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An efficient simulation method for reliability analysis of systems with expensive-to-evaluate performance functions

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Abstract. This paper proposes a novel reliability analysis method which computes reliability index, most probable point and probability of failure of uncertain systems more efficiently and accurately with compared to Monte Carlo, first-order reliability and response surface methods. It consists of Initial and Simulation steps. In Initial step, a number of space-filling designs are selected throughout the variables space, and then in Simulation step, performances of most of samples are estimated via interpolation using the space-filling designs, and only for a small number of the samples actual performance function is used for evaluation. In better words, doing so, we use a simple interpolation function called "reduced" function instead of the actual expensive-to-evaluate performance function of the system to evaluate most of samples. By using such a reduced function, total number of evaluations of actual performance is significantly reduced; hence, the method can be called Reduced Function Evaluations method. Reliabilities of six examples including series and parallel systems with multiple failure modes with truncated and/or non-truncated random variables are analyzed to demonstrate efficiency, accuracy and robustness of proposed method. In addition, a reliability-based design optimization algorithm is proposed and an example is solved to show its good performance.

Keywords: uncertainty; reliability; failure probability; Monte-Carlo simulation; reduced function evaluation

1. Introduction

In the current analysis, design and optimization of structural/mechanical systems, variables are assumed to be deterministic, however, uncertainty exists in their values and they are in reality non-deterministic random variables. Taking not into account these uncertainties simplifies the computation significantly, decreasing computational cost. On contrary to its simplicity, it may result in likely to fail designs. The probability of failure will be significant especially for minimum weight/cost designs. To take in hand both the safety and minimum cost we should take the aleatory or epistemic uncertainties (Kiureghian and Ditlevsen 2009) into account. This type of optimal design approach is called reliability-based design optimization (RBDO).

Reliability of a system can be defined as the distance of origin of random variables standard space from the nearest point of failure region to the origin (called most probable point, MPP), i.e., as reliability index. However, it can best be defined through the probability of failure (P_f) of the

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system. The more the P_f of the system is, the less its reliability is.

There are two main groups of methods for estimating the P_f of a system: approximate methods like first- and second-order reliability (FORM/SORM) methods, and simulation methods like Monte-Carlo Simulation (MCS). The former estimates P_f by approximating the limit-state surface around MPP, while, the latter simulates the problem by evaluating the various possible cases (samples).

Up to now, various reliability analysis methods have been proposed. Among these methods Importance Sampling (Melchers 1990) and Subset Simulation (Au and Beck 2001) have attracted attention of reliability analysis community. Zhang *et al*. (2013) proposed an interval quasi-Monte Carlo method using pseudo-random number generation and probability-box modeling for structural reliability analysis. Shi *et al*. (2014) used maximum entropy method in reliability analysis for probabilistic modeling of structural response. Zhang *et al*. (2014) used evidence theory and response surface approach for the analysis of structural reliability. Recently, Xiao *et al*. (2014a) proposed a method based on back-propagation neural network for reliability analysis of systems with truncated random variables. In another work, Xiao *et al*. (2014b) extended the saddlepoint approximation-based approach for the reliability analysis of structural systems with parameter uncertainties using probability-box models. Basaga *et al*. (2012) presented an improved response surface method (RSM) for structural reliability analysis. Their three-stage algorithm uses FORM, vector projected and SORM methods to approximate reliability index and probability of failure of systems. Moreover, Zhao *et al*. (2013) proposed an efficient response surface method for reliability analysis of structures with highly nonlinear limit-state functions based on direct MCS. In their method, in order to approximate the actual limit-state more accurately, experimental points are selected judiciously. In addition to these works, more valuable details regarding RSM can be found in the works of Jiang *et al*. (2014b), Li *et al*. (2013), wherein, stochastic-RSM (SRSM) with RSM and three collocation point methods for odd order SRSM with each other are compared, respectively. Furthermore, an iterative hybrid random-interval method is proposed by Fang *et al*. (2014) for reliability analysis of structures by strength degradation accounted for. Their method uses first-order second-moment (FOSM) approach based on stress-strength inference theory. Piliounis and Lagaros (2014) used meta-heuristics in combined with FORM to analyze the reliability of geostructures.

On the other hand, as it was stated above, the RBDO provides a cost-safety balance with the design of systems. In literature there are valuable works on the RBDO. Among these works are the works of Jensen and Sepulveda (2014), Jensen and Kusanovic (2014) wherein the RBDO of controlled structures are investigated. Beaurepaire *et al*. (2013) proposed an efficient method for RBDO using importance sampling approach. The use of meta-heuristic optimization algorithms in the RBDO problems can be found in the works of Dimou and Komousis (2009), Yang and Hsieh (2011), wherein, particle swarm optimization (PSO) is used and improved for RBDO of structures. In more details, in Dimou and Komousis (2009) an improved binary-PSO is introduced and used for reliability-based shape and size optimization of truss structures on the basis of reliability index approach, whereas, Yang and Hsieh (2011) proposed an auto-tuning boundary-approaching PSO for solving RBDO problems with discrete design variables and non-smooth performance functions using subset simulation. Reliability-based design approaches have been successfully used in different fields of civil engineering; recently, Kim *et al*. (2014) developed reliability-based design limit-states of AASHTO LRFD code for the case of integral abutment bridges considering variability of abutment support conditions and thermal loading. Also, the application of stochastic finite element, RSM and SRSM methods in reliability analysis of slopes can best be found in Jiang *et al*. (2014a), Li *et al*. (2011, 2015).

In this paper, a novel simulation method is proposed for reliability analysis and RBDO of structural systems. The method is well suited for systems with expensive-to-evaluate performance functions. It consists of two main steps: Initial step and Simulation step. In the Initial step, a number of uniformly distributed space-filling designs are selected throughout the entire variables space or its important region, for systems with truncated and non-truncated random variables, respectively. Then, in Simulation step, performance measures of samples are computed by making use of an interpolation function on the basis of samples evaluated exactly in initial step. This interpolation function is called "reduced" performance function herein, which gives an estimation of performance measure without need to evaluate its exact value using actual performance function. After this simple estimation, we can guess that the sample under evaluation locates at which of regions of failure or safety. Since this estimation can not exactly determine the failure or safety of samples close to limit-state surface, these samples are evaluated using actual performance function and will be used as an exact point for interpolation in the evaluation of remainder of samples during simulation. In addition, for non-truncated random variables we can select an important region, in which most of samples will locate, and then, use actual performance function for the evaluation of rest of samples those locate outside this region. The proposed method can be called Reduced Function Evaluation (RFE) method, since in which the number of required evaluations of performance function is reduced using a "reduced" performance function instead of actual one.

The RFE method can also be used in RBDO of systems. In this paper, we use RFE in combined with recently proposed optimization algorithm, namely HS-PSO, for RBDO problems. The HS-PSO algorithm, introduced by Hadidi and Rafiee (2014), is an efficient and robust algorithm, which performs better than standard harmony search (HS), particle swarm optimization (PSO) and big bang-big crunch (BB-BC) algorithms (Rafiee *et al*. 2013). The proposed RBDO approach is not restricted to HS-PSO and other efficient algorithms like the algorithm proposed by Hadidi and Rafiee (2015) can be used instead, as well. Six numerical examples are used in this work to demonstrate the efficiency, robustness and accuracy of the proposed reliability method in estimating P_f. Moreover, the examples show the ability of RFE method in finding reliability index and MPP.

2. Proposed reliability analysis method

This section of the paper presents proposed reliability analysis method in four subsections as follows: In the first subsection, general concepts regarding reliability measures are briefly reviewed. In next subsection the truncation procedure for random variables is presented. Then, the proposed method is described in subsections 2.3 and 2.4 for reliability analysis of systems with truncated and non-truncated random variables, respectively. Finally, the method is illuminated by providing with a flowchart.

2.1 General remarks

Reliability of a system can be measured by its reliability index or its probability of failure. Reliability index (β) is the minimum distance of failure region from the origin in the standard Gaussian space (U-space). The nearest point of failure region to the origin is called design or most

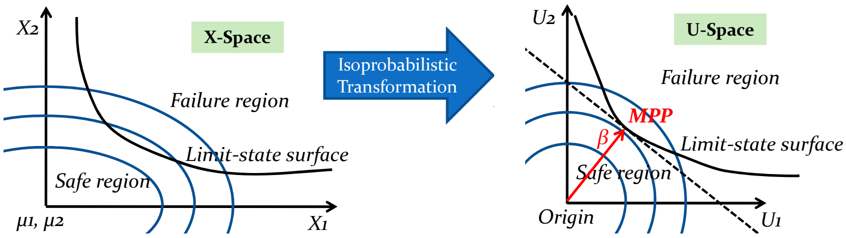


Fig. 1 Isoprobabilistic transformation and reliability measures

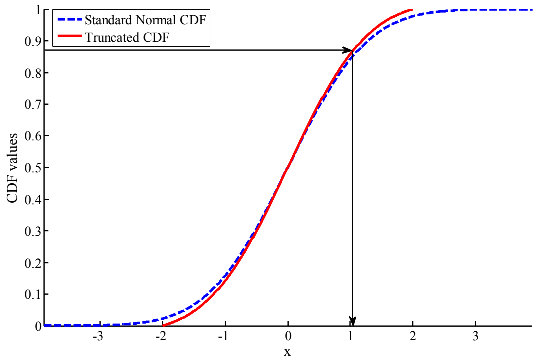


Fig. 2 The CDF of standard normal distribution and its truncated form

probable point (MPP). These are illustrated in Fig. 1 for two-dimensional case. In this figure, blue curves are contours of joint probability density function (JPDF).

On the other hand, the probability of failure (P_f) of a system can be defined as

$$P_f = \Pr(g(X) \leq 0) = \int_F f_{jpdf}(X)dX \tag{1}$$

where, $g(X)$ is performance of system design X, which is negative for a failed design and f_{jpdf} is its probability of occurrence. The integral of Eq. (1) is computed throughout Failure region. It is hard and impossible in most cases to solve this integral analytically. Then, simulation methods like direct Monte Carlo estimate this integral as

$$P_f \cong \frac{1}{N}\sum_{i=1}^{N} I_f(i) = E(I_f) \tag{2}$$

where, N is total number of samples and $I_f(i)$ denotes value of Indicator function for i-th sample. This value is equal to zero and one for safe and failed samples, respectively. $E(\ )$ is mathematical expectation. In this general way, there is no need for isoprobabilistic transformation. However, large number of samples is required for accurate estimation especially in the case of estimation of small failure probability, which is common in engineering problems. Other efficient simulation methods like importance sampling (IS), although need small number of samples, have their disadvantages. For example, the IS is a dangerous method, since all the results are computed based on MPP, and then, an error in finding true MPP results in an unacceptable error in estimation of P_f.

Such an error in search for global MPP is not avoidable in many engineering systems having multiple failure modes.

2.2 Truncated random variables

In most of engineering reliability problems, we deal with random variables whose values cannot cover the range (−∞, +∞). This property of existing random variables forms rational basis for the use of truncated random variables whose values cover a practical range. In this sense, upper and lower bounds are defined for common non-truncated random distributions based on experimental judgment regarding uncertain variables.

Such a truncation changes the cumulative distribution function (CDF) as follows

$$CDF_X^{tr}(x)=\left(CDF_X^{nt}(x)-CDF_X^{nt}(a)\right)/\left(CDF_X^{nt}(b)-CDF_X^{nt}(a)\right) \quad (3)$$

where, superscripts *tr* and *nt* are abbreviations for truncated and non-truncated, respectively. Lower and upper bounds of truncation are denoted by a and b, respectively, while the relationship $-\infty<a\leq x\leq b<+\infty$ holds. In Fig. 2, standard normal CDF is truncated at range [−2, 2] using above procedure. In this truncated form, random number can simply be generated by making use of uniform random generation approach as shown in Fig. 2, as well.

2.3 Illustration of proposed method: Initial and Simulation steps

The proposed method analyzes the reliability of a system within two steps, namely Initial and Simulation steps. In this subsection, RFE method is described for reliability analysis of systems with truncated random variables and its capability for solving systems with non-truncated variables will be illuminated in next subsection.

In the Initial step, a number of space-filling designs (called SFDs) are created throughout the entire variables space to cover it properly. The SFDs should be equally-spaced as far as possible. These designs can be created by two ways: (1) By generating equally-spaced grids (meshes) on the variables space or (2) By using pseudo-random generation tools (e.g., Halton or Faure sequences) which are available in most of programming languages as built-in. The first procedure is suitable for problems with low number of random variables, whereas, the second procedure is well suited for problems with high number of random variables. Then, performances of all the SFDs are evaluated exactly using actual performance function. The evaluated performances are then normalized as follows

$$g_X^{normalized}(x)= g_X^{computed}(x)\Big/\left|g_X^{origin}(\mu)\right| \quad (4)$$

where, $g_X^{computed}(x)$ is the computed performance for design x and $\left|g_X^{origin}(\mu)\right|$ is absolute value of the actual performance measure for the origin of variables space (design corresponding to mean values of variables). By doing so, all the performances are converted to normalized dimensionless values which are now appropriate to be used for interpolation. After this normalization, the space-filling designs and their normalized performances are stored in a memory for use in Simulation step. This memory having NSFD (total number of space-filling designs) designs will be called SFDs memory in this paper, to be concise.

In the Simulation step of the method, on the other hand, direct-MCS is used for reliability analysis. For each sample, first an estimation of normalized performance is computed via interpolation using SFDs memory. In this paper, we do this radial basis interpolation by introducing reduced performance function as follows

$$\tilde{g}_X^{normalized}(sample) = \sum_{i=1}^{NSFD}\left(\frac{r_{\min}{}^2}{r_i^2}\right)^n \times g_X^{normalized}(x_i) \Bigg/ \sum_{i=1}^{NSFD}\left(\frac{r_{\min}{}^2}{r_i^2}\right)^n \qquad (5)$$

where, $\tilde{g}_X^{normalized}(sample)$ is the estimated value of normalized performance for the sample under evaluation; x_i is i-th design in the SFDs memory among total number of *NSFD* designs; $g_X^{normalized}(x_i)$ is the exact value of normalized performance for x_i; n is an arbitrarily chosen positive value adopted to be integer herein; r_i^2 is square value of Euclidean distance of the sample from x_i and $r_{\min}{}^2$ is their minimum as

$$r_{\min}{}^2 = \min(r_i^2), \quad i = 1, 2, \ldots NSFD. \qquad (6)$$

After this simple evaluation, we can guess that in which of regions of failure or safety the sample design locates. Since, this guess cannot be exactly true especially for samples close to limit-state, we determine a "critical region" near the limit-state surface (symmetrically in its both sides), wherein, exact performance measure of samples should be evaluated using actual performance function. The critical region can simply be defined by normalized performance measure, such that, a sample is in this region when absolute value of its estimated performance is smaller than a threshold. This threshold for critical region is called TCR, here, for convenience. After exact evaluation of samples that belong to this region, these performances are normalized (using Eq. (4)) and obtained data are stored in SFDs memory for use in future interpolations for the rest of samples. In Fig. 3, the variables space is shown for two random variables $x1$ (with $\mu 1$ and $\sigma 1$) and $x2$ (with $\mu 2$ and $\sigma 2$), truncated at [$a1$, $b1$] and [$a2$, $b2$] intervals (i.e., the space inside blue rectangle), respectively. The limit-state surface is shown by solid curve, while dashed curves are contours of normalized performance function. It should be noted that the critical region (between g=−TCR and g=TCR) is defined by reduced performance function, whereas, the limit-state is detected by actual performance function. In this figure, the shadowy part is the critical region and space-filling designs are depicted by stars. Moreover, red and green circles are failed and safe samples, respectively.

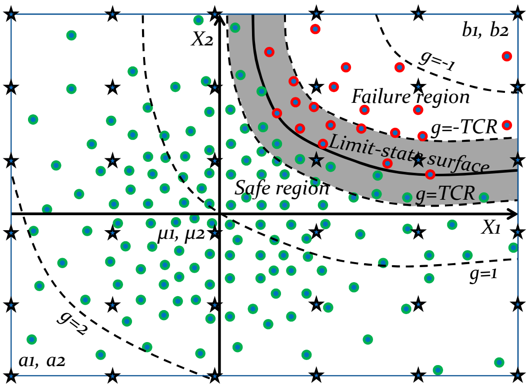


Fig. 3 Details of concepts of the proposed method for truncated random variables

In addition, during Simulation step, SFDs memory will be enriched by adding those samples which are evaluated exactly (i.e., those were in critical region in previous estimations as discussed above). This will improve our next estimations for the rest of samples especially for those which locate near limit-state. So, in this work, to reduce further the need for exact performance evaluations, we suggest an auto-tuning scheme for TCR as follows

$$TCR^{new} = TCR^{current} \times \left(1 - \varepsilon \times sign(\, \tilde{g}_X^{normalized}(c.s.) \times g_X^{normalized}(c.s.)\,)\right) \tag{7}$$

where, ε is TCR auto-tuning parameter (a positive value in the range (0,1)); sign( ) is the sign function having −1, 0 or 1 values; $\tilde{g}_X^{normalized}\left(c.s.\right)$ and $g_X^{normalized}\left(c.s.\right)$ are estimated and actual values of normalized performance for the critical sample (*c.s.*). In better words, doing so, the current critical region will be shrunk or expanded if our guess turns out true or false for the current sample, respectively. However, in rare cases (when sign( )=0) it is not changed.

2.4 Reliability analysis of systems with non-truncated random variables

In most of engineering reliability problems, we deal with truncated random variables. However, the proposed method possesses the capability of solving problems with non-truncated variables. In this case, only a simple additional task is required. For these problems we can define an important region, wherein, most of samples may locate. This region is defined herein based on the PDF of the variables by eliminating their tails in points where their PDF values are small enough. This small value can be chosen arbitrarily. After defining such a region, space-filling designs are selected throughout this part of variables space and SFDs memory is constructed analogous to subsection 2.3. Then in Simulation step, actual performance function is used for samples which locate outside the important region, whereas samples which locate inside are evaluated using the procedure described in previous subsection. In Fig. 4, important region is the part inside red rectangle (within ranges [*c*1, *d*1] for *x*1 and [*c*2, *d*2] for *x*2) while other details are similar to Fig. 3.

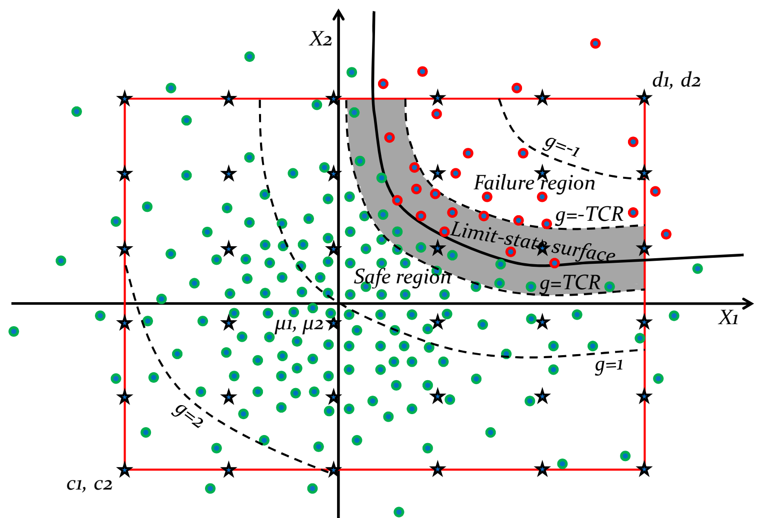


Fig. 4 Details of concepts of the proposed method for non-truncated random variables

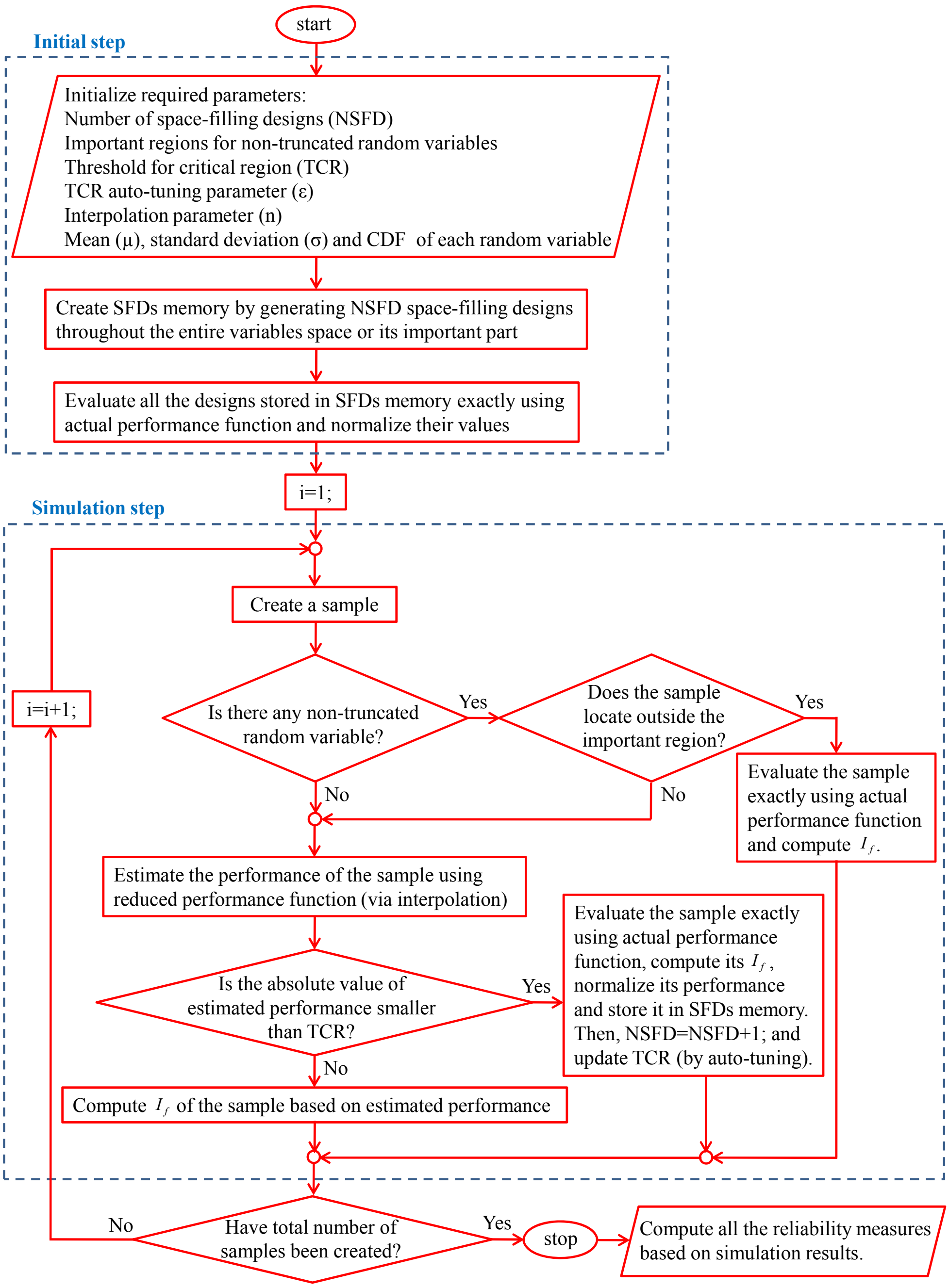


Fig. 5 The flowchart of the proposed reliability analysis method

The flowchart of Fig. 5 provides a better illumination of the proposed method. Using this method one can compute all the reliability measures, i.e., probability of failure (P_f), reliability index (β) and most probable to fail point (MPP). The former is computed using I_f of samples, whereas, two latter measures are computed based on the standard distances of failed samples from

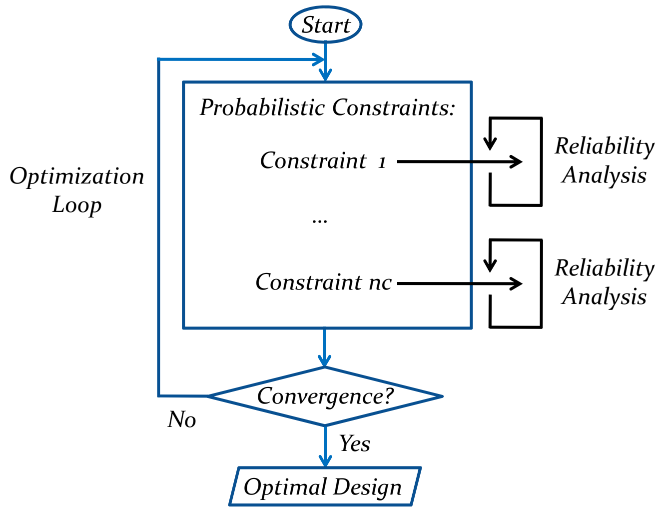


Fig. 6 The flowchart of the proposed RBDO algorithm using HS-PSO and RFE

the origin. In better words, the failed sample which is closest to the origin in the standard space is MPP and its distance from the origin is reliability index (see Fig. 1).

3. Proposed reliability-based design optimization algorithm

The proposed reliability analysis method can also be used in combined with efficient optimization methods for reliability-based design optimization (RBDO) of systems. In this contribution, we use recently proposed harmony search-based, improved particle swarm optimization (HS-PSO) algorithm, which is an efficient optimization method and has proved to perform better than standard big bang-big crunch (BB-BC), harmony search (HS) and particle swarm optimization (PSO) algorithms (Rafiee and coworkers 2013, 2014). The proposed RBDO algorithm can be illustrated by the self-explanatory flowchart of Fig. 6, wherein, sufficient details regarding HS, PSO and HS-PSO algorithms can best be found in Hadidi and Rafiee (2014). In addition, in this flowchart, the optimization loop is the same HS-PSO optimization algorithm, while, the inner loop of reliability analyses is the proposed reliability analysis method. The proposed RBDO algorithm is not limited to HS-PSO and other efficient optimization methods (e.g., the algorithm proposed by Hadidi and Rafiee 2015) can also be used.

4. Numerical examples

In this study, six numerical examples are solved to demonstrate the accuracy, efficiency and robustness of proposed reliability analysis and RBDO methods. The examples are selected such that to include parallel and series systems with multiple failure modes with truncated and/or non-truncated random variables. In addition, in the examples different structural systems with uncertainties in loads and structural parameters are included and different reliability measures are computed via proposed method and other available methods.

Table 1 Results of reliability analysis for Example 1

Method	P_f	Number of samples	Number of evaluations of actual performance
2nd-order SRSM (Jiang *et al*. 2014b)	0.0490	-	6
3rd-order SRSM (Jiang *et al*. 2014b)	0.0060	-	10
RSMncross (Jiang *et al*. 2014b)	0.0040	-	120
RSMcross (Jiang *et al*. 2014b)	0.0056	-	324
MCS (this work)	0.0058	1,000,000	1,000,000
Proposed method (TCR=0.0005; ε=0.2; n=4)	0.0058	1,000,000	25+62+10=97
Proposed method (TCR=0; n=4)	0.0060	1,000,000	25+0+10=35

4.1 Example 1: a nonlinear performance function

A 2D problem is solved in this example, and the results of proposed method are compared with those of direct-Monte Carlo (MCS), Response Surface (RSM) and Stochastic Response Surface (SRSM) methods. The performance function of this example, obtained from literature, is expressed as

$$g_X(x_1, x_2) = x_1^3 + x_1^2 x_2 + x_2^3 - 18 \tag{8}$$

Both x_1 and x_2 follow the Normal distribution. The means of x_1 and x_2 are 10 and 9.9, respectively, and the standard deviation is 5 for both. The results of reliability analyses for this example are listed in Table 1. In this Table RSMncross is the RSM based on a quadratic polynomial chaos expansion (PCE) without cross-terms using the vector projection sampling technique. For more information see Jiang *et al*. (2014b).

For this example, important region is defined as the region constructed by the range $\mu_i-4.5\sigma_i$ to $\mu_i+4.5\sigma_i$ for i=1,2. In addition, SFDs are created using the first procedure described in part 2.3, with 5 equally spaced points in each of ranges of variables in the important region, i.e., as $[\mu_i-4.5\sigma_i \quad \mu_i-2.25\sigma_i \quad \mu_i \quad \mu_i+2.25\sigma_i \quad \mu_i+4.5\sigma_i]$ for $i=1,2$.

In Table 1, the number of evaluations of actual performance function in the proposed method is sum of three terms: first term is the number of SFDs evaluated at Initial step (5^2=25 evaluations); second term shows the number of samples fallen in critical region; and third term is the number of samples which locate outside important region (about 0.001% of total samples). The superiority of the proposed method over the RSM is evident from the results of Table 1, while it gives P_f with the same accuracy of 3rd-order SRSM by 35 evaluations.

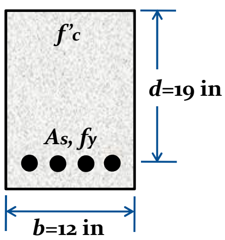


Fig. 7 Cross section of RC beam

Table 2 Random variables of RC beam

Variable	Distribution type	Mean value	Standard deviation
A_s in^2 (cm^2)	Normal	4.08 (26.32)	0.08 (0.516)
f_y ksi (MPa)	Normal	44 (303.38)	4.62 (31.85)
f'_c ksi (MPa)	Normal	3.12 (21.51)	0.44 (3.034)
Q k-in (kN.m)	Normal	2052 (231.88)	246 (27.80)

Table 3 Results of reliability analysis for RC beam

Method	MPP	β	P_f	Number of samples	Number of evaluations of actual performance
FORM (Nowak and Collins 2000)	-	2.350	0.0094	-	-
MCS (this work)	[4.05, 36.83, 2.93, 2465]	2.349	0.0110	100,000	100,000
	[4.05, 36.83, 2.93, 2465]	2.349	0.0110	200,000	200,000
	[4.06, 36.34, 3.07, 2457]	2.346	0.0111	500,000	500,000
	[4.06, 36.34, 3.07, 2457]	2.346	0.0111	1,000,000	1,000,000
Proposed method	[4.05, 36.83, 2.93, 2465]	2.349	0.0105	100,000	625+5,429+29=6,083
	[4.05, 36.83, 2.93, 2465]	2.349	0.0106	200,000	625+8,471+61=9,157
	[4.06, 36.34, 3.07, 2457]	2.346	0.0106	500,000	625+14,836+128=15,589
	[4.06, 36.34, 3.07, 2457]	2.346	0.0106	1,000,000	625+21,671+252=22,548

4.2 Example 2: a reinforced concrete beam

In this example, obtained from literature, reliability of a reinforced concrete (RC) beam, shown in Fig. 7, is analyzed. The limit-state of the beam capacity in bending would be

$$g_X\left(A_s, f_y, f_c', Q\right) = A_s f_y d - 0.59\frac{\left(A_s f_y\right)^2}{f_c' b} - Q \tag{9}$$

where, A_s is the area of steel; f_y is the yield strength of the steel; f'_c is the compressive strength of the concrete; b is the width of the section and d is its depth. Moreover, Q is the moment (load effect) due to the applied loads. The random variables of the problem are listed in Table 2, whereas, b and d values are assumed to be deterministic.

This example has been previously solved in Nowak and Collins (2000) using FORM. The reliability results of this example are shown in Table 3. In this example, we use TCR=0.45, ε=0.0002 and n=3, while important region is defined as the region constructed by the range $\mu_i-4\sigma_i$ to $\mu_i+4\sigma_i$ for i=1,2,3,4 i.e., for four random variables of Table 2. In addition, SFDs are created using the first procedure described in part 2.3, with 5 equally spaced points in each of ranges of variables in the important region, i.e., as $[\mu_i-4\sigma_i \;\; \mu_i-2\sigma_i \;\; \mu_i \;\; \mu_i+2\sigma_i \;\; \mu_i+4\sigma_i]$ for i=1,2,3,4.

In Table 3, the number of evaluations of actual performance function in the proposed method is sum of the number of SFDs evaluated at Initial step (5^4=625 evaluations), the number of samples fallen in critical region (second term) and the number of samples which locate outside important region (third term). As it is seen from the results, the proposed method gives reliability measures close to those of FORM and MCS, while the number of required actual performance evaluations is

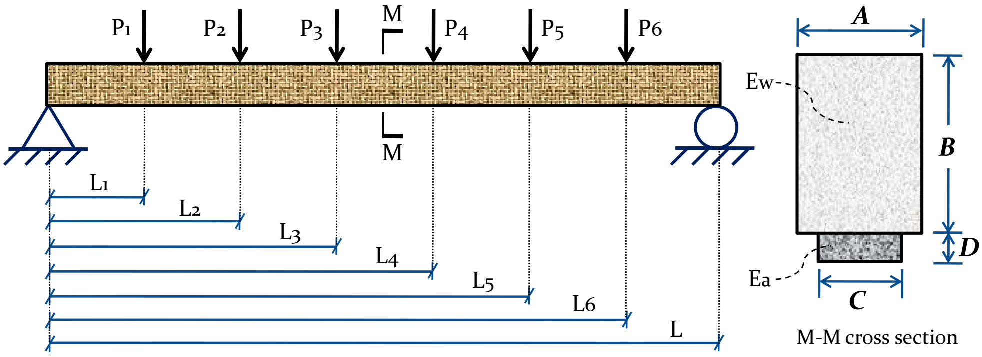


Fig. 8 A composite beam

reduced by more than 97% compared to MCS. It is also seen that 500,000 samples is sufficient for MCS while with this number of samples only 15,589 evaluations are needed with proposed method. It is obvious that for MCS with 15,589 samples, different reliability results will be found for different seeds of random sets. In better words, the results of MCS with same number of evaluations will be very sensitive to random generation seed, whereas, the proposed method is much less sensitive and is a reliable method for analysis of reliability.

On the other hand, by choosing optimum values for parameters of method better results can be obtained. This latter will be examined for Example 4.

4.3 Example 3: a composite beam (a high-dimensional problem with highly nonlinear limit-state function and small failure probability)

A composite beam with twenty independent random variables, as shown in Fig. 8, is employed to demonstrate the efficiency and accuracy of the proposed method in reliability analysis of high dimensional problems with highly nonlinear limit-states. This example is modified from Huang and Du (2008) to show the high performance of RFE in estimating the small failure probabilities.

The maximum stress of the beam is calculated in the middle cross-section M-M as

$$\sigma_{\max}=\frac{\left[\frac{\sum_{i=1}^{6}P_i(L-L_i)}{L}L_3-P_1(L_2-L_1)-P_2(L_3-L_2)\right]\left[\frac{0.5AB^2+\frac{E_a}{E_w}DC(B+D)}{AB+\frac{E_a}{E_w}DC}\right]}{\frac{1}{12}AB^3+AB\left\{\frac{0.5AB^2+\frac{E_a}{E_w}DC(B+D)}{AB+\frac{E_a}{E_w}DC}-0.5B\right\}^2+\frac{1}{12}\frac{E_a}{E_w}CD^3+\frac{E_a}{E_w}CD\left\{0.5D+B-\frac{0.5AB^2+\frac{E_a}{E_w}DC(B+D)}{AB+\frac{E_a}{E_w}DC}\right\}^2} \tag{10}$$

where, A, B, C and D are dimensions of cross-section; L_1, L_2, … L_6 and L are corresponding length from left support; P_1, P_2, … P_6 are concentrated loads applied at six different points along the beam; E_w and E_a are corresponding Young's modulus values. In order to have a safe design, the maximum stress of the beam should not exceed its allowable strength, hence, the limit-state is

$$g_X\left(A,B,C,D,L_1,L_2\ldots L_6,L,P_1,P_2\ldots P_6,E_a,E_w,S\right)=S-\sigma_{\max} \tag{11}$$

Table 4 Random variables of composite beam

Variable	Distribution type	Mean value	Standard deviation	Important range
A	Normal	100	0.2	[99, 101]
B	Normal	200	0.2	[199, 201]
C	Normal	80	0.2	[79, 81]
D	Normal	20	0.2	[19, 21]
L_1	Normal	200	1	[195, 205]
L_2	Normal	400	1	[395, 405]
L_3	Normal	600	1	[595, 605]
L_4	Normal	800	1	[795, 805]
L_5	Normal	1000	1	[995, 1005]
L_6	Normal	1200	1	[1195, 1205]
L	Normal	1400	2	[1390, 1410]
P_1	Gumbel	15	1.5	[0, 20]
P_2	Gumbel	15	1.5	[0, 20]
P_3	Gumbel	15	1.5	[0, 20]
P_4	Gumbel	15	1.5	[0, 20]
P_5	Gumbel	15	1.5	[0, 20]
P_6	Gumbel	15	1.5	[0, 20]
E_a	Normal	70	7	[35, 105]
E_w	Normal	8.75	0.875	[4.375, 13.125]
S	Gumbel	50	3	[12, 60]

Table 5 Results of reliability analysis for the composite beam

Method	P_f	Number of samples	Number of evaluations of actual performance
MCS (this work)	0.0000240	1,000,000	1,000,000
	0.0000219	10,000,000	10,000,000
Proposed method	0.0000230	1,000,000	1,000+78+288= 1,366
	0.0000203	10,000,000	1,000+190+2,848= 4,038

where, S is the allowable strength of the beam.

The distribution information of the random variables is given in Table 4. The reliability results of this example are shown in Table 5. For this example, TCR=0.5, ε=0.05 and n=3 are used, while important region is constructed based on the important ranges defined for the random variables (as given in Table 4). In addition, 1000 SFDs are created using the second procedure described in part 2.3. As it is seen from the results, the proposed method can estimate small failure probabilities of high dimensional problems with highly nonlinear limit-state functions.

4.4 Example 4: a parallel system

This example considers a parallel system with two failure modes, for which following limit-states are defined

$$\begin{cases} g_1(X_1, X_2, X_3) = X_1 X_2 / X_4 + X_3 - 10 \\ g_2(X_1, X_2, X_3) = X_1^2 / X_3 + 2X_2 - X_4 - 6 \end{cases} \tag{12}$$

where, X_1 through X_3 are truncated random variables described in Table 6, whereas X_4 is a deterministic constant.

The reliability results obtained for this system, in three cases of X_4=3, 5 and 7 are shown in Tables 7 and 8. This example has recently been solved by Xiao *et al*. (2014a) using back propagation-neural network (BP-NN) with 250 evaluations of actual performance function,

Table 6 Random variables of parallel system

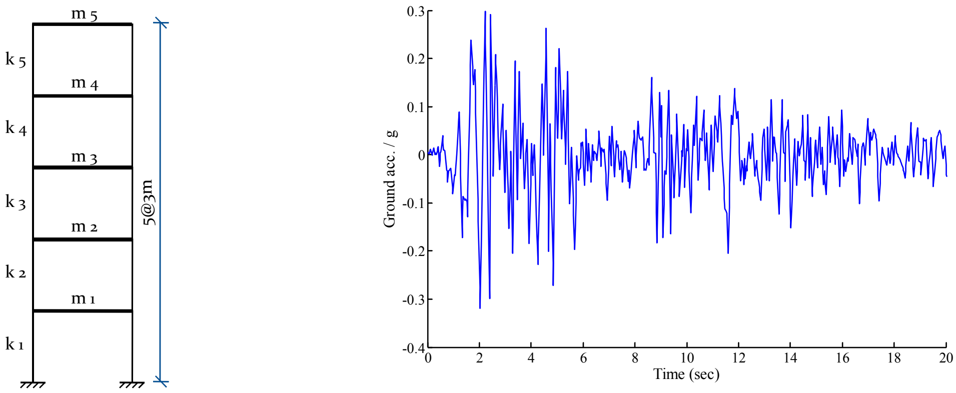
Variable	Distribution type	Mean value	Standard deviation	Truncation range
X_1	Normal	2	1	[0, 5]
X_2	Normal	5	1	[2, 8]
X_3	Normal	10	1	[7, 13]

Table 7 Results of reliability analysis for parallel system with different parameters (X_4=3)

Method	TCR	ε	n	P_f	Number of samples	Number of evaluations of actual performance
MCS (this work)	-	-	-	0.0131	100,000	100,000
Proposed method	0.5	10^{-4}	1	0.0131	100,000	8,580
	0.25	10^{-4}	1	0.0131	100,000	6,396
	0.1	10^{-4}	1	0.0125	100,000	3,978
	0.5	10^{-3}	1	0.0141	100,000	2,736
	0.5	10^{-5}	1	0.0131	100,000	20,251
	0.1	10^{-4}	2	0.0132	100,000	2,349
	0.1	10^{-4}	3	0.0132	100,000	2,094
	0.1	10^{-3}	3	0.0132	100,000	1,187
	0.1	0.01	3	0.0133	100,000	443
	0.05	0.05	3	0.0130	100,000	221
	0.02	0.2	3	0.0129	100,000	149

Table 8 Results of reliability analysis for parallel system

Method	X_4	P_f	Number of samples	Number of evaluations of actual performance
Using BP-NN (Xiao *et al.* 2014a)	3	0.0141	-	250
	5	0.0539	-	250
	7	0.1158	-	250
MCS (this work)	3	0.0131	100,000	100,000
	5	0.0540	100,000	100,000
	7	0.1113	100,000	100,000
Proposed method	3	0.0129	100,000	149
	5	0.0549	100,000	152
	7	0.1152	100,000	148



(a) Shear building structural model (b) Elcentro, Imperial Valley, 1940 earthquake

Fig. 9 Seismically excited building

wherein, 250 uniformly distributed random points are selected throughout the variables space and then used for training of network. Their results are not far from those of MCS, however, it seems that their results are sensitive to seed of random sets used for selection of the points, whereas, RFE uses 100,000 points and then the sensitivity to the random seed will be very low. In addition, the method proposed herein needs no surrogate network model. It should be noted that the results of Table 8 are based on TCR=0.02, ε=0.2 and n=3, which seems to be optimum ones based on results of investigation on these parameters. Such an investigation is accomplished and results are listed in Table 7. Furthermore, SFDs are created using the first procedure described in subsection 2.3, by selecting 5 equally spaced points in each of ranges of variables (i.e., 5^3=125 SFDs).

4.5 Example 5: a seismically excited building (a series system)

In this example, reliability of a series system with expensive-to-evaluate performance function is analyzed. Consider the five-storey building shown in Fig. 9, which is excited by the north-south component of earthquake record of Elcentro, Imperial Valley, 1940. The first 20 seconds of this record is shown in Fig. 9, as well, wherein the peak ground acceleration is equal to −0.319g. In this study, the stiffness values of stories and modal damping ratios of the structure are taken as ten independent truncated random variables of the problem (see Table 9), whereas the mass values of all the stories are assumed to be deterministic and equal to 26.35 kN.sec^2/m. Time history analysis of shear building model is accomplished by preparing a home-made computer program written based on modal analysis and linear acceleration form of Newark's method for first ten seconds of excitation with time steps equal to 0.02 sec. More information on these methods can best be found in Chopra (2007). This system with five failure modes for five stories, has following limit-states

$$g_i(k_1,\ldots k_5,\zeta_1,\ldots\zeta_5)=d_i/h_i-1/300, \quad i=1,2,\ldots 5. \tag{13}$$

where, h_i and d_i values are, respectively, storey height and inter-storey drift values for i-th storey. This limitation is chosen based on AISC (2010) specifications.

Table 9 Random variables of the building

Variable	Distribution type	Mean value	Standard deviation	Truncation range
$k_1, k_2 ... k_5$ (kN/m)	Normal	6,762	1,352	[4,057, 8,114]
$\zeta_1, \zeta_2 ... \zeta_5$	Normal	0.05	0.02	[0.02, 0.10]

Table 10 Results of reliability analysis for the building

Method	P_f	Number of samples	Number of evaluations of actual performance
MCS (this work)	0.0310	10,000	10,000
	0.0328	20,000	20,000
	0.0330	30,000	30,000
	0.0339	40,000	40,000
	0.0333	50,000	50,000
Proposed method	0.0307	10,000	2,833
	0.0319	20,000	4,290
	0.0316	30,000	5,456
	0.0322	40,000	6,515
	0.0313	50,000	7,411

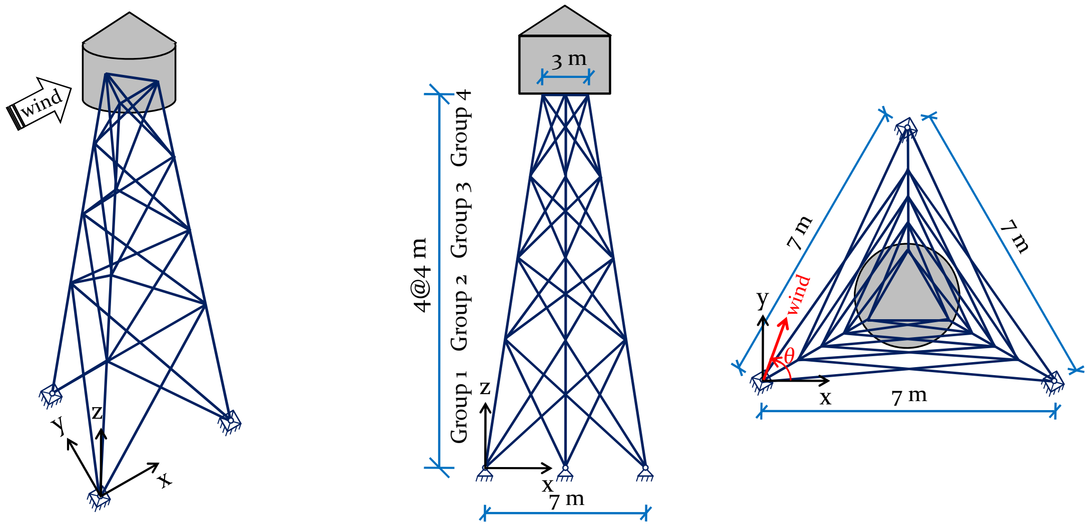
Table 11 Random variables of 3D truss structure

Variable	Distribution type	Mean value	Standard deviation	Truncation range
F_V (kN)	Normal	360	30	[240, 420]
F_H (kN)	Normal	180	40	[100, 260]
θ (rad)	Normal	$\pi/2$	$\pi/18$	[$\pi/3$, $2\pi/3$]

The reliability results of this example using MCS and proposed method (with TCR=0.45, ε=0.0002, n=3 and 1000 SFDs produced by making use of the second procedure described in subsection 2.3), are listed in Table 10. As it is seen from this Table, the proposed method finds P_f with 6% error compared to MCS, however, needs only 15% of actual performance evaluations which are required in MCS. If one solves this problem via MCS with only 7,411 samples, would find different probabilities of failure for different seeds of random sets. For example, for two different seeds we found 0.0301 and 0.0383 with −11% and 13% error, respectively. This is due to the fact that the proposed method is not sensitive to seed of random generation. Moreover, as it was shown in previous example by tuning the parameters of proposed method one can find optimal balance between accuracy and efficiency of the method.

4.6 Example 6: a 3-dimensional 39-bar truss (a RBDO problem)

In this example, a RBDO problem is solved using the RBDO algorithm proposed in Section 3. Consider the truss structure of Fig. 10, which supports an elevated cylindrical reservoir. The total weight of reservoir and its content, denoted as F_V, is assumed to be equally carried by three top nodes of the truss (i.e., $F_V/3$ for each node). In addition to F_V, it is assumed that the speed of wind and its direction be non-deterministic variables. The uncertainty in the speed of wind, herein, is seen as variations of wind load denoted by F_H, which is assumed to be carried by the top nodes



(a) 3D view (b) Side view (c) Top view

Fig. 10 Three-dimensional 39-bar truss

Table 12 Standard Pipe sections list of AISC

Section no.	AISC_Label	w (kg/m)	A_g (mm^2)	r (mm)
1	Pipe8XXS	108	12900	70.6
2	Pipe12XS	97.4	11300	110
3	Pipe10XS	81.5	9740	92.5
4	Pipe6XXS	79.1	9480	52.8
5	Pipe12STD	73.8	8840	112
6	Pipe8XS	64.5	7680	73.4
7	Pipe10STD	60.2	7420	93.5
8	Pipe5XXS	57.4	6900	44.2
9	Pipe8STD	42.5	5060	74.9
10	Pipe6XS	42.5	5050	55.9
11	Pipe4XXS	41.0	4940	35.3
12	Pipe5XS	30.9	3700	47.0
13	Pipe6STD	28.3	3350	57.2
14	Pipe3XXS	27.7	3340	26.9
15	Pipe4XS	22.3	2670	37.6
16	Pipe5STD	21.7	2590	47.8
17	Pipe2-1/2XXS	20.4	2470	21.7
18	Pipe3-1/2XS	18.6	2210	33.3
19	Pipe4STD	16.1	1910	38.4
20	Pipe3XS	15.3	1830	29.0
21	Pipe2XXS	13.4	1620	18.1
22	Pipe3-1/2STD	13.6	1610	34.0
23	Pipe2-1/2XS	11.4	1350	23.6
24	Pipe3STD	11.3	1340	29.7
25	Pipe2-1/2STD	8.62	1040	24.2

equally (i.e., each one carries $F_H/3$). Moreover, the direction of wind is specified by the angle θ, measured from positive part of x-axis in counter-clock-wise (C.C.W.) manner (see Fig. 10). More details on these truncated random variables can be observed from Table 11.

Then, the problem can be formulated as follows: Minimize the weight of the structure by selecting suitable sections for its members, such that, its probability of failure remains smaller than an allowable value, i.e., $P_f^{structure} \leq P_f^{allowable} = 0.01$. In this problem, design variables are the cross-sections of members of the structure. For ease in construction, these 39 cross sections are grouped into four groups with 9 (for group numbers 1, 2 and 3), and 12 (group 4) members. During optimization, these sections are chosen among AISC standard Pipe sections list, which are presented in Table 12.

The weight of the truss can simply be computed as

$$W_{structure} = \sum_{i=1}^{39} L_i w_i \tag{14}$$

where, L_i and w_i are, respectively, length of i-th member and its weight in unit length (presented in Table 12). This weight will be multiplied by the penalty function value as follows

$$W_{structure}^{penalized} = W_{structure} \times [1 + C \times \max\left(0, P_f^{structure} - P_f^{allowable}\right)] \tag{15}$$

where, C is a penalty constant taken equal to 100 in this example. Doing so, the weight value is increased for those designs in which optimization constraints are violated. In this way, the constrained optimization problem is converted to an unconstrained one.

On the other hand, by assuming the structure as a series system, its probability of failure ($P_f^{structure}$) is computed using following limit-states:

a) The displacement of each node in x, y and z directions must be smaller than 200 mm.

b) The axial force carried by each member must be smaller than the corresponding axial load-carrying-capacity for that member. This latter is computed based on AISC (2010) code for tensile members as

$$Tensile\ strength = 0.9 F_y A_g \tag{16}$$

while, for compressive members following formulae (AISC 2010) are used

$$Compressive\ strength = 0.9 F_{cr} A_g \tag{17}$$

$$F_{cr} = \begin{cases} \left[0.658^{\frac{F_y}{F_e}}\right] F_y & when\ \frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}} \\ 0.877 F_e & when\ \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} \end{cases} \tag{18}$$

$$F_e = \pi^2 E \Big/ \left(\frac{KL}{r}\right)^2 \tag{19}$$

In Eqs. (16)-(19), A_g is the cross-sectional area of the member; F_y is the yield stress taken as 248.2 MPa; K is the effective length factor of the member (equal to unity for all the 39 members of

Table 13 The convergence history of RBDO problem and the corresponding reliability results

Iteration no.	Global best sections no.	Global best cost (kg)	P_f (MCS)	P_f (RFE)	Reduction of actual performance evaluations
1, 2	[5, 7, 13, 11]	11,007	0.00000	0.00000	97.4%
3	[6, 6, 13, 12]	10,160	0.00000	0.00000	97.0%
4	[9, 9, 13, 11]	8,256	0.00000	0.00000	97.4%
5	[9, 9, 13, 12]	7,720	0.00000	0.00000	96.9%
6, … 14	[10, 9, 13, 16]	7,232	0.00000	0.00000	94.1%
15, … 18	[9, 9, 12, 19]	7,060	0.00016	0.00016	92.7%
19, 20	[10, 9, 13, 19]	6,935	0.00088	0.00088	91.3%

this example); E is modulus of elasticity equal to 2.1×10^5 N/mm^2; L is unbraced length of the member and r is its radius of gyration (available in Table 12).

In addition to these constraints, according to AISC (2010) specifications, the effective slenderness ratio (*KL/r*) for tensile and compressive members should not exceed 200 and 300, respectively.

To solve the RBDO problem of this example, we use HS-PSO algorithm with *HMCR*=0.7 and *PAR*=0.4 (Hadidi and Rafiee, 2014). The number of individuals (particles) is taken as 30, while, the algorithm is terminated when 20 iterations are performed. Then, during optimization, 20×30=600 points of design space are examined and their probabilities of failure are computed. For each of these reliability analyses, 20,000 samples are used for MCS and the proposed method (with TCR=0.5, $\varepsilon=10^{-4}$, n=3 and 5^3=125 SFDs). It is observed from the results that for same seed of random sets, the proposed RBDO algorithm gives the same results of RBDO with MCS. This optimal design for a seed is obtained as [Pipe6XS, Pipe8STD, Pipe6STD, Pipe4STD] for both of methods, whereas, the method based on MCS requires $600\times20{,}000=1.2\times10^7$ evaluations of actual performance and our method needs 1,109,572 evaluations. So, it is observed that the proposed RBDO algorithm gives same results with only 9.2% of actual performance evaluations needed in RBDO based on MCS. This percent can further be reduced by choosing optimal values for the parameters of the proposed method. The convergence history of RBDO problem is compared for two methods by tabulating details of the global best design (the optimal design up to that iteration) of iterations of the optimization in Table 13. In this Table, the reliabilities of global best designs are re-analyzed with 50,000 samples to provide better comparison, while, it was seen that 20,000 samples are sufficient for this RBDO problem with $P_f^{structure} \le P_f^{allowable} = 0.01$ limitation.

5. Conclusions

Uncertainty exists in values of parameters of systems in dealing with engineering problems. In structural systems, in particular, uncertainties in acting loads and load-carrying-capacity of the structure should be taken into account in structural design process, to design a reliable structure. Considering these uncertainties is of paramount importance especially in minimum cost design problems, where, the designer seeks for a design in which the load-carrying-capacity of the structure is used as far as possible. In such a case, the structure may be designed with a significant probability of failure. Hence, to keep a cost-safety balance, reliability-based design optimization

(RBDO) formulations are given rise. This paper proposes a novel simulation method for reliability analysis and RBDO, advantages of which can be summarized as follows

• It is an accurate method which computes different reliability measures (reliability index, most probable point (MPP) and probability of failure) with high accuracy compared to first-order reliability (FORM), response surface (RSM) and direct-Monte Carlo simulation (MCS) methods.

• It is an efficient method in comparison with MCS, such that, the number of required evaluations of actual performance function in it, is far less than which is needed in MCS.

• It can be tuned, so, one can tune its parameters (TCR and ε) to provide a proper balance between its accuracy and efficiency. This advantage of the proposed method may also be seen as a disadvantage, since this tuning may be time-consuming for real expensive-to-evaluate systems, when the user wants to keep both the high accuracy and maximum efficiency in hand.

• It is a robust method proved to be of high performance in dealing with different problems of reliability analysis.

• It is also a reliable method for analysis of reliability, for two reasons: One reason is the fact that the proposed method does not need MPP to be known for simulation, whereas, in many well-known reliability methods like Importance sampling, the reliability results are computed based on MPP, hence, an error in finding MPP will directly result in error in reliability results. The other reason refers to the fact that although this method needs small number of samples to be evaluated exactly using actual performance function, is not sensitive to seeds of random sets, in comparison to the MCS with same number of evaluations.

• The proposed method for reliability analysis can be used in combined with efficient optimization algorithms to solve RBDO problems, as well.

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