Structural Engineering and Mechanics, *Vol. 55, No. 3 (2015) 525-534*
DOI: http://dx.doi.org/10.12989/sem.2015.55.3.525

Time-variant structural fuzzy reliability analysis under stochastic loads applied several times

Yongfeng Fang[1a], Jianbin Xiong[*2] and Kong Fah Tee[3b]

[1]School of Mechanical Engineering, Guizhou University of Science Engineering, Bijie, 551700, China
[2]School of Computer and Electronic Information, Guangdong University of Petrochemical Technology, Maoming, 525000, China
[3]Department of Civil Engineering, University of Greenwich, Kent, ME4 4TB, UK

(Received February 9, 2015, Revised April 27, 2015, Accepted May 8, 2015)

Abstract. A new structural dynamic fuzzy reliability analysis under stochastic loads which are applied several times is proposed in this paper. The fuzzy reliability prediction models based on time responses with and without strength degeneration are established using the stress-strength interference theory. The random loads are applied several times and fuzzy structural strength is analyzed. The efficiency of the proposed method is demonstrated numerically through an example. The results have shown that the proposed method is practicable, feasible and gives a reasonably accurate prediction. The analysis shows that the probabilistic reliability is a special case of fuzzy reliability and fuzzy reliability of structural strength without degeneration is also a special case of fuzzy reliability with structural strength degeneration.

Keywords: fuzzy; dynamic reliability; probability; stochastic loads

1. Introduction

Structural reliability is an important indicator in structural performance evaluation. One of the challenges in reliability analysis is that loads and structural strength are uncertain (Fröling *et al*. 2014, Kim *et al*. 2015). The context of structural reliability includes both probabilistic reliability and fuzzy reliability. The structural probabilistic reliability has been researched in many practical applications such as buildings, bridges, offshore structures, mechanical structures, underground pipelines, highway infrastructure, etc (He and Wang 1993, Rashid and Ramezan 2013, Laskar *et al*. 2014, Mahmoodian *et al.* 2012, Tee and Lutfor 2014).

A large amount of data or information is needed when the probabilistic reliability of a structure is predicted (Ellishkoff 1995). In practical applications, the predicted reliability results may contain large error due to inaccurate probability distribution functions and are very sensitive to the accuracy of the estimated distributional parameters (Qiu 2005, Ben-Haim 1994). If the data or information is not enough, it is difficult to estimate accurately the probability distribution function and especially to differentiate between binomial distribution and uniform distribution. Even the

∗Corresponding author, Ph.D., E-mail: xiongjianbin@21cn.com
[a]Professor, E-mail: fangyf_9707@126.com
[b]Senior Lecturer, E-mail: K.F.Tee@greenwich.ac.uk

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

probability distribution function is determined accurately, its parameters may not be necessary accurate and this will affect the reliability results. For example, the calculated reliability indices may not be the same using slightly different means and variances of normal distribution.

There are many fuzzy reliability cases for solving real life practical engineering problems as compared to probabilistic reliability problems. In fact, fuzzy reliability problem is considered even more complex because it contains not only random information, but also fuzzy information. Many solving methods of fuzzy reliability problems have been researched (Tutmez *et al.* 2013, Ayyub and Lai 1992, Huang 2000, Jiang and Chen 2003). Among them include structural fuzzy reliability problems under random stress and fuzzy strength (Huang *et al.* 2001, Ma *et al.* 2006, Wang and Liu 2005, Rezazadeh *et al*. 2012). A practical approach has been proposed where a fuzzy problem is converted to a random problem by using an equivalent method and then the structural reliability index is calculated using First-Order Second Moment (FOSM) method (Štemberk and Kruis 2007).

Nevertheless, the proposed approaches in the above articles were only applied to the case of single load and did not consider structural fuzzy strength degeneration. In fact, during the service period of engineering structures, the random loads are normally applied several times and structural strength is often degraded due to vibration, shock, fatigue, corrosion, aging, as well as the combined effects of uncertain inherent and extrinsic factors. Therefore, structural reliability is a function of time. The dynamic reliability of mechanical components and system reliability have also been researched. Most of the probabilistic reliability models of mechanical components are proposed by considering that the applied random loads can be modeled by a probability distribution function. However, in practical applications, the predicted reliability may contain large error due to inaccurate estimation of probability distribution functions. Thus, this approach is very sensitive to the accuracy of the estimated distributional parameters (Fang *et al.* 2014). It is indicated that their failure rates are similar to the well-known "bath-tub" shaped curve (Zhu *et al.* 2014, Wang *et al.* 2010).

In this paper, a new structural dynamic fuzzy reliability analysis under stochastic loads is proposed. The random loads are applied several times and fuzzy structural strength is analyzed. The structural random-fuzzy reliability prediction model based on time responses with and without strength degeneration is established using the stress-strength interference theory. Finally, it is demonstrated that the proposed model is feasible, accurate and practicable by an example.

2. Fuzzy reliability membership function

Suppose $\mu_{\tilde{r}}(x)$ is the membership function of the corresponding structural fuzzy strength $\tilde{r}$, $\mu_{\tilde{r}}(x) \geq 0$ for $x_{\min} \leq x \leq x_{\max}$ where $x_{\min}$ is the lower bound and $x_{\max}$ is the upper bound of x. If the stress of random loads acting on the structure is $s(t)$ at time t, the membership function of the fuzzy safety state can be written as follows.

$$\mu_{\tilde{A}}(s(t)) = \begin{cases} 1 & s(t) < x_{\min} \\ \dfrac{\int_{s(t)}^{x_{\max}} \mu_{\tilde{r}}(x)dx}{\int_{x_{\min}}^{x_{\max}} \mu_{\tilde{r}}(x)dx} & x_{\min} \leq s(t) < x_{\max} \\ 0 & s(t) \geq x_{\max} \end{cases} \tag{1}$$

3. Structural reliability models under random loads applied several times with fuzzy strength

3.1 Structural reliability without fuzzy strength degeneration

Structural reliability analysis considered in this section is for structures which have been subjected to random loads applied several times but their fuzzy strengths are not degraded during its service period. The random load applied on the structure, S and its stress s are assumed as random variables. During its service period, the structure is not subjected to a single continuous load, but multiple series of random loads. If the structure does not fail under the maximum load of these series of random loads, then the structure is considered safe under these series of random loads. Thus, fatigue failure is not considered in this study. Hence, it is assumed that structural reliability under n times of random loads is equivalent to the reliability under the maximum random load. Let the maximum value of n times of random loads is $S_{\max}$ and based on the above assumption, structural reliability under the maximum load $S_{\max}$ can be used to predict structural reliability under n times random loads. In other words, $S_{\max}$ can be used as an equivalent load to predict the structural reliability.

Suppose probability density function of the maximum stress $s_{\max}$ is $f(s_{\max})$ and the cumulative distribution of $s_{\max}$ under n times random loads which is equivalent to the maximum load can be written as follows.

$$f_n(s_{\max}) = n[F(s_{\max})]^{n-1} f(s_{\max}) \tag{2}$$

Therefore, Eq. (2) can also be written as follows.

$$f_n(s(t)) = n[F(s(t))]^{n-1} f(s(t)) \tag{3}$$

Based on Eq. (1) and Eq. (3), structural fuzzy reliability under n times random loads can be derived as follows.

$$\begin{aligned} R_n(t) &= \int_{-\infty}^{+\infty} \mu_{\tilde{A}}(s(t)) f_n(s(t)) ds(t) \\ &= \int_{-\infty}^{x_{\min}} f_n(s(t)) ds(t) + \int_{x_{\min}}^{x_{\max}} \frac{\int_{s(t)}^{x_{\max}} \mu_{\tilde{r}}(x) dx}{\int_{x_{\min}}^{x_{\max}} \mu_{\tilde{r}}(x) dx} f_n(s(t)) ds(t) \\ &= \int_{-\infty}^{x_{\min}} n[F(s(t))]^{n-1} f(s(t)) ds(t) + \int_{x_{\min}}^{x_{\max}} \frac{\int_{s(t)}^{x_{\max}} \mu_{\tilde{r}}(x) dx}{\int_{x_{\min}}^{x_{\max}} \mu_{\tilde{r}}(x) dx} n[F(s(t))]^{n-1} f(s(t)) ds(t) \end{aligned} \tag{4}$$

The applied random loads are considered to obey a Poisson distribution with mean parameter λt. Thus, the probability distribution of the stress s at time t is given as follows.

$$P(N(t) = n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \quad n = 0, 1, 2 \cdots \tag{5}$$

The structural fuzzy reliability based on time response without fuzzy strength degeneration (or strength degeneration is small which can be ignored) under random loads which are applied several times can be obtained by using fully probabilistic theory with Eq. (3) and Eq. (4) as follows.

$$\begin{aligned} R(t) &= \sum_{n=0}^{+\infty} P(N(t)=n)R_n(t) \\ &= \sum_{n=0}^{+\infty} \frac{(\lambda t)^n e^{-\lambda t}}{n!}\Big(\int_{-\infty}^{x_{\min}} n[F(s(t))]^{n-1} f(s(t))ds(t) \\ &\quad + \int_{x_{\min}}^{x_{\max}} \frac{\int_{s(t)}^{x_{\max}} \mu_{\tilde{r}}(x)dx}{\int_{x_{\min}}^{x_{\max}} \mu_{\tilde{r}}(x)dx} n[F(s(t))]^{n-1} f(s(t))ds(t)\Big) \\ &= \sum_{n=0}^{+\infty} \frac{(\lambda t)^n e^{-\lambda t}}{n!}[F(x_{\min})]^n + \sum_{n=0}^{+\infty} \frac{(\lambda t)^n e^{-\lambda t}}{n!} \int_{x_{\min}}^{x_{\max}} \frac{\int_{s(t)}^{x_{\max}} \mu_{\tilde{r}}(x)dx}{\int_{x_{\min}}^{x_{\max}} \mu_{\tilde{r}}(x)dx} n[F(s(t))]^{n-1} f(s(t))ds(t)) \end{aligned} \tag{6}$$

Eq. (6) can be simplified as follows.

$$R(t) = e^{\lambda t(F(x_{\min})-1)} + \frac{(e^{\lambda t(F(x_{\max})-1)} - e^{\lambda t(F(x_{\min})-1)}) \int_{x_{\min}}^{x_{\max}} \int_{s(t)}^{x_{\max}} \mu_{\tilde{r}}(x)dxds(t)}{\int_{x_{\min}}^{x_{\max}} \mu_{\tilde{r}}(x)dx} \tag{7}$$

3.2 Structural reliability with fuzzy strength degeneration

In fact, in addition to random loads which change with time, structural fuzzy strength is also degraded during its service period due to corrosion, vibration, fatigue, aging, etc. The probability of structural strength which is greater than the applied stress at time t can be calculated as follows based on fuzzy reliability membership function in Eq. (1).

$$\begin{aligned} G(s(t) &= P(r(t) > s(t)) = \int_{-\infty}^{+\infty} \mu_A(s(t)) f(s(t))ds(t) \\ &= \int_{-\infty}^{x_{\min}} f(s(t))ds(t) + \int_{x_{\min}}^{x_{\max}} \frac{\int_{s(t)}^{x_{\max}} \mu_{r(t)}(x)dx}{\int_{x_{\min}}^{x_{\max}} \mu_{r(t)}(x)dx} f(s(t))ds(t) \end{aligned} \tag{8}$$

where $r(t)$ is the remainder of structural fuzzy strength at time t, $r(t)=r(0)-[r(0)-s(t)](\frac{t}{T})^c$ (Schaff and Davidson 1997), $r(0)$ is the initial structural fuzzy strength, T is the service life, c is degeneration factor of material strength and $f(s(t))$ is probability density function of the stress due to the applied load at time t.

Based on the Poisson distribution function, the following observations can be deduced.

1) $\{N(t), t \geq 0\}$ is an independent increment process

2) $P(N(t+\Delta t)-N(t)=1)=\lambda\Delta t+o(\Delta t)$
3) $P(N(t+\Delta t)-N(t)\geq 2=o(\Delta t)$
where $N(t)$ is the number of applied load at time $(0,t)$ and λ is parameter of the Poisson distribution. Structural fuzzy reliability at time $t+\Delta t$ can be obtained using fully probabilistic theory as follows.

$$\begin{aligned} R(t+\Delta t) &= R(t)P(r(t)>s(t))\lambda\Delta t + R(t)(1-\lambda\Delta t) \\ &= R(t)+R(t)\lambda\Delta t(P(r(t)>s(t))-1) \\ &= R(t)+R(t)\lambda\Delta t(G(s(t))-1) \end{aligned} \tag{9}$$

where $P(r(t)>s(t))$ is the probability of structural reliability at time t. By arranging Eq. (9), it can be rewritten as follows.

$$R(t+\Delta t)-R(t)=R(t)\lambda\Delta t(G(s(t))-1) \tag{10}$$

$$\frac{R(t+\Delta t)-R(t)}{\Delta t}=R(t)\lambda(G(s(t))-1) \tag{11}$$

$$\frac{dR(t)}{dt}=R(t)\lambda(G(s(t))-1) \tag{12}$$

$$R(t)=e^{\int \lambda(G(s(t))-1)dt+C} \tag{13}$$

The general solution of Eq. (12) is given in Eq. (13). The constant of integration in Eq. (13) can be determined by the initial condition at t=0, $R(t)$=$R(0)$ as follows.

$$R(t)=e^{\int \lambda(G(s(t))-1)dt+\ln R(0)} \tag{14}$$

The finally solution of Eq. (12) is given in Eq. (14) which is the prediction model of structural fuzzy reliability based on time response with fuzzy strength degeneration under random loads which are applied several times.

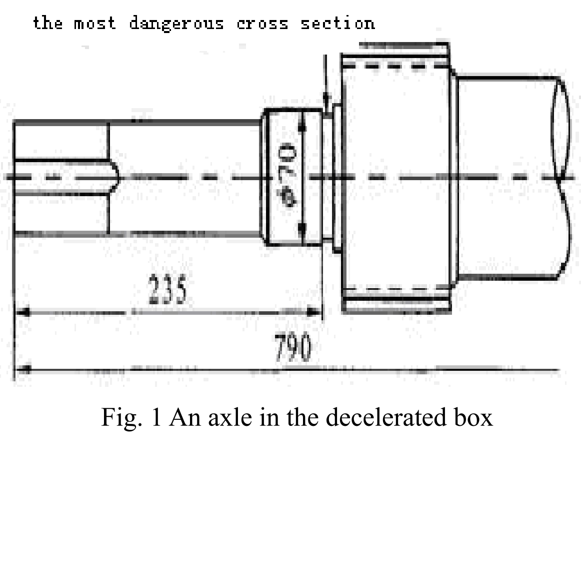

Fig. 1 An axle in the decelerated box

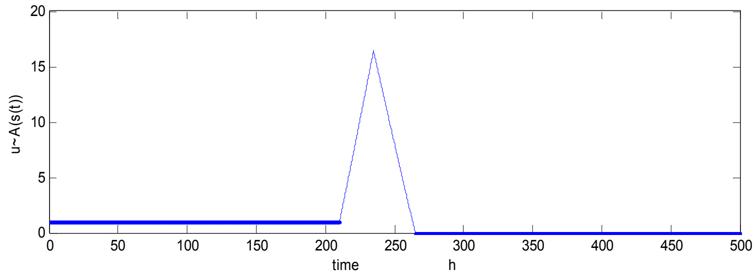

Fig. 2 The membership function of the fuzzy safety state

4. Example

An axle in the decelerated box is used as an example to verify the proposed approach as shown in Fig. 1. The strength of the axle in the decelerated box $\tilde{r}$ is 235 MPa and its membership function is defined as triangular distribution as follows.

$$\mu_{\tilde{r}}(r)=\begin{cases}(r-210)/25 & 0\le r\le 235\\ (265-r)/30 & 235<r\le 265\end{cases} \tag{15}$$

The applied stress is assumed to obey normal distribution $s(t)$~(137, 18) MPa, λ=0.5.

4.1 Case 1: structural reliability without fuzzy strength degeneration

The membership function of the fuzzy safety state as shown in Fig. 2 can be obtained using Eq. (1) as follows.

$$\mu_A(s(t))=\begin{cases}1 & s(t)\le 210\\ -95.10+0.3055s(t)+0.000721s^2(t) & 210<s(t)\le 235\\ 181.89-0.8468s(t)+0.0006061s^2(t) & 235<s(t)\le 265\\ 0 & s(t)>265\end{cases} \tag{16}$$

Then, the probability density function of the applied stress under random loads which are applied 50 times on the structure can be computed using Eq. (3) as follows.

$$f_{50}(s(t))=46.91\times\frac{1}{18\sqrt{2\pi}}\exp[-\frac{(x-137)^2}{2\times18^2}] \tag{17}$$

Finally, the structural fuzzy reliability based on time response without strength degeneration can be obtained using Eq. (7) as follows.

$$\begin{aligned}R(t)=&e^{-0.0000128t}+0.5te^{-0.03885t}\times\frac{1}{18\sqrt{2\pi}}(\int_{210}^{235}(-31.07+0.3055s(t)-0.0007272s^2(t))\,e^{-\frac{(s(t)-137)^2}{2\times18^2}}ds(t)\\ &+\int_{235}^{265}(42.56-0.8154s(t)+0.0006061s^2(t))\,e^{-\frac{(s(t)-137)^2}{2\times18^2}}ds(t))\\ =&e^{-0.0000128t}-0.00011082te^{-0.03885t}\end{aligned} \tag{18}$$

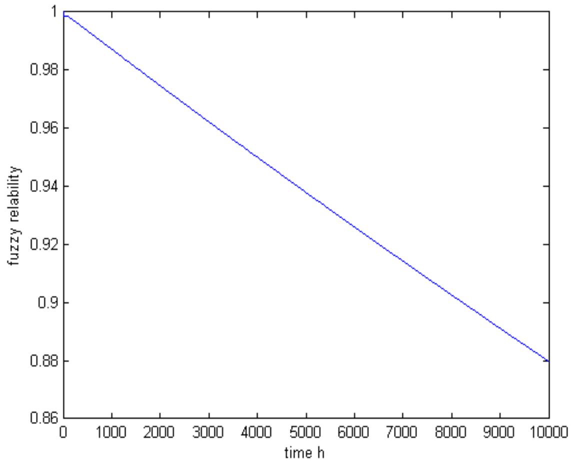

Fig. 3 Dynamic fuzzy reliability without strength degeneration

Eq. (18) is plotted in Fig. 3. Under random loads which are applied 50 times, the results show that structural fuzzy reliability is decreased over time regardless of fuzzy strength degeneration. This is because when the number of applied random loads increases with time, the axle is continuously affected by the random loads, thus the fuzzy reliability is descended over time. This situation is considered to conform to engineering practice.

4.2 Case 2: structural reliability with fuzzy strength degeneration

The similar axle is also used to validate the proposed fuzzy reliability model for the case of strength degeneration. Its service period is designed as T=10000h and the degeneration factor of material strength c=4.108 (Schaff and Davidson 1997). The probability of structural strength which is greater than the applied stress at time t can be obtained using Eq. (8) as follows.

$$G(s(t)=1+\int_{235}^{265}\frac{1170.4-516.8\times(\frac{t}{10000})^{4.108}-s(t)(8.8333-6.3667\times(\frac{t}{10000})^{4.108})}{14.999+59\times(\frac{t}{10000})^{4.108}}$$

$$\frac{\frac{1}{60}s^2(t)(1-(\frac{t}{10000})^{4.108})}{}\times\frac{1}{18\sqrt{2\pi}}\exp(-\frac{(s(t)-137)^2}{2\times18^2})ds(t)$$

$$+\int_{210}^{235}\frac{-869.52+690.92(\frac{t}{10000})^{4.108}-0.04s(t)(191\times(\frac{t}{10000})^{4.108}-210)}{12.5-31.5\times(\frac{t}{10000})^{4.108}}$$

$$\frac{-0.02s^2(t)(1-(\frac{t}{10000})^{4.108})}{}\times\frac{1}{18\sqrt{2\pi}}\exp(-\frac{(s(t)-137)^2}{2\times18^2})ds(t)$$

$$=1-\frac{1}{18\sqrt{2\pi}}\left(\frac{-2.2039+2.2818\times(\frac{t}{10000})^{4.108}}{14999+59000\times(\frac{t}{10000})^{4.108}}+\frac{0.0268-0.08\times(\frac{t}{10000})^{4.108}}{25+63\times(\frac{t}{10000})^{4.108}}\right) \tag{19}$$

Finally, the structural fuzzy reliability based on time response with strength degeneration can be obtained using Eq. (14) as follows.

$$R(t)=e^{h(t)} \tag{20}$$

where

$$\begin{aligned} h(t)=&0.0009\ln\frac{t^2+2.898t+1.494}{t^2-2.898t+1.491}-0.9856\ln\frac{t^2+0.0001t+0.5042}{t^2-0.0001t+0.5042}\\ &-0.0467\arctan(0.0199t+1)-0.0467\arctan(0.0199\mathrm{t}\text{-}1)\\ &+0.0005\ln\frac{\mathrm{t}^2+1.084\,\mathrm{t}+0.6299}{\mathrm{t}^2\text{-}1.084\,\mathrm{t}+0.6299}-0.0314\arctan(0.0111\mathrm{t}+1)\\ &+0.0314\arctan(0.0111-1)+0.5849\ln\frac{\mathrm{t}^2+0.0002\,\mathrm{t}+1}{\mathrm{t}^2-0.0002\,\mathrm{t}+1}\\ &-0.0743\arctan(0.0178\mathrm{t}+1)-0.0743\arctan(0.0178\text{-}1))-0.0004t \end{aligned}$$

Eq. (20) is plotted in Fig. 4. Based on the results from Huang *et al.* (2001), for the case without consideration of fuzzy randomness, without consideration of the number of times the random load is applied and without strength degeneration, the computed reliability by the classical reliability method is 1. On the other hand, for the case with fuzzy randomness but without consideration of the number of times the random load is applied and without strength degeneration, the computed reliability is 0.99975.

In this study, without consideration of strength degeneration, when t=0, $R(0)$=0.99975 based on

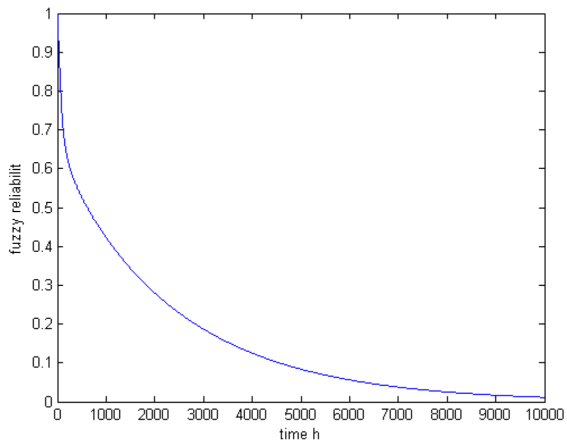

Fig. 4 Dynamic fuzzy reliability with strength degeneration

Eq. (17). This leads to the same result given by Huang *et al.* (2001). However, when t=2h and when random load which is applied one time is considered, R(2)=0.999744. Similarly, with consideration of strength degeneration, R(0)=0.99975 when t=0 but R(2)=0.995 when t=2h using Eq. (19). This again leads to the same result given by Huang *et al.* (2001) when t=0.

Based on the above results, for the case without strength degeneration, the estimated structural fuzzy reliability is descended over time due to the random loads applied several times. On the other hand, for the case with strength degeneration, the estimated structural dynamic fuzzy reliability is also descended over time but to a much greater extent due to both the random loads applied several times and fuzzy strength degeneration. The analysis shows that the probabilistic reliability is a special case of fuzzy reliability and fuzzy reliability of structural strength without degeneration is also a special case of fuzzy reliability with structural strength degeneration.

5. Conclusions

A new structural fuzzy reliability analysis under random loads which are applied several times is proposed in this paper. The proposed approach is based on fuzzy theory. The fuzzy reliability prediction models based on time response with and without strength degeneration are established using the stress-strength interference theory. The proposed model is simple and easy to implement. The proposed model can be used to determine structural service life and maintenance strategy. It can also be used to provide a theoretical basis for structural fuzzy reliability-based design and sensitivity analysis. The efficiency of the proposed model is demonstrated numerically through an axle. The results are consistent with engineering practice and it has been shown that the proposed model is efficient, practicable, feasible and gives reasonable prediction.

Acknowledgments

The work described in this paper was supported in part by a research grant from the National Natural Science Foundation of China (61473331), the Foundation from the Excellent Researcher of Bijie University (G2013017, G2015003), the Science Foundation, China (2014GZ30456, Qian Jiao Ke KY (2014) 238) and the project of Guizhou province experiment demonstration teaching center.

References

Ayyub, B.M. and Lai, K.L. (1992), “Structural reliability assessment with ambiguity and vagueness in failure”, *Naval Eng. J.*, **104**(3), 21-35.

Ben-Haim, Y. (1994), “A non-probabilistic concept of reliability”, *Struct. Saf.*, **14**(4), 227-245.

Ellishkoff, I. (1995), “Essay on uncertainties in elastic and viscoelastic structures: form A M Freudenthal’s criticisms to modern convex modeling”, *Comput. Struct.*, **56**(6), 871-895.

Fang, Y.F., Xiong, J.B. and Tee, K.F. (2014), “An iterative hybrid random-interval structural reliability analysis”, *Earthq. Struct.*, **7**(6), 1061-1070.

Fröling, M., Persson, K. and Austrell, P.E. (2014), “A reduced model for the design of glass structures subjected to dynamic impulse load”, *Eng. Struct.*, **80**(12), 53-60.

He, S.Q. and Wang, S. (1993), *Structural Reliability Analysis and Design*, National Defence Industry Pres, Beijing s.

Huang, H.Z. (2000), “Calculation of fuzzy reliability in the case of random stress and fuzzy fatigue strength”, *Chin. J. Mech. Eng.*, **13**(3), 197-200.

Huang, H.Z., Sun, Z.Q., Guo, D.M., Li, L., Zhou, F. and Tian, Z.F. (2001), “Fuzzy reliability analysis in the case of random stress and fuzzy strength”, *J. Mech. Streng.*, **23**(3), 305-307.

Jiang, Q.M. and Chen, C.H. (2003), “A numerical algorithm of fuzzy reliability”, *Reliab. Eng. Syst. Saf.*, **80**(2), 299-307.

Kim, J.H., Lee, S.H., Paik, I. and Lee, H.S. (2015), “Reliability assessment of reinforced concrete columns based on the P-M interaction diagram using AFOSM”, *Struct. Saf.*, **55**(6), 70-79.

Laskar, A., Lu, L., Qin, F., Mo, Y.L., Hsu, T.C., Lu, X. and Fan, F. (2014), “Constitutive models of concrete structures subjected to seismic shear”, *Earthq. Struct.*, **7**(5), 336-348.

Ma, J., Chen, J.J. and Gao, W. (2006), “Dynamic response analysis of fuzzy stochastic truss structures under fuzzy stochastic excitation”, *Comput. Mech.*, **38**(3), 283-292.

Mahmoodian, M., Alani, A.M. and Tee, K.F. (2012), “Stochastic failure analysis of the gusset plates in the Mississippi river bridge”, *Int. J. Forens. Eng.*, **1**(2), 153-166.

Qiu, Z. (2005), *Convex Method Based on Non-probabilistic Set-theory and Its Application*, National Defence Industry Press, Beijing.

Rashid, E.C. and Ramezan, A.I. (2013), “Demand response modification factor for the investigation of inelastic response of base isolated structures”, *Earthq. Struct.,* **5**(1), 210-226

Rezazadeh, S., Mehrabi, M., Pashaee, T. and Mirzaee, I. (2012), “Using adaptive neuro-fuzzy inference system (ANFIS) for proton exchange membrane fuel cell (PEMFC) performance modeling”, *J. Mech. Sci. Tech.*, **26**(11), 3701-3709

Schaff, J.R. and Davidson, B.D. (1997), “Life prediction methodology for composite structures. Part I-constant amplitude and two-stress level fatigue”, *J. Compos. Mater.*, **31**(2), 128-157.

Štemberk, P. and Kruis, J. (2007), “Fuzzy dynamic structural analysis of two-dimensional frame”, *Struct. Eng. Mech.*, **25**(2), 147-160.

Tee, K.F. and Khan, L.R. (2014), “Reliability analysis of underground pipelines with correlation between failure modes and random variables”, *J. Risk Reliab., Proc. Inst. Mech. Eng., Part O*, **228**(4), 362-370.

Tutmez, B., Cengiz, A.K. and Sarici, D.E. (2013), “Use of uncertain numbers for appraising tensile strength of concrete”, *Struct. Eng. Mech.*, **46**(4), 132-148.

Wang, L. and Liu, W.T. (2005), “Model for structural fuzzy reliability analysis based on fuzzy random parameter”, *J. Beijing Univ. Aeronaut. Astronaut.*, **31**(4), 412-417.

Wang, Z., Kang, R. and Xie, L. (2010), “Reliability modeling of systems with dependent failure when the life measured by the number of loadings”, *J. Mech. Eng.*, **32**(3), 188-197.

Zhu, J.S., Chen, C. and Han, Q.H. (2014), “Vehicle-bridge coupling vibration analysis based fatigue reliability prediction of prestressed concrete highway bridges”, *Struct. Eng. Mech.*, **49**(2), 203-223.

CC