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A novel sensitivity method to structural damage estimation in bridges with moving mass

Akbar Mirzaee*, Mohsenali Shayanfar[a] and Reza Abbasnia[b]

Department of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran, Islamic Republic of Iran

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Abstract. In this research a theoretical and numerical study on a bridge damage detection procedure is presented based on vibration measurements collected from a set of accelerometers. This method, referred to as "Adjoint Variable Method", is a sensitivity-based finite element model updating method. The approach relies on minimizing a penalty function, which usually consists of the errors between the measured quantities and the corresponding predictions attained from the model.

Moving mass is an interactive model and includes inertia effects between the model and mass. This interactive model is a time varying system and the proposed method is capable of detecting damage in this variable system.

Robustness of the proposed method is illustrated by correct detection of the location and extension of predetermined single, multiple and random damages in all ranges of speed and mass ratio of moving vehicle.

A comparative study on common sensitivity and the proposed method confirms its efficiency and performance improvement in sensitivity-based damage detection methods.

In addition various possible sources of error, including the effects of measurement noise and initial assumption error in stability of method are also discussed.

Keywords: damage detection; sensitivity; moving mass; finite element model updating; Ill posed problem; inverse problem; regularization; noise

1. Introduction

Structural Health Monitoring (SHM) is a process of detecting damage in structures. The main object of SHM is to improve the safety and reliability of different structures by detecting damage before it reaches an acute state. SHM is necessary for various aerospace, mechanical, and civil engineering applications for evaluating the fitness of a structure to perform its prescribed tasks (Pawar and Ganguli 2011). To achieve this goal, an analytical method is being developed to replace qualitative visual inspection and time-based maintenance procedures with more

∗Corresponding author, Ph.D., E-mail: mirzaee_akbar@yahoo.com
[a]Associate Professor, E-mail: shayanfar@iust.ac.ir
[b]Associate Professor, E-mail: abbasnia@iust.ac.ir

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quantifiable damage evaluation processes.

Vibration based Damage Detection (DD) methods are commonly based on the relationship between dynamic specifications (i.e., eigen-frequencies, damping ratios and mode shapes) and structural properties (mass, stiffness and damping) and on the possibility to detect the occurred damage, observing changes in dynamic properties.

A subdivision between model based and non-model based techniques is possible in DD methods. The former ones assume a pre-determined structural response, such as modeling the structure by the Finite Element (FE) method, whereas the latter ones are based only on changes of eigen-frequencies, mode shapes (or their derivatives) (Pandey and Biswas 1991, 1995).

A wide class of model-based DD techniques relies on the modification of structural parameters (stiffness, mass and damping) of a structural model capable to reproduce the measured static and dynamic response, as closely as possible. The process that modifies and updates a FE model in order to match, at best, the experimental data is often called Finite Element Model Updating (FEMU) (Friswell and Mottershead 1994, 1995, Maia and Silva 1997)

The FEMU process consists of the minimization of a function of residuals, called objective function, measuring the differences between experimental and numerical data. Obviously the formulation of the objective function and the residuals is of primary importance: They may contain eigen-frequencies, modal displacements, modal curvatures and any other dynamic features such as Frequency Response Functions (FRF), Modal Assurance Criterion (MAC) values and so on (Jaishi and Ren 2005, Kwon and Lin 2004, Lin and Zhu 2006). From the comparison between updated FE models corresponding to different damage levels, it is possible to detect, localize and quantify the damage (Fritzen *et al*. 1998, Teughels 2003, Friswell 2007, Link and Weiland 2009).

The application of vibration-based methods in damage assessment has become very attractive for civil engineers for its great potentialities (even though it is not yet a standard practice): in fact the opportunity to detect and locate the structural damage at an early state allows maintenance and repair works to be properly programmed, minimizing managing costs and preventing failures.

Doebling *et al*. (1996, 1998) have presented comprehensive review of literature mainly focusing on frequency-domain methods for damage detection in linear structures. A discussion on methods of damage detection and location using natural frequency changes has been presented by Salawu (1997).

Alampalli and Fu (1994), Alampalli *et al*. (1995) conducted laboratory and field studies on bridge structures to investigate the feasibility of measuring bridge vibration for inspection and evaluation. These studies focused on sensitivity of measured modal parameters to damage. Cross diagnosis using multiple signatures involving natural frequencies, mode shapes, modal assurance criteria and co-ordinate modal assurance criteria was shown to be necessary to detect the damages. Casas and Aparicio studied concrete bridge structures and investigated dynamic response as an inspection tool to assess bearing conditions and girder cracking (Casas and Aparicio1994). Their study showed the need to investigate more than one natural frequency and also determine mode shapes in order that the damage could be successfully detected and located.

The frequency-domain DD algorithms have been more widely developed and applied as the amount of measured data is reduced dramatically after the transform, thus they can be handled easily. Unfortunately, the effects of local damages on the natural frequencies and mode shapes of higher modes are greater than lower ones, but they are usually difficult to measure from experiments. In addition, structural damping properties cannot be identified in frequency domain DD.

The time-domain DD may be an attractive one to overcome the drawbacks of the frequency-domain DD. For time-domain DD, the forced vibration responses of the structure are needed in the identification. However, in some cases it is either impractical or impossible to use artificial inputs to excite the civil engineering structures, so natural excitation must be measured along with the structural responses to assess the dynamic characteristics (Alvin *et al*. 2003 and Sieniawska *et al.* 2009). In recent years, some researchers have investigated both the problem of load identification (moving load and impact load) and modal parameters identification under operational conditions (Gentile and Saisi 2007, Ren and Zong 2004). In addition, identification of the structural parameters applying a moving load has been considered in many papers. Law *et al*. (2008) presented a novel moving force and pre-stress identification method based on the FE and the wavelet-based methods for a bridge-vehicle system. Jiang *et al*. (2004) identified the parameter of a vehicle moving on multi-span continuous bridges.

Unlike moving load problem, moving mass is an interactive model. This model is the simplification of suspension model, but it includes transverse inertia effects between the beam and the mass. Interaction force between the moving mass and the structure during the traveling time of the mass along the structure considers contribution from the inertia of the mass, the centrifugal force, the Coriolis force and the time-varying velocity-dependent forces. These inertia effects are mainly caused by structural deformations (structure-trolley interaction) and structural irregularities.

Zhu and Law (2007) presented a method for damage detection of a simply supported concrete bridge structure in time domain using the interaction forces from the moving vehicles as excitation. Majumder and Manohar (2003) proposed a time domain approach for damage detection in beam structures using vibration data induced by a vehicle moving on a bridge deck.

In this paper, a novel sensitivity base damage detection method referred to as “adjoint variable method”, is developed. The load environment is modeled by a single vehicle-bridge interaction mass moving at a prescribed velocity (Azimi and Galal 2013), and the bridge is modeled as a plane grid structure. This problem is a time varying model and nonlinear in nature. The proposed method is capable of detecting damage in this time varying system.

Dynamic response can be measured at all accessible Degrees of Freedom (DOF’s) of a structure, and the amount of data is only limited by time. It is directly used in the proposed method as an unlimited source of damage information in the time-domain DD problem.

An error function, defined as the difference between the calculated and measured responses of the structure, is used in the sensitivity equation for the system identification problem. Penalty function method is used for the iterative solution with regularization. The sensitivities of the dynamic responses with respect to the unknown parameters are then calculated with adjoint variable and direct differential method in order to form the sensitivity matrix. Numerical simulations and comparison between the two methods show more efficiency and robustness of Adjoint Variable Method (AVM) and its extremely high solution speed compared with other analytical discrete methods to identify the damage in bridge structure. For modern and practical engineering applications, the cost of damage detection analysis is expensive. So, this method is feasible for large-scale problems.

2. Finite element modeling of bridge vibration under moving mass

Moving load is the simplest model that can be supposed for bridge-vehicle interaction. This model has been frequently adopted by researchers in studying the vehicle-induced bridge

vibrations. Via this model, the fundamental dynamic specifications of the bridge caused by the moving action of the vehicle can be captured with an adequate degree of accuracy.
However, the effect of interaction between the bridge and moving vehicle was just ignored. For this reason, the moving load model is only useful for the case where the mass of the vehicle is small relative to that of the bridge and only when the vehicle response is not of interest.
For cases where the inertia of the vehicle cannot be regarded as small, a moving mass model should be adopted instead.

For a general FE model of a linear elastic time-invariant structure, the equation of motion is given by

$$[\mathrm{M}(t)]\{\mathrm{z}_{,\mathrm{tt}}\}+[\mathrm{C}]\{\mathrm{z}_{,\mathrm{t}}\}+[\mathrm{K}]\{\mathrm{z}\}=[\mathrm{B}]\{\mathrm{F}(\mathrm{t})\} \tag{1}$$

Where $[M(t)]=[M_b]+[M_v(t)]$ is the total mass matrix of system in which $[M_b]$ and $[M_v(t)]$ are mass matrix of bridge and vehicle respectively, $[K]$ and $[C]$ are stiffness and damping matrices. $Z_{,tt}$ and $Z_{,t}$ and Z are the respective acceleration, velocity and displacement vectors for the whole structure and $\{F(t)\}$ is a vector of applied forces with matrix $[B]$ mapping these forces to the associated DOF's of the structure. A proportional damping is assumed to show the effect of damping ratio on the dynamic magnification factor. Rayleigh damping, in which the damping matrix is proportional to the combination of the mass and stiffness matrices, is used.

$$[\mathrm{C}]=\mathrm{a}_0[\mathrm{M}_\mathrm{b}]+\mathrm{a}_1[\mathrm{K}] \tag{2}$$

Where a_0 and a_1 are constants to be determined from two modal damping ratios. If a more accurate estimation of the actual damping is required, a more general form of Rayleigh damping, the Caughey damping model can be adopted.

As Eq. (1) shows, moving masses in a bridge-vehicle system not only excite the supporting structure via their gravities but also modify its inertial properties and the differential equation of motion is time-varying. In order to solve this equation repeated numerical integration of the equation of motion as well as updating the mass matrix in each time step should be used.

3. Finite element model updating and inverse problem

Since many algorithms of damage detection are based on the difference between modified model before occurrence of damage and after that, problems such as parameter identification and DD are closely related to FEMU. Discrepancy between two models is used for detection and quantification of damage.

Unlike other methods, the advantage of DD using model updating is that FEMU is a general method. This method can be used in any quality which is sensitive to damage such as natural frequency, modal shape, structural response or different combination of them. (Lauwagie *et al.* 2002)

A key step in model-based damage identification is the updating of the FE model of the structure in such a way that the measured responses can be reproduced by the FE model. A general flowchart of this operation is given in Fig. 1. The identification procedure presented in this paper is a sensitivity based model updating routine. Sensitivity coefficients are the derivatives of the system responses with respect to the response parameters, and are needed in the cost function of the flowchart of Fig. 1.

$$[\mathrm{C}]=\mathrm{a}_0[\mathrm{M}_\mathrm{b}]+\mathrm{a}_1[\mathrm{K}] \tag{2}$$

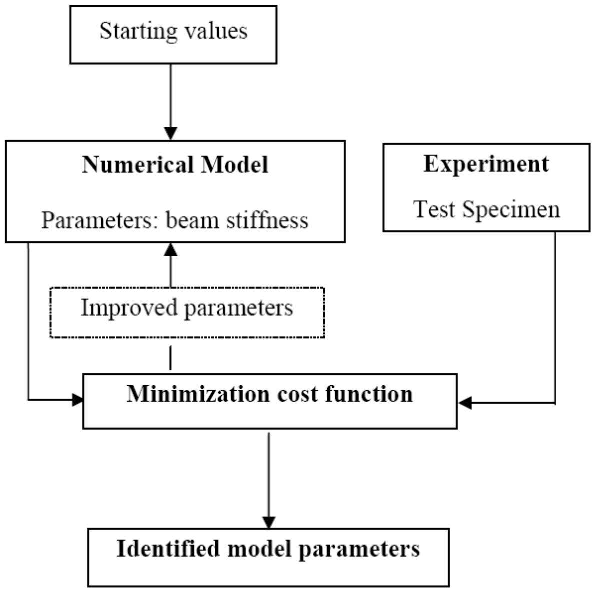


Fig. 1 General flowchart of a FEM-updating

3.1 Objective functions

The approach minimizes the difference between response quantities (usually acceleration response) of the measured data and model predictions. This problem may be expressed as the minimization of *J*, where

$$J(\theta) = \|z_m - z(\alpha)\|^2 = \epsilon^T \epsilon$$
$$\epsilon = z_m - z(\alpha) \tag{3}$$

Here z_m and $z(\alpha)$ are the measured and computed response vectors, α is a vector of all unknown parameters, and $\in$ is the response residual vector.

3.2 Penalty function methods

When the parameters of a model are unknown, they must be estimated using measured data. The measured response is a nonlinear function of the parameters. Therefore, minimization of the error between the measured and predicted response will produce a nonlinear optimization problem.

Penalty function method is generally used for modal sensitivity with a truncated Taylor series expansion in terms of the unknown parameters. In this paper, the truncated series of the dynamic responses in terms of the system parameter α are used to derive the sensitivity-based formulation. The identification problem to find the vector $\{\alpha\}$ such that the calculated response best matches the measured one can be expressed as follows

$$[Q]\{R\} = \{\hat{R}\} \tag{4}$$

Where the selection matrix $[Q]$ is a matrix with elements of zeros or ones, matching the Dofs corresponding to the measured response components. Vector $\{R\}$ can be obtained from Eq. (4) for a given set of $\{\alpha\}$.

Let

$$\{\delta z\} = \{\hat{R}\} - [Q]\{R\} = \{\hat{R}\} - \{R_{cal}\} \tag{5}$$

Where $\{\delta z\}$ is the error vector in the measured output. In the penalty function method, we have

$$\{\delta z\} = [S]\{\delta\alpha\} \tag{6}$$

Where $\{\delta\alpha\}$ is the perturbation in the parameters, $[S]$ is the two-dimensional sensitivity matrix which is one of the matrices at time t in the three-dimensional sensitivity matrix shown in Fig. 2. (Lu and Law 2007) For a FE model with N elements each with M system parameters, the number of unknown parameters is $N\times M$, and $N\times M$ equations are needed to solve the parameters. Matrix $[S]$ is on the parameter-t plane in Fig. 2, and we can select any row of the three-dimensional sensitivity matrix, say, the i^{th} row corresponding to the i^{th} measurement for the purpose. When writing in full, Eq. (5) can be written as

$$\{\delta z\} = \begin{Bmatrix} \hat{R}(t_1) \\ \hat{R}(t_2) \\ \vdots \\ \hat{R}(t_l) \end{Bmatrix} - \begin{Bmatrix} R_{cal}(t_1) \\ R_{cal}(t_2) \\ \vdots \\ R_{cal}(t_l) \end{Bmatrix} \tag{7}$$

With $l \geq N \times M$ to make sure that the set of equation is over-determined. Eq. (6) can be solved by simple least squares method as follows

$$\delta\alpha = [S^T S]^{-1} S^T \delta z \tag{8}$$

$$\alpha_{j+1} = \alpha_j + \left[S_j^T S_j\right]^{-1} S_j^T \left(\hat{R} - R_{cal}\right) \tag{9}$$

The subscript j indicates the iteration number at which the sensitivity matrix is computed. One of the important difficulties in parameter estimation is ill-conditioning. In the worst case this can mean that there is no unique solution for the estimation problem, and many sets of parameters are able to fit the data. Many optimization procedures result in the solution of linear equations for the unknown parameters. The use of the Singular Value Decomposition (SVD) (Golub and van Loan, 1996) for these linear equations enables ill-conditioning to be identified and quantified. The options are then to increase the available data, which is often difficult and costly, or to provide extra conditions on the parameters. These may take the form of smoothness conditions (for example, the truncated SVD), minimum norm parameter values (Tikhonov regularization) or minimum changes from the initial estimates of the parameters (Hansen 1992, 1994).

From experiences gained in model updating with simulated structures, Law and Li (2010) found that Tikhonov regularization can give the optimal solution when there is no noise or very small noise in the measurement.

3.3 Tikhonov regularization

Like many other inverse problems, Eq. (6) is an ill-conditioned problem. In order to provide bounds to the solution, the Damped Least Squares Method (DLSM) is used and SVD is used in the pseudo-inverse calculation. Eq. (8) can be written in the following form

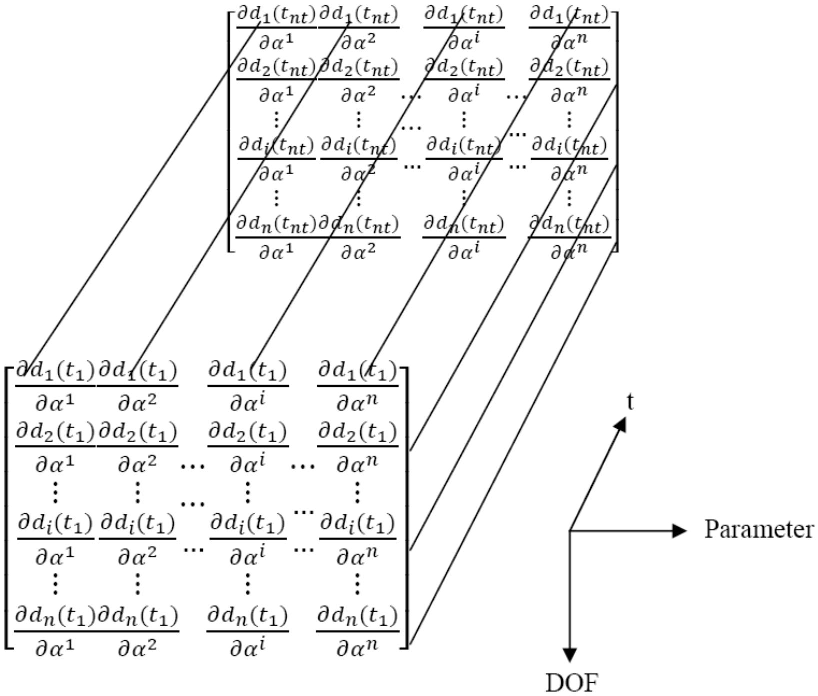


Fig. 2 Three-dimensional sensitivity matrix

$$\delta\alpha = (S^T S + \lambda I)^{-1} S^T \delta z \tag{10}$$

Where λ is the non-negative damping coefficient governing the participation of least-squares error in the solution. The solution of Eq. (10) is equivalent to minimizing the function (Tikhonov 1977)

$$J(\{\delta\alpha\}, \lambda) = \|S\delta\alpha - \delta z\|^2 + \lambda\|\delta\alpha\|^2 \tag{11}$$

With the second term in Eq. (11) provides bounds to the solution. When the parameter λ approaches zero, the estimated vector $\{\delta\alpha\}$ approaches to the solution obtained from the simple least-squares method. *L*-curve method is used in this paper to obtain the optimal regularization parameter λ.

3.4 Element damage index

Many DD methods are based on the assumption that the structure can be modeled with a linear elastic behavior also in the damaged states. In such linear damage identification techniques, the structural response at damage state can still be analyzed using a linear elastic model.

In the inverse problem of damage identification, it is assumed that the stiffness matrix of the whole element decreases uniformly with damage, and the flexural rigidity, EI_i of the i^{th} FE of the beam becomes $\beta_i EI_i$, when there is damage. The fractional change in stiffness of an element can be expressed as (Zhu and Hao 2007)

$$\Delta K_{bi} = \left(K_{bi} - \widetilde{K}_{bi}\right) = (1 - \beta_i) K_{bi} \tag{12}$$

Where K_{bi} and $\widetilde{K}_{bi}$ are the i^{th} element stiffness matrices of the undamaged and damaged beam, respectively. ΔK_{bi} is the stiffness reduction of the element. A positive value of $\beta_i \in [0,1]$ will indicate a loss in the element stiffness. The i^{th} element is undamaged when $\beta_i = 1$ and the stiffness of the i^{th} element is completely lost when $\beta_i = 0$

The stiffness matrix of the damaged structure is the assemblage of the entire element stiffness matrix $\widetilde{K}_{bi}$

$$\mathrm{K_b} = \sum_{i=1}^{N} \mathrm{A_i^T \widetilde{K}_{bi} A_i} = \sum_{i=1}^{N} \beta_i \, \mathrm{A_i^T K_{bi} A_i} \tag{13}$$

Where A_i is the extended matrix of element nodal displacement that facilitates assembling of global stiffness matrix from the constituent element stiffness matrix.

4. Sensitivity analysis of transient dynamic response

The objective of sensitivity analysis is to quantify the effects of parameter variations on calculated results. Terms such as influence, importance, ranking by importance and dominance are all related to sensitivity analysis.

The most important difficulty in sensitivity base System Identification (SI) methods is the calculation of sensitivity matrix. Calculation of this massive matrix is repeated in each iteration and according to its dimensions, is so time-consuming and has a significant effect on the efficiency of method.

4.1 Methods of structural sensitivity analysis

When the parameter variations are small, the traditional way to assess their effects on calculated responses is carried out using perturbation theory via variational principles, either directly or indirectly. The basic aim of perturbation theory is to predict the effects of small parameter variations without actually calculating the perturbed configuration but rather by using solely unperturbed quantities.

Various methods employed in sensitivity analysis are listed in Fig. 3. Three approaches are used to obtain the sensitivity matrix: the approximation, discrete, and continuum approaches.

4.2 Approximation approach

In the approximation approach, sensitivity matrix is obtained by either the forward finite difference or the central finite difference method.

If the design is perturbed to u+Δu, where Δu represents a small change in the design, then the sensitivity of $\psi(u)$ can be approximated as

$$\frac{\mathrm{d}\psi}{\mathrm{du}} \approx \frac{\psi(\mathrm{u} + \Delta u) - \psi(\mathrm{u})}{\Delta \mathrm{u}} \tag{14}$$

Eq. (14) is called the forward difference method since the design is perturbed in the direction of +Δu. If $-\Delta u$ is substituted in Eq. (14) for Δu, then the equation is defined as the backward difference method. Additionally, if the design is perturbed in both directions, such that the design

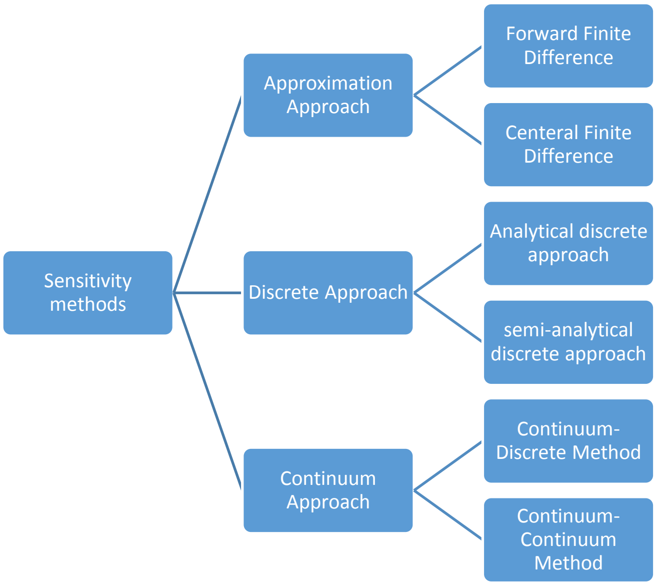


Fig. 3 Different approaches to sensitivity analysis

sensitivity is approximated by

$$\frac{d\psi}{du} \approx \frac{\psi(u + \Delta u) - \psi(u - \Delta u)}{2\Delta u} \tag{15}$$

Then the equation is defined as the central difference method.

4.3 Discrete approach

In the discrete method, sensitivity matrix is obtained by design derivatives of the discrete governing equation. For this process, it is necessary to take the derivative of the stiffness matrix. If this derivative is obtained analytically using the explicit expression of the stiffness matrix with respect to the variable, it is an analytical method. However, if the derivative is obtained using a finite difference method, the method is called a semi analytical method. The design represents a structural parameter that can affect the results of the analysis.

The design sensitivity information of a general performance measure can be computed either with the Direct Differentiation Method (DDM) or with the AVM.

4.3.1 Direct differentiation method

The DDM is a general, accurate and efficient method to compute FE response sensitivities to the model parameters. This method directly solves for the design dependency of a state variable, and then computes performance sensitivity using the chain rule of differentiation. This method clearly shows the implicit dependence on the design, and a very simple sensitivity expression can be obtained.

Consider a structure in which the generalized stiffness and mass matrices have been reduced considering boundary conditions. Let the damping force be represented in the form of $C(b)z_{,t}$ where $z_{,t}=dz/dt$ denotes the velocity vector. Under these conditions, Lagrange’s equation of motion becomes the second-order differential equation, as (Choi and Kim 2005)

$$M(t,b)z_{,tt} + C(b)z_{,t} + K(b)z = F(t,b) \tag{16}$$

With the initial conditions

$$z(0) = z^0 \ \& \ z_{,t}(0) = z_{,t}^0 \tag{17}$$

If design parameters are just related to stiffness matrix, so we have

$$[M(t)]\left\{\frac{\partial z_{,tt}}{\partial b^i}\right\} + [C]\left\{\frac{\partial z_{,t}}{\partial b^i}\right\} + [K]\left\{\frac{\partial z}{\partial b^i}\right\} = -\frac{\partial[K]}{\partial b^i}\{z\} - \alpha_2\frac{\partial[K]}{\partial b^i}\{z_{,t}\} \tag{18}$$

In which $\left\{\frac{\partial z}{\partial b^i}\right\}$, $\left\{\frac{\partial z_{,t}}{\partial b^i}\right\}$ and $\left\{\frac{\partial z_{,tt}}{\partial b^i}\right\}$ are sensitivity vectors of displacement, velocity and acceleration respect to design parameter b^i, respectively. Assume that

$$Y_{,tt} = \frac{\partial z_{,tt}}{\partial b^i} \tag{19a}$$

$$Y_{,t} = \frac{\partial z_{,t}}{\partial b^i} \tag{19b}$$

$$Y = \frac{\partial z}{\partial b^i} \tag{19c}$$

So, by replacing Eq. (19) to Eq. (18) we have

$$[M(t)]\{Y_{,tt}\} + [C]\{Y_{,t}\} + [K]\{Y\} = -\frac{\partial[K]}{\partial b^i}\{z\} - \alpha_2\frac{\partial[K]}{\partial b^i}\{z_{,t}\} \tag{20}$$

Right side of Eq. (20) can be considered as an equivalent force, so Eq. (20) is similar to Eq. (16) and sensitivity vectors can be obtained by Newmark method.

4.4 Continuum approach

In the continuum approach, the design derivative of the variational equation is taken before it is discretized. If the structural problem and sensitivity equations are solved as a continuum problem, then it is called the continuum-continuum method. The continuum sensitivity equation is solved by discretization in the same way that structural problems are solved. Since differentiation is taken at the continuum domain and is then followed by discretization, this method is called the continuum-discrete method.

5. Adjoint variable method and proposed algorithm

Sensitivity analysis can be performed efficiently by using deterministic methods based on adjoint functions. The use of adjoint functions for analyzing the effects of small perturbations in a linear system was introduced by Wigner (1945).

This method, constructs an adjoint problem that solves for the adjoint variable, which contains all implicitly dependent terms.

For the dynamic response of structure, the following form of a general performance measure will be considered

$$\psi = g(z(T), b) + \int_0^T G(z, b)dt \tag{21}$$

Where *g* and *G* are functions which describe the form of performance measure and the final time *T* is determined in the following equation

$$\Omega\big(z(T), z_{,t}(T), b\big) = 0 \tag{22}$$

When final time *T* is prescribed before the response analysis, the relation in Eq. (22), need not be considered.

To obtain the design sensitivity of *Ψ*, define a design variation in the form

$$b_\tau = b + \tau\delta b \tag{23}$$

Design *b* is perturbed in the direction of *δb* with the parameter *τ*. Substituting b_τ into Eq. (21), the derivative of Eq. (21), can be evaluated with respect to *τ* at *τ*=0. Leibnitz's rule of differentiation of an integral may be used to obtain the following expression

$$\psi' = \frac{\partial g}{\partial b}\delta b + \frac{\partial g}{\partial z}\left[z'(T) + z_{,t}(T)T'\right] + G(z(T), b)T' + \int_0^T \left[\frac{\partial G}{\partial z}z' + \frac{\partial G}{\partial b}\delta b\right]dt \tag{24}$$

Where

$$z' = z'(b, \delta b) \equiv \frac{d}{d\tau}z(t, b + \tau\delta b)|_{\tau=0} = \frac{d}{db}[z(t, b)]\delta b$$

$$T' = T'(b, \delta b) \equiv \frac{d}{d\tau}T(b + \tau\delta b)\Big|_{\tau=0} = \frac{dT}{db}\delta b$$

The derivative of performance measures corresponds to the following equation

$$\psi' = [\frac{\partial g}{\partial z} - \left(\frac{\partial g}{\partial z}z_{,t}(T) + G(z(T), b)\frac{1}{\Omega_{,t}}\frac{\partial \Omega}{\partial z}\right]z'(T) - \left[\frac{\partial g}{\partial z}z_{,t}(T) + G(z(T), b)\right]\frac{1}{\Omega_{,t}}\frac{\partial \Omega}{\partial z_{,t}}z'_{,t}(T) + \int_0^T \left[\frac{\partial G}{\partial z}z' + \frac{\partial G}{\partial b}\delta b\right]dt + \frac{\partial g}{\partial b}\delta b - \left[\frac{\partial g}{\partial z}z_{,t}(T) + G(z(T), b)\right]\frac{1}{\Omega_{,t}}\frac{\partial \Omega}{\partial b}\delta b \tag{25}$$

Note that *Ψ*' depends on *z*' and $z'_{,t}$ at *T*, as well as on *z*' within the integration.

In order to write *Ψ*' explicitly in terms of a design variation, the adjoint variable technique can be used. In the case of a dynamic system, all terms in Eq. (16), can be multiplied by $\lambda^T(t)$ and integrated over the interval [0, *T*], to obtain the following identity in *λ*

$$\int_0^T \lambda^T\big[M(b, t)z_{,tt} + C(b)z_{,t} + K(b)z - F(t, b)\big]dt = 0 \tag{26}$$

Since this equation must hold for arbitrary *λ*, which is now taken to be independent of the design, substitute b_τ into Eq. (26), and differentiate it with respect to *τ* in order to obtain the following relationship

$$\int_0^T \left[\lambda^T M(b,t) z'_{,tt} + \lambda^T C(b) z'_{,t} + \lambda^T K(b) z' - \frac{\partial R}{\partial b}\delta b\right] dt = 0 \tag{27}$$

Where

$$R = \tilde{\lambda}^T F(t,b) - \tilde{\lambda}^T M(b,t)\tilde{z}_{,tt} - \tilde{\lambda}^T C(b)\tilde{z}_{,t} - \tilde{\lambda}^T K(b)\tilde{z} \tag{28}$$

With the superposed tilde (~) denoting variables that are held constant during the differentiation with respect to the design in Eq. (27).

Since Eq. (27), contains the time derivatives of *z*', integrate the first two integrands by parts in order to move the time derivatives to λ, as

$$\lambda^T M(b,T) z'_{,t}(T) - \lambda^T_{,t}(T) M(b,T) z'(T) - \lambda^T(T) M_{,t}(b,T) z'(T) + \lambda^T(T) C(b) z'(T)$$
$$+ \int_0^T \left\{\left[\lambda^T_{,tt} M(b,t) - \lambda^T_{,t}(C(b) - 2M_{,t}(b,t)) + \lambda^T(K(b) + M_{,tt}(b,t))\right] z' - \frac{\partial R}{\partial b}\delta b\right\} dt = 0 \tag{29}$$

The AVM expresses the unknown terms in Eq. (25), in terms of the adjoint variable (λ). Since Eq. (29), must hold for arbitrary functions $\lambda(t)$, λ may be chosen so that the coefficients of terms involving $z'(T)$, $z'_{,t}(T)$ and z' in Eq. (25), and Eq. (29), are equal. If such a function $\lambda(t)$ can be found, then the unwanted terms in Eq. (25), involving $z'(T)$, $z'_{,t}(T)$ and z' can be replaced by terms that explicitly depend on δb in Eq. (29), To be more specific, choose a $\lambda(t)$ that satisfies the following

$$M(T)\lambda(T) = -\left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T),b)\right]\frac{1}{\Omega_{,t}}\frac{\partial \Omega^T}{\partial z_{,t}} \tag{30}$$

$$M(b,T)\lambda_{,t}(T) = (C^T(b) - M^T_{,t}(T))\lambda(T) - \frac{\partial g^T}{\partial z} + \left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T),b)\right]\frac{1}{\Omega_{,t}}\frac{\partial \Omega^T}{\partial z} \tag{31}$$

$$M(t)\lambda_{,tt} - \bar{C}^T(b,t)\lambda_{,t} + \bar{K}(b,t)\lambda = \frac{\partial G^T}{\partial z}, \qquad 0 \le t \le T \tag{32}$$

In which

$$\bar{C}(b,t) = C(b) - 2M_{,t}(t)$$

$$\bar{K}(b,t) = K(b) + M_{,tt}(t)$$

The design derivative vector of Ψ is

$$\frac{d\psi}{db} = \frac{\partial g}{\partial b}(z(T),b) + \int_0^T \left[\frac{\partial G}{\partial b}(z,b) + \frac{\partial R}{\partial b}(\lambda(t), z(t), z_{,t}(t), z_{,tt}(t), b)\right] dt$$
$$-\frac{1}{\Omega_{,t}}\left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T),b)\right]\frac{\partial \Omega}{\partial b} \tag{33}$$

The computational algorithm that leads to the determination of $d\Psi/db$ requires the initial-value problem be integrated forward in time from 0 to *T*. Then, the adjoint terminal-value problem presented by Eqs. (30), (31) and (32), must be integrated backward in time from *T* to 0. Once these initial- and terminal-value problems have been solved, the design derivative of Ψ in Eq. (33) can then be evaluated using a numerical integration formula. Although substantial numerical computation is required, it is clear that the design derivatives of the dynamic response can be computed.

5.1 Approximate solution of the effects of a moving mass

C.E Inglis proposed an approximation solution of the effects of vehicles moving over large-span bridges. He introduced an assumption according to which the gravitational effects of the load may be separated from the inertial ones. In the calculation, the force is considered as moving along the beam while the mass of the vehicle acts at a definite, constant point x_0.

Using this method, one can reduce system time dependent matrices to:

$$\mathrm{M}(\mathrm{t}) \approx \mathrm{M}(\frac{\mathrm{T}}{2})$$

$$\bar{\mathrm{C}}(\mathrm{b},\mathrm{t}) \approx \mathrm{C}(\mathrm{b}) - 2\mathrm{M}_{,\mathrm{t}}\left(\frac{\mathrm{T}}{2}\right)$$

$$\bar{\mathrm{K}}(\mathrm{b},\mathrm{t}) \approx \mathrm{K}(\mathrm{b}) + \mathrm{M}_{,\mathrm{tt}}\left(\frac{\mathrm{T}}{2}\right)$$

So, Eq. (32) can be rewritten as

$$\mathrm{M}(\frac{\mathrm{T}}{2})\lambda_{,\mathrm{tt}} - \bar{\mathrm{C}}^{\mathrm{T}}\left(\mathrm{b},\frac{\mathrm{T}}{2}\right)\lambda_{,\mathrm{t}} + \bar{\mathrm{K}}\left(\mathrm{b},\frac{\mathrm{T}}{2}\right)\lambda = 0, \qquad 0 \le \mathrm{t} \le \mathrm{T} \tag{34}$$

That is a linear equation.

5.2 Damage detection using dynamic structural response

While structural vibration responses are used for damage detection, assuming G=0, Eq. (32), is a free vibration of beam with terminal conditions. Solving Eq. (32), for a single degree of freedom system is as follow

$$\lambda_{\mathrm{T}}(\mathrm{t}) = \mathrm{e}^{\xi\omega(\mathrm{t}-\mathrm{T})}\left(\frac{\lambda_{,\mathrm{t}}(\mathrm{T})}{\omega_{\mathrm{D}}}\cos(\omega_{\mathrm{D}}\mathrm{T})\sin(\omega_{\mathrm{D}}\mathrm{t}) - \frac{\lambda_{,\mathrm{t}}(\mathrm{T})}{\omega_{\mathrm{D}}}\sin(\omega_{\mathrm{D}}\mathrm{T})\cos(\omega_{\mathrm{D}}\mathrm{t})\right)$$
$$= \mathrm{P}_{\mathrm{T}}\mathrm{f}(\mathrm{t}) + \mathrm{Q}_{\mathrm{T}}\mathrm{g}(\mathrm{t}) \tag{35}$$

In which

$$\mathrm{P}_{\mathrm{T}} = \mathrm{e}^{-\xi\omega\mathrm{T}}\frac{\lambda_{,\mathrm{t}}(\mathrm{T})}{\omega_{\mathrm{D}}}cos(\omega_{\mathrm{D}}\mathrm{T}) \qquad \mathrm{f}(\mathrm{t}) = \mathrm{e}^{\xi\omega\mathrm{t}}\sin(\omega_{\mathrm{D}}\mathrm{t})$$

$$\mathrm{Q}_{\mathrm{T}} = -\mathrm{e}^{-\xi\omega T}\frac{\lambda_{,\mathrm{t}}(\mathrm{T})}{\omega_{\mathrm{D}}}\sin(\omega_{\mathrm{D}}\mathrm{T}) \qquad \mathrm{g}(\mathrm{t}) = \mathrm{e}^{\xi\omega\mathrm{t}}\cos(\omega_{\mathrm{D}}\mathrm{t})$$

Using Eq. (33), assuming T is known and G=0 because of using structural vibration data, Eq. (36) can be obtained

$$\frac{d\psi}{db} = \int_0^{\mathrm{T}}\frac{\partial\mathrm{R}}{\partial\mathrm{b}}\mathrm{dt} \tag{36}$$

In this equation

$\mathrm{R} = \tilde{\lambda}^{\mathrm{T}}\mathrm{F}(\mathrm{t}) - \tilde{\lambda}^{\mathrm{T}}\mathrm{M}\tilde{\mathrm{z}}_{,\mathrm{tt}} - \tilde{\lambda}^{\mathrm{T}}\mathrm{C}(\mathrm{b})\tilde{\mathrm{z}}_{,\mathrm{t}} - \tilde{\lambda}^{\mathrm{T}}\mathrm{K}(\mathrm{b})\tilde{\mathrm{z}}$ and $\mathrm{C} = \mathrm{a}_0\mathrm{K}(\mathrm{b}) + \mathrm{a}_1\mathrm{M}$ is Rayleigh damping matrix, so

$$\frac{\partial R}{\partial b} = -\widetilde{\lambda^{T}} a_0 \frac{\partial K}{\partial b} \widetilde{z_{,t}} - \widetilde{\lambda^{T}} \frac{\partial K}{\partial b} \tilde{z} \tag{37}$$

And finally the component of sensitivity matrix in time *T* is

$$\frac{d\psi}{db}(T) = \int_0^T (-\widetilde{\lambda^{T}} a_0 \frac{\partial K}{\partial b} \widetilde{z_{,t}} - \widetilde{\lambda^{T}} \frac{\partial K}{\partial b} \tilde{z}) dt \tag{38}$$

In a multi degree of freedom problem, solving the above equations directly is not possible. For this purpose, change the variables as follow

$$\{\lambda\} = [\phi]\{Y\} \tag{39}$$

In this equation matrix $[\varphi]$ forms vibration modes (modal matrix) and terminal conditions of above equations are

$$\{Y(T)\} = M^{-1}[\phi]^T[m]\{\lambda(T)\} \tag{40}$$

$$\{Y_{,t}(T)\} = M^{-1}[\phi]^T[m]\{\lambda_{,t}(T)\} \tag{41}$$

By inserting Eq. (40) in Eq. (32) and multiplying $[\phi]^T$ in both sides, the new equation in modal space is

$$[M]\{Y_{,tt}\} - [C]\{Y_{,t}\} + [K]\{Y\} = \{0\} \tag{42}$$

Each of $[M]$, $[C]$ and $[K]$ matrices is diagonal, so

$$M_i\{Y_{,tt_i}\} - C_i\{Y_{,t_i}\} + K_i\{Y_i\} = \{0\} \tag{43}$$

$$\frac{d\psi}{db}(T) = -\int_0^T \langle Y\rangle \times [\phi]^T \times a_0 \left[\frac{\partial k}{\partial b}\right] \times \{z_{,t}\} + \langle Y\rangle \times [\phi]^T \times \left[\frac{\partial k}{\partial b}\right] \times \{z\} dt \tag{44}$$

Consider: $[\phi]^T \times a_0 \left[\frac{\partial k}{\partial b}\right] \times \{z_{,t}\} = \{zz_{,t}\}$ and $[\phi]^T \times \left[\frac{\partial k}{\partial b}\right] \times \{z\} = \{zz\}$

Eq. (44) can be reduced to Eq. (45)

$$\frac{d\psi}{db}(T) = -\int_0^T \langle Y\rangle \times \{zz_{,t}\} + \langle Y\rangle \times \{zz\} dt \tag{45}$$

From Eq. (35) variable *Y* in modal space can be written as

$$\{Y\} = \{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\} \tag{46}$$

Replacing Eq. (46) in Eq. (45) a new expression is derived to calculate the sensitivity.

$$\frac{d\psi}{db}(T) = -\int_0^T (\{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\})^T \times \{zz_{,t}\} +$$
$$(\{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\})^T \times \{zz\} dt \tag{47}$$

Eq. (47) can be rewritten as follow

$$\frac{d\psi}{db}(T) = -\int_0^T \langle P(T)\rangle \times (\{f(t)\}.\{zz_{,t}\} + \{f(t)\}.\{zz\}) + \langle Q(T)\rangle$$

$$\times \left(\{g(t)\}.\left\{zz_{,t}\right\} + \{g(t)\}.\{zz\}\right)dt \tag{48}$$

Consider following parameters:

$$A = \int_0^T \{f(t)\}.\left\{zz_{,t}\right\} dt \quad B = \int_0^T \{g(t)\}.\left\{zz_{,t}\right\} dt \quad C = \int_0^T \{f(t)\}.\{zz\} dt \quad D = \int_0^T \{g(t)\}.\{zz\} dt$$

So, Eq. (48) is presented as

$$\frac{d\psi}{db}(T) = -\langle P(T)\rangle \times (\{A\} + \{C\}) - \langle Q(T)\rangle \times (\{B\} + \{C\}) \tag{49}$$

Solution of Eq. (49) directly is too time-consuming, because in each time step all terms in Eq. (49) should be recalculated. Therefore, an incremental solution is developed as follow

$$\{A_{T+\Delta T}\} = \int_0^{T+\Delta T} \{f(t)\}.\left\{zz_{,t}\right\} dt = \int_0^{T} \{f(t)\}.\left\{zz_{,t}\right\} dt + \int_T^{T+\Delta T} \{f(t)\}.\left\{zz_{,t}\right\} dt \tag{50}$$

$$\{A_{T+\Delta T}\} = \{A_T\} + \{\delta A\} \ ,\{\delta A\} = \int_T^{T+\Delta T} \{f(t)\}.\left\{zz_{,t}\right\} dt \cong \left\{f\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{zz_{,t}\left(T + \frac{\Delta T}{2}\right)\right\} \tag{51}$$

Similar to Eq. (51) for other parameters we have

$$\{\delta B\} = \int_T^{T+\Delta T} \{g(t)\}.\left\{zz_{,t}\right\} \tag{52}$$

$$\{\delta C\} = \int_T^{T+\Delta T} \{f(t)\}.\{zz\} dt \cong \left\{f\left(T + \tfrac{\Delta T}{2}\right)\right\}.\left\{zz\left(T + \tfrac{\Delta T}{2}\right)\right\} \tag{53}$$

$$\{\delta D\} = \int_T^{T+\Delta T} \{g(t)\}.\{zz\} dt \cong \left\{g\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{zz\left(T + \frac{\Delta T}{2}\right)\right\} \tag{54}$$

And finally the sensitivity expression in time $T + \Delta T$ is

$$\frac{d\psi}{db}(T + \Delta T) = -\langle P(T + \Delta T)\rangle \times (\{A_{T+\Delta T}\} + \{C_{T+\Delta T}\}) - \langle Q(T + \Delta T)\rangle$$
$$\times (\{B_{T+\Delta T}\} + \{D_{T+\Delta T}\}) \tag{55}$$

5.3 Proposed algorithm

The computational algorithm that leads to the determination of sensitivity matrix is as follow:

- Step1: Calculate $\lambda_{,t}(T)$
- Step2: Calculate $\omega, \omega_D \ and \ \phi$ from and consider i=1
- Step3: For the i^{th} element calculate $\frac{\partial K}{\partial b}$ and $zz_{,t}, zz$ and consider j=1
- Step4: For the j^{th} sensor and corresponding DOF calculate $\lambda_{,t}(T)$ from step1 and $Y_{,t}(T)$ from Eq. (41) and T_n=Δt and T_o=0
- Step5: Consider A=B=C=D=0
- Step6: Calculate $T_m = T_0 + \frac{\Delta t}{2}$ and Calculate $P(T_n) - Q(T_n) - f(T_m) - g(T_m)$ from Eq. (35)

• Step7: Calculate $\{\delta A\}, \{\delta B\}, \{\delta C\}$ and $\{\delta D\}$ from Eq. (51)-(54)

• Step8: Calculate $\frac{d\psi}{db}(T_n)$ from Eq. (55)

• Step9: If $T_n < T_{final}$ Consider $T_0 = T_n$ and $T_n = T_n + \Delta t$ and go to step5 otherwise go to next step

• Step10: If $j < number\ of\ sensors$ Consider j=j+1 and go to step 4 otherwise go to next step

• Step11: If $i < number\ of\ elements$ Consider i=i+1 and go to step 3 otherwise finish.

5.4 Procedure of iteration for damage detection

The initial analytical model of a structure deviates from the true model and measurement from the initial intact structure is used to update the analytical model. The improved model is then treated as a reference model, and measurement from the damaged structure will be used to update the reference model.

When response measurement from the intact state of the structure is obtained, the sensitivities are computed from proposed algorithm or DDM (Eq. (20)) based on the analytical model of the structure and well knowing input force and velocity. The vector of parameter increments is then obtained from Eqs. (8) or (10) using the computed and experimentally obtained responses. The analytical model is then updated and the corresponding response and its sensitivity are again computed for the next iteration. When measurement from the damaged state is obtained, the updated analytical model is used in the iteration in the same way as that using measurement from the intact state. Convergence is considered to be achieved when the following criteria are met

$$\frac{||\mathrm{E}_{\mathrm{i+1}} - \mathrm{E}_{\mathrm{i}}||}{||\mathrm{E}_{\mathrm{i}}||} \times 100\% \ \leq \mathrm{Tol1} \tag{56}$$

$$\frac{||\mathrm{Response}_{\mathrm{i+1}} - \mathrm{Response}_{\mathrm{i}}||}{||\mathrm{Response}_{\mathrm{i}}||} \times 100\% \ \leq \mathrm{Tol2} \tag{57}$$

The final vector of identified parameter increments corresponds to the changes occurring in between the two states of the structure. The tolerance is set equal to 1×10^{-6} in this study except otherwise specified.

Eq. (6) has been popularly used in the form of the first-order approximation of the increment on the left side of the equation. The higher order term of the Taylor expansion has been omitted in the computation. The iterative computation described above on the updating of the sensitivity and the system aiming error reduction due to such omission, particularly with large local damages.

5.5 Sensitivity method selection

The advantage of the finite difference method is obvious. If structural analysis can be performed and the performance measure can be obtained as a result of structural analysis, then the expressions in Eq. (14) and Eq. (15) are virtually independent of the problem types considered.

Major disadvantage of the finite difference method is the accuracy of its sensitivity results. Depending on perturbation size, sensitivity results are quite different. For a mildly nonlinear performance measure, relatively large perturbation provides a reasonable estimation of sensitivity results. However, for highly nonlinear performances, a large perturbation yields completely inaccurate results. Thus, the determination of perturbation size greatly affects the sensitivity result.

Although it may be necessary to choose a very small perturbation, numerical noise becomes dominant for a too-small perturbation size. That is, with a too-small perturbation, no reliable difference can be found in the analysis results.

The continuum-continuum approach is so limited and is not applicable in complex engineering structures because very simple, classical problems can be solved analytically.

The discrete and continuum-discrete methods are equivalent under the conditions given below. (Choi and Kim 2005).

First, the same discretization (shape function) used in the FE method must be used for continuum design sensitivity analysis. Second, an exact integration (instead of a numerical integration) must be used in the generation of the stiffness matrix and in the evaluation of continuum-based design sensitivity expressions. Third, the exact solution (and not a numerical solution) of the FE matrix equation and the adjoint equation should be used to compare these two methods. Fourth, the movement of discrete grid points must be consistent with the design parameterization method used in the continuum method.

In this paper two different analytical discrete methods, including DDM and AVM are presented and efficiency of the proposed method is investigated with compared to DDM.

6. Numerical results

To illustrate the formulations presented in the previous sections, we consider the system shown in Figs. 4 and 10, and Capabilities of proposed method are investigated.

The Relative Percentage Error (RPE) in the identified results is calculated from Eq. (58), where $\|.\|$ is the norm of matrix, $\mathrm{E_{Identified}}$ and $\mathrm{E_{True}}$ are the identified and the true elastic modulus respectively.

$$\mathrm{RPE} = \frac{\|\mathrm{E_{Identified}} - \mathrm{E_{True}}\|}{\|\mathrm{E_{True}}\|} \times 100\% \tag{58}$$

Since the true value of elastic modulus is unknown, RPE can just be used for investigating the efficiency of method.

6.1 Multi span model

A three-span bridge as shown in Fig. 4 is studied to illustrate the proposed method. It consists of 30 Euler-Bernoulli beam elements with 31 nodes each with two DOF's. The mass density of material is 7.8×10^{-9} kg/mm^3 and the elastic modulus of material is 2.1×10^{-5} N/ mm^2. The total length of bridge is 30 m and height and width of the frame section are respectively 200 and 200 mm. The first five un-damped natural frequencies of the intact bridge are 37.73, 55.17, 66.97, 134.2 and 196.485 Hz. Rayleigh damping model is adopted with the damping ratios of the first two modes taken equal to 0.05. The equivalent Rayleigh coefficients a_0 and a_1 are respectively 0.1 and 4.804×10^{-5}.

The transverse point mass M has a constant velocity, $V = L/T$, Where T is the traveling time across the bridge and L is the total length of the bridge.

For the forced vibration analysis, an implicit time integration method, called as the "Newmark integration method" is used with the integration parameters $\beta = 1/4$ and $\gamma = 1/2$, which lead to constant-average acceleration approximation.

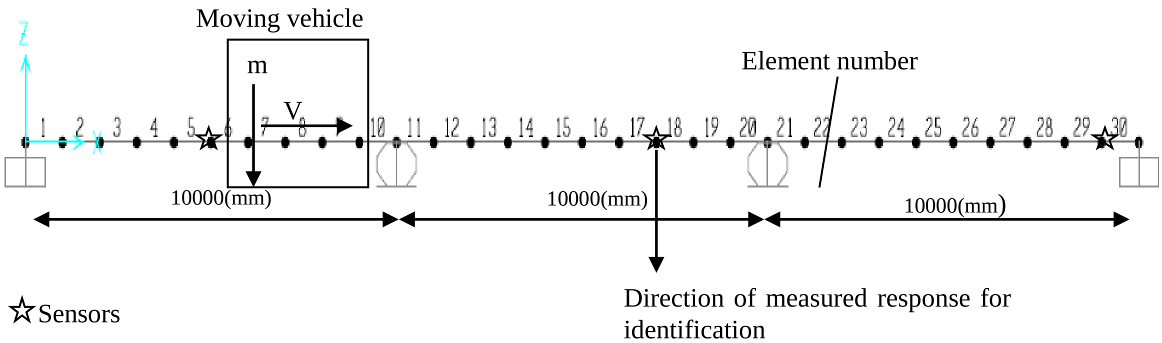


Fig. 4 Multi span bridge model used in detection procedure

Table 1 Damage scenarios for multi-span bridge

Damage scenario	Damage type	Damage location	Reduction in elastic modulus	Noise
M1-1	Single	17	12%	Nil
M1-2	Multi	3,7,19,25,28	11%,6%,5%,2%,18%	Nil
M1-3	Random	All elements	Random damage in all elements with an average of 7%	Nil
M1-4	Estimation of undamaged state	All elements	5% reduction in all elements	Nil

Speed parameter is defined as

$$\alpha = \frac{V}{V_{cr}} \tag{59}$$

In which V_{cr} is critical speed ($V_{cr} = \frac{\pi}{l}\sqrt{\frac{EI}{\rho}}$), V is moving load speed and ρ is mass per unit length of beam.

6.1.1 Damage scenarios

Three damage scenarios of single, multiple and random damages in the bridge without measurement noise are studied and they are shown in Table 1.

Local damage is simulated with a reduction in the elastic modulus of material of an element. The sampling rate is 10000 Hz and 1050 data of the acceleration response (degree of indeterminacy is 35) collected along the *z*-direction at nodes 5, 15 and 25 are used in the identification.

Scenario 1 studies the single damage state. The iterative solution converges in all speed parameters and mass ratio of moving vehicle to bridge ranges, but changes of relative error significantly increase with increases in relative speed and mass of moving vehicle.

In both methods, the minimum error is related to least relative mass and moving speed, as shown in Fig. 6. In the AVM method, the RPE in this case is equal to 0.08 and it increases with increasing speed. As shown in Fig. 6 for relative mass ratio of 0.15 the RPE for speed parameter of 0.95 is equal to 0.1. It’s remarkable that the error ratio significantly increases with increasing

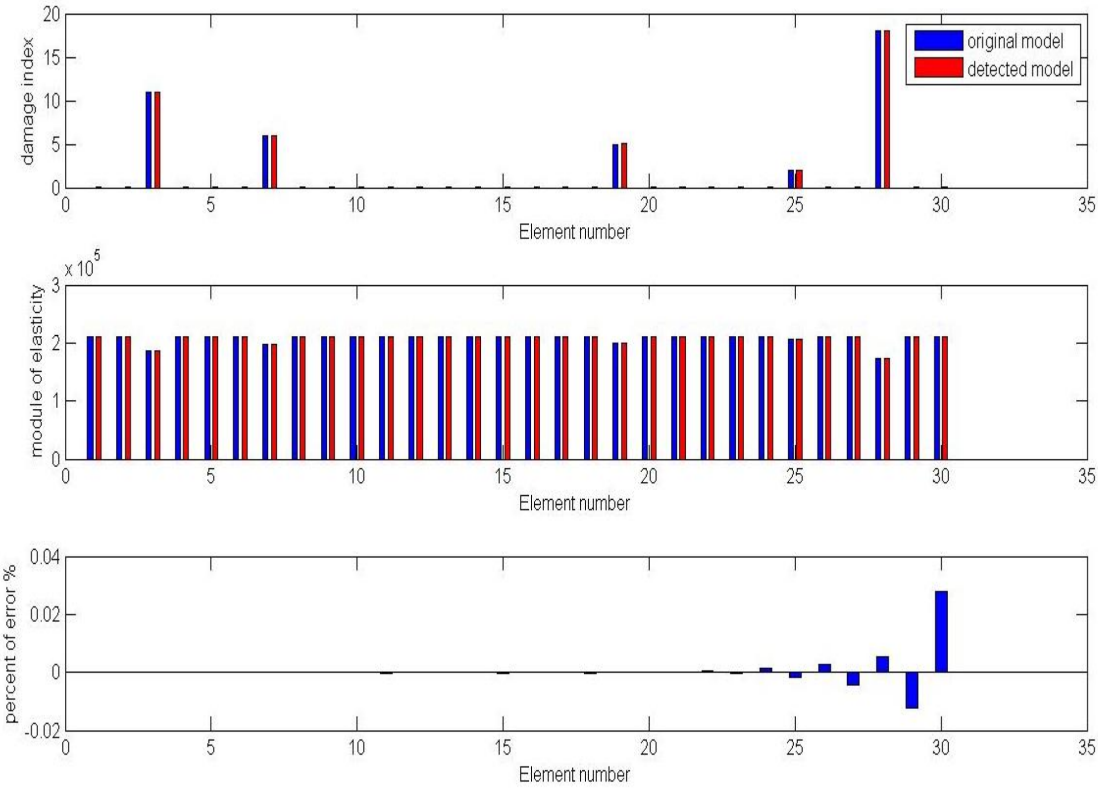
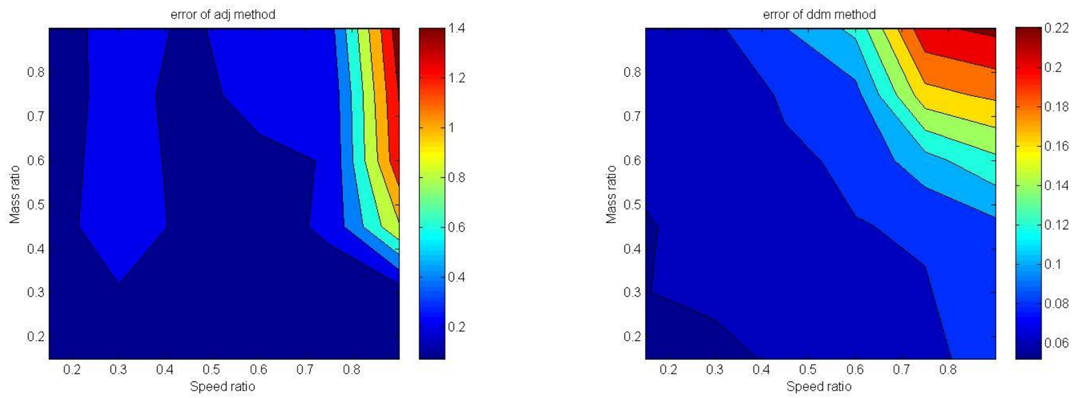


Fig. 5 Detection of damage location and amount in elements 3, 7, 19, 25 and 28



(a) Scenario1-AVM

(b) Scenario1-DDM

Fig. 6 RPE contours with respect to speed and mass ratio in model1

mass. As for mass ratio of 0.75 and relative speed equal to 0.15; the RPE is equal to 0.1% but in relative speed equal to 0.9 it reaches to 0.32% whereas the RPE for relative speed of 0.9 and relative mass of 0.9 significantly increases to 1.5%.

The RPE change for DDM method is more stable with respect to mass and speed ratio variation. For range of mass ratio equal to 0.75, the error of two methods is almost identical but with increasing mass ratio to 0.9 and relative speed greater than 0.45, the error of AVM is significantly greater than DDM.

Scenario 2 is on multiple damages with different amount of measured responses for the identification and scenario 3 is on random damages for the identification. These scenarios also converge in all speed parameter ranges and their results are similar to the first scenario. One more scenario with model error is also included as scenario 4. This scenario consists of no simulated damage in the structure, but with the initial elastic modulus of material of all the elements under-estimated by 5% in the inverse identification. Using both described methods, including DDM and the proposed method, the damage location and amount are identified correctly in all the scenarios (Fig. 5) and the RPE parameter is shown in Fig. 6.

Further studies on scenario 4 shows that both methods are sensitive to initial model error and for maximum 20% initial error can be converged and a relatively qualified FE model is therefore needed for DD procedure.

6.1.2 Effect of noise

Noise is the random fluctuation in the value of measured or input that causes random fluctuation in the output value. Noise at the sensor output is due to either internal noise sources, such as resistors at finite temperatures, or externally generated mechanical and electromagnetic fluctuations. (Alampalli and Fu 1994)

To evaluate the sensitivity of results to such measurement noise, noise-polluted measurements are simulated by adding to the noise-free acceleration vector a corresponding noise vector whose Root Mean Square (RMS) value is equal to a certain percentage of the RMS value of the noise-free data vector. The components of all the noise vectors are of Gaussian distribution, uncorrelated and with a zero mean and unit standard deviation. Then on the basis of the noise-free acceleration $Z_{,tt_{nf}}$; the noise-polluted acceleration $Z_{,tt_{np}}$ of the bridge at location x can be simulated by

$$\mathrm{Z}_{,\mathrm{tt}_{\mathrm{np}}} = \mathrm{Z}_{,\mathrm{tt}_{\mathrm{nf}}} + RMS\left(\mathrm{Z}_{,\mathrm{tt}_{\mathrm{nf}}}\right) \times \mathrm{N}_{\mathrm{level}} \times \mathrm{N}_{\mathrm{unit}} \tag{60}$$

Where RMS ($Z_{,tt_{nf}}$) is the RMS value of the noise-free acceleration vector $Z_{,tt_{nf}} \times N_{level}$ is the noise level, and N_{unit} is a randomly generated noise vector with zero mean and unit standard deviation. (Jiang *et al.* 2004).

In order to study the effect of noise in stability of sensitivity methods, scenario2 (Speed ratio of moving load is considered to be fix and equal with 0.3 and mass ratio is also equal to 0.3) is considered and different levels of noise pollution are investigated, and RPE changes with increasing number of loops for iterative procedure has been studied.

Results are illustrated in Fig. 7 for DDM and AVM. These contours show that both AVM and DDM are sensitive to noise and if noise level becomes greater than 1.6% these methods lose their effectiveness and are not able to detect the damage. So, in cases with noise level greater than 1.6%, a de-noising tool alongside sensitivity methods should be used.

6.1.3 Efficiency of proposed method

In order to compare and quantification the performance of different methods and evaluate the proposed method, Relative Efficiency Parameter (REP) is defined as

$$\mathrm{REP} = \mathrm{ST}_{\mathrm{DDM}} / ST_{\mathrm{AVM}} \tag{61}$$

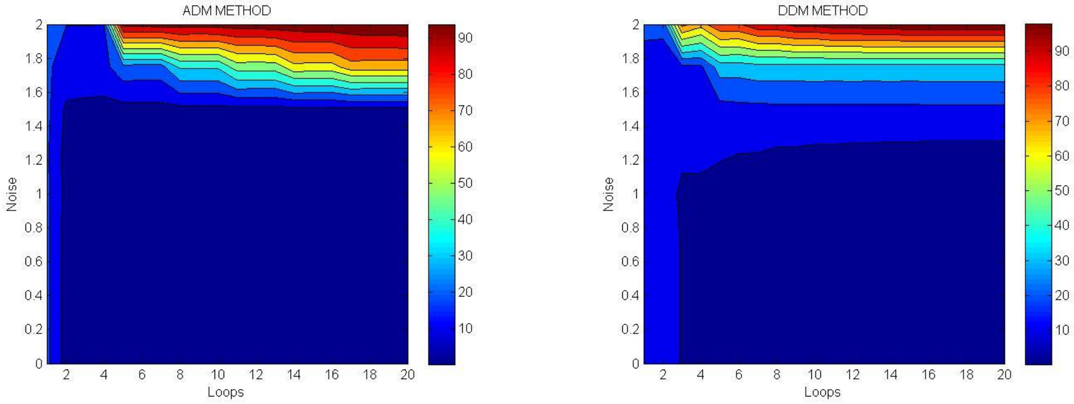
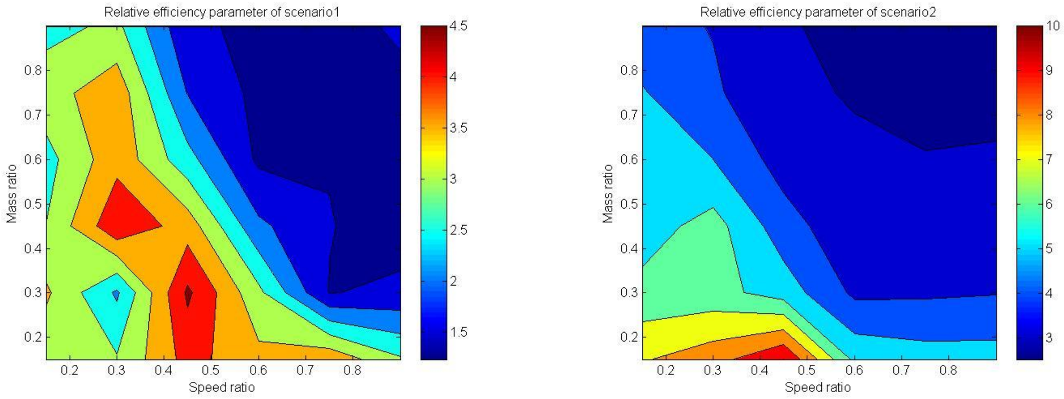


Fig. 7 RPE contours with respect to noise level and loops

Table 2 REP ranges in different scenarios for model1

Damage scenario	Max REP	Min REP	average
M1-1	4.5928	1.2327	2.5669
M1-2	10.0151	2.5146	4.5142
M1-3	10.9170	1.9474	4.4732
M1-4	9.2783	2.7705	5.2750
Total	10.9170	1.2327	4.2073



(a) Scenario1 (b) Scenario2

Fig. 8 REP contours with respect to speed and mass ratio in model1

In which, ST is the solution time of DD method. In fact this parameter represents the computation cost of method.

In Fig. 8 changes of REP parameter with respect to velocity and mass ratio is illustrated. As shown in this figure, as much as velocity and mass ratio decreases, The REP parameter increases. Summary of this figure is shown in Table 2. According to this table, the efficiency parameter is between 1.2327 to 10.9170 and its average is 4.2073.

Variations of REP average with respect to velocity and mass ratio are shown in Fig. 9. As

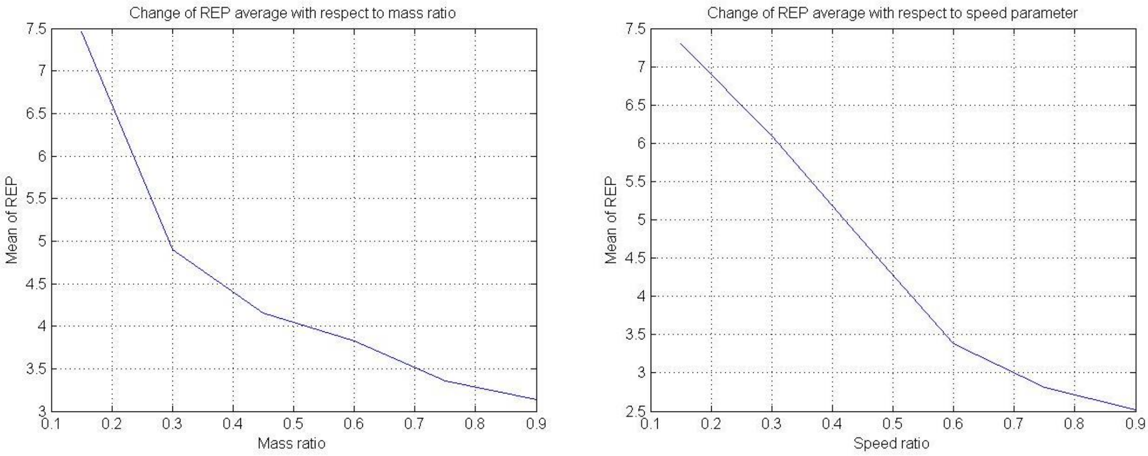


Fig. 9 Average of REP variations with respect to speed and mass ratio in model1-scenario3

illustrated in this figure, increasing these two ratios, the REP parameter decreases almost linear.For example, in mass ratio equal to 0.15 average of REP is about 7.5 but increasing mass ratio to 0.9 causes this amount reduces to 3.2.

In addition, accuracy of AVM reduces significantly for mass ratio greater than 0.9 and in velocity ratio greater than 0.45. So, in this range, using AVM is not recommended. It is noteworthy that in real bridges, Including highways and railway bridges, maximum ratio of moving vehicle to bridge is much lower than this ratio. So the AVM is extremely successful for real time structures and computational cost for this method is about 23.8% of other sensitivity-based FEMU method.

6.2 Plane grid model

A plane grid model of bridge is studied as another numerical example to illustrate the effectiveness of the proposed method. The FE model of the structure is shown in Fig. 10 the structure is modeled by 46 frame elements and 32 nodes with three DOF's at each node for the translation and rotational deformations. The mass density of material is 7.8×10^{-9} kg/mm^3 and the elastic modulus of material is 2.1×10^{-5} N/ mm^2. The first five un-damped natural frequencies of the intact bridge are 45.59, 92.77, 181.74, 259.73 and 399.07 Hz. Rayleigh damping model is adopted with the damping ratios of the first two modes taken equal to 0.05. The equivalent Rayleigh coefficients a_0 and a_1 are respectively 0.1 and 2.364×10^{-5}.

6.2.1 Damage scenarios

Three damage scenarios of single, multiple and random damages in the bridge without measurement noise are studied and they are shown in Table 3.

The sampling rate is 14000 Hz and 1150 data of the acceleration response (degree of indeterminacy is 25) collected along the z-direction at nodes 4, 11, 21 and 27 are used.

Similar to the previous model, scenario 1 studies the single damage scenario and the iterative solution converges in all speed parameters and mass ratio and changes of relative error significantly increase with increases in relative speed and mass of moving vehicle.

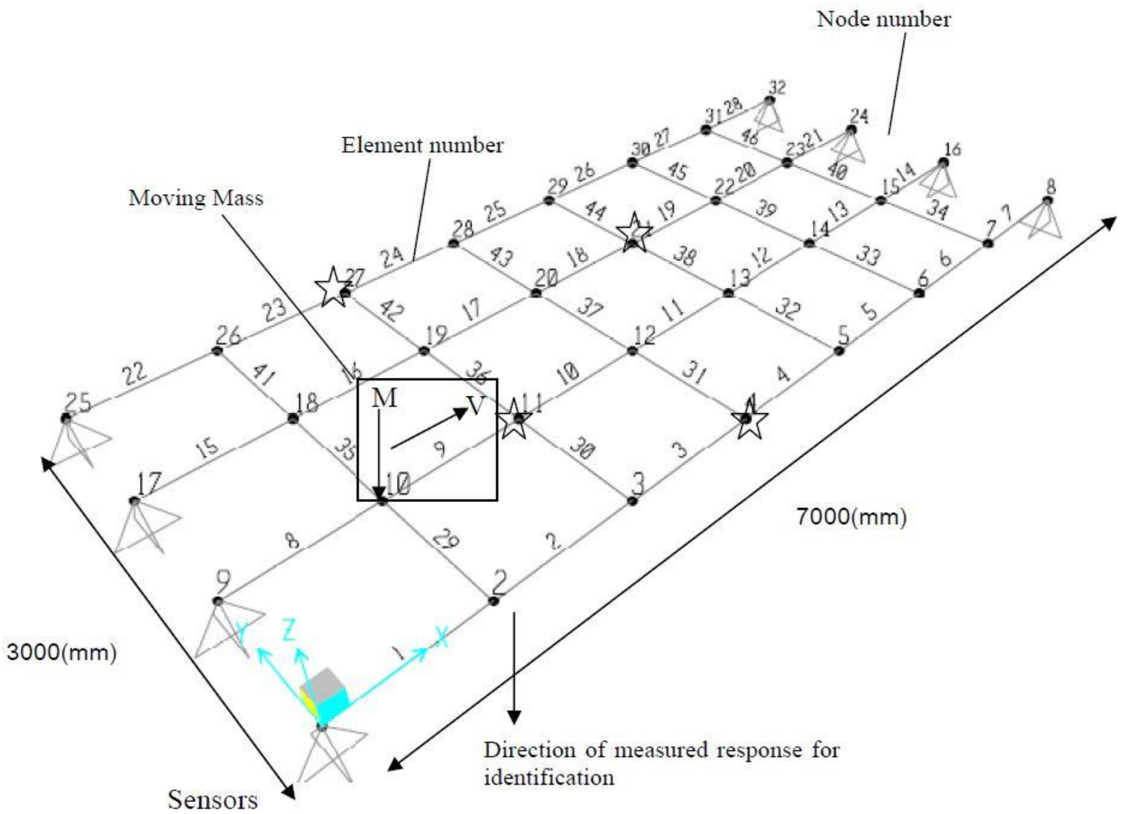


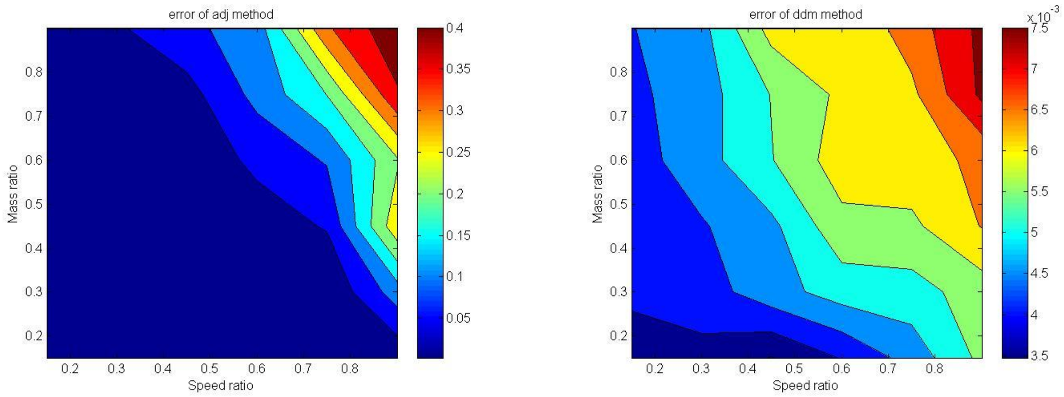
Fig. 10 Plane grid bridge model used in detection procedure

Table 3 Damage scenarios for grid model

Damage scenario	Damage type	Damage location	Reduction in elastic modulus	Noise
M2-1	Single	41	7%	Nil
M2-3	Multi	5,7,12,15,24,37	4%,11%,6%,2%,10%,16%	Nil
M2-4	Random	All elements	Random damage in all elements with an average of 5%	Nil
M2-6	Estimation of undamaged state	All elements	6% reduction in all elements	Nil

Fig. 11 shows that in both methods, the minimum error is related to least relative mass and moving speed and for high amounts of mass ratio; it increases significantly for AVM method. For DDM method, The RPE parameter is more stable with respect to mass and speed ratio variation. For range of mass ratio equal to 0.75, the error of two methods are almost identical but with increasing mass ratio to 0.9 and relative speed greater than 0.45, the error of AVM is significantly greater than DDM.

Scenario 2 is on multiple damages with different amount of measured responses for the identification and scenario3 is on random damages for the identification. These scenarios also converge in all speed parameter ranges and similar results with the first scenario were obtained.



(a) Scenario1-AVM (b) Scenario1-DDM

Fig. 11 RPE contours with respect to speed and mass ratio in model2

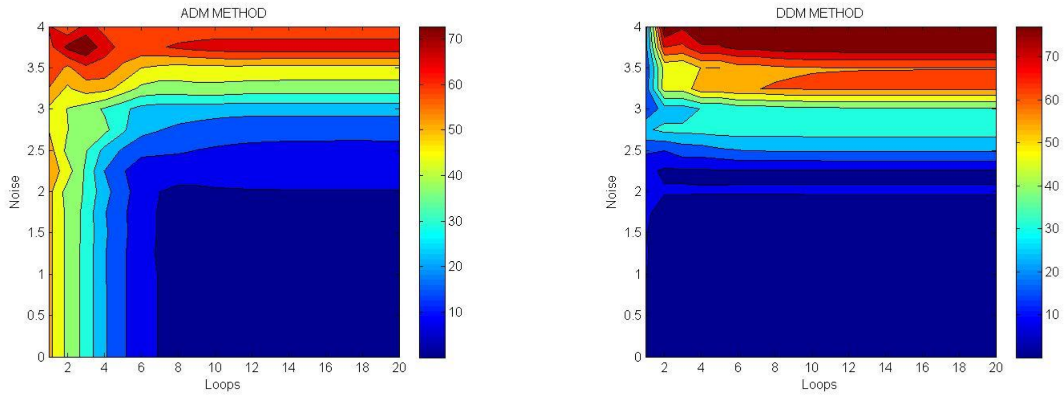


Fig. 12 RPE contours with respect to noise level and loops

One more scenario with model error is also included as scenario 4. This scenario consists of no simulated damage in the structure, but with the initial elastic modulus of material of all the elements under-estimated by 6% in the inverse identification.

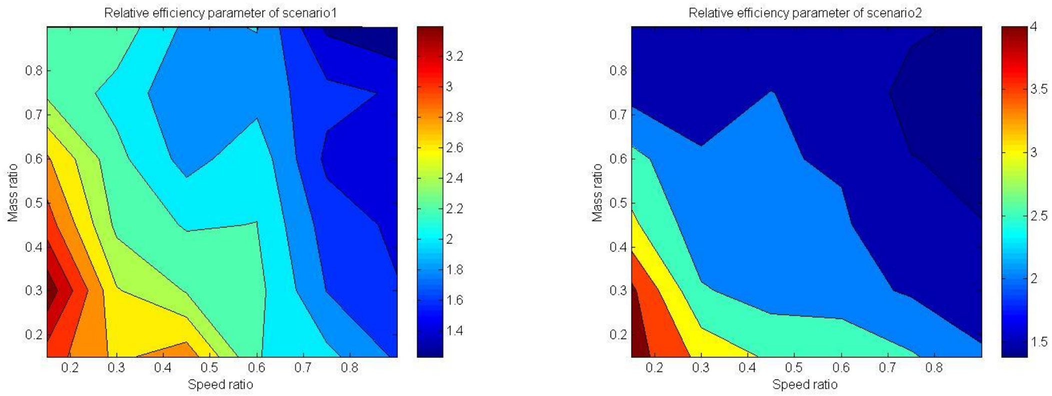
6.2.2 Effect of noise

In order to study the effect of noise in stability of sensitivity methods, scenario3 (Speed ratio of moving load is considered to be fix and equal with 0.3 and mass ratio is equal to 0.3) is considered and different levels of noise pollution are investigated, and RPE changes with increasing number of loops for iterative procedure has been studied.

Fig. 12 shows that both AVM and DDM are sensitive to noise and if noise level becomes greater than 2.8% and 2.5% for AVM and DDM respectively, these methods lose their effectiveness and are not able to detect damage. So, in cases with noise level greater than the mentioned values, a de-noising tool such as wavelet transform alongside sensitivity methods should be used. The wavelet transform is mainly attractive because of its ability to compress and encode information to reduce noise or detect any local singular behavior of a signal (Solís *et al.* 2013).

Table 4 REP ranges in different scenarios for model2

Damage scenario	Max REP	Min REP	average
M2-1	3.5441	1.2303	2.1395
M2-2	4.2359	1.3822	2.1382
M2-3	3.2535	1.3183	1.8880
M2-4	2.9503	1.1897	1.7480
Total	4.2359	1.1897	1.9784



(a) Scenario1 (b) Scenario2

Fig. 13 REP contours with respect to speed and mass ratio in model2

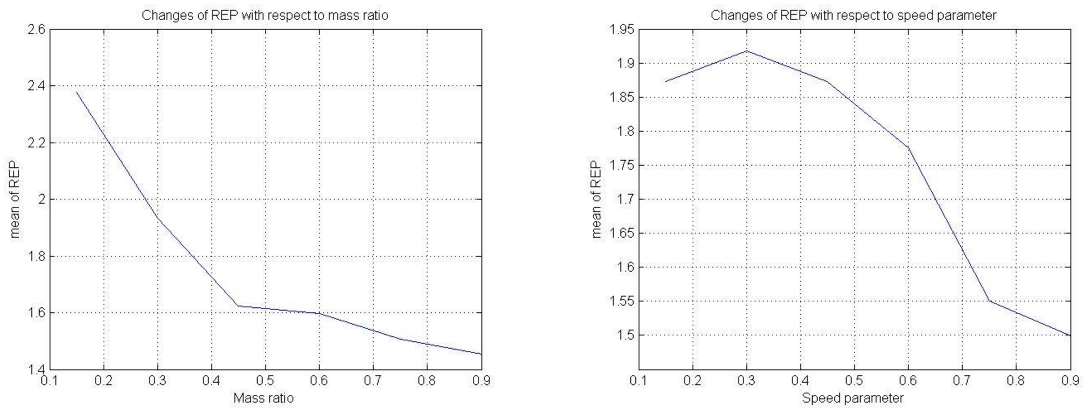


Fig. 14 Average of REP changes with respect to speed and mass ratio in model2-scenario4

6.2.3 Efficiency of proposed method

In Fig. 13 changes of REP parameter with respect to velocity and mass ratio are illustrated. As shown in this figure, as much as velocity and mass ratio decreases, The REP parameter increases. Summary of this figure is shown in Table 4. According to this table, the efficiency parameter is between 1.1897 to 4.2359 and its average equals to 1.9784,

In Fig. 14 changes of REP average with respect to velocity and mass ratio is shown. As illustrated in this figure, by increasing these two ratios, the REP parameter decreases. Furthermore, accuracy of AVM reduces significantly for mass ratio greater than 0.9 and in velocity ratio greater than 0.45. So, for second model similar to the first, in this range, using AVM is not recommended. Outside of this limit and for real time structures, the AVM is extremely successful and computational cost for this method is about 50.5% of DDM.

7. Conclusions

A new damage detection method based on FEMU and sensitivity technique using acceleration time history data of bridge deck affected by a moving vehicle with specified mass, named "AVM" is presented. The updating procedure can be regarded as a parameter identification technique which aims to fit the unknown parameters of an analytical model such that the model behavior corresponds as closely as possible to the measured behavior.

In this paper, an incremental solution for adjoint variable equation is developed which calculates each elements of sensitivity matrix separately. Through this work, an analytical solution found which significantly increases the speed and accuracy of the solution. Also, the proposed method has the advantage that only short duration of dynamic response measurement from as few as one sensor is needed to solve the inverse analysis. Numerical simulations demonstrate the efficiency, accuracy and stability of the method to identify location and intensity of single, multiple and random damages of different bridge models. Compared with existing and well established techniques, it is demonstrated that the proposed technique is found to be computationally simpler and more flexible. Different studies confirmed that computational costs for this method are 23.8% and 50.5% compared to other sensitivity methods for multi-span and grid models, respectively. The drawback of the proposed method is its accuracy and efficiency reduction in mass and speed ratios near to unity. It's notable that in real structures, this range of speed and mass ratio is not accessible.

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