Structural Engineering and Mechanics, *Vol. 56, No. 2 (2015) 317-340*
DOI: http://dx.doi.org/10.12989/sem.2015.56.2.317

Minimum shear reinforcement ratio of prestressed concrete members for safe design

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(Received December 1, 2014, Revised August 13, 2015, Accepted October 20, 2015)

Abstract. Design codes have specified the minimum shear reinforcement requirement for reinforced concrete (RC) and prestressed concrete (PSC) members to prevent brittle and premature shear failure. They are, however, very different from one another, and particularly, ACI318 code allows the required minimum shear reinforcement to be reduced in PSC members, compared to that in RC members, by specifying the additional equation for PSC members whose basis is not clear. In this paper, the minimum shear reinforcement ratio for PSC members was proposed, which can provide a sufficient reserved shear strength and deformation capacity. The proposed equation was also verified by the test results of PSC specimens lightly reinforced in shear, comparing to design codes and other proposed equations from previous studies.

Keywords: prestressed concrete, minimum shear reinforcement, cracking angle, shear cracking, reserved shear strength, reserved shear deformation

1. Introduction

National design codes, such as ACI318 building code (ACI Committee 318 2011), and AASHTO-LRFD bridge design specification (American Association of State Highway and Transportation Officials 2004) in the United States, CSA-04 (CSA Committee A23.3-04 2004) in Canada, AIK(Architectural Institute of Japan 1998, 1991) in Japan, KCI(KCI-M-07 2007) in South Korea, and MC-90 (Comite Euro-International du Beton 1990) in Europe, regulate the minimum shear reinforcement ratio ($\rho_{v,\min}$), the maximum spacing of shear reinforcement ($s_{\max}$), and the maximum shear reinforcement ratio ($\rho_{v,\max}$) for reinforced concrete (RC) and prestressed concrete (PSC) members. The minimum shear reinforcement ratio ($\rho_{v,\min}$) is required to prevent abrupt shear failure right after diagonal shear cracking, which is considered as the amount of transverse

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

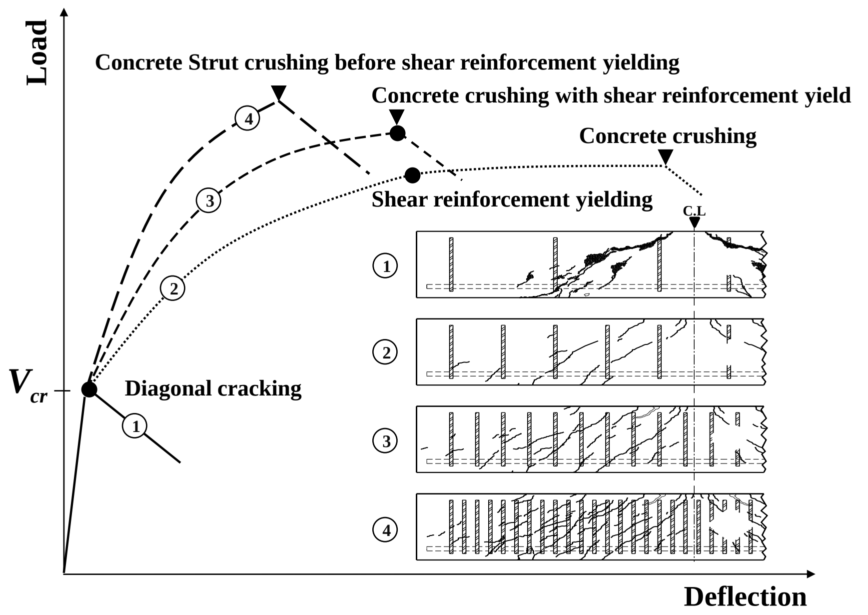


Fig. 1 Shear failure modes of RC members
(Yoon *et al.* 1996, Lee and Kim 2008, Lee and Hwang 2010)

reinforcement in web that can support at least the shear cracking force. In addition, the maximum shear reinforcement spacing is to have one or more stirrups pass through the diagonal tension cracks with a certain cracking angle so that it can prevent a sudden propagation of shear cracks and the brittle shear failure of concrete members. The allowable maximum amount of shear reinforcement is to prevent the shear-compression failure of concrete struts in the web before the shear reinforcement yields, which occurs when the shear reinforcement ratio is larger than the balanced shear reinforcement ratio (ρ_{vb}), the reinforcement ratio wherein the crushing failure of the inclined concrete compression strut and the yielding of the shear reinforcement occur simultaneously, and also to prevent the overgrowth of diagonal tension crack width (ACI Committee 318 2011, American Association of State Highway and Transportation Officials 2004, CSA Committee A23.3-04 2004, KCI-M-07 2007).

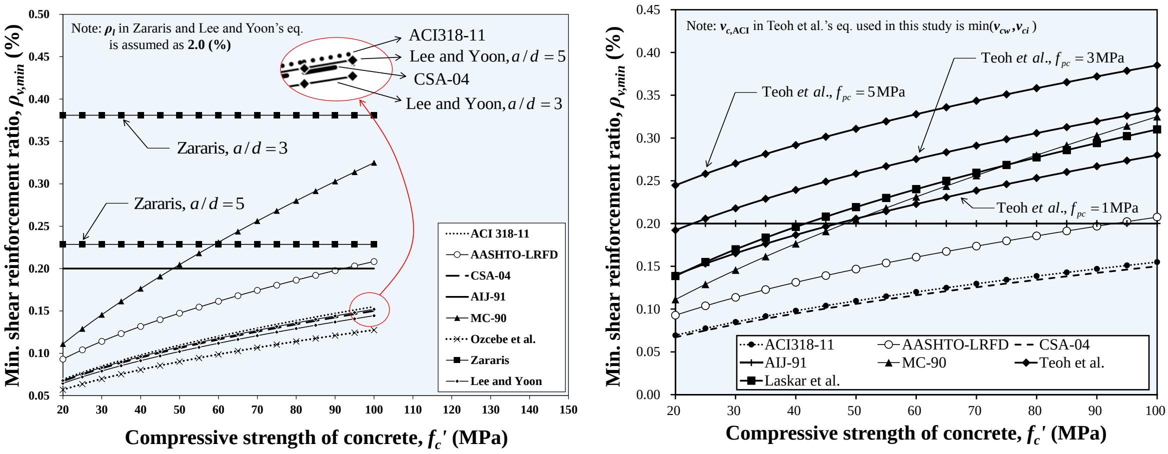
The shear failure modes of RC and PSC members are significantly affected by not only the shear reinforcement ratio (ρ_v) but also many other influential parameters, such as the longitudinal tension reinforcement ratio (ρ_l), the concrete compressive strength (f_c'), the inclination angle of diagonal tension cracks (θ). Particularly, the shear failure mode in PSC members is affected substantially by the magnitude of the prestress as well. Fig. 1 shows the typical shear failure modes of RC and PSC members (Lee and Kim 2008, Lee and Hwang 2010, Yoon *et al.* 1996). The line ① shows the load-deflection behavior of concrete members failing in a extremely brittle manner right after development of diagonal tension crack because the amount of shear reinforcement provided in web is too small to sustain the diagonal-shear cracking load. In order to prevent such a brittle shear failure mode developed in lightly reinforced members, as aforementioned, design codes specify the minimum shear reinforcement requirements. The line ② shows a case in which the shear reinforcement ratio is larger than the minimum shear reinforcement ratio ($\rho_{v,\min}$) and smaller than the balanced shear reinforcement ratio (ρ_{vb}). This is the most desirable mode of failure because its ductile behavioural characteristics resulting from the yielding of the shear reinforcements before the web concrete crushes. The line ③ shows the shear balance failure mode, in which the shear reinforcement yields at the same time with the compressive failure of diagonal concrete. The line ④ shows the shear failure mode typically developed in over-reinforced members against shear, in which shear reinforcement is provided more than the balanced shear reinforcement ratio (ρ_{vb}). In this case, the compressive concrete struts crush before the stirrup yielding. Therefore, as mentioned, the current code provisions on maximum shear reinforcement ratio ($\rho_{v,\max}$) is aimed to secure sufficient ductility by limiting the amount of shear reinforcement provided in web below the balanced reinforcement ratio (ρ_{vb}), which can guarantee the yielding of shear reinforcements before the web concrete is crushed.

Previous studies on minimum and maximum shear reinforcement ratios have mostly focused on RC members (Lee and Kim 2008, Lee and Hwang 2010, Yoon *et al.* 1996, Johnson and Ramirez 1989, Ozcebe *et al.* 1999, Zararis 2010, Angelakos *et al.* 2001, Rahal and Al-Shaleh 2004, Roller and Russell 1990, Lee and Yoon 2003, Juchma and Kim 2001, Appa Rao and Injaganeri 2013) and few limited studies have been conducted on the shear reinforcement ratio of PSC members. Particularly, the studies on the minimum shear reinforcement ratio of PSC members can be rarely found (Ghosh 1986, 1987, Teoh *et al.* 2002, Avendaño and Bayrak 2010, Avendaño and Bayrak 2011, Laskar *et al.* 2010). Furthermore, recent studies have pointed out that the minimum shear reinforcement ratio of PSC members specified in the design standards of North America, such as ACI318-11 (ACI Committee 318 2011), could result in unsafe shear design for prestressed concrete members (Teoh *et al.* 2002, Avendaño and Bayrak 2010, Avendaño and Bayrak 2011, Laskar *et al.* 2010). Therefore, this study examines the appropriateness of the minimum shear reinforcement requirements for PSC members presented in national design codes (ACI Committee 318 2011, American Association of State Highway and Transportation Officials 2004, CSA Committee A23.3-04 2004, Architectural Institute of Japan 1998, Architectural Institute of Japan 1991, KCI-M-07 2007, Comite Euro-International du Beton 1990) and other studies (Teoh *et al.* 2002, Avendaño and Bayrak 2010, Avendaño and Bayrak 2011, Laskar *et al.* 2010) and proposes a minimum shear reinforcement ratio for PSC members that can lead to safe shear design with a sufficient reserved shear strength and deformation capacity.

2. Background

Figs. 2(a) and (b) show the comparisons of the minimum shear reinforcement ratios for RC and PSC members, respectively, specified in the design standards (ACI Committee 318 2011, American Association of State Highway and Transportation Officials 2004, CSA Committee A23.3-04 2004, Architectural Institute of Japan 1998, Architectural Institute of Japan 1991) and suggested in the previous studies (Ozcebe *et al.* 1999, Zararis 2010, Lee and Yoon 2003, Teoh *et al.* 2002, Avendaño and Bayrak 2010, 2011, Laskar *et al.* 2010). It is shown in Fig. 2 that these models require significantly different amount of minimum shear reinforcement, and that they also account the effect of concrete compressive strength (f_c') on the minimum shear reinforcement with a very large difference. This means that the minimum shear reinforcement equations contain a high level of uncertainties. Some could be conservative or some may lead to an unsafe design.

As shown in Fig. 2, the design standards in North America require a lower amount of minimum shear reinforcement than those presented in other standards or existing studies (ACI Committee



(a) Reinforced concrete (RC) members (b) Prestressed concrete (PSC) members

Fig. 2 Minimum shear reinforcement ratios in national codes and previous studies

318 2011, American Association of State Highway and Transportation Officials 2004, CSA Committee A23.3-04 2004). Particularly, ACI318-11 (ACI Committee 318 2011) specifies that the amount of the minimum shear reinforcement for PSC members ($\rho_{v,\min}$) can be determined from the smaller value of the following two equations

$$\rho_{v,\min} = \frac{0.062\sqrt{f_c'}}{f_{yt}} \geq \frac{0.35}{f_{yt}} \tag{1}$$

$$\rho_{v,\min} = \frac{A_{ps} f_{pu}}{80 f_{yt} b_w d}\sqrt{\frac{d}{b_w}} \tag{2}$$

The minimum shear reinforcement ratio required in Canadian concrete design code (CSA Committee A23.3-04 2004) is $0.06\sqrt{f_c'}/f_{yt}$, which is very similar to Eq. (1). Eq. (1) reflects the effect of the concrete compressive strength (f_c') on the minimum shear reinforcement ratio based on the studies of Roller and Russell (1990), Johnson and Ramirez (1989), Ozcebe *et al.* (1999) and Yoon *et al.* (1996), which has been implemented in ACI318 building code since 2005. On the other hand, Eq. (2) was first appeared ACI 318 code in 1971 (ACI Committee 318), and it requires the amount of minimum shear reinforcement for PSC members less than that of RC members in most cases.

Unfortunately, the origin of Eq. (2) cannot be found in any literature, but some speculations on the derivation of Eq. (2) were introduced in the research report by Olessen *et al.* (1965). According to this report, any possible imperfections caused by the erection of the prestressed concrete member or other similar reasons could reduce the tensile strength of concrete, and thus, decrease the shear cracking strength. In order to replace the part of the concrete contribution with web reinforcement, the amount of shear reinforcement should satisfy the following relationship (Olessen *et al.* 1965)

$$\rho_{v,\min} f_{yt} b_w d \geq k_1 b_w d \tag{3}$$

where, k_1 is a measure of the reduction in the tensile strength of concrete. However, Eq. (3) requires larger amount of shear reinforcement for rectangular beams than I-shaped beams, which is unreasonable because the imperfections are less likely in rectangular members. Also, the required shear reinforcement should be related to the amount of the longitudinal tensile reinforcement, which is not the case in Eq. (3). Thus, relating the minimum shear reinforcement ratio to the amount of longitudinal reinforcement, another proposal has been established, as follows (Olessen *et al.* 1965)

$$\rho_{v,\min} = \frac{A_{ps} f_u}{k_2 f_{yt} b_w d} \sqrt{\frac{d}{b_w}} \tag{4}$$

where, k_2 is the ratio of the depth of the resulting compressive force to the neutral axis depth. By comparing of Eqs. (2) and (4), it seems that k_2 in Eq. (4) was determined to be the constant coefficient of 80, but it has no clear valid reason. Due to this lack of information on Equation (2), however, more detailed reviews would be necessary for better understanding on this. Shear force at diagonal tension cracking should be sustained by the stirrups, if any, and the concrete, and therefore, the shear strength of concrete member with shear reinforcement (V_n) can be expressed, as follows

$$V_n = V_s + V_c \geq V_{cr} \tag{5}$$

where, V_c and V_s are the shear contribution of concrete and shear reinforcement, respectively. By rearranging the Eq. (5) in terms of the shear reinforcement ratio ($\rho_{v,\min}$), the following relationships can be obtained for RC members and PSC members, respectively:

$$\rho_{v,\min} f_{yt} \geq v_{cr,RC} - v_{c,RC} \tag{6}$$

$$\rho_{v,\min} f_{yt} \geq v_{cr,PSC} - v_{c,PSC} \tag{7}$$

where, $v_{cr,RC}$ and $v_{cr,PSC}$ are the shear cracking strength of RC and PSC members, respectively, and $v_{cr,RC}$ and $v_{cr,PSC}$ are the shear contribution of concrete in RC and PSC members at the ultimate state, respectively. Eqs. (6) and (7) indicate that difference between the shear cracking strength and the shear contribution of concrete at ultimate should be sustained by the minimum shear reinforcement. In Eq. (6), as the concrete compressive strength (f_c') increases, also the difference between the shear cracking strength and shear contribution of concrete at ultimate ($v_{cr,RC}-v_{c,RC}$) increases, and therefore, the required amount of the minimum shear reinforcement ($\rho_{v,\min}$) would be increased. This simple principal is also applicable to Eq. (7), and particularly, the shear cracking strength ($v_{cr,PSC}$) in PSC members would be increased not only by the compressive strength of concrete (f_c') but also by the effective prestress (f_{pe}). Therefore, the larger f_c' and f_{pe} is, the greater the shear stress ($v_{cr,PSC}-v_{c,PSC}$) resisted by the minimum shear reinforcement ($\rho_{v,\min}$) should be (Lyngberg 1976). In other words, the effective prestress (f_{pe}) would result in a greater shear cracking strength ($v_{cr,PSC}$) in PSC members, compared to RC members, it is anticipated that a greater amount of minimum shear reinforcement ($\rho_{v,\min}$) would be required in PSC members than in RC members.

As mentioned previously, ACI318-11 (ACI Committee 318 2011) have specified that the minimum shear reinforcement ratio of PSC members can be determined to be the smaller value of Eqs. (1) and (2). Fig. 3 shows the minimum shear reinforcement ratio by Eqs. (1) and (2) as specified in ACI 318-11. As the concrete compressive strength (f_c') in PSC members increases, the

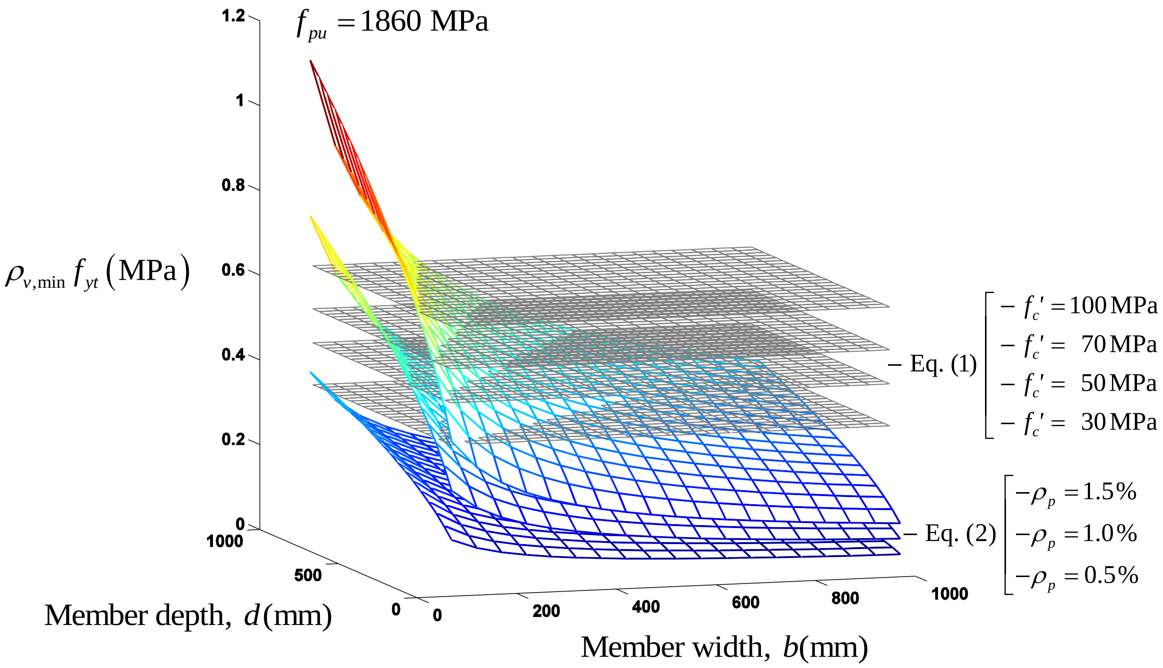


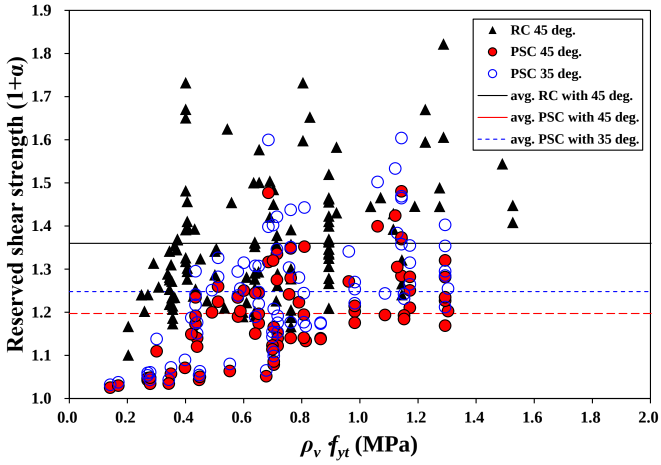
Fig. 3 Minimum shear reinforcement ratio specified by Eq. (1) and Eq. (2) in ACI318-11

required reinforcement ratio estimated by Eq. (1) increases as well, but the one by Eq. (2) does not. Thus, as the concrete strength gets higher than a certain level, Eq. (2) governs the minimum shear reinforcement ratio in PSC members. In such a case, the minimum shear reinforcement required for PSC members is lower than that required for RC members. When the effective prestress introduced to concrete (f_{pe}) is relatively small, which means that prestressing steel ratio (ρ_p) is, for instance, 0.5% in Fig. 3, the required amount of minimum shear reinforcement in PSC members is smaller than that of RC members, even for the deep members. When the prestressing steel ratio (ρ_p) becomes higher in Fig. 3, the required amount of minimum shear reinforcement in PSC members increases, but it is still smaller than that of RC members unless the member height is significantly deep. This is the opposite of what was found in the recent studies on the minimum shear reinforcement ratio for PSC members (Teoh *et al.* 2002, Avendaño and Bayrak 2010, 2011, Laskar *et al.* 2010). Thus, it is necessary to examine the minimum shear reinforcement requirement for PSC members specified in the current ACI318-11 provision.

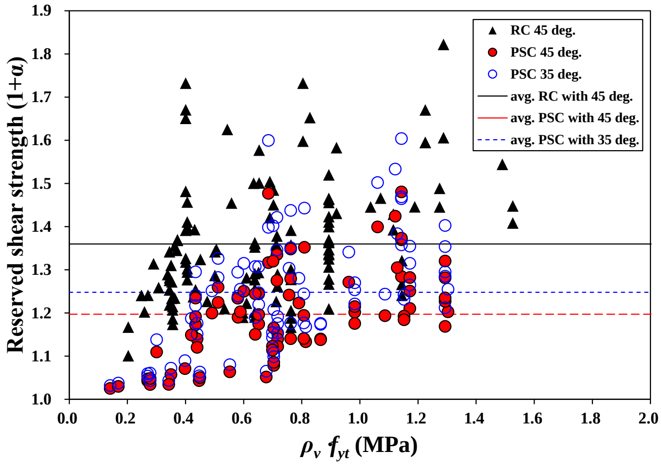
Figs. 4(a) and (b) show the reserved shear strength of RC and PSC beam specimens with respect to the concrete compressive strength (f_c') and the amount of the shear reinforcement ($\rho_v f_{yt}$), respectively. Since the shear cracking strength of RC and PSC members (V_{cr}) is generally proportional to the shear contribution of concrete (V_c), Eq. (5) can be expressed in a simplified manner by introducing the reserved shear strength factor (α) (Ozcebe *et al.* 1999, Kuchma and Kim 2001, Teoh *et al.* 2002, Avendaño and Bayrak 2011, Sneed and Ramirez. 2009), as follows

$$V_n = V_c + V_s \geq (1+\alpha) V_c \tag{8}$$

Therefore, the reserved shear strength (1+ α) can be expressed as



(a) Reserved shear strength vs. f_c'



(b) Reserved shear strength vs. $\rho_v f_{yt}$

Fig. 4 Reserved shear strength of RC and PSC members

$$(1+\alpha)=\frac{V_n}{V_c} \tag{9}$$

and the reserved shear strength factor (α) is

$$\alpha=\frac{V_s}{V_c} \tag{10}$$

In Eq. (10), V_s is significantly affected by the cracking angle, and for the cases of RC members with very small amounts of shear reinforcement, the inclination angle of the concrete compressive diagonals can be assumed to be 45 degree as it is typical in RC members without shear reinforcement. On this basis, the contribution of the shear reinforcement can be estimated by 45-degree truss analogy model specified in ACI318-11. Also, V_c can be calculated by subtracting the

Table 1 Minimum shear reinforcement ratio for RC and PSC members proposed in previous studies

	Sources	Minimum shear reinforcement ratio
RC member	Ozcebe *et al*. (1999)	$\rho_{v,\min} = 0.3 v_{c,\text{ACI-1}}$
	Zararis (2010)	$\rho_{v,\min} = \dfrac{\rho_l}{1.75}\dfrac{d}{a}$
	Lee and Yoon (2003)	$\rho_{v,\min} = 0.035\dfrac{\sqrt{f_c'}}{f_{yt}}\left(\dfrac{d}{\rho_l a}\right)^{0.1}$
PSC member	Teoh *et al*. (2002)	$\rho_{v,\min} = \dfrac{0.35 v_{c,\text{ACI-2}}}{f_{yt}}$
	Avendaño and Bayrak (2010, 2011)	$\rho_{v,\min} = \dfrac{0.25 v_{c,\text{ACI-2}}}{f_{yt}}$
	Laskar *et al*. (2010)	$\rho_{v,\min} = \dfrac{0.062\sqrt{f_c'}}{f_{yt}}$ when a/d<2, a/d>4 $= \dfrac{0.124\sqrt{f_c'}}{f_{yt}}$ when 2 < a/d<4

note : $v_{c,\text{ACI-1}}$ is the shear strength provided by concrete for nonprestressed members in ACI building code (ACI318-11). In Fig. 2(a), $0.17\lambda\sqrt{f_c'}$ is used.

$v_{c,\text{ACI-2}}$ is the shear strength provided by concrete for prestressed members in ACI building code (ACI318-11). In Fig. 2(a), the minimum value of v_{cw} and v_{ci} is used.

Table 2 Summary of properties and test results of RC specimens lightly reinforced in shear

Beam Name	f_c' (MPa)	a/d	Longitudinal Reinforcement		Transverse Reinforcement			$\rho_v f_{yt}$ (MPa)	$V_{u,test}$ (kN)	$V_{s,45deg}$ (kN)	V_c (kN)	$V_s/V_c=\alpha$	Reserved shear strength	Reserved shear deformation
			f_y (MPa)	ρ_l (%)	s (mm)	f_{yt} (MPa)	ρ_v (%)							
Yoon *et al.* (1996)														
M1-N	67.0	3.2	399.9	2.800	325.0	429.9	0.082	0.353	405.00	86.64	318.36	0.27	1.27	17.64
M2-N	67.0	3.2	399.9	2.800	230.0	429.9	0.165	0.708	689.00	173.85	515.15	0.34	1.34	14.89
M2-S	67.0	3.2	399.9	2.800	325.0	429.9	0.117	0.501	552.00	123.03	428.97	0.29	1.29	14.43
N1-N	36.0	3.2	399.9	2.800	325.0	429.9	0.082	0.353	457.00	86.64	370.36	0.23	1.23	14.37
N2-N	36.0	3.2	399.9	2.800	325.0	429.9	0.117	0.501	483.00	123.03	359.97	0.34	1.34	10.99
N2-S	36.0	3.2	399.9	2.800	465.0	429.9	0.081	0.350	363.00	85.99	277.01	0.31	1.31	8.83
Sarsam and Al-Musawi (1992)														
AL2-H	75.3	4.0	494.9	2.229	150.0	819.8	0.093	0.763	122.60	32.28	90.32	0.36	1.36	-
AL2-N	40.4	4.0	494.9	2.229	150.0	819.8	0.093	0.763	114.70	32.28	82.42	0.39	1.39	-
AS2-H	75.5	2.5	494.9	2.258	150.0	819.8	0.093	0.763	201.00	31.86	169.14	0.19	1.19	-
AS2-N	39.0	2.5	494.9	2.229	150.0	819.8	0.093	0.763	189.30	32.28	157.02	0.21	1.21	-
AS3-H	71.8	2.5	494.9	2.229	100.0	819.8	0.140	1.145	199.10	48.41	150.69	0.32	1.32	-
BL2-H	75.7	4.0	542.9	2.816	150.0	819.8	0.093	0.763	138.30	32.00	106.30	0.30	1.30	-
BS2-H	73.9	2.5	542.9	2.816	150.0	819.8	0.093	0.763	223.50	32.00	191.50	0.17	1.17	-
BS3-H	73.4	2.5	542.9	2.816	100.0	819.8	0.140	1.145	228.10	48.00	180.10	0.27	1.27	-
BS4-H	80.1	2.5	542.9	2.816	75.0	819.8	0.186	1.526	206.90	64.00	142.90	0.45	1.45	-
CL2-H	70.1	4.0	542.9	3.505	150.0	819.8	0.093	0.763	147.20	32.00	115.20	0.28	1.28	-
CS2-H	70.2	2.5	542.9	3.505	150.0	819.8	0.093	0.763	247.20	32.00	215.20	0.15	1.15	-
CS3-H	74.2	2.5	542.9	3.505	100.0	819.8	0.140	1.145	247.20	48.00	199.20	0.24	1.24	-
CS4-H	75.7	2.5	542.9	3.505	75.0	819.8	0.186	1.526	220.70	64.00	156.70	0.41	1.41	-

Table 2 Continued

Roller and Russell (1990)														
No.1	120.1	2.5	472.3	1.649	215.9	406.8	0.076	0.308	297.30	61.14	236.16	0.26	1.26	-
No.6	72.4	3.0	464.0	1.733	381.0	445.4	0.081	0.363	665.36	126.44	538.92	0.23	1.23	-
No.7	72.4	3.0	483.3	1.881	196.9	445.4	0.158	0.702	787.87	244.72	543.14	0.45	1.45	-
No.8	125.3	3.0	483.3	1.881	381.0	445.4	0.081	0.363	482.81	126.44	356.37	0.35	1.35	-
No.9	125.3	3.0	483.3	2.352	196.9	445.4	0.158	0.702	749.44	244.72	504.71	0.48	1.48	-
No.10	125.3	3.0	464.0	2.889	133.4	445.4	0.233	1.037	1172.19	361.26	810.94	0.45	1.45	-
Ozcebe *et al*. (1999)														
ACI36	75.0	3.0	449.9	2.594	120.0	254.9	0.140	0.356	105.30	16.55	88.75	0.19	1.19	2.11
ACI39	73.0	3.0	424.9	3.081	120.0	254.9	0.140	0.356	111.80	16.55	95.25	0.17	1.17	2.22
ACI56	58.0	5.0	449.9	3.459	120.0	254.9	0.140	0.356	93.60	16.55	77.05	0.21	1.21	-
ACI59	82.0	5.0	424.9	4.432	120.0	254.9	0.140	0.356	96.50	16.55	79.95	0.21	1.21	-
TH36	75.0	3.0	449.9	2.594	100.0	254.9	0.168	0.427	141.00	19.86	121.14	0.16	1.16	2.96
TH39	73.0	3.0	424.9	3.081	80.0	254.9	0.209	0.534	142.90	24.83	118.07	0.21	1.21	3.75
TH56	63.0	5.0	449.9	3.459	100.0	254.9	0.168	0.427	103.50	19.86	83.64	0.24	1.24	-
TH59	75.0	5.0	424.9	4.432	90.0	254.9	0.186	0.475	119.30	22.07	97.23	0.23	1.23	-
TS36	75.0	3.0	449.9	2.594	70.0	254.9	0.239	0.610	155.90	28.37	127.53	0.22	1.22	-
TS39	73.0	3.0	424.9	3.081	60.0	254.9	0.279	0.712	179.20	33.10	146.10	0.23	1.23	-
TS56	61.0	5.0	449.9	3.459	70.0	254.9	0.239	0.610	129.20	28.37	100.83	0.28	1.28	-
TS59	82.0	5.0	424.9	4.432	60.0	254.9	0.279	0.712	125.40	33.10	92.30	0.36	1.36	-
Moayer and Regan (1974)														
P20	40.7	3.5	641.2	1.920	152.4	310.3	0.210	0.652	120.10	27.28	92.82	0.29	1.29	-
P21	42.8	5.4	641.2	1.920	228.6	310.3	0.140	0.434	89.85	18.19	71.67	0.25	1.25	-
P22	43.3	5.4	641.2	1.920	152.4	255.1	0.280	0.714	108.98	29.91	79.07	0.38	1.38	-
P5	43.0	3.5	641.2	1.450	101.6	255.1	0.420	1.071	145.01	46.09	98.92	0.47	1.47	-
Collins and Kuchma (1999)														
BM100	47.0	2.9	549.9	0.757	600.0	507.9	0.079	0.401	342.00	111.18	230.82	0.48	1.48	-
BM100D	47.0	2.9	549.9	1.910	600.0	507.9	0.079	0.401	461.00	111.18	349.82	0.32	1.32	-
SE100A-M-69	71.0	2.5	482.9	1.032	440.0	521.9	0.154	0.804	516.30	218.24	298.06	0.73	1.73	-
SE100B-M-69	75.0	2.5	482.9	1.363	440.0	521.9	0.154	0.804	583.20	218.24	364.96	0.60	1.60	-
SE50A-M-69	74.0	2.7	482.9	1.031	276.0	592.9	0.110	0.653	138.50	50.68	87.82	0.58	1.58	-
SE50B-M-69	74.0	2.7	482.9	1.160	276.0	592.9	0.110	0.653	151.80	50.68	101.12	0.50	1.50	-
Krefeld and Thurston (1966)														
Ss2-213.5-1	38.9	4.0	386.1	2.228	342.9	341.3	0.073	0.248	148.13	28.73	119.40	0.24	1.24	-
Ss2-213.5a-2	37.0	4.0	386.1	2.228	342.9	372.3	0.073	0.271	161.47	31.34	130.13	0.24	1.24	-
Ss2-218a-2	37.6	4.0	386.1	2.228	457.2	372.3	0.055	0.203	164.14	23.50	140.64	0.17	1.17	-
Ss2-218b-2	34.6	4.0	386.1	2.228	457.2	372.3	0.055	0.203	256.17	23.50	232.66	0.10	1.10	-
Ss2-26-1	40.1	4.0	386.1	2.228	152.4	341.3	0.164	0.558	206.84	64.64	142.21	0.45	1.45	-
Ss2-29-3	34.3	4.0	386.1	2.228	228.6	237.2	0.109	0.259	177.93	29.95	147.98	0.20	1.20	-
Ss2-29a-1	38.8	4.0	386.1	2.228	228.6	341.3	0.109	0.372	159.69	43.09	116.60	0.37	1.37	-
Ss2-29a-2	37.2	4.0	386.1	2.228	228.6	372.3	0.109	0.406	216.63	47.01	169.62	0.28	1.28	-
Ss2-29b-1	37.6	4.0	386.1	2.228	228.6	341.3	0.109	0.372	160.14	43.09	117.04	0.37	1.37	-
Ss2-29b-2	41.4	4.0	386.1	2.228	228.6	372.3	0.109	0.406	202.39	47.01	155.39	0.30	1.30	-
Ss2-29c-2	24.1	4.0	386.1	2.228	228.6	372.3	0.109	0.406	161.47	47.01	114.46	0.41	1.41	-
Ss2-29d-2	30.4	4.0	386.1	2.228	228.6	372.3	0.109	0.406	165.03	47.01	118.02	0.40	1.40	-
Ss2-29e-2	48.5	4.0	386.1	2.228	228.6	372.3	0.109	0.406	206.40	47.01	159.39	0.29	1.29	-
Ss2-29g-2	15.7	4.0	386.1	2.228	228.6	372.3	0.109	0.406	149.91	47.01	102.90	0.46	1.46	-
Ss2-313.5-3	42.7	4.0	386.1	2.228	342.9	275.8	0.164	0.451	213.51	52.23	161.28	0.32	1.32	-
Ss2-318-1	40.5	4.0	386.1	2.228	457.2	517.1	0.123	0.635	220.19	73.45	146.74	0.50	1.50	-
Ss2-318-2	38.9	4.0	386.1	2.228	457.2	351.6	0.123	0.432	177.04	49.95	127.09	0.39	1.39	-
Ss2-318-3	43.0	4.0	386.1	2.228	457.2	275.8	0.123	0.338	174.82	39.17	135.64	0.29	1.29	-
Ss2-321-1	38.7	4.0	386.1	2.228	533.4	517.1	0.105	0.544	163.69	62.96	100.74	0.62	1.62	-
Ss2-321-2	38.0	4.0	386.1	2.228	533.4	351.6	0.105	0.370	166.81	42.81	124.00	0.35	1.35	-
Ss2-321-3	43.0	4.0	386.1	2.228	533.4	275.8	0.105	0.290	140.56	33.58	106.99	0.31	1.31	-

Table 2 Continued

Kong and Rangan (1998)														
S1-1	63.6	2.5	451.9	2.803	100.0	568.9	0.157	0.893	228.30	65.23	163.07	0.40	1.40	-
S1-2	63.6	2.5	451.9	2.803	100.0	568.9	0.157	0.893	208.30	65.23	143.07	0.46	1.46	-
S1-3	63.6	2.5	451.9	2.803	100.0	568.9	0.157	0.893	206.10	65.23	140.87	0.46	1.46	-
S1-4	63.6	2.5	451.9	2.803	100.0	568.9	0.157	0.893	277.90	65.23	212.67	0.31	1.31	-
S1-5	63.6	2.5	451.9	2.803	100.0	568.9	0.157	0.893	253.30	65.23	188.07	0.35	1.35	-
S1-6	63.6	2.5	451.9	2.803	100.0	568.9	0.157	0.893	224.10	65.23	158.87	0.41	1.41	-
S2-1	72.5	2.5	451.9	2.803	150.0	568.9	0.105	0.597	260.30	43.49	216.81	0.20	1.20	-
S2-2	72.5	2.5	451.9	2.803	125.0	568.9	0.126	0.717	232.50	52.19	180.32	0.29	1.29	-
S2-3	72.5	2.5	451.9	2.803	100.0	568.9	0.157	0.893	253.30	65.23	188.07	0.35	1.35	-
S2-4	72.5	2.5	451.9	2.803	100.0	568.9	0.157	0.893	219.40	65.23	154.17	0.42	1.42	-
S2-5	72.5	2.5	451.9	2.803	75.0	568.9	0.209	1.189	282.10	86.98	195.13	0.45	1.45	-
S3-1	67.4	2.5	449.9	1.659	100.0	631.9	0.101	0.638	209.20	47.16	162.04	0.29	1.29	-
S3-2	67.4	2.5	449.9	1.659	100.0	631.9	0.101	0.638	178.00	47.16	130.84	0.36	1.36	-
S3-3	67.4	2.5	451.9	2.793	100.0	631.9	0.101	0.638	228.60	46.53	182.07	0.26	1.26	-
S3-4	67.4	2.5	451.9	2.793	100.0	631.9	0.101	0.638	174.90	46.53	128.37	0.36	1.36	-
S3-5	67.4	2.4	441.9	3.692	100.0	631.9	0.101	0.638	296.60	47.48	249.12	0.19	1.19	-
S3-6	67.4	2.4	441.9	3.692	100.0	631.9	0.101	0.638	282.90	47.48	235.42	0.20	1.20	-
S4-1	87.3	2.4	451.9	3.020	100.0	568.9	0.157	0.893	354.00	121.08	232.92	0.52	1.52	-
S4-2	87.3	2.4	432.9	2.959	100.0	568.9	0.157	0.893	572.80	99.19	473.62	0.21	1.21	-
S4-3	87.3	2.4	449.9	2.849	100.0	568.9	0.157	0.893	243.40	77.29	166.11	0.47	1.47	-
S4-4	87.3	2.5	451.9	2.803	100.0	568.9	0.157	0.893	258.10	65.23	192.87	0.34	1.34	-
S4-6	87.3	2.5	441.9	2.788	100.0	568.9	0.157	0.893	202.90	44.23	158.67	0.28	1.28	-
S5-1	89.4	3.0	451.9	2.803	100.0	568.9	0.157	0.893	241.70	65.23	176.47	0.37	1.37	-
S5-2	89.4	2.7	451.9	2.803	100.0	568.9	0.157	0.893	259.90	65.23	194.67	0.34	1.34	-
S5-3	89.4	2.5	451.9	2.803	100.0	568.9	0.157	0.893	243.80	65.23	178.57	0.37	1.37	-
S6-3	68.9	2.7	451.9	2.793	100.0	631.9	0.101	0.638	178.40	46.53	131.87	0.35	1.35	-
S6-4	68.9	2.7	451.9	2.793	100.0	631.9	0.101	0.638	214.40	46.53	167.87	0.28	1.28	-
S7-1	74.8	3.3	432.9	4.468	150.0	568.9	0.105	0.597	217.20	43.79	173.42	0.25	1.25	-
S7-2	74.8	3.3	432.9	4.468	125.0	568.9	0.126	0.717	205.40	52.54	152.86	0.34	1.34	-
S7-3	74.8	3.3	432.9	4.468	100.0	568.9	0.157	0.893	246.50	65.68	180.82	0.36	1.36	-
S7-4	74.8	3.3	432.9	4.468	80.0	568.9	0.196	1.115	273.60	82.10	191.50	0.43	1.43	-
S7-5	74.8	3.3	432.9	4.468	70.0	568.9	0.224	1.274	304.40	93.83	210.58	0.45	1.45	-
S7-6	74.8	3.3	432.9	4.468	60.0	568.9	0.262	1.490	310.60	109.46	201.14	0.54	1.54	-
S8-1	74.6	2.5	451.9	2.803	150.0	568.9	0.105	0.597	272.10	43.49	228.61	0.19	1.19	-
S8-2	74.6	2.5	451.9	2.803	125.0	568.9	0.126	0.717	251.00	52.19	198.82	0.26	1.26	-
S8-3	74.6	2.5	451.9	2.803	100.0	568.9	0.157	0.893	309.60	65.23	244.37	0.27	1.27	-
S8-4	74.6	2.5	451.9	2.803	100.0	568.9	0.157	0.893	265.80	65.23	200.57	0.33	1.33	-
S8-5	74.6	2.5	451.9	2.803	80.0	568.9	0.196	1.115	289.20	81.54	207.66	0.39	1.39	-
S8-6	74.6	2.5	451.9	2.803	70.0	568.9	0.224	1.274	283.90	93.19	190.71	0.49	1.49	-
Johnson and Ramirez (1989)														
1	36.4	3.1	524.7	2.490	137.2	479.2	0.144	0.690	338.06	113.31	224.75	0.50	1.50	-
2	36.4	3.1	524.7	2.490	279.4	479.2	0.072	0.345	222.41	56.66	165.76	0.34	1.34	-
3	72.3	3.1	524.7	2.490	279.4	479.2	0.072	0.345	262.44	56.66	205.79	0.28	1.28	-
4	72.3	3.1	524.7	2.490	279.4	479.2	0.072	0.345	315.82	56.66	259.17	0.22	1.22	-
5	55.8	3.1	524.7	2.490	137.2	479.2	0.144	0.690	382.55	113.31	269.24	0.42	1.42	-
7	51.3	3.1	524.7	2.490	279.4	479.2	0.072	0.345	280.24	56.66	223.58	0.25	1.25	-
8	51.3	3.1	524.7	2.490	279.4	479.2	0.072	0.345	258.00	56.66	201.34	0.28	1.28	-
Clark (1951)														
D5-1	27.7	2.4	320.6	3.420	254.0	331.1	0.370	1.225	146.16	58.66	87.50	0.67	1.67	-
D5-2	29.0	2.4	320.6	3.420	254.0	331.1	0.370	1.225	157.28	58.66	98.62	0.59	1.59	-
D5-3	27.1	2.4	320.6	3.420	254.0	331.1	0.370	1.225	157.28	58.66	98.62	0.59	1.59	-

Table 2 Continued

Angelakos *et al*. (2001)														
DB120M	21.0	2.9	549.9	1.009	600.0	507.9	0.079	0.401	282.00	111.18	170.82	0.65	1.65	10.80
DB140M	38.0	2.9	549.9	1.009	300.0	507.9	0.079	0.401	277.00	111.18	165.82	0.67	1.67	8.59
DB165M	65.0	2.9	549.9	1.009	300.0	507.9	0.079	0.401	452.00	111.18	340.82	0.33	1.33	20.31
DB180M	80.0	2.9	549.9	1.009	300.0	507.9	0.079	0.401	395.00	111.18	283.82	0.39	1.39	12.79
DBO530M	32.0	2.9	549.9	0.505	300.0	507.9	0.079	0.401	263.00	111.18	151.82	0.73	1.73	17.29
Adebar and Collins (1996)														
ST18	49.8	2.9	535.9	1.950	173.3	459.9	0.200	0.920	246.30	74.15	172.15	0.43	1.43	-
ST19	50.8	2.9	535.9	1.950	173.3	459.9	0.200	0.920	201.40	74.15	127.25	0.58	1.58	-
ST4	49.3	2.9	535.9	1.950	315.2	459.9	0.110	0.506	158.20	40.78	117.42	0.35	1.35	-
ST5	49.3	2.9	535.9	1.950	192.6	459.9	0.180	0.828	169.00	66.74	102.26	0.65	1.65	-
ST6	49.3	2.9	535.9	1.950	123.8	459.9	0.280	1.288	230.10	103.81	126.29	0.82	1.82	-
ST7	49.3	2.9	535.9	1.950	123.8	459.9	0.280	1.288	275.10	103.81	171.29	0.61	1.61	-
Rahal and Al-Shaleh (2004)														
A65-200	60.9	2.8	440.0	2.192	200.0	240.0	0.141	0.339	175.00	22.04	152.97	0.14	1.14	-
A65-140	62.1	2.8	440.0	2.192	140.0	240.0	0.202	0.485	150.00	31.53	118.48	0.27	1.27	-
A65-110	60.9	2.8	440.0	2.192	110.0	240.0	0.257	0.617	188.00	40.11	147.90	0.27	1.27	-
A65-95	62.1	2.8	440.0	2.192	95.0	240.0	0.298	0.714	220.00	46.41	173.59	0.27	1.27	-
B65-200	64.3	3.0	440.0	3.958	200.0	305.0	0.120	0.366	195.00	21.96	173.04	0.13	1.13	-
B65-160	65.1	3.0	440.0	3.958	160.0	305.0	0.150	0.458	208.00	27.48	180.52	0.15	1.15	-
B65-140	65.1	3.0	440.0	3.958	140.0	305.0	0.171	0.523	235.00	31.38	203.62	0.15	1.15	-
B65 125	66.4	3.0	440.0	3.958	125.0	305.0	0.192	0.586	242.00	35.16	206.84	0.17	1.17	-
B65-110	66.4	3.0	440.0	3.958	110.0	305.0	0.218	0.665	270.00	39.90	230.10	0.17	1.17	-
Lee and Kim (2008)														
L1-B	40.8	3.0	525.0	1.790	80.0	215.0	0.175	0.376	214.00	53.99	160.01	0.34	1.34	1.82
L2-B	40.8	3.0	525.0	3.210	80.0	215.0	0.175	0.376	254.50	52.68	201.83	0.26	1.26	1.74
L3-B	40.8	3.0	525.0	4.760	80.0	215.0	0.175	0.376	316.50	50.70	265.80	0.19	1.19	3.98
L4-B	30.5	3.0	550.0	0.930	140.0	215.0	0.160	0.344	91.50	21.19	70.31	0.30	1.30	-
L5-B	30.5	3.0	550.0	1.860	140.0	215.0	0.160	0.344	103.00	21.19	81.81	0.26	1.26	-
L6-B	30.5	3.0	550.0	2.790	140.0	215.0	0.160	0.344	127.00	19.68	107.32	0.18	1.18	-
S1-B	40.8	2.0	525.0	2.240	77.0	215.0	0.183	0.393	348.50	56.46	292.04	0.19	1.19	2.95
S2-B	40.8	3.0	525.0	2.240	77.0	215.0	0.183	0.393	221.50	56.46	165.04	0.34	1.34	2.29
S3-B	40.8	4.0	525.0	2.240	77.0	215.0	0.183	0.393	231.00	56.46	174.54	0.32	1.32	-
S4-B	30.5	3.0	550.0	1.400	140.0	215.0	0.160	0.344	101.50	21.19	80.31	0.26	1.26	-
S5-B	30.5	4.0	550.0	1.400	140.0	215.0	0.160	0.344	92.50	21.19	71.31	0.30	1.30	-
S6-B	30.5	5.0	550.0	1.400	140.0	215.0	0.160	0.344	82.50	21.19	61.31	0.35	1.35	-
C1-B	19.7	3.0	520.0	2.240	110.0	215.0	0.128	0.275	204.00	39.49	164.51	0.24	1.24	1.80
														Total 150 EA

shear contribution of the stirrups (V_s) from the observed shear strength of the specimens (V_n). In the case of PSC members, the angle of inclined compressive diagonals is smaller than that of RC members due to the effect of prestress (Collins and Mitchell 1991, Nawy 2006). Thus, for a more reasonable evaluation, the reserved shear strengths ($1+\alpha$) of PSC members were estimated for the two different cases with 35 and 45 degree angles of inclined compressive struts, respectively. The RC and PSC specimens with relatively small amount of shear reinforcement were extracted from database reported in the previous studies (Kuchma and Kim 2001, Reineck *et al.* 2003, Kuchma *et al.* 2005), and their detailed information and estimated reserved shear strengths ($1+\alpha$) are shown in Table 2 and Table 3, respectively. The RC and PSC specimens collected in this study had the $\rho_v f_{yt}$ values under 1.6 MPa and 1.3 MPa, respectively. As shown in Fig. 4(a), the reserved shear strengths of the PSC members estimated with assumption of 35° and 45° angle of inclined cracks were relatively smaller than those of the RC members with the identical concrete compressive

Table 3 Summary of properties of PSC specimens lightly reinforced in shear

Beam Name	f_c' (MPa)	a/d	Longitudinal Reinforcement		Prestressing Reinforcement		Transverse Reinforcement			Reserved shear strength	Reserved shear deformation
			f_y (MPa)	ρ_l (%)	ρ_p (%)	f_{se} (MPa)	s (mm)	f_{yt} (MPa)	ρ_v (%)		
Cederwall *et al*. (1974)											
824-1B	41.7	2.5	637.4	0.722	303.7	0.722	200.0	495.2	0.214	1.69	-
824-2B	25.7	2.5	637.4	0.731	551.1	0.731	200.0	519.8	0.216	1.74	-
803-2S	28.6	2.6	882.6	0.371	455.6	0.743	200.0	235.4	0.218	1.42	-
803-1S	24.6	2.6	882.6	0.371	407.9	0.743	200.0	235.4	0.218	1.35	-
842-6	46.1	2.5	637.4	1.462	529.4	0.731	200.0	529.6	0.216	1.63	-
842-7B	35.7	2.6	637.4	1.468	555.4	0.734	200.0	529.6	0.216	1.63	-
842-10	50.8	2.6	637.4	1.468	520.7	0.734	200.0	353.0	0.216	1.45	-
842-11	50.5	4.2	637.4	1.462	520.7	0.731	200.0	353.0	0.216	1.58	-
842-12	53.5	1.7	637.4	1.462	473	0.731	200.0	353.0	0.216	1.21	-
842-13	53.5	3.4	637.4	1.439	473	0.719	200.0	353.0	0.214	1.38	-
842-14	40.9	2.5	637.4	1.462	182.2	0.731	200.0	529.6	0.216	1.86	-
842-16	51.6	2.6	637.4	1.481	312.4	0.74	200.0	529.6	0.216	1.46	-
Elzanaty *et al*. (1986)											
CW17	69.7	3.8	434.5	1.155	1086.1	3.014	254.0	434.5	0.300	1.32	-
CI17	69.7	5.8	434.5	1.100	1084.5	2.583	203.2	434.5	0.250	1.30	-
Moayer and Regan (1974)											
P8	42.7	3.6	641.4	0.980	1109.4	0.729	152.4	310.3	0.210	1.27	-
P9	40.4	5.5	641.4	0.980	1109.4	0.729	228.6	310.3	0.140	1.27	-
P13	39.4	3.5	641.4	0.950	1139.4	0.351	152.4	310.3	0.210	1.39	-
P14	44.1	5.3	641.4	0.950	1139.4	0.351	228.6	310.3	0.140	1.37	-
P18	44.5	3.7	641.4	0.500	1109.4	0.736	152.4	310.3	0.210	1.31	-
P19	45.4	5.6	641.4	0.500	1109.4	0.736	228.6	310.3	0.140	1.30	-
P27	45.4	5.6	641.4	0.500	1109.4	0.736	152.4	255.2	0.280	1.56	-
P29	46.6	5.5	641.4	0.980	1109.4	0.729	152.4	255.2	0.280	1.45	-
Bennett and Debaiky (1974)											
NL-6-240	41.2	3.0	410.0	2.064	1485.0	1.523	240.0	280.0	0.256	1.23	-
NM-6-240	38.5	3.0	410.0	2.064	1486.0	1.523	240.0	418.0	0.235	1.32	-
NH-6-240	35.4	3.0	410.0	2.064	1487.0	1.523	240.0	545.0	0.215	1.40	-
PL-6-240	44.2	3.0	410.0	2.064	1500.0	1.523	240.0	280.0	0.256	1.21	-
PM-6-240	43.1	3.0	410.0	2.064	1501.0	1.523	240.0	418.0	0.235	1.34	-
PH-6-240	42.1	3.0	410.0	2.064	1502.0	1.523	240.0	545.0	0.215	1.46	-
CL-6-240	56.5	3.0	410.0	2.064	1509.0	1.523	240.0	280.0	0.256	1.18	-
CM-6-240	57.0	3.0	410.0	2.064	1510.0	1.523	240.0	418.0	0.235	1.27	-
CH-6-240	57.8	3.0	410.0	2.064	1511.0	1.523	240.0	545.0	0.215	1.33	-
Durrani and Robertson (1987)											
3	46.1	3.5	0.0	0.000	958.6	0.956	152.4	503.4	0.139	1.19	-
4	44.1	3.5	0.0	0.000	1062.1	0.956	152.4	503.4	0.139	1.17	-
5	44.6	3.5	0.0	0.000	1055.2	0.956	152.4	443.8	0.183	1.20	-
6	41.9	3.5	0.0	0.000	1034.5	0.956	304.8	503.4	0.069	1.08	-
8	39.4	3.5	0.0	0.000	1055.2	0.956	152.4	389.2	0.222	1.21	-
9	41.8	3.5	0.0	0.000	1048.3	0.956	152.4	518.3	0.222	1.30	-
10	42.0	3.5	0.0	0.000	1048.3	0.956	152.4	389.2	0.222	1.21	-
11	41.8	3.5	0.0	0.000	1041.4	0.956	152.4	518.3	0.222	1.29	-
12	41.6	3.5	0.0	0.000	1082.8	0.956	152.4	347	0.078	1.06	-
13	41.3	3.5	0.0	0.000	1048.3	0.956	152.4	347	0.078	1.07	-
MacGregor (1958)											
M10	27.6	3.6	0.0	0.000	779.3	0.812	127.0	248.3	0.276	1.86	1.47
M11	29.0	3.6	0.0	0.000	889.7	0.812	127.0	248.3	0.276	1.52	2.90
Kaufuman and Ramirez (1988)											
I-2	57.5	2.4	0.0	0.000	1207.4	0.873	710.5	338.6	0.238	1.21	5.87
I-3	57.7	2.4	0.0	0.000	1245.1	0.873	285.1	294.5	0.327	1.44	2.74
I-4	57.7	2.4	0.0	0.000	1266.8	0.873	710.5	338.6	0.238	1.30	2.87
II-1	62.7	2.5	0.0	0.000	1237.2	1.002	508.0	338.6	0.333	1.50	2.50

Table 3 Continued

					MacGregor (1960)						
BV.14.30	27.7	3.6	0.0	0.000	848.3	0.812	127.0	253.8	0.276	1.53	2.49
BV.14.32	26.2	3.6	0.0	0.000	772.4	0.838	114.3	253.8	0.319	1.59	2.49
BW.14.34	24.6	3.6	0.0	0.000	845.5	0.809	266.7	234.5	0.128	1.16	3.00
BW.14.38	21.4	3.6	0.0	0.000	827.6	0.811	63.5	300.0	0.193	1.37	2.74
BW.14.58	21.8	3.6	0.0	0.000	754.5	1.262	127.0	296.6	0.196	1.30	1.61
BW.14.60	20.9	3.6	0.0	0.000	757.2	1.269	127.0	296.6	0.199	1.32	1.97
CW.14.17	21.7	3.4	0.0	0.000	868.3	0.655	127.0	300	0.164	1.31	5.75
CW.14.23	18.6	3.4	0.0	0.000	867.6	0.660	127.0	253.8	0.237	1.40	1.57
					Laskar *et al.* (2010)						
B1-North	72.4	1.6	0.0	0.000	1256.9	1.364	254.0	413.8	0.170	1.12	1.43
B1-South	72.4	1.6	0.0	0.000	1256.9	1.364	254.0	413.8	0.170	1.13	1.46
B4-South	71.0	4.3	0.0	0.000	1256.9	1.364	254.0	413.8	0.170	1.25	4.98
					Hernandez (1958)						
G5	22.3	3.6	358.3	0.401	832.3	1.417	63.5	244.6	0.140	1.05	3.51
G6	20.7	3.6	358.3	0.397	821.3	1.375	63.5	292.8	0.095	1.05	2.56
G7	32.1	3.6	358.3	0.400	837.8	1.405	63.5	231.5	0.193	1.06	3.94
G10	17.5	3.6	358.3	0.396	818.5	1.413	127.0	231.5	0.194	1.07	3.05
G13	21.6	3.4	358.3	0.192	867.5	0.660	127.0	292.8	0.048	1.04	3.51
G14	21.4	3.6	358.3	0.398	826.8	0.817	63.5	292.8	0.095	1.07	2.60
G20	16.5	3.5	358.3	0.397	837.1	1.371	127.0	211.5	0.261	1.09	4.00
G21	18.5	3.4	358.3	0.192	866.8	0.664	127.0	244.6	0.069	1.04	2.27
G28	26.7	3.6	358.3	0.402	804.8	0.810	254.0	270.8	0.147	1.10	5.00
G29	29.8	2.8	358.3	0.402	816.5	1.387	63.5	244.6	0.277	1.08	10.00
					Teoh *et al*. (2002)						
A3-8	88.9	2.7	521.0	2.990	602.8	0.603	315.0	365.5	0.120	1.21	2.60
A6-8	93.9	2.7	521.0	2.990	602.8	0.603	315.0	365.5	0.120	1.18	2.02
B3-8	84.3	2.7	521.0	2.990	602.8	0.603	215.0	365.5	0.175	1.39	1.81
B6-4	42.9	2.7	521.0	2.990	602.8	0.603	325.0	365.5	0.115	1.23	1.47
B6-8	92.1	2.7	521.0	2.990	602.8	0.603	215.0	365.5	0.175	1.23	2.95
B6-12	99.9	2.7	521.0	2.990	602.8	0.603	175.0	365.5	0.216	1.35	3.23
					Mattock and Kaar (1961)						
S9	44.9	1.0	0.0	0.000	1300.2	1.635	190.5	340.4	0.380	1.26	2.61
S10	43.2	2.0	0.0	0.000	1300.2	1.635	190.5	340.4	0.380	1.36	4.21
S11	43.2	3.3	0.0	0.000	1300.2	1.635	190.5	340.4	0.380	1.46	3.61
S12	45.4	4.5	0.0	0.000	1300.2	1.635	190.5	340.4	0.380	1.53	-
S21	46.3	2.0	0.0	0.000	1300.2	1.635	190.5	340.4	0.380	1.37	-
											Total 79 EA

strengths (f_c'). As shown in Fig. 4(b), the reserved shear strengths of the PSC members were also smaller than those of the RC members with the identical amount of shear reinforcement ($\rho_v\ f_{yt}$). These analytical investigations shown in Fig. 4 indicate that the greater amount of shear reinforcement should be required for the PSC members than the RC members to achieve the same margin of safety (or the same reserved shear strength). Such a trend of the low reserved strengths of the PSC members compared to the RC members was also pointed out by Teoh *et al.* (2002) based on their shear test results, and they emphasized that the sufficient tied arch-action cannot be developed right after the shear cracking for the cases of PSC members with small amount of shear reinforcement. This is because the shear cracking strengths (V_{cr}) of the PSC members were very close to their shear-compression capacity. Consequently, the reserved shear strength factors (α) of the PSC members are lower than those of the RC members, and thus, greater amount of minimum shear reinforcement would be required for the PSC members than the RC members. Based on such test observations by Ozcebe *et al.* (1999), Teoh *et al.* (2002) proposed the minimum shear reinforcement ratio for PSC members ($\rho_{v,\min}$), which was determined in order that the shear contribution of stirrups (V_s) should be at least 30% larger than the concrete contribution estimated by ACI318 code model, as follows

$$\rho_{v,\min} = \frac{0.35 v_{c,ACI}}{f_{yt}} \tag{11}$$

where, $v_{c,ACI}$ is the shear contribution provided by concrete in PSC members according to ACI318-11 (ACI Committee 318 2011). It is noted that minimum shear reinforcement requirement proposed by Teoh et al. shown in Fig. 2(b) were estimated by the Eq. (11-9) in ACI318-11(ACI Committee 318 2011), which is as follows

$$v_{v,ACI} = 0.05\lambda\sqrt{f_c'} + 4.8V_u d_p / M_u \tag{12}$$

As shown in Fig. 2(b), Eq. (11) proposed by Teoh *et al.* (2002) results in about 2.3 times larger amount of shear reinforcement than ACI318-11(ACI Committee 318 2011) (Eq. (1)) when *a/d* is 3.0, and about 3.3 times larger when a/d is 2.0. Similarly, Avendaño and Bayrak (2010, 2011) proposed the minimum shear reinforcement ratio for PSC members by modifying the multiplier in Teoh *et al*.'s model from 0.35 to 0.25, based on their shear database, as follows

$$\rho_{v,\min} = 0.25\frac{v_{c,ACI}}{f_{yt}} \tag{13}$$

where, $v_{c,ACI}$ is, as in Teoh *et al.* (2002) the smaller of the web-shear cracking strength and the flexure-shear strength estimated by ACI318-11. The required amount of shear reinforcement determined by this equation is also greater than that estimated by ACI318-11.

Laskar *et al.* (2010) also argued that Eq. (2) provided very unsafe design results, and to ensure sufficient ductility (or shear deformation capacity) in the PSC members with a shear-span ratio (*a/d*) ranging from 2.0 to 4.0, they proposed the following equation

$$\rho_{v,\min} = \frac{0.124\sqrt{f_c'}}{f_{yt}} \tag{14}$$

As shown in Fig. 2(b), Eq. (14) mostly, say when $a/d \geq 3.0$, requires larger amount of minimum shear reinforcement than Teoh *et al.* (2002)'s proposal.

AIJ-91 design standards for prestressed concrete structures in Japan (Architectural Institute of Japan 1991, 1998) have recommended the constant minimum shear reinforcement ratio for both RC and PSC members ($\rho_{v,\min}$), as follows

$$\rho_{v,\min} = 0.002 \tag{15}$$

This is very simple to be applied to practical design, but it provides very conservative results for RC members (Lee and Kim 2008, Lee and Yoon 2003). Also, as shown in Fig. 2(a), it does not reflect the effect of the magnitude of prestress and the concrete compressive strength (f_c') in PSC members at all.

In MC-90 (Comite Euro-International du Beton 1990), the minimum shear reinforcement ratio ($\rho_{v,\min}$) for both RC and PSC members is presented, as follows

$$\rho_{v,\min} = \frac{0.2}{f_{yt}} f_{ctm} \tag{16}$$

where, f_{ctm} is the mean tensile strength of concrete, which is $1.4(f_c'/10)^{2/3}$. As shown in Figs. 2(a) and (b), MC-90(Comite Euro-International du Beton 1990) requires much larger amount of minimum shear reinforcement than ACI318-11 (ACI Committee 318 2011), and such a difference would be even bigger when the Eq. (2) governs. AASHTO-LRFD (American Association of State Highway and Transportation Officials 2004) specification requires the minimum shear reinforcement ratio ($\rho_{v,\min}$) for RC and PSC members, about 30% larger than ACI318-11(ACI Committee 318 2011), as follows

$$\rho_{v,\min} = \frac{0.083\sqrt{f_c'}}{f_{yt}} \geq \frac{0.35}{f_{yt}} \tag{17}$$

AASHTO-LRFD (American Association of State Highway and Transportation Officials 2004) requires lower amount of minimum shear reinforcement than AIJ (Architectural Institute of Japan 1998) and Teoh *et al.* (2002) for normal-strength concrete members, but it requires a much higher level of minimum shear reinforcement ratio for high-strength concrete members.

Based on the discussions in above, it is clear that reducing the amount of the minimum shear reinforcement for PSC members compared to that for RC members is unreasonable, considering the basic concept of the minimum shear reinforcement ratio and the reserved shear strength. Also, Eq. (2), the minimum shear reinforcement for PSC members presented in ACI318-11, has been used for almost 30 years without clear reasoning or sufficient verification. Additionally, according to the recent test results reported by Kuchma *et al.* (2008a, 2008b) and Hawkins *et al.* (2005), a large shear cracking width were observed right after diagonal tension cracking at the web concrete of the full-scaled PSC test specimen that was designed with a similar level of the minimum shear reinforcement ratio required in ACI318 (Eq. (1) in this paper), which showed that the large PSC members lightly reinforced in shear based on the current design code (ACI Committee 318 2011) would lead to concerns on serviceability. Therefore, in designing PSC members, determining the minimum amount of shear reinforcement using the smaller value of Eqs. (1) and (2) would not guarantee sufficient safety margin or serviceability. Therefore, more studies are still necessary for yielding a reasonable minimum shear reinforcement ratio in PSC members.

3. Formulation of minimum shear reinforcement ratio for prestressed concrete members

As mentioned earlier, the minimum shear reinforcement required for RC members specified in the current ACI318-11 was revised in 2005 to achieve sufficient reserved shear strength (ACI Committee 318 2005). To attain the level of reserved shear strength identical to RC members even for the PSC members lightly reinforced in shear, the shear cracking strength of the PSC members that increased significantly by the effective prestress compared to the RC members should be considered. This can be simply done by multiplying the ratio between web shear cracking strength of PSC members and that of RC members to Eq. (1). On this basis, the minimum amount of the shear reinforcement ($\rho_{v,\min}$) can be expressed, as follows

$$\begin{aligned} \rho_{v,\min} f_{yt} &= 0.0625\sqrt{f_c'}\left(\frac{0.29\sqrt{f_c'}+0.3 f_{pc}}{0.17\sqrt{f_c'}}\right) \\ &= 0.0625\sqrt{f_c'}\left(1.7+\frac{0.3 f_{pc}}{0.17\sqrt{f_c'}}\right) \end{aligned} \tag{18}$$

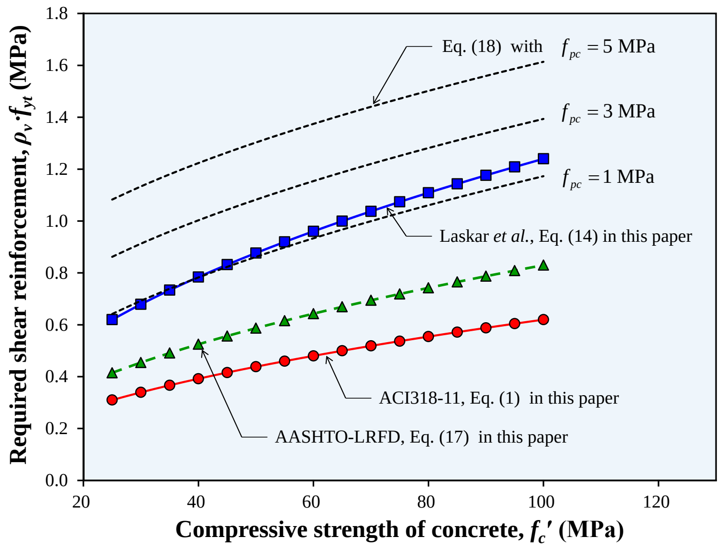
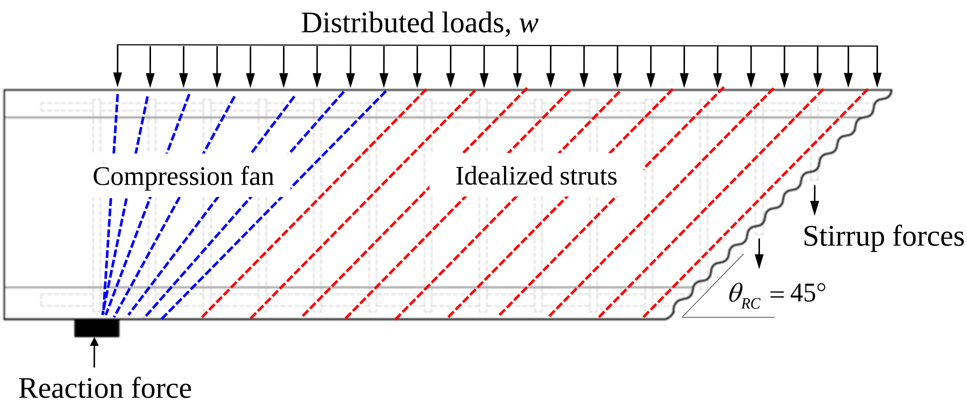
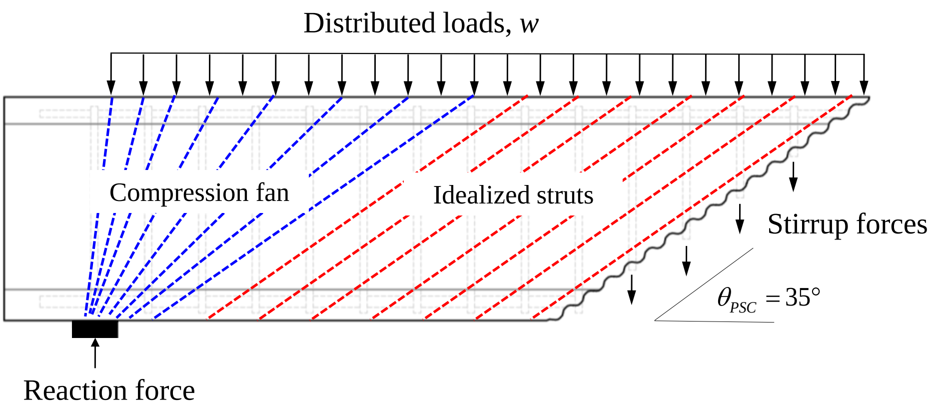


Fig. 5 Comparisons of Eq. (18) and code requirements



(a) Reinforced concrete member



(b) Prestressed concrete member

Fig. 6 Cracking behavior of reinforced and prestressed concrete members

In Eq. (18), the web-shear cracking strength of ACI318-11 (v_{cw}) (Eq. (11-12) in ACI318-11) was used for the shear cracking strength of PSC members to simplify derivation process, and this is reasonable assumption because typical PSC members have relatively thin concrete web. Fig. 5 shows the comparisons of minimum shear reinforcement ratio estimated by Eq. (18) with those by Laskar *et al.* (2010) (Eq. (14)), AASHTO-LRFD (2004) (Eq. (17)) and the ACI318-11 (2011) (Eq. (1)). As shown in Fig. 5, Eq. (18) differs greatly according to the magnitude of the effective prestress (f_{pc}) and requires an irrationally large amount of minimum shear reinforcement at the high level of f_{pc}. Also, as shown in Fig. 6, Eq. (18) does not reflect the marked difference of the angles of the critical shear resistance plane between RC and PSC members, which determine the number of effective stirrups across the cracks. Therefore, to reflect the difference of the shear cracking angles between the RC and PSC members, Eq. (18) can be modified, as follows

$$\rho_{v,\min} f_{yt} = 0.0625\sqrt{f_c'}\left(1.7 + \frac{0.3 f_{pc}}{0.17\sqrt{f_c'}}\right)\frac{\tan\theta_{PSC}}{\tan\theta_{RC}} \tag{19}$$

where, $\tan\theta_{PSC}$ and $\tan\theta_{RC}$ are the shear cracking angles of PSC members and RC members, respectively. Kuchma *et al.* (2005, 2008a, 2008b) and Hawkins *et al.* (2005) proposed the shear cracking angles of PSC members ($\tan\theta_{PSC}$) as $1/\left(1+1.14 f_{pc}/\sqrt{f_c'}\right)$, which was adopted in the simplified shear strength model of ASHTTO-LRFD (American Association of State Highway and Transportation Officials 2004). By adopting this, Eq. (19) can be re-expressed, as follows

$$\rho_{v,\min} f_{yt} = 0.0625\sqrt{f_c'}\left(\frac{0.286 + 0.3 f_{pc}/\sqrt{f_c'}}{0.17 + 0.19 f_{pc}/\sqrt{f_c'}}\right) \tag{20}$$

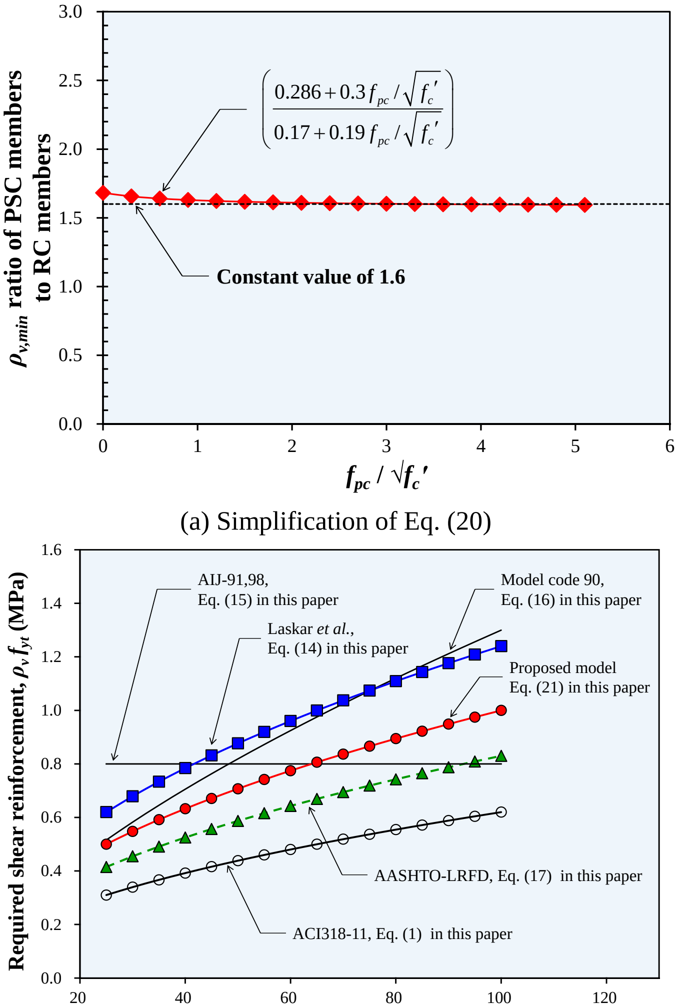
The term in the parenthesis of Eq. (20) actually converges to 1.6, as shown in Fig. 7(a), which is because the effect of the concrete shear cracking ratio on the minimum shear reinforcement of PSC members is more significant than that of the crack inclination ratio. Thus, Eq. (20) can be simplified, as follows

$$\rho_{v,\min} f_{yt} = 0.1\sqrt{f_c'} \tag{21}$$

Fig. 7(b) shows the comparisons of the minimum shear reinforcement ratio estimated by the code equations and the ones presented in Fig. 5 with those estimated by Eq. (21) proposed in this study. The minimum shear reinforcement ratio proposed in this study is higher than the amount required in AASHTO-LRFD and ACI318-11 for RC members, and is lower than the amount proposed by Laskar *et al.* (2010) for PSC members. Also, the proposed Eq. (21) expresses the minimum shear reinforcement increased in PSC members in a very simple manner, compared to RC members. Furthermore, it can be expressed as a unified form for both RC and PSC members in the same way as in the ACI318-11, as follows

$$\rho_{v,\min} = \alpha_p \frac{0.062\sqrt{f_c'}}{f_{yt}} \geq \frac{0.35}{f_{yt}} \tag{22}$$

where, α_p is the coefficient to account for the member type, which is 1.0 for RC members and 1.6 for PSC members.



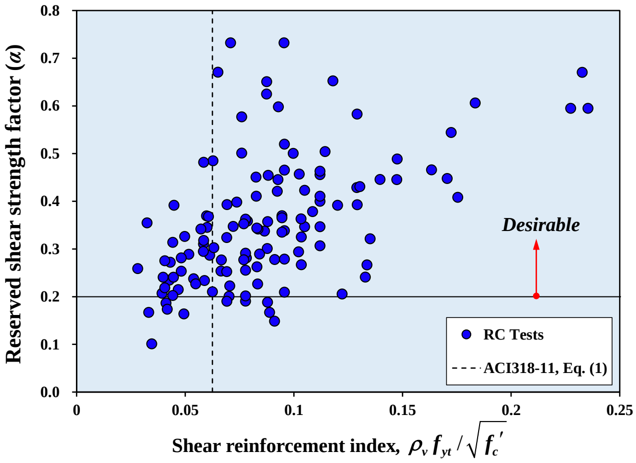
(a) Simplification of Eq. (20)

(b) Simplified Equation

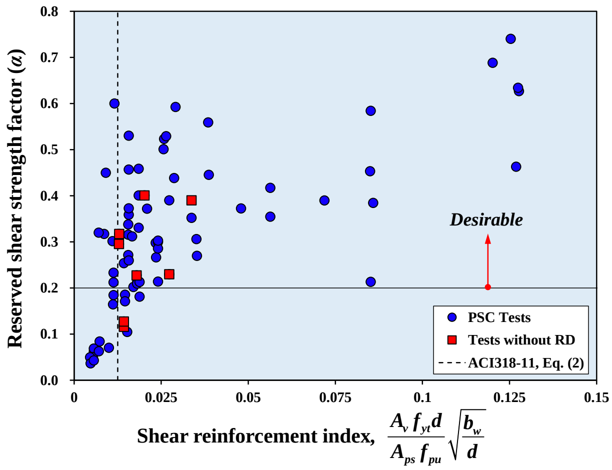
Fig. 7 Minimum shear reinforcement ratio and its simplification

4. Verification of proposed minimum shear reinforcement ratio

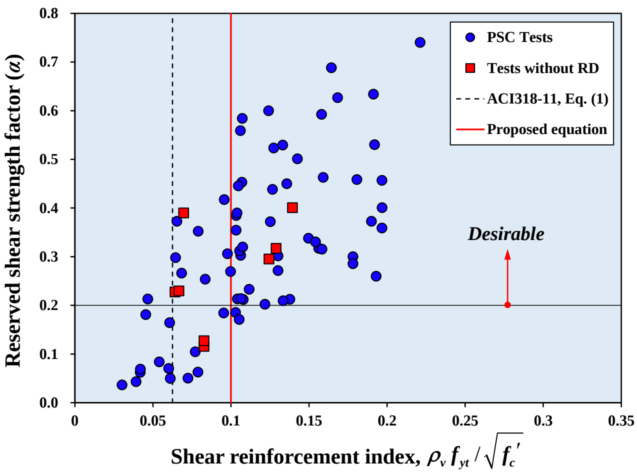
Fig. 8 shows the comparisons of Eq. (21) proposed in this study and the ACI equations for the minimum shear reinforcement ratio with the test results of the PSC specimens presented in Table 3. In this figure, the reserved shear strength (V_u/V_{cr}) were calculated by using the actual test results when the shear cracking strength (V_{cr}) and ultimate shear strength (V_u) were reported in original literature. If there is no information on the load-displacement relationship or shear cracking strength, however, the reserved shear strengths of these specimens were estimated by Eq. (10), in which the cracking angle was assumed to be 35 degree as shown in Fig. 4. Also, the specimens failed in an extremely brittle manner whose reserved shear deformation (RD), which is defined as the ratio of the deflection at the ultimate (Δ_u) to that at shear cracking (Δ_{cr}), is smaller than 2.0



(a) Reserved shear strength of reinforced concrete members determined by ACI318-11



(b) Evaluation of Eq. (2)

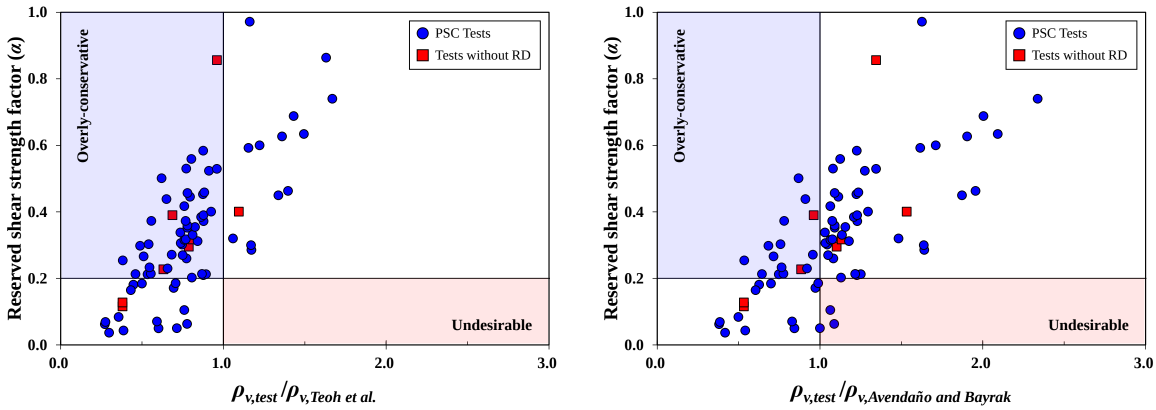


(c) Proposed equation

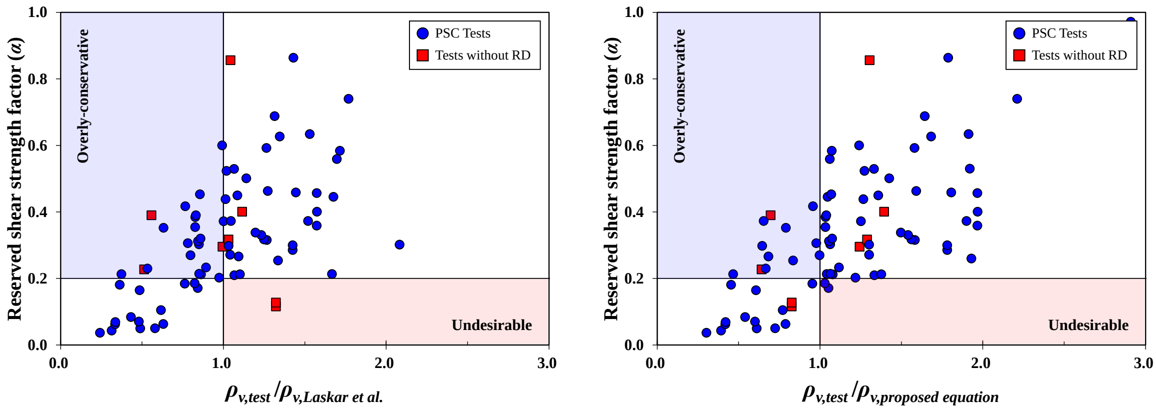
Fig. 8 Evaluation of code equations and proposed equation were rectangular-marked in Fig. 8.

Fig. 8(a) compares the reserved shear strength factors (α) of the RC specimens and the minimum shear reinforcement ratio required in ACI318-11, which can be useful to determine the appropriate reserved shear strength level of PSC members comparable to those of RC members. It can be seen that the RC members with greater amount of shear reinforcement than the minimum shear reinforcement ratio presented in ACI318-11 have at least about 20 % of reserved shear strength. On this basis, it would be proper to secure at least 20 % of the reserved shear strength for PSC member as well.

Fig. 8(b) compares the reserved shear strength factors (α) of the PSC specimens and the minimum shear reinforcement ratio required in ACI318-11, which is presented in Eq. (2) of this paper. As pointed out by Laskar *et al.* (2010) and Avendano and Bayrak (2010, 2011), some prestressed concrete specimens with the shear reinforcement more than that estimated by Eq. (2) did not have sufficient reserved shear strength. In particular, Eq. (2) did not appropriately assess the shear reinforcement ratio of the specimens failed in an extremely brittle manner without sufficient reserved deformation (RD) capacity. Fig. 8(c) shows the minimum shear reinforcement equation (Eq. (1)) specified in ACI318-11 and the equation proposed in this study (Eqs. (21) or



(a) Teoh *et al*. (2002) (b) Avendaño and Bayrak (2010, 2011)



(c) Laskar *et al*. (2010) (d) Proposed equation

Fig. 9 Comparison of proposed equation and other approaches

(22)) with test results. Eq. (1) presented in ACI318-11 provided better estimations on the minimum shear reinforcement than Eq. (2) as more data belong to the safe side when Eq. (1) is applied. But, it still shows that the reserved shear strengths (V_u/V_{cr}) for the PSC members are lower than those of the RC members. After all, it implies that the minimum shear reinforcement calculated by the two equations specified in ACI318-11, i.e., Eqs. (1) and (2), do not provide sufficient reserved shear strength. On the other hand, as shown in Fig. 8(c), the specimens reinforced by the shear reinforcement more than the amount proposed by Eqs. (21) or (22) in this study showed the reserved shear strength at least 20 %, which means that a proper margin of safety can be guaranteed when proposed equation is applied.

Fig. 9 shows the comparisons of the reserved shear strength factors (α) for the PSC specimens estimated by Eqs. (11), (13), (14) that were proposed by Teoh *et al*., Avendaño and Bayrak, Laskar *et al*., respectively, and Eq. (21) proposed in this study. The data points included in the shaded area on the right-bottom side in the graph means that their minimum shear reinforcement ratio were unsafely estimated, which is undesirable. On the other hand, the data points included in the shaded area on the left-top side in the graph means that their minimum shear reinforcement ratio were estimated in an overly-conservative manner. Eq. (11) proposed by Teoh *et al*., as shown in Fig. 9(a), did not give any unsafe estimation on the minimum shear reinforcement ratio of the PSC members, but provided overly-conservative results for most cases, which means that the minimum shear reinforcement ratio required by Eq. (11) can be reduced. Compared to this, Eq. (13) proposed by Avendaño and Bayrak, as shown in Fig. 9(b), provided better results on the minimum shear reinforcement ratio of the PSC members, but there were some unsafe results. Eq. (14) proposed by Laskar *et al*., as shown in Fig. 9(c), provided a similar trend with Eq. (13), but there were unsafe data that had no reserved shear deformation capacity, which is of course undesirable. Compared to the approaches mentioned above, the proposed equation, as shown in Fig. 9(d), provided reasonable estimations on the minimum shear reinforcement ratio of the PSC members, and there was no unsafe estimation on the test specimens with insufficient reserved shear deformation capacity.

5. Conclusions

The minimum shear reinforcement requirements differ substantially between current design codes. In particular, the basis of Eq. (2) presented in ACI318 code for the minimum shear reinforcement ratio of PSC members is not clear. Therefore, based on the ratio of the shear contribution of concrete between PSC and RC members and the concept of effective stirrups with respect to the difference of cracking angles between PSC and RC members, this study proposed a simple minimum shear reinforcement ratio that is suitable for PSC members, and compared it to the test results collected from previous researches. From this study, the following conclusions can be drawn:

1. The design standards of different countries and the proposals by researchers have presented very different amount of minimum shear reinforcement ratios for PSC members, and showed up to three times difference one another.

2. The assessment results of the reserved shear strength using the shear test data of the RC and PSC members collected from previous studies showed that the reserved shear strengths of the PSC members were lower than those of the RC members, whose compressive strength of concrete (f_c') and shear reinforcement ratio (ρ_v) were the same. Such a result indicates that a greater amount of

minimum shear reinforcement is required for PSC members compared to RC members.

3. ACI318-11 provided much lower reserved shear strengths than the other design standards examined in this study.

4. The minimum shear reinforcement ratio required for PSC members is about 1.6 times that for RC members because of the higher shear cracking strength of PSC members.

5. The minimum shear reinforcement ratio proposed in this study is very simple and can be applied to both RC and PSC members. The specimens with shear reinforcement greater than the amount calculated by the proposed equation showed sufficient reserved shear strengths and reserved deformation capacities.

6. Although the minimum shear reinforcement ratio proposed in this study can be applied to both RC and PSC members, there is no smooth transition between the minimum shear reinforcement ratios required for RC members and PSC members. As it is not desirable, this issue should be addressed in future study.

Acknowledgments

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (NRF-2013R1A1A1A05007001).

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CC

Notations

α = reserved shear strength factor
α_p = coefficient to account for the member type
Δ_{cr} = deflection at shear cracking
Δ_u = deflection at failure
θ = angle of inclination of compressive stress(or strut)
ρ_l = reinforcement ratio of non-prestressed steel in longitudinal direction
ρ_p = reinforcement ratio of prestressed steel in longitudinal direction
ρ_v = reinforcement ratio in transverse direction
ρ_{vb} = shear reinforcement ratio at yielding of shear reinforcements with concrete crushing
$\rho_{v,\max}$ = maximum shear reinforcement ratio
$\rho_{v,\min}$ = minimum shear reinforcement ratio
A_{ps} = area of prestressing tendons
a/d = shear-span ratio
b_w = web width
d = effective depth of member
f_c' = specified compressive strength of concrete
f_{ctm} = tensile strength specified in CEB-FIP MODEL CODE 90
$_{pc}$ = compressive stress in concrete at centroid of cross section due to prestress forces
f_{pe} = effective prestress forces
f_{pu} = ultimate tensile strength of prestressing steel
f_{yt} = yield stress of shear reinforcement
k_1 = a measure of the reduction in tensile strength of concrete
k_2 = ratio of depth of the resultant compressive force to depth of neutral axis
M_u = ultimate design flexural moment
$S_{\max}$ = the maximum spacing of shear reinforcement
$v_{c,ACI}$ = shear strength provided by concrete for nonprestressed/prestress members in ACI building code (ACI318-11)
v_c = concrete shear stress of RC/PSC member at the ultimate state
v_{cr} = shear cracking stress of RC/PSC member
V_c = nominal shear strength provided by concrete
V_{cr} = shear cracking strength of RC/PSC member
V_n = nominal shear strength
V_s = shear strength provided by shear reinforcement
$V_{s,35}$ = shear contribution of shear reinforcement to shear strength of member with 35° crack angle
$V_{s,45}$ = shear contribution of shear reinforcement to shear strength of member with 45° crack angle
V_u = ultimate shear strength