Structural Engineering and Mechanics, *Vol. 54, No. 1 (2015) 135-148*
DOI: http://dx.doi.org/10.12989/sem.2015.54.1.135

Effect of the rotation on a non-homogeneous infinite cylinder of orthotropic material with external magnetic field

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(Received August 29, 2013, Revised December 1, 2014, Accepted January 3, 2015)

Abstract. The present investigation is concerned with a study effect of magnetic field and non-homogenous on the elastic stresses in rotating orthotropic infinite circular cylinder. A certain boundary conditions closed form stress fields solutions are obtained for rotating orthotropic cylinder under initial magnetic field with constant thickness for three cases: (1) Solid cylinder, (2) Cylinder with a circular hole at the center, (3) Cylinder mounted on a circular rigid shaft. Analytical expressions for the components of the displacement and stress fields in different cases are obtained. The effect of rotation and magnetic field and non-homogeneity on the displacement and stress fields are studied. Numerical results are illustrated graphically for each case. The effects of rotating and magnetic field and non-homogeneity are discussed.

Keywords: magnetic field; rotation; free vibrations; non-homogeneous; orthotropic cylinder

1. Introduction

In the past, with increasing application of composite and orthotropic materials in rotating components machinery and structures (e.g., flywheels, turbines, etc.), study of plane elastic wave propagation in a non-rotating medium is receiving considerable attention in recent years. In the past, accident of rotating cylinder wheels due to flexural vibration has frequently occurred in rot-dynamic machinery such as steam turbines and gas turbines. At the present time, applied mathematicians are exhibiting considerable interest in dynamical methods of elasticity, since the usual quasi-static approach ignores certain very important features of the problems. That approach is based on the assumption that the inertia terms may be omitted from the equations of motion. This assumption holds good but arise number of problems in engineering and technology. When this assumption may not hold good, the inertia terms in the equations of motion may lead to cases of considerable mathematical complications, which increase application of composite and orthotropic materials in rotating components of machinery and structures (e.g., flywheels and

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

turbines). This mater has attracted the attention of many researchers such as Abid and Khan (2013), Selvamani and Ponnusamy (2013), Marin *et al*. (2013), Abd-Alla and Yahya (2013), Abd-Alla *et al*. (2013), Abd-Alla and Mahmoud (2013), Abd-Alla *et al*. (2011). The recent trend of research concerning non-homogeneous elasticity may be found in the works of all Noda *et al*. (1986), Tsai (1993), Chandrasekharaiah and Keshavan (1991). Generalized thermoelastic infinite medium with cylindrical cavity subjected to moving heat source was investigated by Youssef (2009). Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section was studied by Ponnusamy (2007). Stress concentration in elastic bodies, i.e., local accumulation of stresses arise in the presence of material discontinuities such as those due to inclusions of materials with elastic properties which differ from those of the surrounding matters, may be found in the works of Mahmoud (2014, 2012), Mahmoud *et al.* (2011), Mahmoud *et al*. (2011), Mahmoud (2010, 201). McGeeIII and Kim (2010) investigated the three-dimensional vibrations of cylindrical elastic solids with v-notches and sharp radial cracks. Vibration and radial wave propagation velocity in functionally graded thick hollow cylinder was studied by Shakeri, *et al*. (2006). Buchanan (2003) investigated the free vibration of an infinite magneto-electro-elastic cylinder. Exact analysis of the plane-strain vibrations of thick-walled hollow poroelastic cylinders has been studied by Reddy and Tajuddin (2000). Wang *et al.* (2010) investigated the three-dimensional exact solutions for free vibrations of simply supported magneto-electro-elastic cylindrical panels. Heyliger and Jilani (1992) studied the free vibrations of inhomogeneous elastic cylinders and spheres. Abd-Alla and Mahmoud (2012), Abd-Alla *et al.* (2011, 2012) studied analytical solution of wave propagation in non-homogeneous orthotropic rotating elastic media, wave propagation modeling in cylindrical human longe wet bones with cavity, the problem of transient coupled thermoelasticity of an annular fin. Chen *et al*. (2005, 2004) investigated the free vibration and general solution of non-homogeneous transversely isotropic magneto-electro-elastic hollow cylinders. Zhou and Lo (2012) studied the three-dimensional vibrations of annular thick plates with linearly varying thickness. Toudeshky *et al*. (2009) studied sound transmission into a thick hollow cylinder with the fixed and boundary condition. Buchanan (2003) investigated the free vibration of an infinite magneto-electro-elastic cylinder. Mofakhami *et al*. (2006) investigated the finite cylinder vibration with different end boundary conditions.

The present investigation is concerned with a study effect of non-homogenous on the elastic stresses in rotating orthotropic infinite circular cylinder subjected to magnetic field. The solutions have been obtained in analytical form. In both cases, the stresses have been calculated. Finally, comparisons between both cases are clarified by figures. The stresses in rotating cylinder made of orthotropic materials will be found for the following three cases:

(1) a solid cylinder ;
(2) a cylinder mounted on a circular rigid shaft ;
(3) a cylinder with a circular hole at the center.

2. Formulation of the problem

Let us consider a cylindrical coordinate systems (r,θ,z) with z-axis coinciding with the axis of cylinder. We consider the strains symmetric about z-axis. The radial displacement $u_r=u$, u is a function of r and t only, the circumferential displacement $u_\theta=0$ and the longitudinal displacement $u_z=0$, which are independent of θ and z. let $\vec{H}$ $(0,0,H_z)$, the analysis is based on the following assumptions (1) the infinitesimal elasticity theory of an orthotropic body (2) the materials are macroscopically homogeneous and cylindrically orthotropic; (3) stress-strain relations obey a generalized Hook's law; (4) the condition of plane strain is used. In plane strain in the plane perpendicular to the *z*-axis, *u* is a function of *r* alone the stresses components are given by

$$\sigma_{rr} = c_{11}\frac{du}{dr} + c_{12}\frac{u}{r},$$

$$\sigma_{\theta\theta} = c_{12}\frac{du}{dr} + c_{22}\frac{u}{r},$$

$$\sigma_{zz} = c_{13}\frac{du}{dr} + c_{23}\frac{u}{r},$$

$$\tau_{rr} = \mu_e H_z^2(\frac{du}{dr} + \frac{u}{r}),$$

$$\tau_{rz} = \tau_{\theta z} = 0. \tag{1}$$

The equilibrium equation in the direction of *r* is given by

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + \rho\Omega^2 r + F_r = 0, \tag{2}$$

where c_{ij} are the elastic constants, Ω is the uniform angular velocity about z-axis, $\vec{\Omega}(0,0,\Omega)$ and ρ is the density of the cylinder. F_r the radial component of Lorentz's force.

$$F_r = \mu_e H_z^2 \frac{d}{dr}(\frac{du}{dr} + \frac{u}{r}). \tag{3}$$

We characterize the elastic constants c_{ij} and ρ and μ_e of non-homogeneous material , the elastic constants and the density are power functions of the radial coordinate

$$c_{ij} = \alpha_{ij} r^{2m}\ ,\ \rho = \rho_0 r^{2m}, \qquad \mu_e = \mu_{e_o} r^{2m} \text{ at } i=1,2;\ j=1,2,3 \tag{4}$$

where α_{ij}, ρ_o, μ_{e_o} are constants and m is rational number. The elastic matrix of the elastic constants is

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}$$

3. Solution of the problem

We seek the solution of Eq. (2) as

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + \rho\Omega^2 r + \mu_e H_z^2 \frac{d}{dr}(\frac{du}{dr} + \frac{u}{r}) = 0. \tag{5}$$

Multiple Eq. (5) by *r*

$$r\frac{d\sigma_{rr}}{dr} + \sigma_{rr} - \sigma_{\theta\theta} + \rho\Omega^2 r^2 + \mu_e H_z^2 \frac{d}{dr}(\frac{du}{dr} + \frac{u}{r}) = 0,$$

$$\frac{d}{dr}(r\sigma_{rr}) - \sigma_{\theta\theta} + \rho\Omega^2 r^2 + \mu_e H_z^2 \frac{d}{dr}(\frac{du}{dr} + \frac{u}{r}) = 0,$$

$$\frac{d}{dr}(r\sigma_{rr}) - \sigma_{\theta\theta} + \rho\Omega^2 r^2 + \mu_e H_z^2 r \frac{d}{dr}(\frac{du}{dr} + \frac{u}{r}) = 0, \tag{6}$$

by using Eqs. (1), (2), (6)

$$\frac{d}{dr}(r\alpha_{11}r^{2m}\frac{du}{dr} + \alpha_{12}r^{2m}u) - (\alpha_{12}r^{2m}\frac{du}{dr} + \alpha_{22}r^{2m}\frac{u}{r})$$
$$+\rho_o r^{2m}\Omega^2 r^2 + \mu_{e_o}H_z^2 r^{2m} r \frac{d}{dr}(\frac{du}{dr} + \frac{u}{r}) = 0.$$

$$(2m+1)r^{2m}\alpha_{11}\frac{du}{dr} + \alpha_{11}r^{2m+1}\frac{d^2u}{dr^2} + 2m\alpha_{12}r^{2m-1}u + \alpha_{12}\frac{du}{dr}r^{2m}$$
$$-\alpha_{12}r^{2m}\frac{du}{dr} - \alpha_{22}r^{2m}\frac{u}{r} + \rho_o r^{2m}\Omega^2 r^2$$
$$+\mu_{e_o}H_z^2 r^{2m+1}\left[\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}\right] = 0\,, (r^{2m-1} \neq 0)$$

$$r^2(\alpha_{11} + \mu_{e_o}H_z^2)\frac{d^2u}{dr^2} + [(2m+1)\alpha_{11} + \mu_{e_o}H_z^2]r\frac{du}{dr}$$
$$+\left(2m\alpha_{12} - \alpha_{22} - \mu_{e_o}H_z^2\right)u + \rho_o r^3\Omega^2 = 0\left(\div\left(\alpha_{11} + \mu_{e_o}H_z^2\right)\right), \tag{7}$$

one obtain

$$r^2\frac{d^2u}{dr^2} + \frac{[(2m+1)\alpha_{11} + \mu_{e_o}H_z^2]}{[\alpha_{11} + \mu_{e_o}H_z^2]}r\frac{du}{dr} + \frac{(2m\alpha_{12} - \alpha_{22} - \mu_{e_o}H_z^2)}{(\alpha_{11} + \mu_{e_o}H_z^2)}u$$
$$= -\frac{\rho_o\Omega^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}r^3. \tag{8}$$

The homogenous solution of Eq. (8)

$$r^2\frac{d^2u}{dr^2} + \frac{[(2m+1)\alpha_{11}+\mu_{e_o}H_z^2]}{[\alpha_{11}+\mu_{e_o}H_z^2]}r\frac{du}{dr} + \frac{(2m\alpha_{12}-\alpha_{22}-\mu_{e_o}H_z^2)}{(\alpha_{11}+\mu_{e_o}H_z^2)}u = 0, \tag{9}$$

This equation is Euler's equation the solution is $u=r^{\lambda}$, $\lambda \notin R$ substitution in Eq. (9) we obtain

$$\lambda^2 + \frac{2m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}\lambda + \frac{(2m\alpha_{12}-\alpha_{22}-\mu_{e_o}H_z^2)}{(\alpha_{11}+\mu_{e_o}H_z^2)} = 0\ ,$$

$$\to \lambda = \frac{1}{2}[\frac{-2m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)} \pm \sqrt{\frac{4m^2\alpha_{11}^2}{(\alpha_{11}+\mu_{e_o}H_z^2)^2} - \frac{4(2m\alpha_{12}-\alpha_{22}-\mu_{e_o}H_z^2)}{(\alpha_{11}+\mu_{e_o}H_z^2)}}]\ ,$$

$$\lambda = \frac{-m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)} \pm \sqrt{\frac{m^2\alpha_{11}^2}{(\alpha_{11}+\mu_{e_o}H_z^2)^2} - \frac{(2m\alpha_{12}-\alpha_{22}-\mu_{e_o}H_z^2)}{(\alpha_{11}+\mu_{e_o}H_z^2)}}\ ,$$

$$u_H = Ar^{-\frac{2m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k} + Br^{-\frac{2m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k}\ ,$$

where $k^2 = \frac{m^2\alpha_{11}^2}{(\alpha_{11}+\mu_{e_o}H_z^2)^2} - \frac{(2m\alpha_{12}-\alpha_{22}-\mu_{e_o}H_z^2)}{(\alpha_{11}+\mu_{e_o}H_z^2)}$,

The particular solution of Eq. (8) $u_p=R\,r^3$ by substitution in Eq. (8) we obtain

$$R = \frac{-\rho_o\Omega^2}{9\alpha_{11}+8\mu_{e_o}H_z^2+6m\alpha_{11}+2m\alpha_{12}-\alpha_{22}}, \tag{10}$$

Hence

$$u(r) = Ar^{-\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k} + Br^{-\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k} + Rr^3, \tag{11}$$

where *A*, *B* are arbitrary constants, *R* given from Eq. (10). Substituting Eq. (11) into Eqs. (1), (2) to find the components of stresses, we obtain

$$\sigma_{rr} = A\beta_1 r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k-1} + B\beta_2 r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k-1} + \beta_3 r^{2m+2}, \tag{12}$$

where $\beta_1 = \alpha_{11}(\frac{-m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)} + k) + \alpha_{12}$,

$$\beta_2 = \alpha_{12} - \alpha_{11}(\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)} + k),\ \ \beta_3 = (3\alpha_{11} + \alpha_{12})R,$$

$$\sigma_{\theta\theta} = A\gamma_1 r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k-1} + B\gamma_2 r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k-1} + \gamma_3 r^{2m+2}, \tag{13}$$

where $\gamma_1 = \alpha_{22} + \alpha_{12}(\frac{-m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)} + k)$, $\gamma_2 = \alpha_{22} - \alpha_{12}(\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)} + k)$, $\gamma_3 = R(3\alpha_{12} + \alpha_{22})$.

3.1 Case I: a solid cylinder with radius b

The constant *A*, *B* are determined by the continuity condition at the center of cylinder

$$u = 0 \text{ at } r = 0 \tag{14}$$

If there are no forces applied there, then

$$\sigma_{rr} = 0 \text{ at } r = b \tag{15}$$

Substituting Eq. (14) into Eq. (11) we obtain

$$B = 0, \qquad k > m. \tag{16}$$

Eq. (12) become

$$\sigma_{rr} = A\beta_1 r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k-1} + \beta_3 r^{2m+2}. \tag{17}$$

Then, by substituting (15) into Eq. (17) we obtain

$$A = -\frac{\beta_3}{\beta_1} b^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+3}. \tag{18}$$

Substituting Eqs. (18), (16) into Eqs. (11), (13) and (17)

$$u = -\frac{\beta_3}{\beta_1} b^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+3} r^{-\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k} + Rr^3, \tag{19}$$

$$\sigma_{rr} = -\beta_3 b^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+3} r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k-1} + \beta_3 r^{2m+2},$$

$$\sigma_{\theta\theta} = \frac{-\beta_3}{\beta_1}\gamma_1 b^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+3} r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k-1} + \gamma_3 r^{2m+2},$$

3.2 Case II: a hollow circular cylinder with internal and external radius a and b, respectively

The boundary conditions

$$\sigma_{rr} = 0 \text{ at } r = a,\ \sigma_{rr} = 0 \text{ at } r = b \tag{20}$$

Substituting Eq. (20) into Eq. (12) we obtain

$$A\beta_1 a^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k-1} + B\beta_2 a^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k-1} + \beta_3 a^{2m+2} = 0,$$

$$A\beta_1 b^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k-1} + B\beta_2 b^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k-1} + \beta_3 b^{2m+2} = 0. \tag{21}$$

Solving Eq. (21) together we obtain

$$A = \frac{\beta_3}{\beta_1} a^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} \frac{[(\frac{b}{a})^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} - (\frac{b}{a})^{-2k}]}{[(\frac{b}{a})^{-2k}-1]}, \tag{22}$$

$$B = \frac{\beta_3}{\beta_2} a^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k+2m+3} \frac{[(\frac{b}{a})^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} - 1]}{[1-(\frac{b}{a})^{-2k}]}, \tag{23}$$

Substituting Eqs. (22) and (23) into Eqs. (11)-(13) we obtain

$$u = \frac{\beta_3}{\beta_1} a^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} \frac{[(\frac{b}{a})^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} - (\frac{b}{a})^{-2k}]}{[(\frac{b}{a})^{-2k}-1]}$$

$$r^{\frac{-m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k} + \frac{\beta_3}{\beta_2} a^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k+2m+3} r^{\frac{-m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k}$$

$$\frac{[(\frac{b}{a})^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} - 1]}{[(\frac{b}{a})^{-2k}-1]} + Rr^3,$$

$$\sigma_{rr} = \beta_3 a^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} \frac{[(\frac{b}{a})^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} - (\frac{b}{a})^{-2k}]}{[(\frac{b}{a})^{-2k}-1]}$$

$$r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k-1} + \beta_3 a^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k+2m+3} r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k-1}$$

$$\frac{[(\frac{b}{a})^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3}-1]}{[1-(\frac{b}{a})^{-2k}]} + \beta_3 r^{2m+2},$$

$$\sigma_{\theta\theta} = \gamma_1 \frac{\beta_3}{\beta_1} a^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} \frac{[(\frac{b}{a})^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3}-(\frac{b}{a})^{-2k}]}{[(\frac{b}{a})^{-2k}-1]}$$

$$r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k-1} + \gamma_2 \frac{\beta_3}{\beta_2} a^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k+2m+3} r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k-1}$$

$$\frac{[(\frac{b}{a})^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3}-1]}{[1-(\frac{b}{a})^{-2k}]} + \gamma_3 r^{2m+2}.$$

3.3 Case III: a hollow circular cylinder with external radius b mounted on a rigid shaft of radius a (b > a)

The constants A and B are determined by the conditions such that no displacements are allowed between the shaft and cylinder. Also no applied forces at the periphery of the cylinder then

$$u = 0 \text{ at } r = a,\ \sigma_{rr} = 0 \text{ at } r = b \tag{24}$$

and substituting in Eqs. (11), (12) we obtain

$$A = \frac{[\beta_3 b^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} - R\beta_2(\frac{b}{a})^{-2k} a^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+3}]}{[\beta_2(\frac{b}{a})^{-2k}-\beta_1]}, \tag{25}$$

$$B = \frac{[\beta_3 b^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} a^{2k} - \beta_1 R a^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k+3}]}{[\beta_1-\beta_2(\frac{b}{a})^{-2k}]}. \tag{26}$$

Substituting Eqs. (25) and (26) into Eqs. (11)-(13) we obtain

$$u = r^{\frac{-m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k} \frac{[\beta_3 b^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} - R\beta_2(\frac{b}{a})^{-2k} a^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+3}]}{[\beta_2(\frac{b}{a})^{-2k}-\beta_1]}$$

$$+r^{\frac{-m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k} \frac{[\beta_3 b^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_z^2}{(\alpha_{11}+\mu_{e_o}H_z^2)}-k+2m+3} a^{2k} - R\beta_1 a^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_z^2)}+k+3}]}{[\beta_1-\beta_2(\frac{b}{a})^{-2k}]}$$

$$+Rr^3$$

$$\sigma_{rr} = \beta_1 r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_Z^2}{(\alpha_{11}+\mu_{e_o}H_Z^2)}+k-1} \frac{[\beta_3 b^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_Z^2}{(\alpha_{11}+\mu_{e_o}H_Z^2)}-k+2m+3} - R\beta_2(\frac{b}{a})^{-2k} a^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_Z^2)}-k+3}]}{[\beta_2(\frac{b}{a})^{-2k}-\beta_1]}$$

$$+\beta_2 \mathrm{r}^{\frac{m\alpha_{11}+2m\mu_{e_o}H_Z^2}{(\alpha_{11}+\mu_{e_o}H_Z^2)}-k-1} \frac{[\beta_3 b^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_Z^2}{(\alpha_{11}+\mu_{e_o}H_Z^2)}-k+2m+3} a^{2k} - R\beta_1 a^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_Z^2)}+k+3}]}{[\beta_1-\beta_2(\frac{b}{a})^{-2k}]}$$

$$+\beta_3 r^{2m+2}$$

$$\sigma_{\theta\theta} = \gamma_1 r^{\frac{m\alpha_{11}+2m\mu_{e_o}H_Z^2}{(\alpha_{11}+\mu_{e_o}H_Z^2)}+k-1} \frac{[\beta_3 b^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_Z^2}{(\alpha_{11}+\mu_{e_o}H_Z^2)}-k+2m+3} - R\beta_2(\frac{b}{a})^{-2k} a^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_Z^2)}-k+3}]}{[\beta_2(\frac{b}{a})^{-2k}-\beta_1]}$$

$$+\gamma_2 \mathrm{r}^{\frac{m\alpha_{11}+2m\mu_{e_o}H_Z^2}{(\alpha_{11}+\mu_{e_o}H_Z^2)}-k-1} \frac{[\beta_3 b^{\frac{-m\alpha_{11}-2m\mu_{e_o}H_Z^2}{(\alpha_{11}+\mu_{e_o}H_Z^2)}-k+2m+3} a^{2k} - R\beta_1 a^{\frac{m\alpha_{11}}{(\alpha_{11}+\mu_{e_o}H_Z^2)}+k+3}]}{[\beta_1-\beta_2(\frac{b}{a})^{-2k}]}$$

$$+\gamma_3 r^{2m+2}$$

4. Numerical results and discussion

For the numerical calculations of u, σ_{rr} and $\sigma_{\theta\theta}$ in different cases, we use the data for orthotropic material. In order to illustrate theoretical results obtained in the preceding section, we now present some numerical results. Numerical calculations are carried out for the displacement and the stress components along the r-direction at different values of the rotation Ω and magnetic field in the case of non-homogeneous material. In order to get magnetic field and non-homogeneity influence on the stress fields distribution in a rotating cylinder, the elastic constants are taken from Hearmon (Abd-Alla *et al.* 2011). α_{11}=0.331, α_{12}=0.203, α_{13}=0.192, α_{22}=0.276, α_{23}=0.241, α_{33}=0.393 Dyne.cm^2 (Units=10^{11}), a=2 cm, b=4 cm. In all figures u, σ_{rr} and $\sigma_{\theta\theta}$ denote the corresponding case when the cylinder is completely orthotropic material.

The cases (1) and (3) in Figs. 1-18 denoting the components of displacement and stresses of orthotropic material. Figs. 1, 4 ,7, 10, 13 and 16 show the components of radial displacement, which it changes with the effect of magnetic field, rotation and non-homogeneity, respectively, in all cases. Figs. 2, 5, 8, 11, 14 and 17 show the components of radial stress fields, which it changes with magnetic field, rotation and non-homogeneity, respectively. Figs. 3, 6, 9, 12, 15 and 18 show the components of tangential stress, which it changes with the effect of magnetic field, rotation and non-homogeneity, respectively, in all cases. Figs. 1, 4, 7, 10, 13 and 16 show the components of displacement which it changes with the magnetic field, rotation and non-homogeneity, respectively, in cases (1) and (3). Figs. 1, 4 ,7, 10, 13 and 16 show the variation of the components of displacement increase and decrease with increasing r in all cases and satisfied the boundary conditions for two problems, while it increase with increasing of rotation, as well it decreases with increasing of magnetic field and non-homogeneity. Figs. 2, 5, 8, 11, 14 and 17 show the variation of the components of radial stress, it is notice that in Fig. 2 the radial stress decreases with increasing of the radial r, while in Figs. 5, 8, 13, 17 the radial stress increase and decrease with

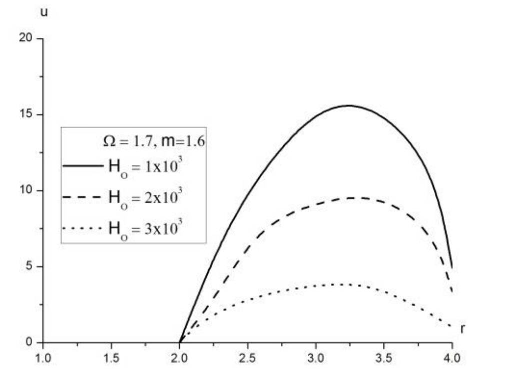

Fig. 1 Variation of the radial component of the displacement u with the radial r. when rotation Ω=0.6, the non-homogeneity m=1.6 at different values for magnetic field H_0, in case I

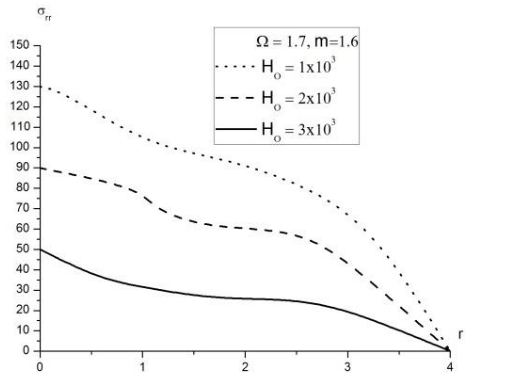

Fig. 2 Variation of the radial component of the stress σ_{rr} with the radial r. when rotation Ω=0.6, the non-homogeneity m=1.6 at different values for magnetic field H_0, in case I.

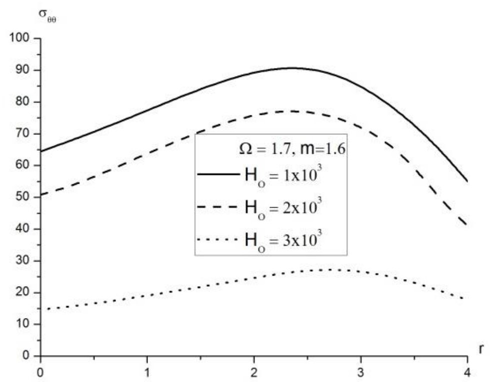

Fig. 3 Variation of the tangential component of the stress $\sigma_{\theta\theta}$ with the radial r. when rotation Ω=0.6, the non-homogeneity m=1.6 at different values for magnetic field H_0, in case I

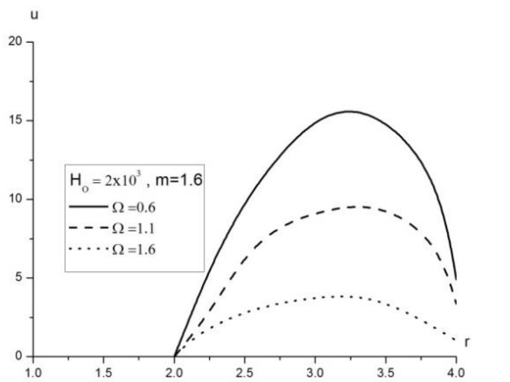

Fig. 4 Variation of the radial displacement with the radial, when magnetic field H_0=2×10^3, the non-homogeneity m=1.6 at different values for rotation Ω=0.6, 1.1, 1.6, in case I

increasing of the radial r, as well it increases with increasing of magnetic field, rotation and non-homogeneity, respectively, and satisfied the boundary conditions for two problems. Figs. 3, 6, 9, 12, 15 and 18 show the variation of the components of tangential stress, in Fig. 3 the tangential stress increases with increasing of the radial r, in Fig. 6 the tangential stress decreases with increasing of the radial r, while in Figs. 9, 12, 15, 18) the tangential stress increase and increase with increasing of the radial r, as well it increase with increasing of the rotation, magnetic field and non-homogeneity.

The variations of the stresses σ_{rr} and $\sigma_{\theta\theta}$ and displacement u, are due to the effect of rotation and magnetic field and non-homogeneity. It can be seen that the components of displacement and the stress satisfy the boundary conditions. It is evident that orthotropic it has a significant influence on the stresses. Also, the influence of the rotation and magnetic field and non-homogeneity on displacement and stresses is very pronounced. The results are specific for the example considered,

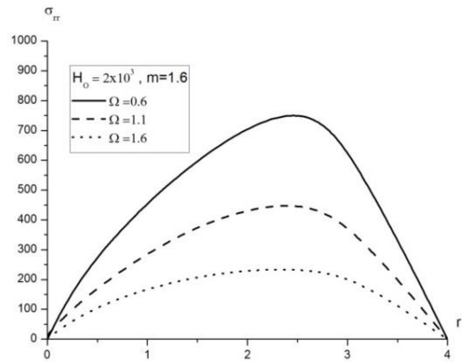

Fig. 5 Variation of the radial stress with the radial, when magnetic field $H_0=2\times10^3$, the non-homogeneity m=1.6 at different values for rotation Ω=0.6, 1.1, 1.6, in case I

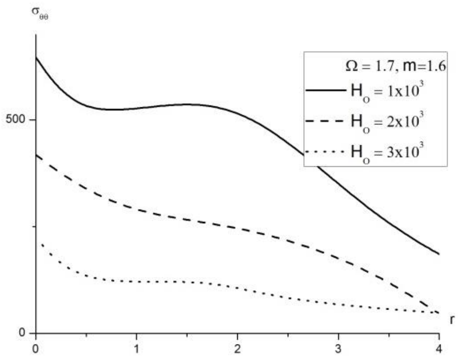

Fig. 6 Variation of the tangential stress with the radial, when magnetic field $H_0=2\times10^3$, the non-homogeneity m=1.6 at different values for rotation Ω=0.6, 1.1, 1.6, in case I

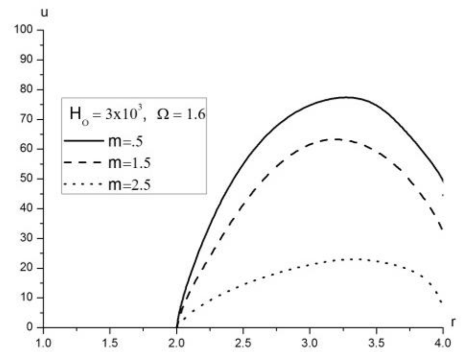

Fig. 7 Variation of the radial displacement with the radial, when magnetic field $H_0=2\times10^3$, rotation Ω=0.6 at different values for the non-homogeneity m=0.5, 1.5, 2.5, in case I

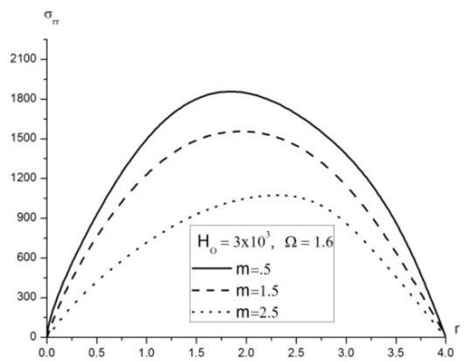

Fig. 8 Variation of the radial displacement with the radial, when magnetic field $H_0=2\times10^3$, rotation Ω=0.6 at different values for the non-homogeneity m=0.5, 1.5, 2.5, in case I

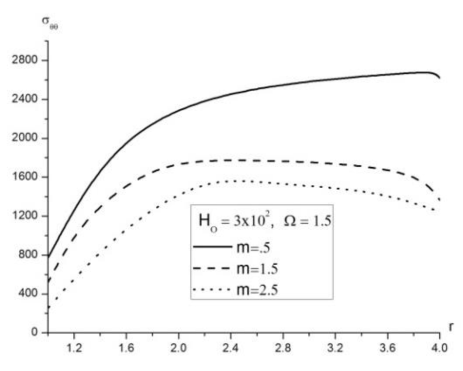

Fig. 9 Variation of the tangential stress with the radial, when magnetic field $H_0=2\times10^3$, rotation Ω=0.6 at different values for the non-homogeneity m=0.5, 1.5, 2.5, in case I

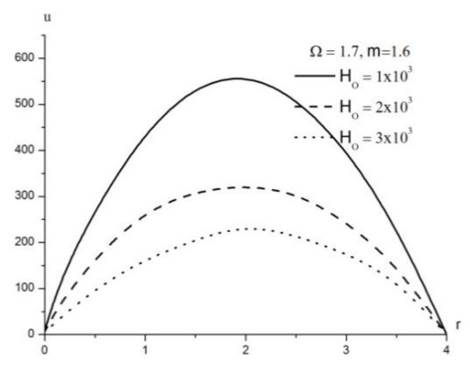

Fig. 10 Variation of the radial component of the displacement u with the radial r, when rotation Ω=0.6, the non-homogeneity m=1.6 at different values for magnetic field H_0, in case II

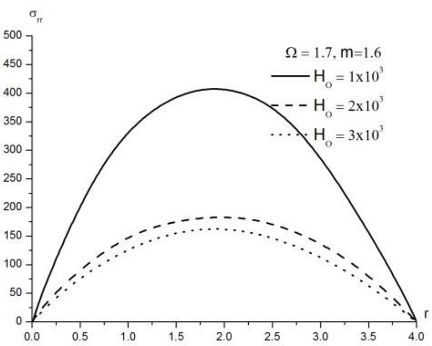

Fig. 11 Variation of the radial component of the stress σ_{rr} with the radial r, when rotation Ω=0.6, the non-homogeneity m=1.6 at different values for magnetic field H_0, in case II

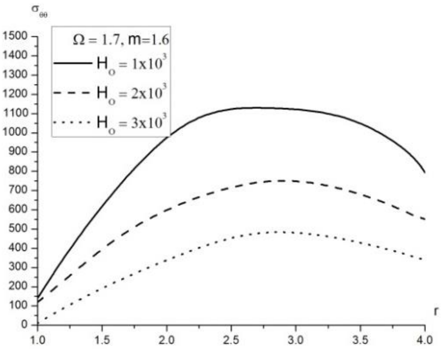

Fig. 12 Variation of the tangential component of the stress $\sigma_{\theta\theta}$ with the radial r, when rotation Ω=0.6, the non-homogeneity m=1.6 at different values for magnetic field H_0,in case II

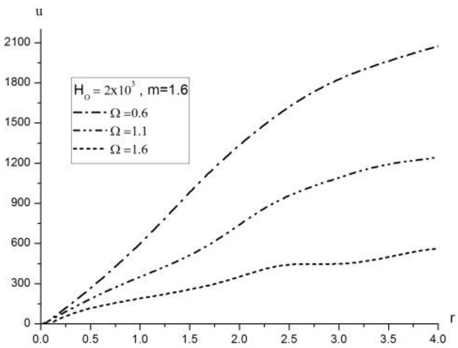

Fig. 13 Variation of the radial displacement with the radial, when magnetic field H_0=2×10^3, the non-homogeneity m=1.6 at different values for rotation Ω=0.6, 1.1, 1.6, in case II

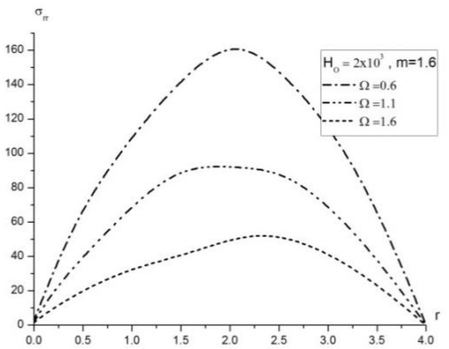

Fig. 14 Variation of the radial stress with the radial, when magnetic field H_0=2×10^3, the non-homogeneity m=1.6 at different values for rotation Ω=0.6, 1.1, 1.6, in case II

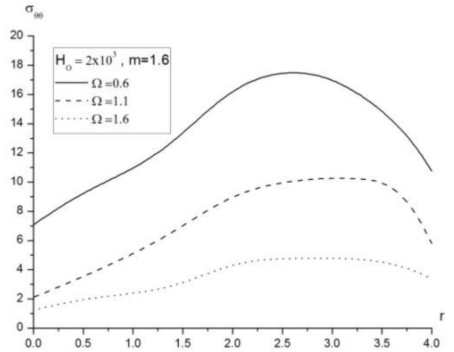

Fig. 15 Variation of the tangential stress with the radial, when magnetic field H_0=2×10^3, the non-homogeneity m=1.6 at different values for rotation Ω=0.6, 1.1, 1.6, in case I

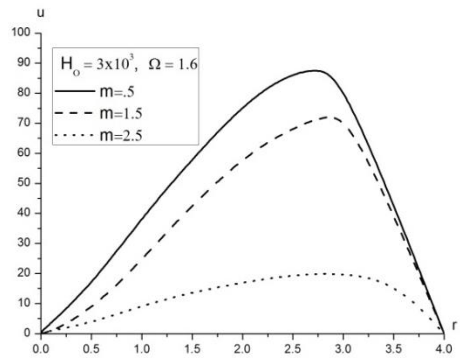

Fig. 16 Variation of the radial displacement with the radial, when magnetic field H_0=2×10^3, rotation Ω=0.6 at different values for the non-homogeneity m=0.5, 1.5, 2.5, in case II

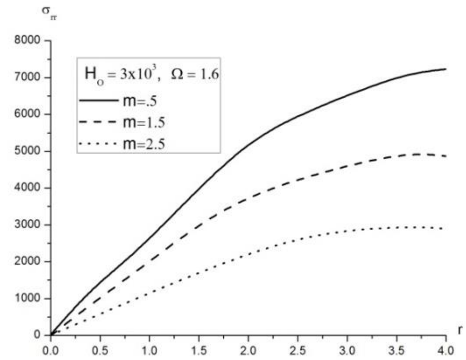

Fig. 17 Variation of the radial displacement with the radial, when magnetic field $H_0=2\times10^3$, rotation Ω=0.6 at different values for the non-homogeneity m=0.5, 1.5, 2.5, in case II

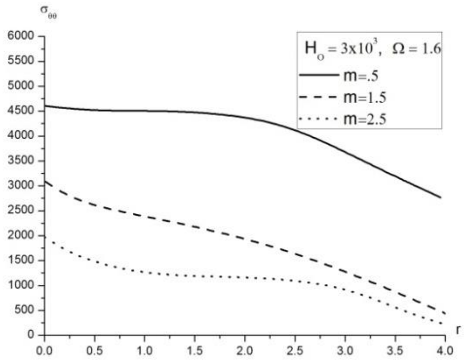

Fig. 18 Variation of the tangential stress with the radial, when magnetic field $H_0=2\times10^3$, rotation Ω=0.6 at different values for the non-homogeneity m=0.5, 1.5, 2.5, in case II

other cases may have different trends because of the dependence of the results on the mechanical properties of the material in many researches such as (Lukic *et al.* 2010, Aslani and Natoori 2013, Aydemir 2013, Bouiadjra *et al*. 2013, Berrabah *et al*. 2013) that have many applications in scientific and technical disciplines from cosmology to materials science.

5. Conclusions

Due to the complicated nature of the governing equations of the magneto-elastic theory, the done works in this field are unfortunately limited. The method used in this study provides a quite successful in dealing with such problems. This method gives exact solutions in the elastic medium without any restrictions on the actual physical quantities that appear in the governing equations of the considered problem. Important phenomena are observed in these computations.

• It was found that for large values of magnetic field give close results. The case is quite different when we consider small value of rotation. The solutions obtained in the context of elasticity theory.

• Comparing Figs. 1-18 for two problems, it was found that u, σ_{rr}, $\sigma_{\vartheta\theta}$ have the same behavior in both problems. But with the passage of magnetic field, non-homogeneity and rotation, the numerical values of u, σ_{rr}, $\sigma_{\vartheta\theta}$ in problem I are large in comparison with those in problem II due to the influences of magnetic field and rotation and non-homogeneity.

• The results presented in this paper will be very helpful for researchers concerning with material science, designers of new materials, as well as for those working on the development of a theory of hyperbolic propagation. Study of the phenomenon of rotation, non-homogeneity and magnetic field are also used to improve the conditions of oil extractions.

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