Structural Engineering and Mechanics, *Vol. 55, No. 4 (2015) 687-702*
DOI: http://dx.doi.org/10.12989/sem.2015.55.4.687

Simultaneous identification of stiffness and damping based on derivatives of eigen-parameters

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(Received January 12, 2015, Revised April 8, 2015, Accepted May 15, 2015)

Abstract. A method based on derivatives of eigen-parameters is presented for damage detection in discrete systems with dampers. The damage is simulated by decrease on the stiffness coefficient and increase of the damping coefficient. In the forward analysis, the derivatives of eigen-parameters are derived for the discrete system. In the inverse analysis, a derivative of eigen-parameters based model updating approach is used to identify damages in frequency domain. Two numerical examples are investigated to illustrate efficiency and accuracy of the proposed method. Studies in this paper indicate that the proposed method is efficient and robust for both single and multiple damages and is insensitive to measurement noise. And satisfactory identified results can be obtained from few numbers of iterations.

Keywords: damage detection; stiffness; damping; derivatives of eigen-parameters; model updating; discrete system

1. Introduction

Some forms of damage (such as corrosion, fatigue induced crack and creep, etc.) cause a change in the structural physical parameters (stiffness or damping, etc.). And the change of stiffness or damping would affect the modal parameters of a structure. So estimation of the modal parameters can find such localized damages. Among the numerous researches of damage detection, the techniques based on damage-induced changes in the modal parameters have gained increasing attention in the last few years (Cawley and Adams 1979, Salawu 1997, Pandey *et al*. 1991, Nobahari and Seyedpoor 2013).

Hassiotis (1999) presented a method to find the location and magnitude of damage in a structure using data from dynamic tests. The method uses the sensitivity of the flexibility matrix to changes in the natural frequencies of the structure to identify the damage. A statistical damage identification algorithm based on frequency changes is developed to account for the effects of random noise in both the vibration data and finite element model by Xia and Hao (2003). Liu and Yang (2006) proposed a three-step (number of damaged elements, damage localization and

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

quantification) damage identification method, which makes use of a subset of measured eigenvalues and eigenvectors. The system matrices of a frame element are decomposed into their static eigenvalues and eigenvectors and their inspection is used to derive the damages by Wu and Law (2007). Frequency shifts for the first few fundamental modes is used to generate the Fourier coefficients of stiffness variation caused by damage, in a rod and beam element by Morassi (2008). Raghuprasad *et al*. (2008) presented the eigenvalue sensitivity equations, derived from first-order perturbation technique for typical infra-structural systems. Gopalakrishnan *et al*. (2010) proposed two methodologies for damage identification from measured natural frequencies of a contiguously damaged reinforced concrete beam, idealised with distributed damage model.

Eigen-pair derivatives have also been used in damage detection. Method of this kind can detect structural damage by directly using the incomplete modal parameters without any eigenvector expansion or model reduction, such as the eigenvalue sensitivity and the eigenvector sensitivity. Wong *et al*. (1995) proposed a perturbation method to detect damage of a multi-storey building by combining the eigenvalue sensitivity with the eigenvector sensitivity. Zhao and DeWolf (1999) investigated the sensitivity coefficient of the natural frequencies, mode shapes and modal flexibilities with respect to the elements of the stiffness matrix. Li *et al*. (2007) derived sensitivity coefficient expressions of modal parameters of shear-type frame structures for damage detection. Yang (2009) presented a mixed sensitivity method to identify structural damage by combining the eigenvalue sensitivity with the flexibility sensitivity.

Real vibration structures are approximately modeled as discrete systems. Discrete vibration systems are common in structural dynamics. In lots of situations, finite element method is used to model the dynamic behavior of the continuous system. Zhu *et al*. (2011) developed a method to identify damage in shear buildings using the change in the first mode shape slopes. Dilena and Morassi (2006) dealt with the identification of a single defect in a discrete spring-mass or beam-like system. The dynamics of a discrete structural system is characterized by its eigen-pairs, i.e., eigenvalues and eigenvectors. Parameter changes in the system will result in variations in its dynamic characteristics. The effective calculation of eigen-pair derivatives is essential in determining the impact of parameter changes upon the system's dynamic behavior.

To the best knowledge of the authors, most of the damage detection in frequency domain assume that the damages only caused by the changes in stiffness but neglecting the increase of damping. It often remains considerable doubt on how the damping behavior should be represented. In this paper, an eigen-pair derivatives based method is presented for damage detection. The advantage of the method is that it can consider the changes in stiffness and damping both and is very sensitive to damage. Two discrete systems are used in evaluating the proposed method. And the method has also been applied to the systems with dampers in which damage is represented by changes in both stiffness and damping coefficients. The effects of artificial measurement noise on the identified results are investigated. The proposed method is computationally efficient and numerically robust.

2. Theory

2.1 Mass-spring systems

A mass-spring-damper system consists of n masses m_i, (i=1,2,...,n), which are connected consecutively by linear elastic springs k_i, (i=1,2,...,n) and dampers c_i, (i=1,2,...,n), as shown in Fig. 1.

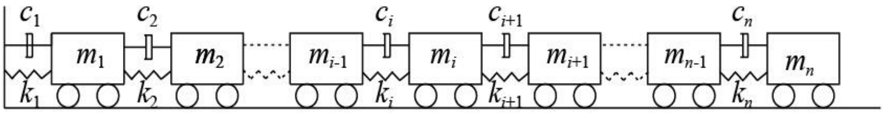

Fig. 1 Mass-spring system

The equation of motion for the discrete structural system can be expressed as

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{C}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{F}(t) \tag{1}$$

where $\ddot{\mathbf{d}}$, $\dot{\mathbf{d}}$ and $\mathbf{d}$ are the acceleration, velocity and displacement response vectors of the structure, respectively. $\mathbf{F}(t)$ is a vector of applied forces. The symmetrical matrix $\mathbf{M}$ is diagonal with mass values m_i, e.g., $\mathbf{M}$=diag $(m_1, m_2,...,m_n)$. $\mathbf{K}$ and $\mathbf{C}$ are the tri-diagonal positive semi-definite matrices as

$$\mathbf{K} = \begin{pmatrix} k_1+k_2 & -k_2 & 0 & \cdots & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & \cdots & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & k_{n-1}+k_n & -k_n \\ 0 & 0 & 0 & \cdots & -k_n & k_n \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} c_1+c_2 & -c_2 & 0 & \cdots & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & \cdots & 0 & 0 \\ 0 & -c_3 & c_3+c_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & c_{n-1}+c_n & -c_n \\ 0 & 0 & 0 & \cdots & -c_n & c_n \end{pmatrix} \tag{3}$$

Let the state vector be

$$\mathbf{x} = \begin{Bmatrix} \mathbf{d} \\ \dot{\mathbf{d}} \end{Bmatrix} \tag{4}$$

and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{n\times n} & \mathbf{I}_{n\times n} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{n\times 1} \\ \mathbf{M}^{-1} \end{bmatrix} \tag{5}$$

Eq. (1) can be rewritten as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{F} \tag{6}$$

2.2 Eigen-pair derivatives of the discrete structural system

For the simplicity, we assume the eigenvalues are distinctive. The situation of repeated eigenvalues can be found in Liu (2013). For the ith eigen-pair $\{\boldsymbol{\lambda}_i, \phi_i\}$ equation of the matrix **A**, we have

$$\mathbf{A}\boldsymbol{\phi}_i = \lambda_i \boldsymbol{\phi}_i . \tag{7}$$

By differentiating Eq. (7) with respect to a system parameter α

$$\frac{\partial \mathbf{A}}{\partial \alpha}\boldsymbol{\phi}_i + \mathbf{A}\frac{\partial \boldsymbol{\phi}_i}{\partial \alpha} = \frac{\partial \boldsymbol{\phi}_i}{\partial \alpha}\lambda_j + \boldsymbol{\phi}_i \frac{\partial \lambda_j}{\partial \alpha} . \tag{8}$$

For a proportionally damped system, we have

$$\boldsymbol{\phi}_j^{\mathrm{H}} \mathbf{W} \frac{\partial \boldsymbol{\phi}_j}{\partial \alpha} = 0 . \tag{9}$$

where **W** is a weighting matrix, which is taken as a unit matrix in this study.

Combing Eqs. (8)-(9), we have

$$\begin{bmatrix} \mathbf{A} - \lambda_i \mathbf{I} & -\boldsymbol{\phi}_i \\ \boldsymbol{\phi}_i^{\mathrm{H}} \mathbf{W} & 0 \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \boldsymbol{\phi}_i}{\partial \alpha} \\ \dfrac{\partial \lambda_i}{\partial \alpha} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial \mathbf{A}}{\partial \alpha}\boldsymbol{\phi}_i \\ 0 \end{Bmatrix} . \tag{10}$$

From the above equation, the first-order eigen-pair derivatives can be obtained

$$\begin{Bmatrix} \dfrac{\partial \boldsymbol{\phi}_i}{\partial \alpha} \\ \dfrac{\partial \lambda_i}{\partial \alpha} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} - \lambda_i \mathbf{I} & -\boldsymbol{\phi}_i \\ \boldsymbol{\phi}_i^{\mathrm{H}} \mathbf{W} & 0 \end{bmatrix}^{-1} \begin{Bmatrix} -\dfrac{\partial \mathbf{A}}{\partial \alpha}\boldsymbol{\phi}_i \\ 0 \end{Bmatrix} . \tag{11}$$

2.3 Inverse problem

In the forward analysis, the eigen-pair derivatives of a discrete structural system can be calculated by Eq. (11). In the inverse problem, the system parameters are required to be identified from the measured modal parameters. In other words, the parameters are chosen to best fit the experiment data.

Penalty function method is generally used for modal sensitivity with a truncated Taylor series expansion in terms of the unknown parameters (Friswell and Mottershead 1995). In this paper, the truncated series of the dynamic responses in terms of the system parameter α are used to derive the sensitivity-based formulation. In the penalty function method, the identification problem can be shown as follows

$$\mathbf{S}\{\Delta\alpha\} = \Delta\mathbf{E} , \tag{12}$$

where $\{\Delta\alpha\}$ is the perturbation in the parameters, **S** is the two-dimensional sensitivity matrix.

$$S = \begin{vmatrix} \dfrac{\partial \boldsymbol{\phi}_1}{\partial \alpha_1} & \dfrac{\partial \boldsymbol{\phi}_1}{\partial \alpha_2} & \cdots & \dfrac{\partial \boldsymbol{\phi}_1}{\partial \alpha_m} \\ \dfrac{\partial \lambda_1}{\partial \alpha_1} & \dfrac{\partial \lambda_1}{\partial \alpha_2} & \cdots & \dfrac{\partial \lambda_1}{\partial \alpha_m} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial \boldsymbol{\phi}_n}{\partial \alpha_1} & \dfrac{\partial \boldsymbol{\phi}_n}{\partial \alpha_2} & \cdots & \dfrac{\partial \boldsymbol{\phi}_n}{\partial \alpha_m} \\ \dfrac{\partial \lambda_n}{\partial \alpha_1} & \dfrac{\partial \lambda_n}{\partial \alpha_2} & \cdots & \dfrac{\partial \lambda_n}{\partial \alpha_m} \end{vmatrix}. \tag{13}$$

When writing in full, $\mathbf{\Delta E}$ can be expressed as

$$\Delta \mathbf{E} = \begin{Bmatrix} \boldsymbol{\phi}_1 \\ \lambda_1 \\ \vdots \\ \boldsymbol{\phi}_n \\ \lambda_n \end{Bmatrix} - \begin{Bmatrix} \boldsymbol{\phi}_{1(\mathrm{cal})} \\ \lambda_{1(\mathrm{cal})} \\ \vdots \\ \boldsymbol{\phi}_{n(\mathrm{cal})} \\ \lambda_{n(\mathrm{cal})} \end{Bmatrix} = \begin{Bmatrix} \Delta \boldsymbol{\phi}_1 \\ \Delta \lambda_1 \\ \vdots \\ \Delta \boldsymbol{\phi}_n \\ \Delta \lambda_n \end{Bmatrix} \tag{14}$$

where $\Delta\mathbf{E}$ is the error vector in the measured output.

And Eq. (12) means finding the $\{\Delta\alpha\}$, which the calculated modal parameters best match the measured ones. It can be solved by simple least-squares method as follows

$$\{\Delta\alpha\} = \left[\mathbf{S}^{\mathrm{T}}\mathbf{S}\right]^{-1}\mathbf{S}^{\mathrm{T}}\Delta\mathbf{E}\,. \tag{15}$$

Like many other inverse problems, Eq. (15) is an ill-conditioned problem. In order to provide bounds to the solution, the damped least-squares method (DLS) (Tikhonov 1963) is used and singular-value decomposition is used in the pseudo-inverse calculation. Eq. (15) can be written in the following form in the DLS method

$$\{\Delta\alpha\} = \left(\mathbf{S}^{\mathrm{T}}\mathbf{S} + \beta\mathbf{I}\right)^{-1}\mathbf{S}^{\mathrm{T}}\Delta\mathbf{E}\,. \tag{16}$$

where β is the non-negative damping coefficient governing the participation of least-squares error in the solution. The solution of Eq. (16) is equivalent to minimizing the function

$$J\left(\{\Delta\alpha\},\beta\right) = \left\|\mathbf{S}\Delta\alpha - \Delta\mathbf{E}\right\|^2 + \beta\left\|\Delta\alpha\right\|^2 \tag{17}$$

with the second term in Eq. (17) provides bounds to the solution. When the parameter β approaches zero, the estimated vector $\{\Delta\alpha\}$ approaches to the solution obtained from the simple least-squares method. L-curve method (Hansen 1992) is used in this paper to obtain the optimal regularization parameter β.

Then the updated parameter α_{i+1} of the ith iteration can be obtained in the next iteration as follow

$$\alpha_{i+1} = \alpha_i + \Delta\alpha_i \tag{18}$$

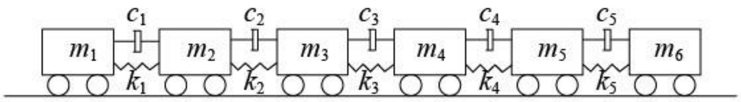

Fig. 2 6 DOFs mass–spring–damper system

The convergence is recognized as accomplished when the following criterion is met

$$\frac{\|\alpha_{i+1} - \alpha_i\|}{\|\alpha_i\|} \le \text{Tol} . \tag{19}$$

In this study, the tolerance Tol is taken as 1×10^{-8}.

2.4 Effect of artificial measurement noise

The numerical examples will study the effect of artificial measurement noise on the identification. Numerically, white noise is added to the mode shapes to simulate the noisy data with

$$\phi_{\text{noise},ij} = \phi_{ij} + E_{\text{p}} \bullet N_{\text{noise}} \bullet \sigma\left(\phi_{ij}\right) \tag{20}$$

where $\phi_{\text{noise},ij}$ and ϕ_{ij} are the mode shape components of the jth mode at the ith degrees of freedom with noise and without noise, respectively; E_p is the percentage noise level, N_{noise} is a standard normal distribution with zero mean and unit standard deviation, $\sigma(\phi_{ij})$ is the standard deviation of the calculated acceleration response. The effect of artificial measurement noise on the identified extent results is studied.

3. Numerical examples

3.1 Example 1: A mass–spring–damper system

As shown in Fig. 2, a mass-spring-damper system with 6 DOFs is studied as an example to illustrate the proposed method. The parameters are m_i=2 kg, k_j=3e5 N/m, c_j=0.5 Ns/m (i=1,2,...,6; j=1,2,...,5). The first 4 modes are used to identify the damage. Local damage is introduced as changes in stiffness and damping coefficients, but the other properties remain unchanged.

Study case 1: Identification of a single damage

It is assumed that a single damage located at element 4 with parameter k_4 perturbed by −30% or with parameter c_4 perturbed by +20%. The identified result is shown in Figs. 3-4. Even with 5% of artificial measurement noise, the results of change of k_4 and c_4 are −29.87% and +19.89%, respectively. It is obvious that the single damage, together with the change of damping, has been identified accurately.

Study case 2: Identification of multiple small damages

In this case, two damages in the system are studied. These two damages locate at 3rd and 4th element with a change in parameters k_3 and c_4 by -3% and +2%, respectively. The identified

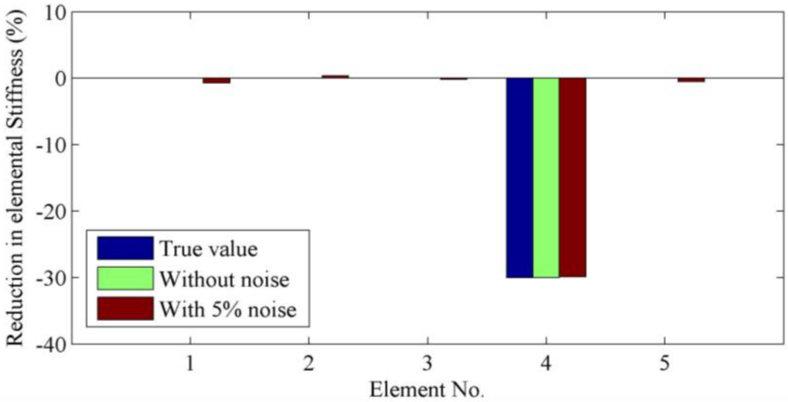

Fig. 3 Reduction in stiffness of a single damage

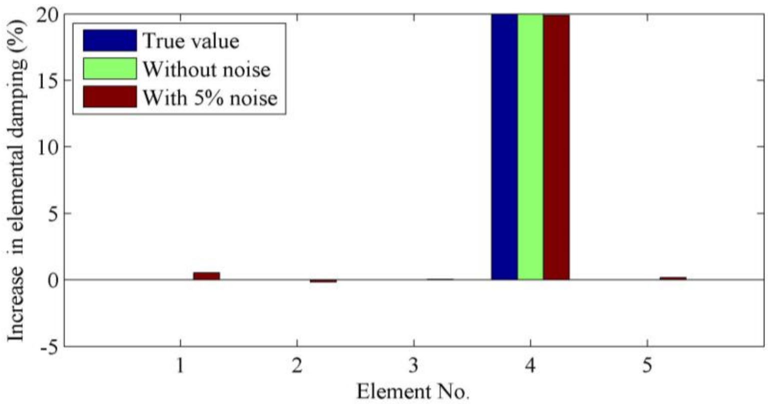

Fig. 4 Increase in damping of a single damage

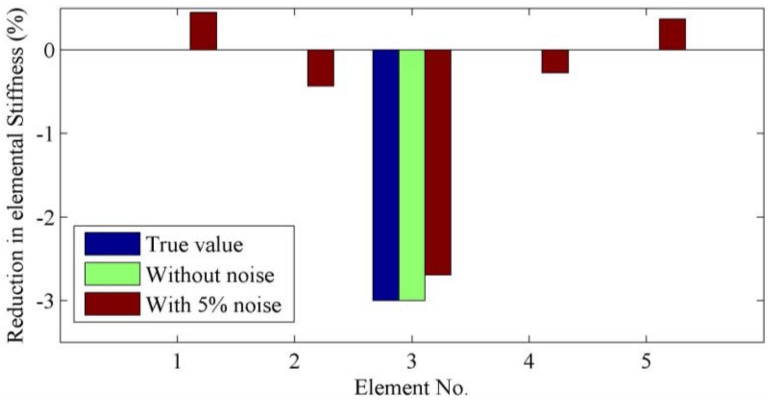

Fig. 5 Reduction in stiffness of multiple small damages

results are presented in Figs. 5-6. Because it is small damages identification in this case, the undamaged elements are identified to have about 0.5% reduction of stiffness parameters when the

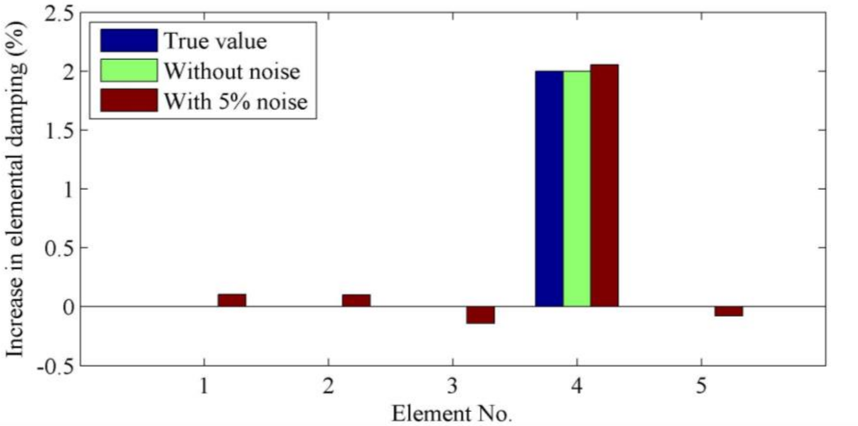

Fig. 6 Increase in damping of multiple small damages

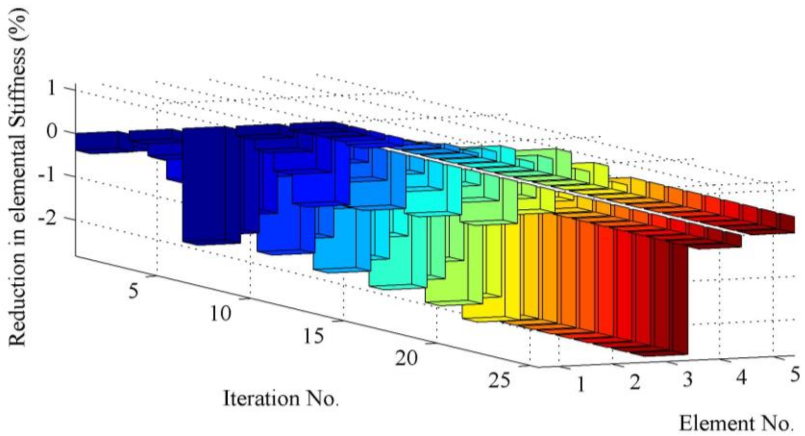

Fig. 7 Damage identification result for each iteration (stiffness parameter)

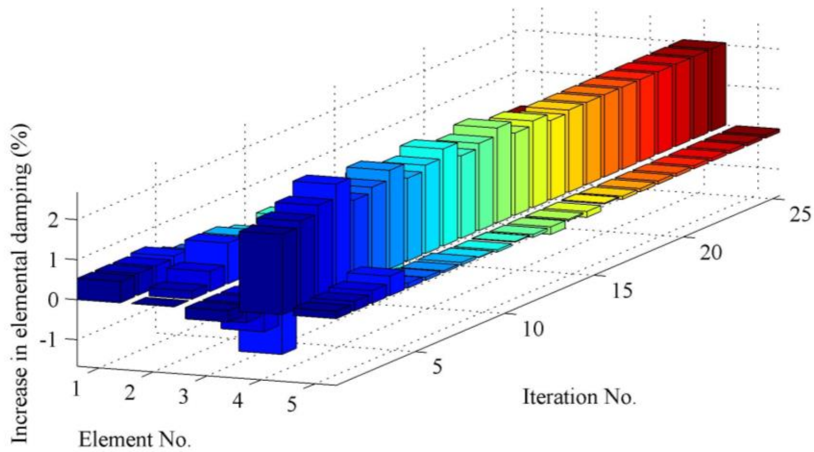

Fig. 8 Damage identification result for each iteration (damping parameter)

noise level is 5%. But comparing to the value of damaged element, it still can eliminate the undamaged elements and identified the damage extents successfully. After 20 iterations as shown in Figs. 7-8, both of stiffness and damping parameters are identified successfully from the proposed method.

Study case 3: Identification of multiple large damages

It is assumed that four damages located at 2nd, 3rd and 4th elements with parameters k_2, k_4, c_3 and c_4 perturbed by −10%, −20%, +30% and +40%, respectively. Comparing with case 2, the errors of these identified results are more accurate as shown in Figs. 9-10. With 5% noise level, the identified changes of stiffness and damping parameters are −9.66%, −19.85%, +30.18% and +39.96%, respectively. In this case, the results are identified accurately from the proposed method after 10 iterations as shown in Figs. 11-12. Comparing to the above case, it is easier to identify the large damages through the proposed method.

Study case 4: Effect of measurement noise

In this study case, the assumption of the damages is the same as the case 3, and there are 4 different noise levels. The identified results are shown in Table 1. It reveals that the proposed

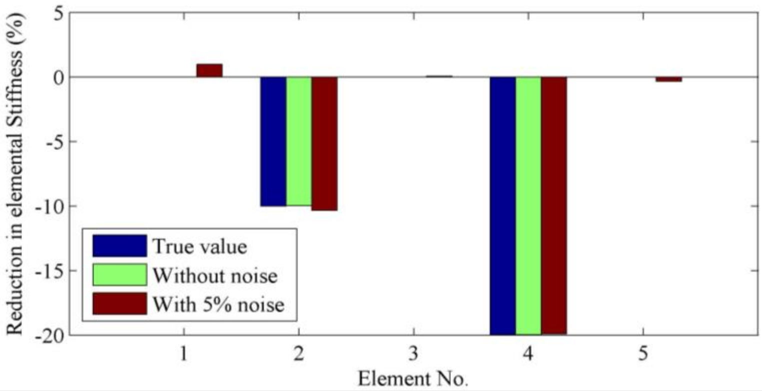

Fig. 9 Reduction in stiffness of multiple large damages

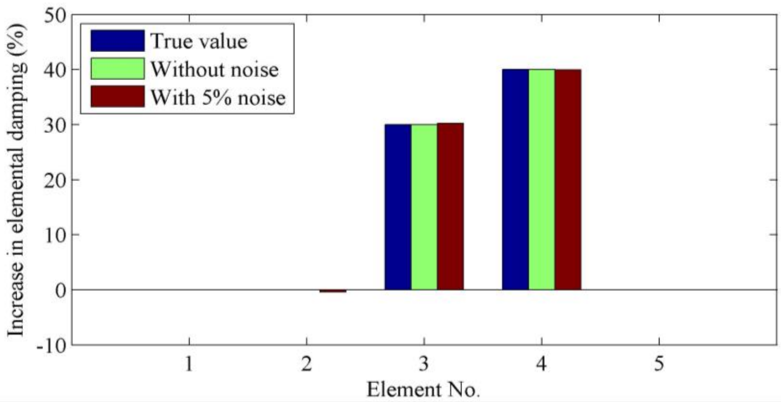

Fig. 10 Increase in damping of multiple large damages

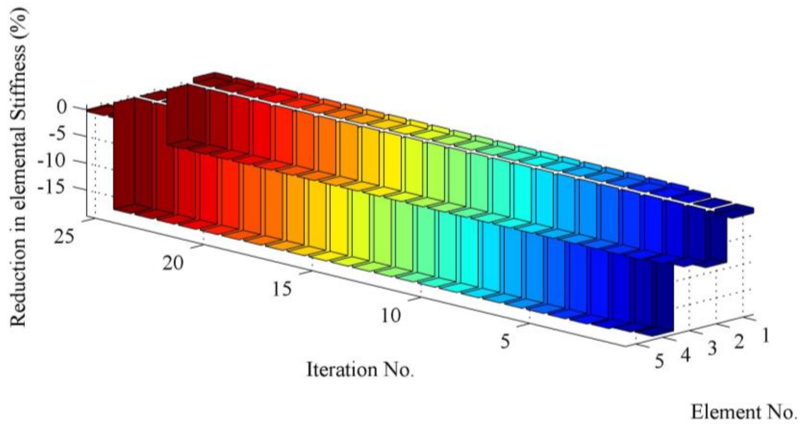

Fig. 11 Damage identification result for each iteration (stiffness parameter)

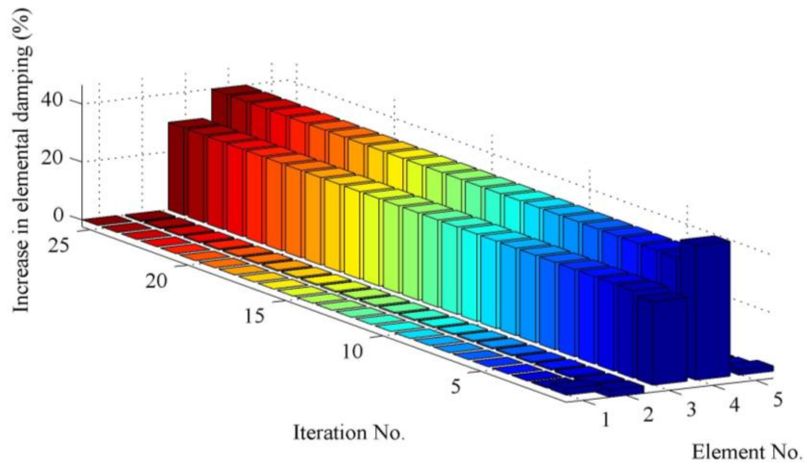

Fig. 12 Damage identification result for each iteration (damping parameter)

Table 1 Results for damage identification of multiple damages

Scenarios	Noise	Identified stiffness parameters				Identified damping parameters			
		Element No. 2		Element No. 4		Element No. 3		Element No. 4	
		Red. (%)	Res. (%)	Red. (%)	Res. (%)	Inc. (%)	Res. (%)	Inc. (%)	Res. (%)
1	Nil	10	0	20	0	30	0	40	0
2	5%	9.66	0.34	19.85	0.15	30.18	0.18	39.96	0.04
3	10%	9.41	0.59	19.80	0.20	30.35	0.35	39.93	0.07
4	15%	9.17	0.83	19.74	0.26	30.52	0.52	39.89	0.11

Note: “Red.” denotes Reduction, “Inc” denotes Increase, “Res.” denotes Relative errors.

method can identify the damages accurately when there is no noise. With the increase of the measurement noise level, identification errors become larger. But even with 15% noise level, the identified errors are still under 1%. These illustrate the efficiency and accuracy of the proposed method.

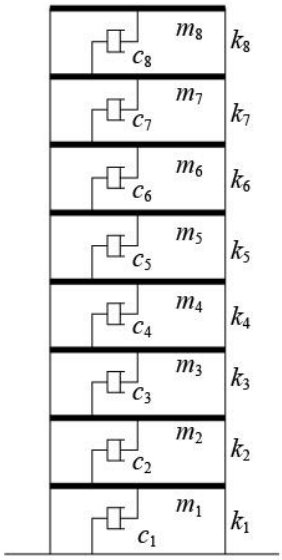

Fig. 13 An eight-storey shear building

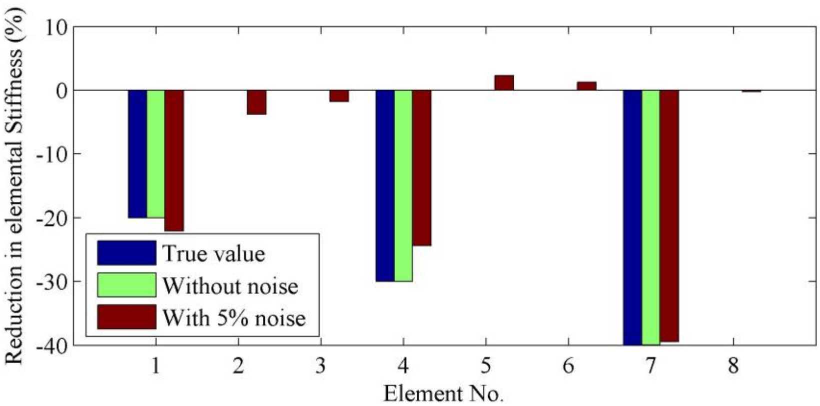

Fig. 14 Reduction only in stiffness of multiple damages

3.2 Example 2: An eight-storey shear building

The second example concerns an eight-storey shear building, as shown in Fig. 13. The column masses are considered negligible in comparison with the floor masses, which have been assumed equal to m_i=8 kg (i=1,2,...,8), the shear stiffness and damping of each story of the undamaged system are equal to k_i=1.2e6 N/m, and c_i=2 Ns/m (i=1,2,...,8), respectively. Local damage is introduced as change in stiffness or damping coefficients, but the other properties remain unchanged as the first example.

Study case 5: Identification of multiple damages in stiffness

In this case, only reductions of stiffness parameter will be considered as damages. It is assumed

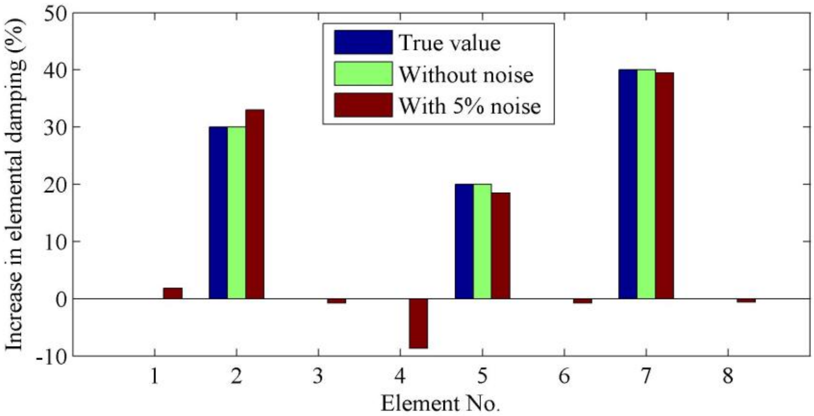

Fig. 15 Increase only in damping of multiple damages

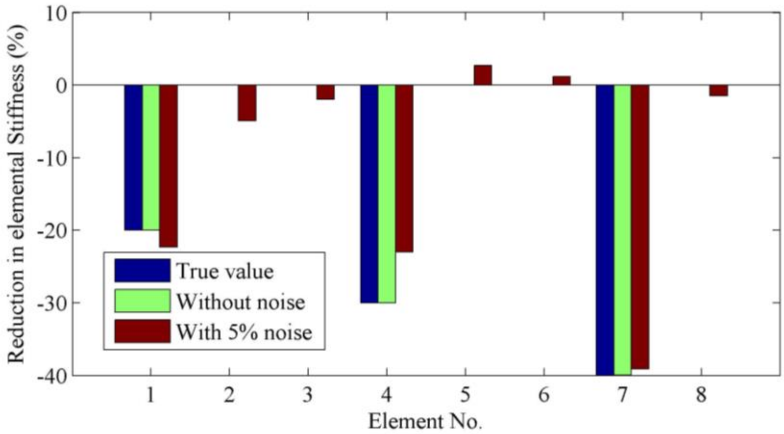

Fig. 16 Reduction in stiffness of multiple damages

that three damages located at 1st, 4th and 7th elements corresponding to reductions of 20%, 30% and 40% of the initial value of stiffness k_1, k_4 and k_7, respectively. The results of identification are presented in Fig. 14. When there is no measurement noise, the three damages have been identified with good accuracy. And the identified results are still acceptable with 5% noise level. The max identified error is 5.6% in the 4th element.

Study case 6: Identification of multiple damages in damping

In this case, only changes of damping parameter will be considered as damages. Three damages located at 2nd, 5th and 7th elements corresponding to increase of 30%, 20% and 40% of the initial value of damping c_2, c_5 and c_7, respectively. Fig. 15 shows that the identified results, even with 5% noise level, all the damages have been identified accurately. The deviations from the exact severity of the damage are negligible for the case considered.

Study case 7: Identification of multiple damages

To illustrate the effectiveness and accuracy of the proposed method, this case is characterized

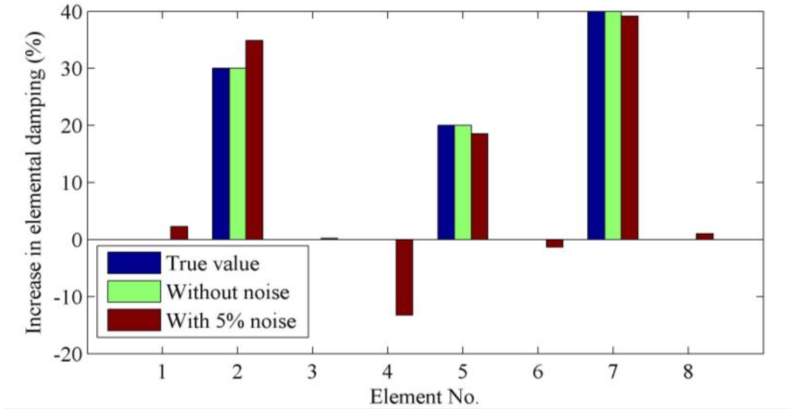

Fig. 17 Increase in damping of multiple damages

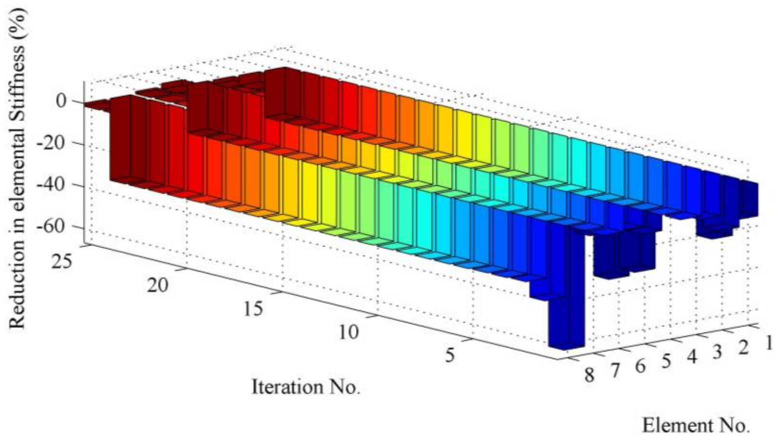

Fig. 18 Damage identification result for each iteration (stiffness parameter)

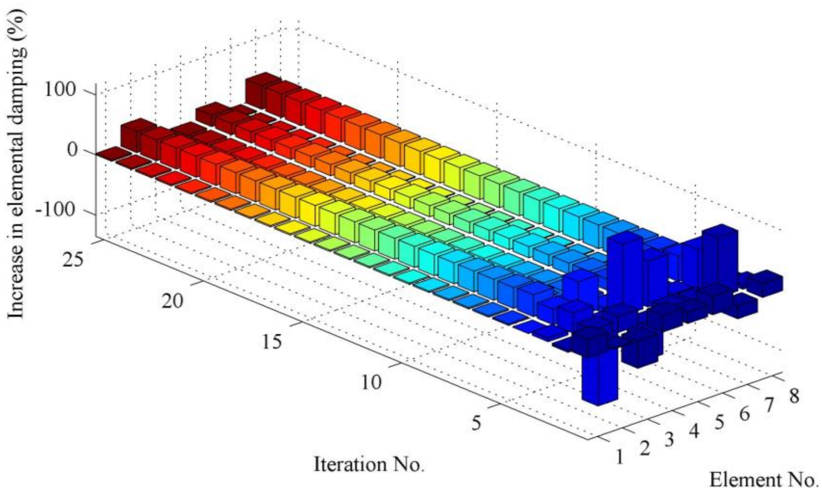

Fig. 19 Damage identification result for each iteration (damping parameter)

Table 2 Identified results obtained from different number of modal data

Mode number	Identified stiffness parameters						Identified damping parameters					
	Element No. 1		Element No. 4		Element No. 7		Element No. 2		Element No. 5		Element No. 7	
	Red. (%)	Res. (%)	Red. (%)	Res. (%)	Red. (%)	Res. (%)	Inc. (%)	Res. (%)	Inc. (%)	Res. (%)	Inc. (%)	Res. (%)
2	28.30	8.30	25.93	4.07	27.50	12.50	27.68	2.32	19.42	0.58	28.59	11.41
4	22.30	2.30	22.86	7.14	39.10	0.90	34.87	4.87	18.57	1.43	39.17	0.83
6	20.30	0.30	29.49	0.51	39.75	0.25	29.00	1.00	20.18	0.18	39.29	0.71

Note: “Red.” denotes Reduction, “Inc” denotes Increase, “Res.” denotes Relative errors.

by the changes in stiffness and damping simultaneously. It is assumed that five damages located at 1st, 2nd, 4th, 5th and 7th elements corresponding to reductions of 20%, 30% and 40% of the initial value of stiffness k_1, k_4 and k_7, and increase of 30%, 20% and 40% of the initial value of damping c_2, c_5 and c_7, respectively. The identified results are shown in Figs. 16-17. Comparing to the previous case, errors of identified results are amplified when the numbers of damages increased in this case. The 4th element may be considered as a damaged element because the increase of damping in this element is identified to be 13.24% as shown in Fig. 17. But except this situation, the identified results are acceptable with max identified error 7.14% at 4th element. Figs. 18-19 show the evolution of the changes in stiffness and damping parameters with iterations for all 8 elements of the building. One can find that the results begin to converge after 10 iterations.

Study case 8: Comparison of identified results from different number of modal data

In this case, the assumption of the damages is the same as the case 7. The mode shapes are contaminated with 5% noise. The identified results obtained from different number of modal data are listed in Table 2. In three different scenarios, when only two modes are used, it is difficult to localize the damage site because the undamaged elements also have large change value of stiffness or damping. Apparently, more modal data can achieve satisfactory results. In the last scenario, the errors of identified results are under 1% with good accuracy.

4. Conclusions

An eigen-pair derivatives method for damage detection in a discrete system has been developed in this study, which considers the sensitivities of the eigenvalue and eigen vector. Two numerical examples are studied to verify the presented method effective in identifying both stiffness and damping parameters. Studies in this paper indicate that the proposed method is efficient and robust for both single and multiple damages. And ideal identified results can be obtained from few numbers of iterations. The results are satisfactory even with measurement noise. And more modal data can improve the accuracy of identified results.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (11272361, 11172333), Doctoral Program Foundation of Ministry of Education of China (20130171110039), the Guangdong Province Science and Technology Program (2012A030200011), and the Fundamental Research Funds for the Central Universities (13lgzd06). Such financial aids are gratefully acknowledged.

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