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Torsion strength of single-box multi-cell concrete box girder subjected to combined action of shear and torsion

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Abstract. A model has been proposed that can predict the ultimate torsional strength of single-box multi-cell reinforced concrete box girder under combined loading of bending, shear and torsion. Compared with the single-cell box girder, this model takes the influence of inner webs on the distribution of shear flow into account. According to the softening truss theory and thin walled tube theory, a failure criterion is presented and a ultimate torsional strength calculating procedure is established for single-box multi-cell reinforced concrete box girder under combined actions, which considers the effect of tensile stress among the concrete cracks, Mohr stress compatibility and the softened constitutive law of concrete. In this paper the computer program is also compiled to speed up the calculation. The model has been validated by comparing the predicted and experimental members loaded under torsion combined with different ratios of bending and shear. The theoretical torsional strength was in good agreement with the experimental results.

Keywords: reinforced concrete; multi-cell box girder; torsional strength; softening truss theory; failure criterion

1. Introduction

Reinforced concrete box girder, with the characteristic of closed section, has excellent spatial mechanical behavior. Therefore, they are widely used in bridge structure. Normally, the reinforced concrete box girder mainly supports the transverse loading through the flexural moment and the vertical shear. However, in some cases, torsional response of girders may also control the overall structure behavior (Fuad 2012), such as curved bridges, spandrel beams and the box girder widened by the steel cantilever (Wang 2011). For this reason, the torsional strength of box girders should be studied and comprehended as well as its shear or flexural strength.

Many researches have been conducted on the torsion strength of single-cell box girder or rectangular solid girder. The representative theoretical approaches for the prediction of the torsional strength are the elastic theory (Popov 1990), the plastic theory (Nilson 1985, Popov 1990), the space truss theory (Collions 1972); and skew bending theory (Elfgren 1974). Based on

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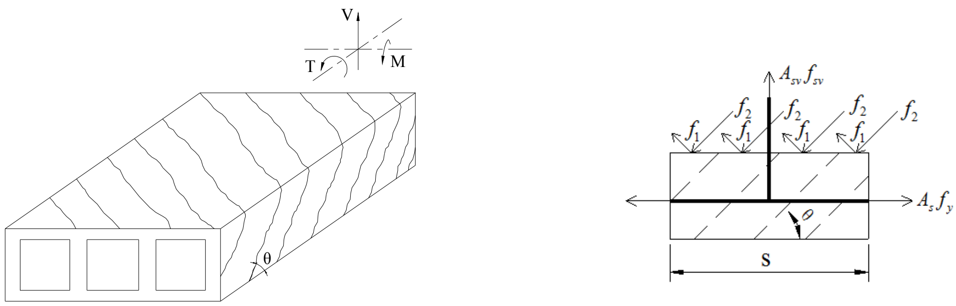
plastic theory, Wang and Hsu (1997) proposed a limit analysis method for predicting the ultimate strength of prismatic square reinforced concrete beams subjected to pure torsion. Greene and Belarbi (2006, 2009) extended the softened truss model and developed combined-action softened truss model that can predict the load-deformation response of a reinforced concrete member subjected to torsion combined with bending and shear to spalling or ultimate capacity. Based on the rotating angle softened truss model, Mullapudi and Ayoub (2009) developed a fiber beam element to analyze the reinforced concrete structure with combined loadings. The proposed model is able to capture the torsional failure and provides reasonable results. Husem gave a formulae to calculate the cracking moments of reinforced concrete beams under the effect of pure torsion, in his formulae, the torsional moment carried with whole section is composed of the torsional moment for plain concrete, torsion carried by longitudinal reinforcement and torsion carried by transverse reinforcement (Husem *et al*. 2011). Ju *et al*. (2013) proposed a tensile behavior model of steel fiber reinforced concrete members, and the proposed constitutive model was applied to the modified fixed-angle softened truss model in order to estimate the torsion behavior of steel fiber reinforced concrete members. According to the skew bending model, Wafa and Akhtaruzzaman (1989) built the strength equations to predict the torsional strength of prestressed concrete beams containing a circular transverse opening and subjected to combined torsion and bending. Huang and Liu (2006) developed a modified skew bending model to calculate the load- carrying capacity of segmental bridges subjected to combined bending, shear and torsion. These studies contribute to the development of the torsion theory of reinforced concrete box girder. However, In the current studies, the study on the torsional strength of multi-cell concrete box girder is scant.

The current codes (ACI 318-05 (ACI Committee 2005), EC2 (Eurocode 2 2002), JTG D62-2004 (MOT 2004)) for torsional design of reinforced concrete box girders have been developed based on the spaced truss analogy and the thin walled tube theory. The torsional strength calculate by the current codes is limited to the single-box girder that the effective thickness of top or bottom slab is greater than 0.1 times of box girder's width and effective thickness of web is greater than 0.1 times of box girder's height (JTG D62-2004). As to wide multi-cell box girder, the methods in these codes are not applicable.

Based on the existed studies, this paper focuses on the torsional strength of multi-cell concrete box girder subjected to combined loads. The model is depending on the softened truss model which is widely adopted in the torsion analysis of the reinforced concrete member after cracking (Hsu 1988, 1998, Yi 2012) and thin walled tube theory. The model considers the effect of tensile stress among the concrete cracks and idealizes the applied loads are distributed to the panels as uniform shear stress. The angle of diagonal struts can be calculated according to conformability of deformation and the relationship of girder's stress and strain conforms with the Mohr's circle. Compared with the single-cell box girder, this model takes the influence of the inner web on the distribution of shear flow into account. By simplifying, the model also can apply to single-cell box girder.

2. Analysis model

Based on the softened truss theory, the reinforced concrete box girder can be seen as a space truss which is consist of longitudinal reinforcement, stirrup and concrete diagonal struts after cracking. Meanwhile, The softened truss theory uses average stress-strain relationships to model the constitutive material laws for concrete and reinforcement. The tensile stress between the



(a) Cracked theory model (b) Force diagram

Fig. 1 Truss model for reinforced concrete multi-cell box girder under combined loading

concrete cracks and the reduction of the peak compressive stress of concrete due to the net tensile strain in the reinforcement acting orthogonal to the concrete strut, known as concrete softening, are considered in the model. This model also assumes that the torque resisted by the section acts as shear stress that flows around the perimeter of the section as described by Bredt's thin-tube theory.

Under bending, shear and torsion, the units in the reinforced concrete member are all in the state of triaxial stress. To simplify the analysis, we assume any unit is in the longitudinal uniaxial stress state (System I) and transverse biaxial stress state (System II), respectively (Rahal and Collins 1995, Xu 2000). In system I, the section is idealized to carry the longitudinal force caused by the axial force, moment and shear. As to System II, the section is idealized to carry shear and torsion. The two systems are connected by the longitudinal strain and the longitudinal stress generated by shear and torsion. In order to get satisfactory results, the model still needs to comply with the equilibrium condition, deformation compatibility condition, constitutive equation of steel and concrete and failure criterion.

For multi-cell box girder, the inner webs scatter the distribution of shear flow. Under the same loading of shear and torsion, the nominal shear stress in the external web of the multi-cell box girder will be obviously smaller than that of the single-cell box girder which has the same outline dimensions. The cracked truss model for the multi-cell box girder is shown in Fig. 1.

3. Equilibrium equations in the biaxial stress state

The shear force V and T torsion induce shear flow in each web of the box girder. When the girder is about to destroy, the nominal shear stress v in the external web can be expressed as

$$v = V_u \tau_V + T_u \tau_T \tag{1}$$

In Eq. (1), V_u is the ultimate shear force, T_u is the ultimate torsion, τ_V ($\tau_V = \int_l \dot{q}_Q / t \cdot l$) and τ_T ($\tau_T = \int_l \dot{q}_T / t \cdot l$) are the average shear stress caused by unit shear and unit torsion, where the shear flow $\dot{q}_Q$ and $\dot{q}_T$ due to unit shear and unit torsion can be calculated by the thin-walled box girders theory and $\dot{q}_Q$ is adopted for the approximations of the torsion shear flow. While t and l

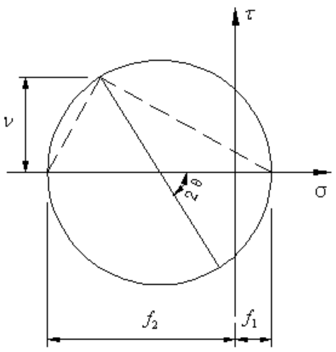


Fig. 2 Mohr's circle for average stresses

stand for the thickness and length of the external web, respectively. Considering the influence of inner webs, the shear flow ($\dot{q}_Q$ and $\dot{q}_T$) in the outer web of the multi-cell box girder is smaller than the single-box girder with the same external profile, because the inner webs share the shear.

This model considers the effect of tensile stress among the concrete cracks and assumes the cracks are at angle θ to the x axis, which are normal to the principle tensile stress f_1 and parallel to the principal compressive stress f_2, as can be seen in Fig. 1.

By Mohr's circle for average stresses shown in Fig. 2, the relationship of f_1, f_2 and nominal shear stress v is deduced in Eq. (2).

$$f_1 + f_2 = (\tan\theta + \cot\theta)v \tag{2}$$

As shown in Fig. 1, the vertical component of diagonal tension and pressure can be balanced by the tension of stirrup. The equilibrium relationship is given by Eq. (3), where A_{sv} is the area of stirrup, f_{sv} is the yielding strength of stirrup, and s is the stirrup spacing.

$$A_{sv}f_{sv} = (f_2\sin^2\theta - f_1\cos^2\theta)\cdot ts \tag{3}$$

Eq. (4) is deduced by Eqs. (1) and (5) is got from Eq. (3).

$$v = (f_1 + f_2)\cdot\sin\theta\cdot\cos\theta \tag{4}$$

$$f_2\sin\theta\cos\theta = \frac{A_{sv}f_{sv}\cot\theta}{t\cdot s} + f_1\cos^2\theta\cot\theta \tag{5}$$

Putting Eq. (5) into Eq. (4), the nominal shear stress v is expressed as Eq. (6).

$$v = f_1\cot\theta + \frac{A_{sv}f_{sv}\cot\theta}{t\cdot s} \tag{6}$$

According to Eq. (1) and (6), Eq. (7) is deduced. The ultimate torsion for the multi-cell box girder is shown in Eq. (8), where η is the torsion-shear ratio ($\eta = T/Vb$).

$$V_u\tau_V + T_u\tau_T = f_1\cot\theta + \frac{A_{sv}f_{sv}\cot\theta}{t\cdot s} \tag{7}$$

$$T_u = \frac{\eta b}{\tau_V + \eta b \tau_T} \cdot (f_1 \cot\theta + \frac{A_{sv} f_{sv} \cot\theta}{t \cdot s}) \tag{8}$$

Thus, if f_1 and θ are known, the ultimate torsion T_u can be calculated by Eq. (8). The principle tensile stress f_1 can be got by compatibility equations and the constitutive laws of materials.

4. Stress state stress-strain relationship of concrete

4.1 principle tensile stress-strain relationship of cracked concrete

Vecchio and Collins (1986, 1996) determined the stress-strain relationships for the cracked concrete by testing 30 reinforced concrete panels under biaxial stress including pure shear. According to their research, after cracking, the principle tensile stress f_1 is related to the principle tensile strain ε_1 as Eq. (9), where f_{cr} is the cracking stress of concrete and equal to $0.33\sqrt{f_c^{'}}$ (MPa), and $f_c^{'}$ is the maximum cylinder strength of concrete.

$$f_1 = \frac{f_{cr}}{1+\sqrt{500\varepsilon_1}} \tag{9}$$

The relationship of the $f_c^{'}$ and the concrete cubic compression strength f_{cr} specified in JTG D62-2004 is given by Eq. (10).

$$f_c^{'} = 0.79 f_{cu} \tag{10}$$

Thus, the Eq. (9) can be changed as

$$f_1 = \frac{0.33\sqrt{0.79 f_{cu}}}{1+\sqrt{500\varepsilon_1}} \tag{11}$$

4.2 Softened principle compressive stress-strain relationship of cracked concrete

Due to the net tensile strain in the reinforcement acting orthogonal to the concrete strut, the peak compressive stress reduced, which is known as concrete softening. The cracked concrete is softer and weaker than concrete in a standard cylinder test. The principle compressive stress f_2 in the concrete is found to be a function not only of the principal compressive strain ε_2 but also of the co-existing principal tensile strain ε_1 (Vecchio and Collins 1986, 1996). The relationship is suggested as

$$f_2 = \xi f_c^{'} \cdot [2(\frac{\varepsilon_2}{\varepsilon_c^{'}}) - (\frac{\varepsilon_2}{\varepsilon_c^{'}})^2] \tag{12}$$

where $\varepsilon_c^{'}$ is the concrete strain at the peak compression stress, and the ξ is softening coefficient and determined by

$$\xi = \frac{1}{0.8+170\varepsilon_1} \tag{13}$$

When the average principle compressive stress f_2 equal or more than $\xi f'_c$, the girder is thought to get the ultimate capacity.

5. Strain compatibility equation

According to the principle of minimum potential energy, the angle of cracks should parallel to the principle compression strain. By Mohr circle for average strain (as shown in Fig. 3) and the trigonometric function formula, the relationship of longitudinal strain ε_x, transverse strain ε_y, principle tensile strain ε_1, principle compressive strain ε_2 and the angle θ of diagonal crack for cracked concrete member is shown in Eqs. (14)-(15).

$$\cos 2\theta = \frac{\frac{\varepsilon_y - \varepsilon_x}{2}}{\frac{\varepsilon_1 - \varepsilon_2}{2}} = \frac{\varepsilon_y - \varepsilon_x}{\varepsilon_1 - \varepsilon_2} = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} \tag{14}$$

$$\varepsilon_x + \varepsilon_y = \varepsilon_1 + \varepsilon_2 \tag{15}$$

From Eq. (14), Eq. (16) is got.

$$(\varepsilon_y - \varepsilon_x + \varepsilon_1 - \varepsilon_2)\tan^2\theta = \varepsilon_1 - \varepsilon_2 - \varepsilon_y + \varepsilon_x \tag{16}$$

Based on Eqs. (15)-(16), we can got

$$\varepsilon_y - \varepsilon_2 = \varepsilon_1 - \varepsilon_x \tag{17}$$

$$2(\varepsilon_y - \varepsilon_2)\tan^2\theta = 2(\varepsilon_x - \varepsilon_2) \tag{18}$$

Bring Eq. (17) into Eq. (18), Eq. (19) can be derived, which gives the expression for ε_1.

$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x - \varepsilon_2)\cot^2\theta \tag{19}$$

When the girder is about to destroy, the principle compression ε_2 reaches the ultimate value ε_u. From the point of safety, the longitudinal strain ε_x of reinforced concrete box girder under

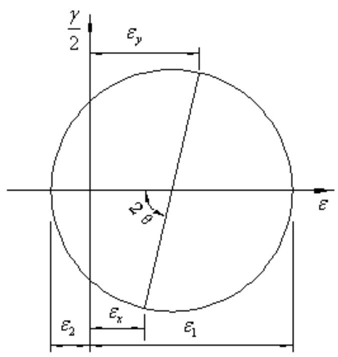


Fig. 3 Mohr's circle for average strains

combined loading of bending, shear and torsion is expressed as the bigger strain of the steel in the tensile region ε_{xl} and that in the compression region ε_{xa}.

$$\varepsilon_x = \max\{\varepsilon_{xl}, \varepsilon_{xa}\} \tag{20}$$

The longitudinal strain ε_{xl} of the steel in the tensile region and the longitudinal strain ε_{xa} of the steel in the compression region are given by Eqs. (21) and (22) (Collins 1991), where M_u is ultimate bending moment ($M_u=T_u/\varphi$, φ is the torsion-bending ratio), z stands for the distance between the tensile reinforcement and compressive reinforcement, E_s is the elastic modulus of steels, A_s and $A_s^{'}$ represent the area of steels in the tensile region and compression region respectively, and N_v is the longitudinal tension caused by the shear and torsion, which can be expressed by Eq. (23) (Collins 1991).

$$\varepsilon_{xl} = \frac{\frac{M_u}{z} + 0.5N_v}{E_s A_s} \tag{21}$$

$$\varepsilon_{xa} = \frac{-\frac{M_u}{z} + 0.5N_v}{E_s A_s^{'}} \tag{22}$$

$$N_v = \cot\theta (V_u + \frac{T_u U_{cor}}{2A_{cor}}) \tag{23}$$

In Eq. (23), A_{cor} is the area enclosed by the hear flow centerline on the outer ring of the section and the U_{cor} is the perimeter of which.

6. Failure criterion

Under combined loading of bending, shear and torsion, suppose the stirrups in the outer web yield first, there are three failure modes for the single-box multi-cell reinforced concrete box girder:

(a) The first one is called rare-reinforced failure. In this case, the longitudinal reinforcement yields, while the principle compression of concrete doesn't reach the softened ultimate compression strength, which can be expressed by $\varepsilon_x \approx \frac{f_s}{E_s}$, $f_2 < f_{2\max}$, where f_s is the yielding strength of longitudinal reinforcement.

(b) The second is balanced-reinforced failure. In this case, the longitudinal reinforcement yields and at the same time the principle compression of concrete reaches the softened ultimate compression strength, which can be expressed by $\varepsilon_x \approx \frac{f_s}{E_s}$, $f_2 \approx f_{2\max}$.

(c) The third is called over-reinforced failure. In this case, the longitudinal reinforcement doesn't yield, while the principle compression of concrete reaches the softened ultimate compression strength, which can be expressed by $\varepsilon_x < \frac{f_s}{E_s}$, $f_2 \approx f_{2\max}$.

7. Solution procedure

Based on the theory analysis mentioned above, the torsion strength of multi-cell concrete box girder under combined loading of bending, shear and torsion can be calculated as follows.

First, calculate τ_V and τ_T by the section property; Second, assume θ and T_u (use $T_{u\text{-}asu}$ to stand for the assumed value), and then calculate ε_x and ε_1 by Eqs. (17)-(21); third, depending on ε_1, derive f_1 by Eq. (11); fourth, solve T_u (used $T_{u\text{-}cal}$ to stand for the calculated value) according to Eq. (8), adjust $T_{u\text{-}asu}$ until the $T_{u\text{-}cal}$ is equal to the $T_{u\text{-}asu}$; fifth, judge whether the girder meets the failure criterion, if not, reassume θ until $\varepsilon_x \approx \dfrac{f_s}{E_s}$ or $f_2 \approx f_{2\max}$. At this time, T_u reaches the torsion strength of multi-cell box girder. The solution flow chart is shown in Fig. 4.

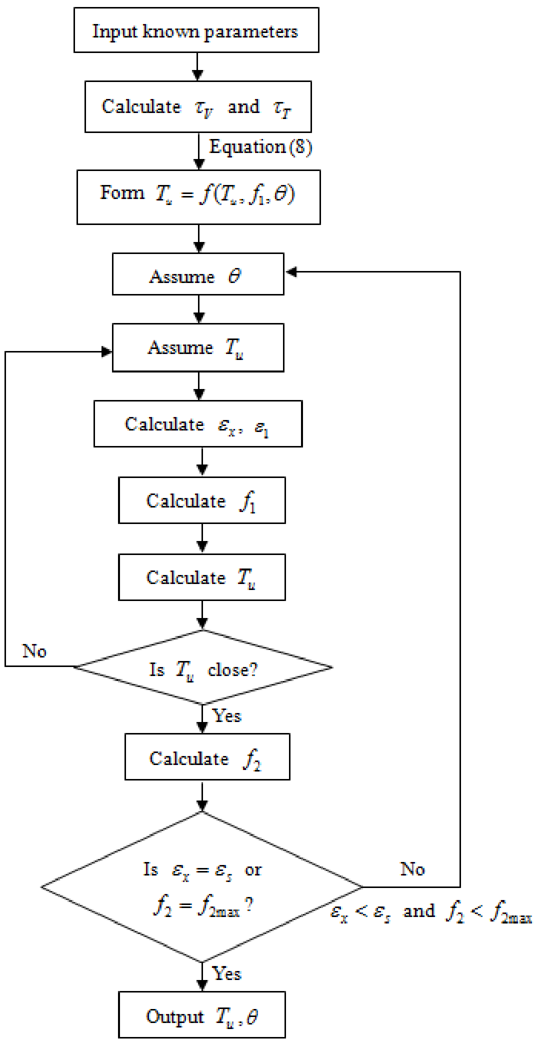


Fig. 4 Solution flow chart for torsional strength of multi-cell box girder

8. Comparison of experimental and predicted values

The proposed model is validated by comparing the predicted torsional strength to the experimental values.

The cross section of beams is shown in Fig. 5, which simulates an existing box girder located in Dalian, China. The reinforcement ratios of longitudinal reinforcement and stirrups are the same to the existing bridge.

The average concrete strength is 32 MPa and the reinforcement yield strength is 340 MPa. The test beams with three points simply support are loaded by eccentric concentrated load and the transverse loading point is selected on the web next to the middle web as shown in Fig. 6. By adjusting the longitudinal position of the loading point, the different ratios of torsion and bending are gotten as shown in Table 1. The strain and displacement of key points are measured, by comparing with the theoretical values, the results shows that in the elastic stage, the experimental values tested agree with the theoretical values, and the maximum error is lower than 8%, which stands the numerical model is suitable to the test beams.

During the loading process, the external web with the effect of superimposing by shear and torsion starts to crack, and then the cracks extend to the top and bottom slab of box girder. The cracks on the external webs develop homogeneously, which are inclined at approximately 40~60° to the beam axis. The cracks on the external web where shear and torsion subtracted are rare. The cracks on the top slab of specimen 3 are most and occur earliest. A trend appears that with the decrease of torsion-bending ratio (φ), the cracks on the top slab decrease. When the cracking sound is detected, we remove the strain and displacement gauges and increase the load gradually. Based

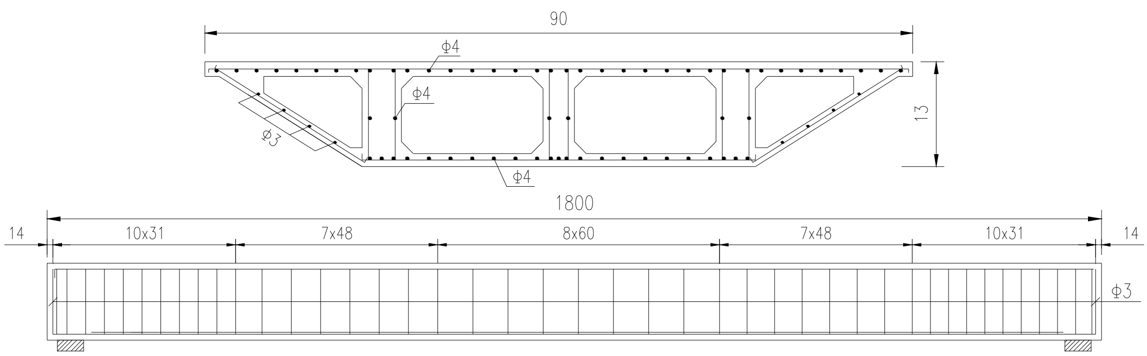


Fig. 5 Overall dimensions of beams (Unit: cm)

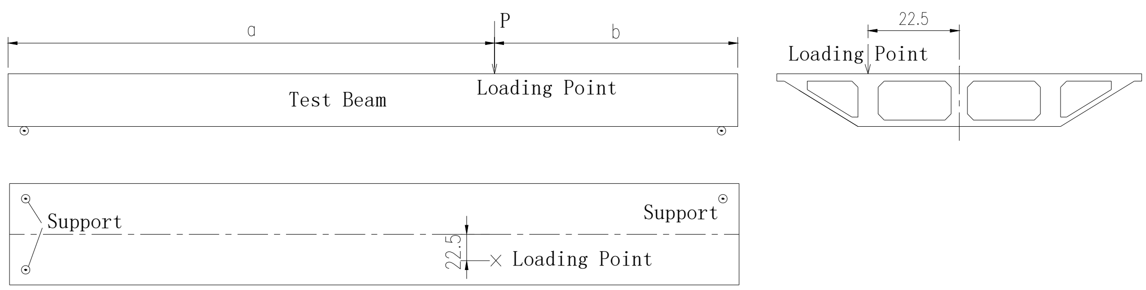


Fig. 6 Loading schematic diagram (Unit: cm)

Table 1 Ultimate torque comparison of theoretical value and experimental results (Unit: KN·m)

Test specimen	φ	η	f_{cu} (MPa)	θ (rad)	T_{ucal}	$T_{u\,\exp}$	$T_{ucal}/T_{u\,\exp}$
1	0.250	0.328	32	1.139	4.03	5.34	0.755
2	0.375	0.328	31.3	1.009	5.50	6.73	0.817
3	0.50	0.328	32.5	0.905	6.87	8.28	0.830

Table 2 Ultimate torque comparison of theoretical value and experimental results (Unit: KN·m)

Test specimen	φ	η	f_{cu} (MPa)	θ (rad)	T_{ucal}	$T_{u\,\exp}$	$T_{ucal}/T_{u\,\exp}$
1	0.535	2.677	43.9	0.564	41.52	58.9	0.705
2	0.535	2.677	41.2	0.572	40.81	58.9	0.693
3	0.535	2.677	35.7	0.592	39.06	54.4	0.718
4	0.535	2.677	50.2	0.544	43.46	54.4	0.799
5	0.535	2.667	40.0	0.579	40.15	58.9	0.682
6	0.285	1.426	39.1	0.59	33.07	35.2	0.940
7	0.285	1.426	28.0	0.643	29.53	36.1	0.818
8	0.285	1.426	41.5	0.574	31.55	38.5	0.819
9	0.255	1.18	33.2	0.613	26.80	35.3	0.760
10	0.255	1.18	30.9	0.627	28.53	38.9	0.733
11	0.255	1.18	40.8	0.582	31.39	34.4	0.912
12	0.144	0.45	52.4	0.542	18.37	20.5	0.896

on three groups of destruction tests of multi-cell box girder under combined loads, the failure load of each group are gotten as 23.7 KN, 29.9 KN and 36.8 KN, respectively. The corresponding ultimate torque are 5.34 KN·m, 6.73 KN·m and 8.28 KN·m. The results show that the torsion strength of specimens increases with the increase of torsion-bending ratio.

The ratio of the predicted to the experimental value is calculated for each specimen as shown in Table 1. The values in the table show that the model proposed made close predictions of the ultimate torque, with average $T_{ucal}/T_{u\,\exp}$ ratio of 0.801. According to the failure criterion, the multi-cell box girder tested satisfies the condition of balanced-reinforced failure. When the ultimate torsion is reached, both of the criterions are satisfied simultaneously ($\varepsilon_x=\varepsilon_s$, $f_2=f_{2\max}$).

The model proposed is also applicable to calculate the torsion strength for single-cell box girder under combined loads, which is validated by comparing the predicted and experimental results of 12 specimens available in Xu's literature (Xu 2000) as shown in Table 2.

As can be seen in Table 2, based on the comparison between the torsional strengths of 12 test beams reported in Xu's literature and those obtained by Eq. (8) and solution flow detailed this paper, it is demonstrated that the theoretical results showed reasonable agreement with the test results, with the average $T_{ucal}/T_{u\,\exp}$ ratio of 0.79. In the calculate process, all the 12 specimens failure mode are rare-reinforced failure by the failure criterion, which is consistent with the description of Xu's (2000) paper.

During the calculation, with the decrease of θ, f_2 and ε_x increase, while $f_{2\max}$ decreases. A effect is when θ is smaller than 15°, f_2 will be much greater than $f_{2\max}$ and ε_x is also much bigger than ε_s. Thus, θ is suggested to be assumed greater than 45° at the start to ensure $\varepsilon_x<\varepsilon_s$ and $f_2<f_{2\max}$, and then adjust T_{uasu} equal to the T_{ucal}. The increase of T_u bring the increase of f_2 and ε_x and the decrease of $f_{2\max}$. The proper assumption of θ is important, which help to got the ultimate torsion T_u efficiently.

9. Conclusions

Based on the softened truss theory and thin walled tube theory, a model is proposed to predict the torsion strength of multi-cell box girder subjected to combined loading of bending, shear and torsion. The proposed model considers the influence of inner webs on the distribution of shear flow and the effect of tensile stress among the concrete cracks into account. Applying the principle of minimum potential energy, the angle of diagonal crack for cracked concrete is calculated by strain compatibility condition. Supposing the stirrups in the outer web yield first, three failure modes for the single-box multi-cell reinforced concrete box girder are established. According to the theory model, the solution flow is detailed and a computer program is compiled to speed up the calculation. Comparisons between the test and calculated torsional moments of the multi-cell box girder detailed in this paper and 12 beams reported in other literature shows reasonable agreement.

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