPL) model.

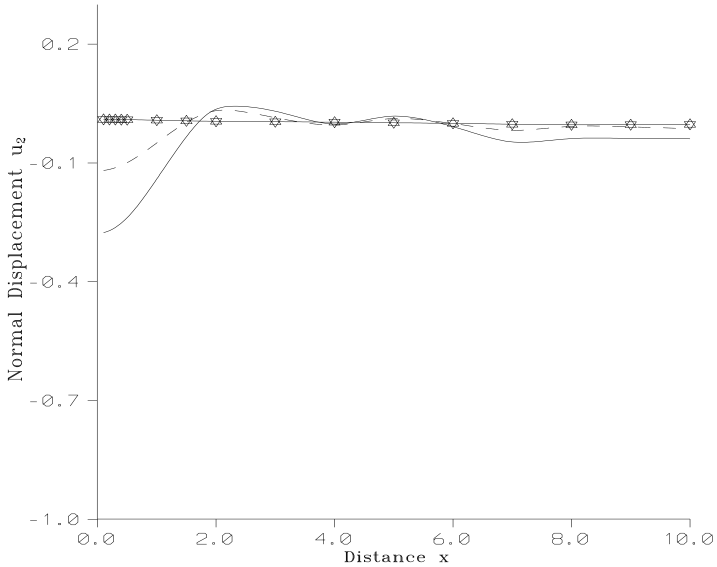

Fig. 1 Variation of normal displacement u_2 with distance x for $\tau_q>\tau_\theta$

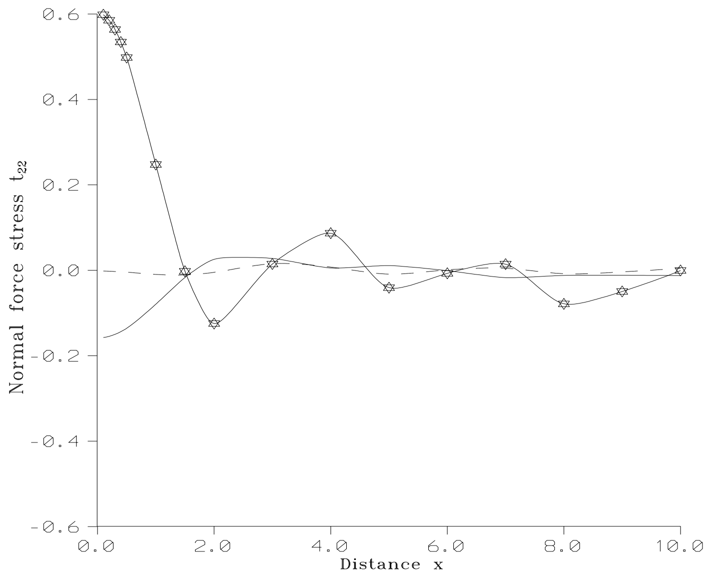

Fig. 2 Variation of normal force stress t_{22} with distance x for $\tau_q>\tau_\theta$

(f) Isotropic generalized thermoelastic medium with internal heat source ($\tau_q=\tau_\theta=0.1$) by solid line with centered symbol (*) i.e., Classical Fourier's law (CFL).

7. Discussions

7.1 Effect of phase lag of heat flux (τ_q) and temperature gradient (τ_θ) when $\tau_q>\tau_\theta$

Figs. 1 to 3 shows the effect of the phase-lag of the heat flux τ_q and Temperature gradient for $\tau_q>\tau_\theta$

Fig. 1 depicts the variations of normal displacement u_2 with distance x. The variations of normal displacement for DPL model and SPL model show similar patterns with different degree of sharpness. i.e., the values for DPL model and SPL model increases and decreases alternately with distance x. Further normal displacement u_2 shows small variations close to zero value in the whole range for CFL.

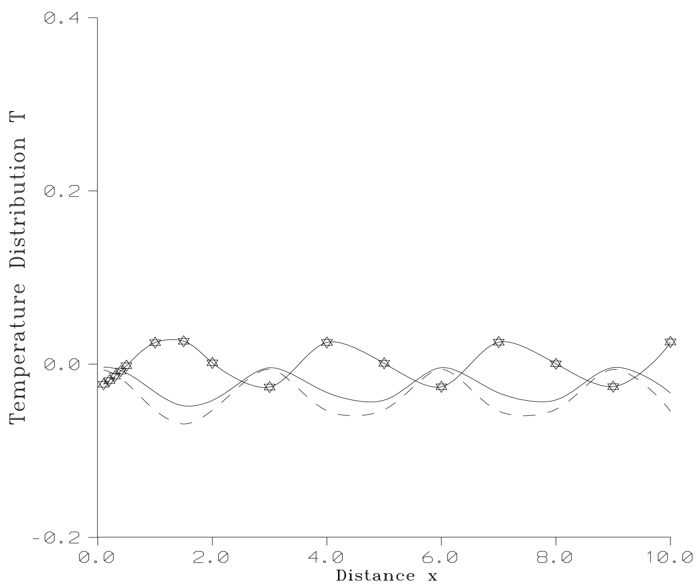

Fig. 3 Variation of temperature distribution T with distance x for $\tau_q>\tau_\theta$

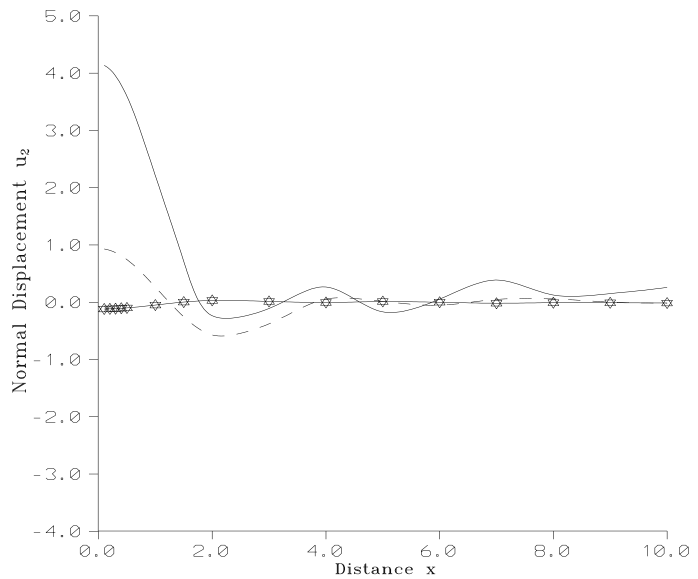

Fig. 4 Variation of normal displacement u_2 with distance x for $\tau_\theta>\tau_q$

The variations of normal force stress t_{22} with distance x is depicted in Fig. 2. The pattern observed for DPL model and CFL are opposite in nature with fluctuating values which clearly reveals the effect of phase lag of the heat flux (τ_q) and temperature gradient (τ_θ). The value of normal force stress are very less in magnitude for SPL model.

Fig. 3 shows the variations of temperature distribution T with distance x. The behaviour of variations of temperature distribution T with reference to x is same for i.e., oscillatory for DPL model and SPL model with difference in their magnitude.

7.2 Effect of phase lag of heat flux (τ_q) and temper ature gradient (τ_θ) when $\tau_q>\tau_\theta$

Figs. 4 to 6 presents the effect of the phase-lag of heat flux and temperature gradient τ_θ for $\tau_\theta>\tau_q$.

The variations of normal displacement u_2 with distance x is depicted in Fig. 4. Initially the value of normal displacement for DPL model decreases in the range $0\leq x\leq 1.5$ and then follow an oscillatory pattern with reference to x. Also, the variations of normal displacement u_2 for CFL lie in a very short range.

Fig. 5 depicts the variations of normal force stress t_{22} with distance x. The pattern observed for DPL model and SPL model are opposite in nature near the point of application of source. The values of normal force stress t_{22} for CFL decreases, then follow an oscillatory pattern with decreasing magnitude.

The variations of temperature distribution T with distance x is depicted in Fig. 6. The pattern observed for SPL model and CFL are opposite in nature with fluctuating values which clearly reveals the effect of phase lag of heat flux and temperature gradient. The variation of temperature distribution T for DPL model is oscillatory to large extent.

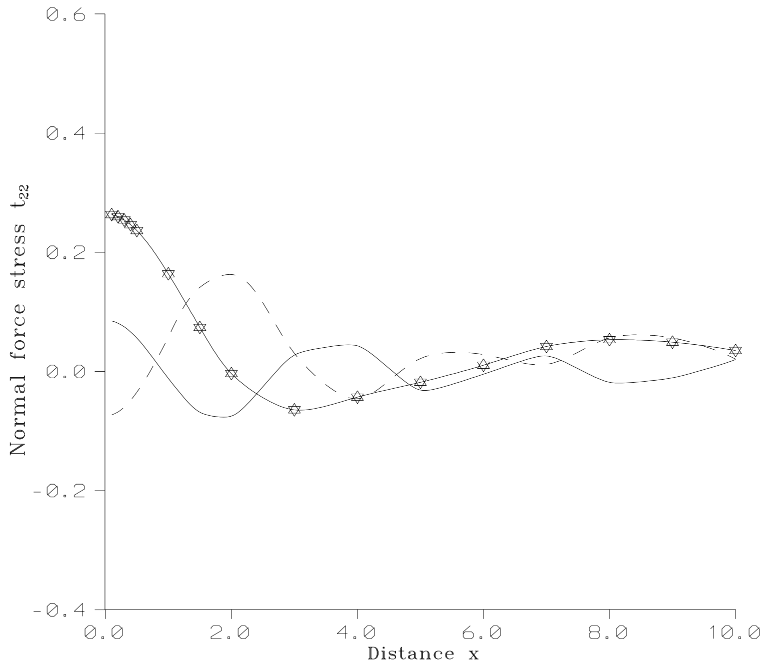

Fig. 5 Variation of normal force stress t_{22} with distance x for $\tau_\theta>\tau_q$

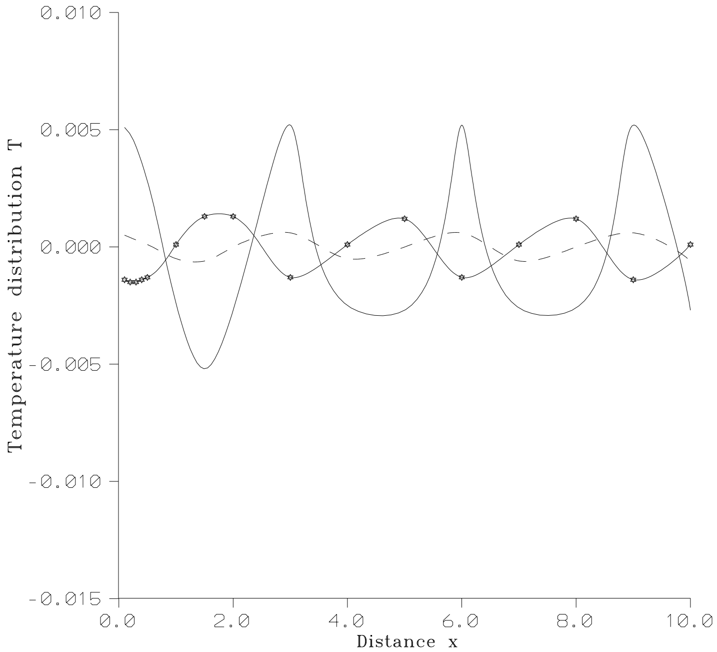

Fig. 6 Variation of temperature distribution *T* with distance x for $\tau_\theta > \tau_q$

8. Conclusions

a. Appreciable effect of dual-phase-lag (DPL) model i.e., effect of phase-lag of heat flux (τ_q) and effect of phase-lag of temperature gradient (τ_θ) is observed on the components of displacement, force stress and temperature distribution.

b. The variations of normal displacement are similar in nature for DPL model and SPL model with difference in magnitude.

c. The variations of all the quantities is significant with phase lag modl (dual and single) with Classical Fourier's law.

d. The normal mode analysis used in this article is applicable to wide range of problems in different branches. This method gives exact solutions without any assumed restrictions on either the temperature or stress distributions.

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Appendix A.

$$\Delta = (L_1 - L_2)[b_2 a_1 k_3 - b_1 a_2 k_3 + N_1 ia(a_2 - a_1)], \Delta_1 = L_1[N_1 ia(a_2 l_1 + l_2) - b_2 l_2 k_3 - P k_3 a_2],$$

$$\Delta_2 = L_1[P a_1 k_3 + b_1 l_2 k_3 - N_1 ia(l_2 + l_1 a_1)], \Delta_3 = L_1[Pia(a_2 - a_1) - b_1 ia(l_2 + l_1 a_2) + b_2 ia(l_1 a_1 + l_2)]$$

$$\Delta_4 = L_2[P a_2 k_3 + b_2 l_2 k_3 - N_1 ia(l_2 + l_1 a_2)], \Delta_5 = L_2[N_1 ia(l_1 a_1 + l_2) - b_1 l_2 k_3 - P a_1 k_3],$$

$$\Delta_6 = L_2[b_2 ia(l_2 + l_1 a_1) - b_1 ia(l_2 + l_1 a_2) + Pia(a_2 - a_1)], L_1 = [c_2 a_1 iak_1 - a_2 c_1 iak_2 + N_3 k_1 k_2 (a_1 - a_2)],$$

$$L_2 = [a_2 c_1 iak_2 - a_1 c_2 iak_1 + N_3 k_1 k_2 (a_2 - a_1)], P = -(P_1 + N_2).$$