Structural Engineering and Mechanics, *Vol. 56, No. 1 (2015) 107-121*
DOI: http://dx.doi.org/10.12989/sem.2015.56.1.107

Sizing, shape and topology optimization of trusses with energy approach

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(Received August 18, 2014, Revised October 2, 2015, Accepted October 5, 2015)

Abstract. The main objective of this research is to present the procedures of combining topology, shape & sizing optimization for truss structure by employing strain energy as objective function under the constraints of volume fractions which yield more general solution than that of total weight approach. Genetic Algorithm (GA) is used as searching engine for the convergence solution. A number of algorithms from previous research are used for evaluating the feasibility and stability of candidate to accelerate convergence and reduce the computational effort. It is followed by solving problem for topology & shape optimization and topology, shape & sizing optimization of truss structure to illustrate the feasibility of applying the objective function of strain energy throughout optimization stages.

Keywords: truss optimization; topology optimization; strain energy; Kinematic stability; genetic algorithm

1. Introduction

In recent decades, the field of structural optimization has received much attention from researchers. It is considered as an inverse procedure of mechanical law (Maute *et al.* 1999). Structural optimization is mainly categorized into sizing, shape and topology optimization. While sizing and shape optimization is to choose the most appropriate cross-sectional area and nodal locations yielding the optimal structure in term of minimizing the objective function, topology optimization relates eliminating of the unnecessary member from the ground structure without concerning element's cross-section area and node coordination. For truss structure optimization problem, it can be a pure topology optimization (Kawamura *et al.* 2002, Richardson *et al.* 2012, Hajela and Lee 1995, Ohsaki 1995, Ruiyi *et al.* 2009) or sizing and shape optimization (Wei *et al.* 2005) or even sizing, shape and topology simultaneously (Rajan 1995, Deb and Gulati 2001). Most of the researchers, however, have just employed total weight as objective function and stress, displacement, buckling or frequency of structure as constraints for the problem of truss optimization (Rajeev and Krishnamoorthy 1992, Hajela and Lee 1995, Jenkins 1991, Lin and Hajela 1992, Wang *et al.* 2004). In practical concept design procedure at which some constraints of stress, displacement or buckling have not indicated, the designers usually expect a number of

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

candidate designs based on initial requirements of boundary conditions (constrained node and external loads). The total weight optimization approach may not work well for this case since it always yields a unique solution.

In this study, the truss optimization problem will be defined by employing strain energy and volume fraction of the truss structure as objective function and constraint, respectively (Sigmund 2001). This approach yields more general solutions which are different with respect to each range of given volume fraction. As consequence, the designer can choose one solution among them with desired volume fraction as the concept design. In the procedure of optimizing truss system, the evaluation of candidate for feasibility and stability in GA process is al so applied. Follow that, unless the candidate is evaluated to be feasible and stable, it would be assigned a very low amount of GA's fitness function without proceeding with analyser. This will reduce the computational cost.

GA was developed by John Holland and his co-workers from 1960s (Holland 1995). This algorithm which was a search heuristic that mimicked the process of natural selection and chromosomal processing in nature genetics was inspired by Charles Darwin’s Theory of Evolution. GA generates the initial population whose each individual represents a candidate solution which, in this current study, corresponds a possible truss structure, and its chromosome consists of the information of topology, node coordinate and cross-sectional area. The fitness rating which reflects the objective function is used to evaluate the quality of individual. It determines the candidate design to be eliminated or selected as the parents for the next generation. From the chosen ones, the new set of population will be created using gene crossover and gene mutation operators. This process is repeated for many iterations in order to obtain the convergence optimal solution. For its advantages, GA has become a widely used tool and proved the effectiveness especially for discrete optimization problem such as truss optimization (Rajan 1995, Kawamura *et al.* 2002, Wei *et al.* 2005).

The outline of this study as follow: The evaluation of candidate topology is described in Section 2. It is followed by presenting the procedure of optimization of trusses in Section 3. Some numerical examples of the stages of pure topology & shape optimization and topology, shape & sizing optimization are conducted in Section 4. Finally, Section 5 shows the conclusions of this study.

2. Evaluation of candidate topology

This section presents an effective procedure which completely checks the feasibility and stability of a candidate topology before proceeding with analyzer. The proposed procedure is based on the assumption that all essential (geometry) and natural (force) boundary conditions would be fixed which means they must be existed in all qualified candidate. The truss shown in Fig. 1 is considered as ground structure to all demonstrations in this section.

2.1 Necessary conditions

A set of three necessary conditions which is used to assess the candidate topology will be presented in this subsection.

Once one of necessary condition is not satisfied, the candidate topology will be indicated as unfeasible or unstable system. If it is the case, this kind of structure will not be analyzed and immediately assigned a very low amount of fitness function when GA is used. The major

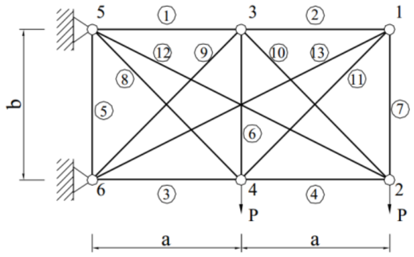

Fig. 1 Ground structure system

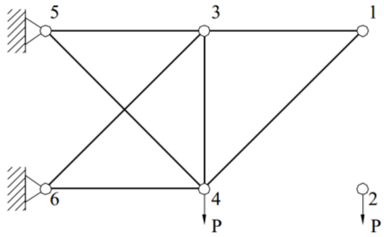

Fig. 2 Unfeasible topology due to necessary condition 1

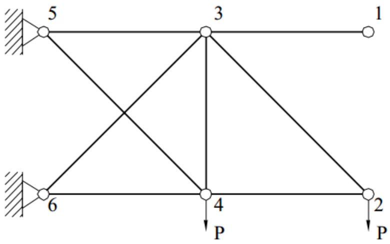

Fig. 3 Unfeasible topology due to necessary condition 2

advantages of these necessary conditions are to early point out an unfeasible or unstable candidate topology. This candidate will not be evaluated by sufficient conditions which take more to assess. As consequence, it saves time and computing effort and faster convergent solutions would be obtained.

If all three necessary conditions are satisfied, the sufficient condition will be employed to verify the feasibility and stability of the candidate topology.

2.1.1 The presence of load-applied node (necessary condition 1)

In this condition, the nodes at which external loads apply are checked for the presence. The candidate topology will be indicated as unfeasible system if one of load-applied node does not exist (Deb and Gulati 2001).

Although it assemblies a stable truss, the topology shown in Fig. 2, follow the necessary condition 1, is unfeasible because of the absence of the load-applied node number 2.

2.1.2 The number of elements connected to a node (necessary condition 2)

By employing this condition, all given nodes are checked by counting the number of elements connected to. The candidate topology will be indicated as unstable system if there are less than two elements, for plane truss structure, or less than three elements, for spatial truss structure, connected to specific unconstrained node.

The topology shown in Fig. 3 is unstable, due to the necessary condition 2, since there is just one element connected to unconstrained node number 1.

2.1.3 The relation between connectivity and constraints (necessary condition 3)

Being a stable structure, all degree of freedom of the system would be treated by the elements and constraints (Deb and Gulati 2001, Ghosh and Mallik 1998). Consequently, it can be drawn another necessary condition to check the stability of the candidate topology which is

$$s - m = b + c - dn \geq 0 \qquad (1)$$

Where b is the number of elements, c represents the number of constraints, dn refer to the number of degree of freedoms of the system which is the product of the number of degrees of freedom of each node and the total number of nodes, s is the degree of statically indeterminacy, m is the degree of kinematically indeterminacy.

The candidate topology will be indicated as unstable if its parameters violate the Eq. (1). Consider the system shown in Fig. 4, by employing the Eq. (1) with the set of parameter of b=6, c= 4 and dn=2×6=12, it is readily got the result of -2 which is smaller than 0. Therefore, the candidate topology is unstable due to the necessary condition 3.

Although the three necessary conditions covers almost all unfeasible or unstable case of the candidate topology, there still has some topology is truly unstable in spite of satisfying all three necessary conditions which can be seen in the Fig. 5. Since all the nodes at which external loads apply are available, it satisfies the necessary condition 1. It also does not violate the necessary condition 2 because all unconstrained nodes are connected by at least two separate elements and all constrained nodes are connected by at least one element. It can be seen to be satisfied the third

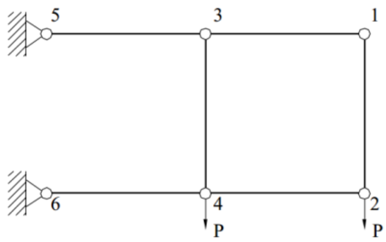

Fig. 4 Unfeasible topology due to necessary condition 3

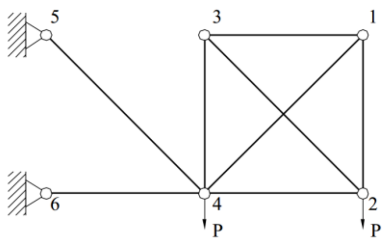

Fig. 5 Unstable topology in spite of satisfying all three necessary conditions

necessary condition since the candidate topology contains the set of value of b=8, c=4 and dn=12 which gives the result of 0 after substituting into the left hand term of the Eq. (1). As consequence, this candidate satisfies all three necessary conditions. Nevertheless, as can be seen, the candidate topology shown in Fig. 5 is not a stable truss.

For completely checking the feasibility and stability of the candidate topology when using GA, a sufficient condition which is based on Force Method needs to be developed.

2.2 Sufficient condition

By investigating equilibrium matrix of structural assembly, some problem of static, kinematic which can be indicate the kinematic stability status of candidate truss topology would be solve (Pellegrino 1993). This advantage of the equilibrium matrix's property is employed as a sufficient condition for the procedure of candidate evaluation.

In the force method, the equilibrium equations (Tran and Lee 2010) of the nodes in each direction of a general pin-jointed structure which includes truss structure can be stated as

$$\boldsymbol{A}\boldsymbol{f} = \boldsymbol{p} \tag{2}$$

Where $\boldsymbol{p}$ is the vector of external loads applied at the free nodes, and $\boldsymbol{A}$ is known as the equilibrium direction-cosine matrix that transforms the vector of member forces $\boldsymbol{f}$ of the system to the vector of external loads $\boldsymbol{p}$ of the free nodes, defined by

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{C}^T\boldsymbol{L}_x \\ \boldsymbol{C}^T\boldsymbol{L}_y \\ \boldsymbol{C}^T\boldsymbol{L}_z \end{pmatrix} \boldsymbol{L}^{-1} \tag{3}$$

Where $\boldsymbol{C}$ is connectivity matrix which describes the connectivity of the members to the nodes; $\boldsymbol{L}_x$, $\boldsymbol{L}_y$, $\boldsymbol{L}_z$, $\boldsymbol{L}$ are the diagonal square matrices of coordinate difference vector in *x*-, *y*-, *z*-directions and the member length vector, respectively (Tran and Lee 2010).

The degree of kinematic indeterminacy m, can be calculated by employing the left hand side term of the Eq. (1) where $s = b - rank(\boldsymbol{A})$ and $rank(\boldsymbol{A})$ is mathematical rank of $\boldsymbol{A}$ matrix. The structures whose degree of kinematic indeterminacy m is equal to 0 are kinematically determinate (Pellegrino 1993). Consequently, this condition of m could be sufficient condition for a candidate topology to be a stable truss.

3. Trusses optimization procedure

3.1 Terminology definition and expression

For the ease of understanding some terminologies which are used in the next section, this section will explain it in detail before going ahead with the problem formulation section. Some important terminologies can be expressed as follow

$\emptyset$ presents topology optimization variable which indicates the presence (value of 1) or absence (value of 0) of element from the ground structure; c and A refer to shape and sizing optimization variable which indicate the coordinate of nodes and cross-sectional area of elements, respectively.

V_0 is the material volume of ground structure with original input value of node coordinate and cross-sectional area; $V_x^T(\emptyset)$,$V_x^{TS}(\emptyset, c)$ and $V_x^{TSS}(\emptyset, c, A)$ present the material volume of truss candidate for updated pure topology, updated topology & shape and updated topology, shape & sizing, respectively; $V_f^T(\emptyset)$, $V_f^{TS}(\emptyset, c)$ and $V_f^{TSS}(\emptyset, c, A)$ are the volume fraction of truss candidate for updated pure topology, updated topology & shape and topology, updated shape & sizing structure, respectively.

f is upper bound volume fraction which is assigned by user receiving the value from 0 to 1.

Some expressions can be drawn as follow

$$V_f^T(\emptyset) = \frac{V_x^T(\emptyset)}{V_0} \tag{4}$$

$$V_f^{TS}(\emptyset, c) = \frac{V_x^{TS}(\emptyset, c)}{V_0} \tag{5}$$

$$V_f^{TSS}(\emptyset, c, A) = \frac{V_x^{TSS}(\emptyset, c, A)}{V_0} \tag{6}$$

3.2 Problem formulation

By employing the objective function of strain energy, the formulation for pure topology, topology & shape and topology, shape & sizing optimization of truss can be posed as follow

Minimize

$$\mathcal{U} = \frac{1}{2}\mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U} \tag{7}$$

Subjected to

$$V_f^T(\emptyset) \le f \tag{8}$$

or

$$\mathrm{V_f^{TS}}(\emptyset, \mathrm{c}) \le \mathrm{V_f^T}(\emptyset) \le \mathrm{f} \tag{9}$$

or

$$\mathrm{V_f^{TSS}}(\emptyset, \mathrm{c}, \mathrm{A}) \le \mathrm{V_f^T}(\emptyset) \le \mathrm{f} \tag{10}$$

Where $\mathcal{U}$ refers to the strain energy of the system, $\mathbf{U}$ presents the global displacement vector, $\mathbf{K}$ is global stiffness matrix; the constrained Equations (8), (9) and (10) are used for pure topology, topology & shape and topology, shape & sizing truss optimization problem, respectively.

By using the strain energy and volume fractions as objective function and constraints, we have accepted an assumption that the yield stress of the elements and the constraint of displacement of nodes are large enough for the availability of the truss structure under the external loads.

To be applied in GA process, the above constrained problem formulation should be transferred to unconstrained problem which can be expressed as

Minimize

$$L_1(\emptyset, c, \gamma) = \mathcal{U} + \gamma_1 (V_f^T - f)^2 \quad (11)$$

$$L_2(\emptyset, c, \gamma) = \mathcal{U} + \gamma_2 [(V_f^T - f)^2 + (V_f^{TS} - V_f^T)^2] \quad (12)$$

$$\mathrm{L}_3(\emptyset, \mathrm{c}, \mathrm{A}, \gamma) = \mathcal{U} + \gamma_3 [(\mathrm{V_f^T} - \mathrm{f})^2 + (\mathrm{V_f^{TSS}} - \mathrm{V_f^T})^2] \quad (13)$$

Where $\gamma_1, \gamma_2, \gamma_3$ is penalty parameters for optimization problems and L refers to the combination of objective function and penalty function; Eqs. (11), (12) and (13) are used for pure topology, topology & shape and topology, shape & sizing truss optimization problem, respectively.

In case of unfeasible or unstable candidate topology which had been assessed by candidate evaluation procedure, their fitness would be automatically assigned a very low value of fitness function without running the analyzer or computing the objective functions as well as concerning the penalty function. Otherwise, it is proceed with analyzing and assessing process. This process will significantly reduce the computational cost. The flowchart of main procedures in a generation is shown in Fig. 6.

4. Numerical examples & discussion

In this section, three different runs for 6-node truss with 13-bar ground structure optimization problem which shown in Fig. 7 will be executed. In the first run, the problem of pure topology

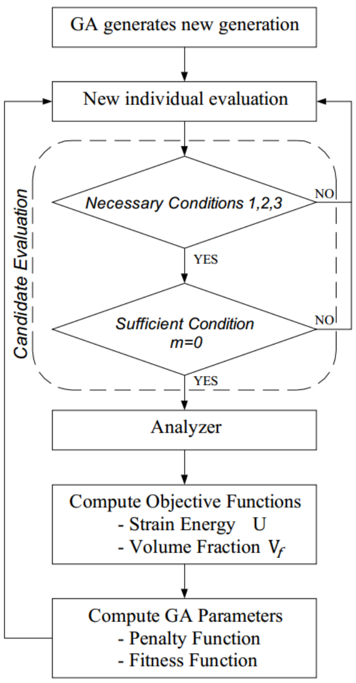

Fig. 6 The flowchart of main procedures in a generation

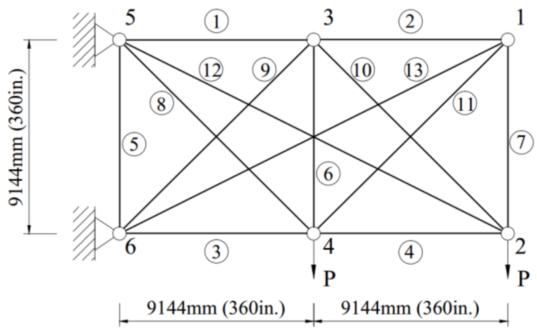

Fig. 7 13-bar truss ground structure

Table 1 Input data for 6-node truss topology optimization problem

Item	Value
Constant cross-sectional area	7419.34 mm^2 (11.5 in^2)
Young's modulus	68947.59 N/mm^2 (10^7 psi)
External load P	444822.162 N (10^5 pound)

optimization will be presented. After that, the problem of topology & shape optimization will be investigated. At last, we will consider the topology, shape & sizing optimization simultaneously. It is followed by a big picture showing the effectiveness of the presented problem definition and algorithms throughout three stages of truss optimization which are pure topology, topology & shape and topology, shape & sizing optimization.

4.1 Topology optimization

In this stage, the pure topology optimization of truss will be considered. The formulation of the problem is presented in the Eqs. (7), (8) and (11). The result plot shown in the Fig. 8 indicated that for different ranges of the upper bound volume fraction f, which is from 0 to 1, there had different solutions with chromosome string attached or even no solution. For the range of f which is smaller than 0.32, there was no solution due to the lack of feasible and stable candidate topology whose volume fraction V_f satisfied the constraint of f. It also could be seen, the unique solution which was connected to black dot (•) could be found in the ranges of f of [0.55,0.61], [0.61,0.66], [0.66,0.72], [0.72,0.77], [0.77,0.80] and [0.80,0.85]. There were two solutions with the same strain energy and volume fraction in the range of f of [0.39,0.42]. In the ranges of f of [0.32,0.39], [0.42,0.55], [0.85,0.94] and [0.94,1.00], there were various topology solutions which connected to black dot (•) and white triangle (Δ).

The reason for this phenomena of getting some different solutions with the same strain energy $\mathcal{U}$ within a range of f was those topology solutions connected to white triangle (Δ) contained some needless element which resulted in the structure with the same amount of strain energy $\mathcal{U}$ but different volume fraction V_f. In addition, those solutions got the same fitness function since they did not violate the constraint of upper bound of volume fraction f. As a result, one of them could be randomly chosen by GA as optimal topology solution. To avoid the various solution phenomena in some range of f, an additional condition of volume fraction V_f were employed. For

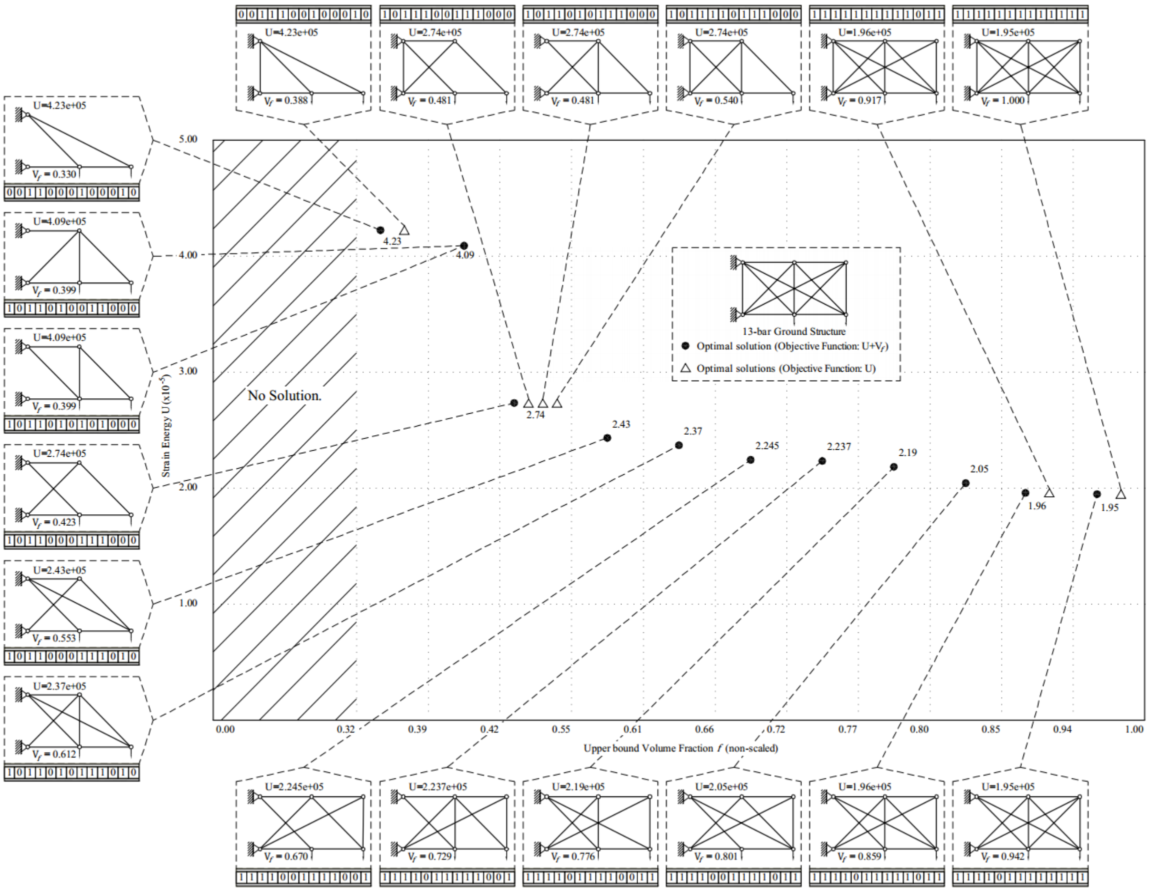

Fig. 8 Solutions for pure topology optimization for 13-bar truss ground structure

those solutions whose strain energy are identical but volume fractions are different, it would be resulted in a unique solution with the minimum volume fraction V_f. The solution for the range of [0.42,0.55] was similar to unique solution of many previous research (Rajan 1995, Richard *et al.* 2012, Rajeev and Krishnamoorthy 1992) which employed weight as objective function with suitable constraint of stress or/and displacement.

Observing the trends plotted in Fig. 8, in the first two solution-available ranges of f and from third to the last range, the strain energy of solution fell gradually while the volume fraction rose steadily. However, in the range of f from 0.39 to 0.55, there was a plummeted decrease in strain energy of the solution, measured 33%, while volume fraction just went up slightly 6%. This was a sign of getting the solution which was identical to the one obtained by most of previous research.

As can be seen, with the input data in Table 2, the graph in Fig. 9 shown result of some GA's result parameters, it was recognized the effectiveness of the candidate evaluation procedure in term of the convergence and the number of generated feasible and stable candidate topology over the generations in GA. On one hand, the convergence criteria of the solution, represented by the best fitness function series, is quite fast, GA obtained the optimal solution just for some first generation. On the other hand, the number of feasible and stable candidate topology was raised

Table 2 GA parameters for pure topology optimization problem

Parameter	Value
Upper bound Volume fraction f	0.55
Population size	100
Max. generation	50
Penalty Parameter γ	10^{10}
Crossover Rate	0.5
Mutation Rate	0.01

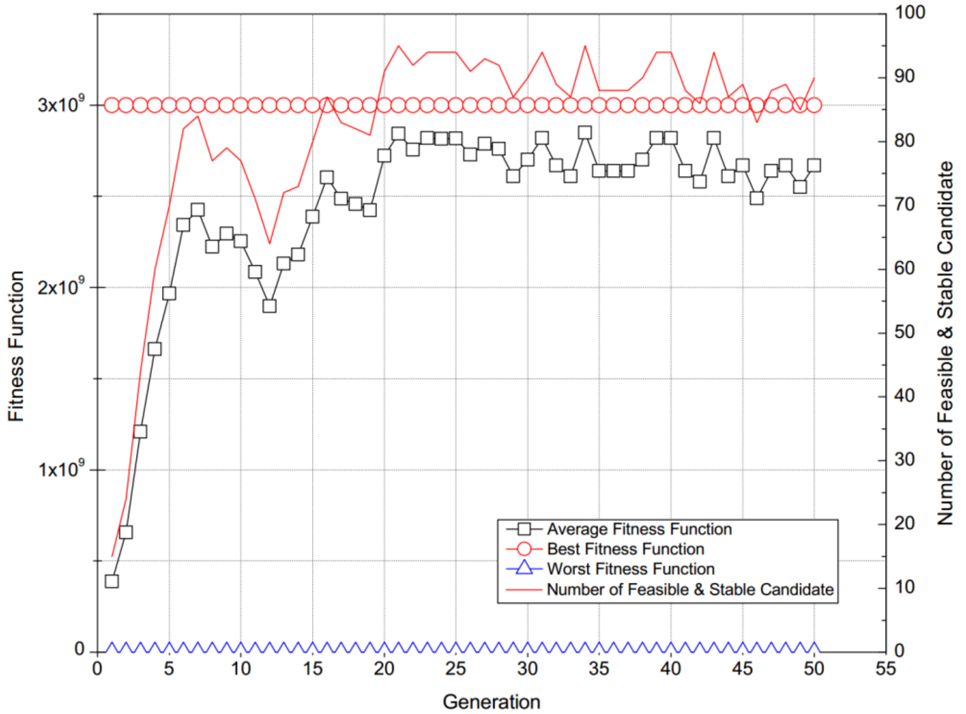

Fig. 9 GA convergence history of pure topology optimization problem

sharply over iterations which reached 80% of the total candidate within 7 generations and hit 90% after 20 generations. This shown the effectiveness of assigning a quite small value of fitness function for the candidate topology which was unfeasible or unstable, consequently, those topology have a little opportunity to be selected by GA for crossover to produce new individual for the next the generation. The average fitness series have got similar trend compared to that of the number of feasible and stable candidate series. Besides, the series of worst fitness was always 0 due to the presence of unfeasible or unstable candidate topology generated by GA in a generation.

4.2 Topology & shape optimization

In this stage, the topology and shape optimization of truss will be performed simultaneously. All elements are considered as Boolean topology variables. The coordinate in y-direction of node

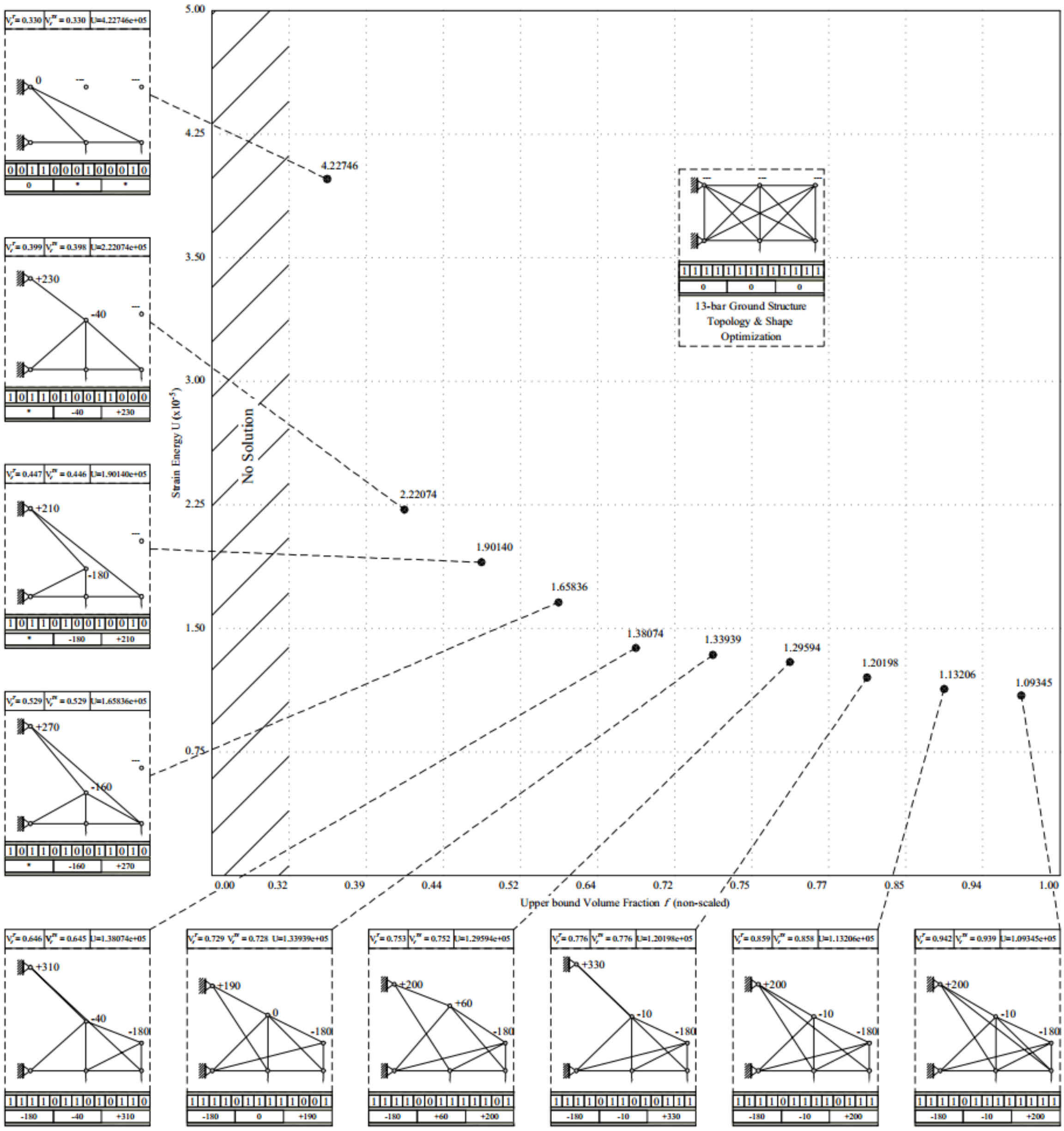

Fig. 10 Solutions for topology & shape optimization for 13-bar truss ground structure

1, 3 and 5 is allowed to vary from -4572 mm (-180 in.) to 16256 mm (640 in.) interval of 254 mm (10 in.) with respect to original position which resulted in changing of its coordinate from 4572 mm (180 in.) to 25400 mm (1000 in.) with a increment of 254 mm (10 in.) These come up with the chromosome length of 34 which are 1 for each element topology design variable and 7 for each node coordinate in y direction.

As can be seen from Fig. 10, there was no solution for range of f smaller than 0.32 which was similarly as previous stage of pure topology optimization. For the first solution for range of f of

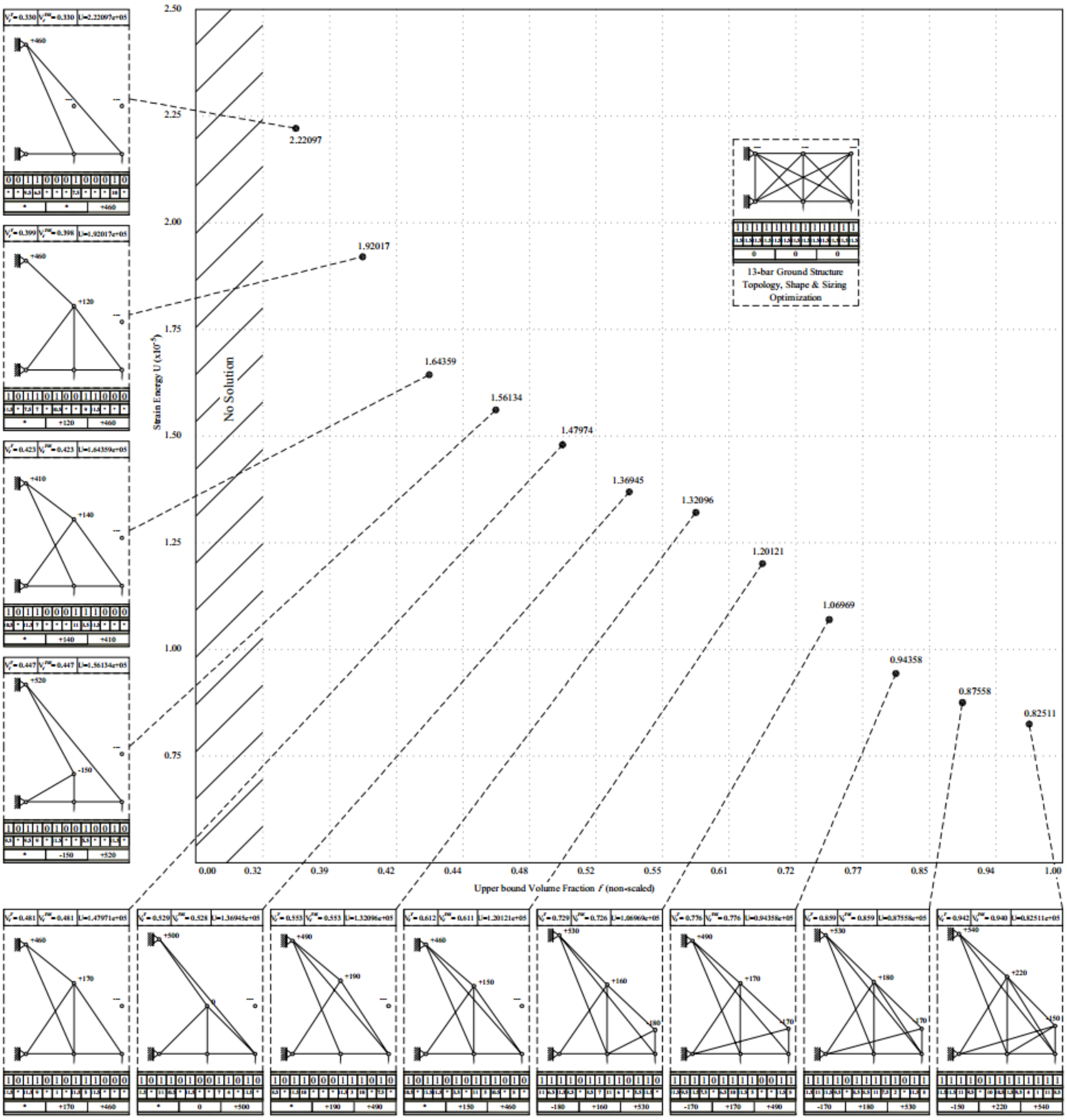

Fig. 11 Solutions for topology, shape & sizing optimization for 13-bar truss ground structure

[0.32,0.39], GA yielded exactly same truss system as that of pure topology optimization. It was reasonable because by changing the coordinate in *y*-direction of node 3 up or down, the truss would be violate the constraint between volume fraction of truss for pure topology and topology & shape which was mentioned in the left part of Eq. (9) or resulting in higher amount of strain energy which was objective function, respectively. There was readily seen that the element 5 connected to two constrained nodes have not existed in any solution across the whole range of f. This, obviously, proved that the element 5 was useless from the ground structure. Observing the trend in the graph, for ranges of f which was greater than 0.39, due to the presence or absence of

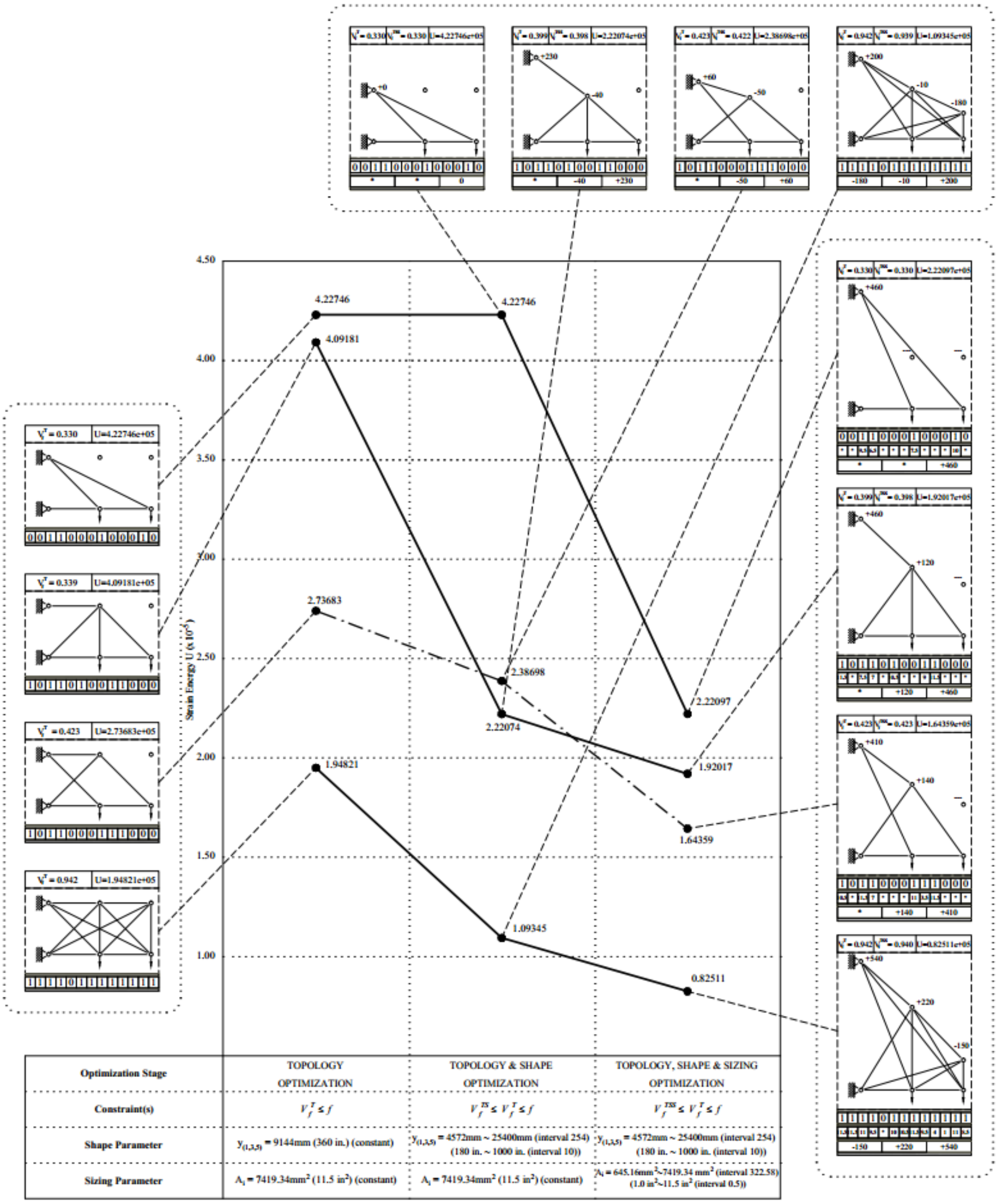

Fig. 12 Solutions over stages of truss optimization with respect to strain energy

elements and the variation of nodes coordinate, the strain energy $\mathcal{U}$ which was objective function declined slightly while the volume fraction of updated topology & shape structure grew gradually and strain energy hit the bottom at the near ground structure topology whose volume fraction was around 0.94.

4.3 Topology, shape & sizing optimization

In this stage, topology, shape & sizing optimization will be performed at the same time. The same topology and shape design variables were used for this stage as that of previous one. In addition, the cross-sectional area of each element was allowed to change by taking the set of 22 discrete values which are from 645.16 mm^2 (1.0 in^2) to 7419.34 mm^2 (11.5 in^2) with the increment of 322.58 mm^2 (0.5 in^2). The sizing variables were required additional a 5-bit string for each element. As consequence, the total chromosome length of 99 would represent each individual candidate for the problem of 13-bar-ground-structure truss topology, shape & sizing optimization simultaneously.

The plotting in the Fig. 11 showed the set of 12 different solutions instead of 10 as previous one. The same manner in term of the decreasing of strain energy over the increasing of volume fraction as that of topology & shape optimization is obtained. The values of strain energy which is objective function, however, were quite smaller since the sizing design variables had been added.

By giving the value of upper bound volume fraction f of 0.43, GA yields the third solution in Fig. 3 whose volume fraction for updated topology V_f^T and updated topology, shape & sizing V_f^{TSS} were both around 0.423. These volume fractions were complied with the constraints mentioned in Eq. (10).

After successfully obtaining solutions for the three different stages of optimization which were pure topology, topology & shape optimization and topology, shape & sizing optimization of truss, a comparison could be drawn throughout the optimization procedures. As could be seen in Fig. 12, four topology solutions were chosen for demonstration. The first, third and fourth one which connected by the solid line were appeared throughout three stages of optimization. For the demonstration purpose, the second solution in Fig. 12 which similar to the solution obtaining by using weight objective function (Rajan 1995, Kawamura *et al.* 2002, Richardson *et al.* 2012, Hajela and Lee 1995, Ohsaki 1995, Deb and Gulati 2001) has been presented. This solution which connected by the dash-dot line was not existed for all three stages of optimization procedure.

It was seen, for the first topology solution, the same strain energy was obtained for pure topology optimization and topology & shape optimization procedure. However, the others fell gradually over stages. In general, the solutions showed the decreasing trend of strain energy over the stages of optimization procedure.

5. Conclusions

This paper has suggested a method of using strain energy as objective function for the problem of truss optimization. The full optimization stages of topology, shape and sizing have been applied successfully. It yields quite reasonable solutions throughout the stages. The energy approach results in more general solutions over the ranges of volume fraction than that of weight approach.

Additionally, the findings prove that this approach of strain energy and volume fraction could also be useful for some optimization problems whose constraints of stress or/and displacement have not been indicated at the first steps of the design procedure, e.g. concept design.

Besides, the candidate evaluation procedure which completely assesses the feasibility and stability of a candidate topology reduces the computer effort, prevents the procedure from singularity phenomena and significantly reduce the computational cost to reach the convergent solution while using GA but still preserve the searching space.

Acknowledgments

This research was supported by National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology through 2015R1A2A1A01007535. The support is gratefully acknowledged.

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