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Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories

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Abstract. In this work, various higher-order shear deformation plate theories for wave propagation in functionally graded plates are developed. Due to porosities, possibly occurring inside functionally graded materials (FGMs) during fabrication, it is therefore necessary to consider the wave propagation in plates having porosities in this study. The developed refined plate theories have fewer number of unknowns and equations of motion than the first-order shear deformation theory, but accounts for the transverse shear deformation effects without requiring shear correction factors. The rule of mixture is modified to describe and approximate material properties of the functionally graded plates with porosity phases. The governing equations of the wave propagation in the functionally graded plate are derived by employing the Hamilton's principle. The analytic dispersion relation of the functionally graded plate is obtained by solving an eigenvalue problem. The effects of the volume fraction distributions and porosity volume fraction on wave propagation of functionally graded plate are discussed in detail. The results carried out can be used in the ultrasonic inspection techniques and structural health monitoring.

Keywords: wave propagation; functionally graded plate; porosity; higher-order plate theory

1. Introduction

Functionally graded materials (FGMs) are a class of composites that have continuous variation of material properties from one surface to another, and thus eliminating the stress concentration found in laminated composites. A typical FGM is made from a mixture of ceramic and metal. The FGM is widely used in many structural applications such as aerospace, nuclear, civil, and automotive. When the application of the FGM increases, more accurate plate theories are required to predict the response of functionally graded (FG) plates. Since the shear deformation has significant effects on the responses of functionally graded (FG) plates, shear deformation theories

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are used to capture such shear deformation effects. The first-order shear deformation theory (FSDT) accounts for the shear deformation effects by the way of linear variation of in-plane displacements through the thickness. Since the FSDT violates the conditions of zero transverse shear stresses on the top and bottom surfaces of the plate, a shear correction factor which depends on many parameters is required to compensate for the error due to a constant shear strain assumption through the thickness (Benguediab *et al.* 2014, Semmah *et al.* 2014, Bouremana *et al.* 2013, Tounsi *et al.* 2013a, Benzair *et al.* 2008, Heireche *et al.* 2008). The higher-order shear deformation theories (HSDTs) account for the shear deformation effects, and satisfy the zero transverse shear stresses on the top and bottom surfaces of the plate, thus, a shear correction factor is not required (Ould Larbi *et al.* 2013, Tounsi *et al.* 2013b, Berrabah *et al.* 2013). Generally, HSDTs are proposed assuming a higher-order variations of in-plane displacements (Reddy 2000, Pradyumna and Bandyopadhyay 2008, Zenkour 2006, Xiang *et al*. 2011) or both in-plane and transverse displacements through the thickness (Matsunaga 2008, Chen *et al*. 2009, Talha and Singh 2010, Reddy 2011). Some of these HSDTs are computational costs because with each additional power of the thickness coordinate, an additional unknown is introduced to the theory (e.g., theories by Pradyumna and Bandyopadhyay (2008) with nine unknowns, Reddy (2011) with eleven unknowns, Talha and Singh (2010) with thirteen unknowns). Although some well-known HSDTs have the same five unknowns (e.g., third-order shear deformation theory (Reddy 2000), sinusoidal shear deformation theory (Zenkour and Alghamdi 2010)), their equations of motion are much more complicated than those of FSDT. Thus, needs exist for the development of HSDTs which are simple to use. Recently, some new plate theories that contain only four unknown functions (Benachour *et al*. 2011, El Meiche *et al.* 2011, Bourada *et al*. 2012, Bachir Bouiadjra *et al*. 2012, Tounsi *et al*. 2013c, Kettaf *et al.* 2013, Khalfi *et al.* 2014, Attia *et al.* 2015, Bachir Bouiadjra *et al*. 2013, Bakhti *et al*. 2013, Bouderba *et al.* 2013, Ait Amar Meziane *et al.* 2014, Zidi *et al.* 2014, Draiche *et al.* 2014, Nedri *et al.* 2014, Sadoune *et al.* 2014) are developed to study the mechanical behaviors of FG plates. Yaghoobi and Torabi (2013) investigated the buckling behavior of FG plates resting on two-parameter Pasternak's foundations under thermal loads. Using an analytical formulation, Yaghoobi and Yaghoobi (2013) analyzed the mechanical buckling response of sandwich plates with FGM face sheets resting on elastic foundation with various boundary conditions. Yaghoobi and Fereidoon (2014) presented a simple refined *nth*-order shear deformation theory for mechanical and thermal buckling analysis of FG plates resting on elastic foundations. In the same way, Klouche Djedid *et al.* (2014) used the refined *nth*-order shear deformation theory for bending and free vibration of FG graded plates.

The study of the wave propagation in the FG structures has received also much attention from various researchers. Chen *et al.* (2007) studied the dispersion behavior of waves in a functionally graded plate with material properties varying along the thickness direction. Han and Liu (2002) investigated SH waves in FG plates, where the material property variation was assumed to be a piecewise quadratic function in the thickness direction. Han *et al*. (2001) proposed an analytical-numerical method for analyzing the wave characteristics in FG cylinders. Han *et al*. (2002) also proposed a numerical method to study the transient wave in FG plates excited by impact loads. Sun and Luo (2011a) also studied the wave propagation and dynamic response of rectangular functionally graded material plates with completed clamped supports under impulsive load. Considering the thermal effects and temperature-dependent material properties, Sun and Luo (2011b) investigated the wave propagation of an infinite functionally graded plate using the higher-order shear deformation plate theory. However, in FGM fabrication, micro voids or porosities can occur within the materials during the process of sintering. This is because of the

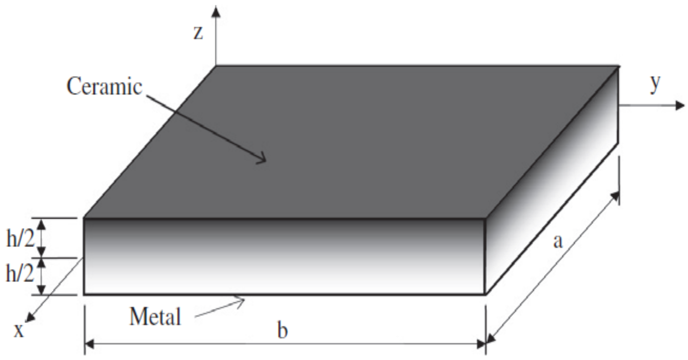


Fig. 1 Geometry and coordinates of FG plates

large difference in solidification temperatures between material constituents (Zhu *et al*. 2001). Wattanasakulpong *et al*. (2012) also gave the discussion on porosities happening in side FGM samples fabricated by a multi-step sequential infiltration technique. Therefore, it is important to take in to account the porosity effect when designing FGM structures subjected to dynamic loadings. Recently, Wattanasakulpong and Ungbhakorn (2014) studied linear and nonlinear vibration problems of elastically end restrained FG beams having porosities.

Considering FG structural members, it is evident from the above discussed literature that there is no study on wave propagation in FG plates having porosities. Thus, the objective of this work is to investigate the wave propagation of an infinite FG plate having porosities using various simple higher-order shear deformation theories. The displacement fields of the proposed theories are chosen based on cubic, sinusoidal, hyperbolic, and exponential variation in the in-plane displacements through the thickness. By dividing the transverse displacement into the bending and shear parts and making further assumptions, the number of unknowns and equations of motion of the proposed theories is reduced and hence makes them simple to use. The governing equations of the wave propagation in the FG plate are derived by using the Hamilton's principle. The analytic dispersion relations of the FG plate are obtained by solving an eigenvalue problem. The dispersion, phase velocity and group velocity curves of the wave propagation in FG plates having porosities are plotted. The influences of the volume fraction index and porosity volume fraction on the dispersion and phase velocity of the wave propagation in the FG plate are clearly discussed.

2. Functionally graded plates with porosities

A FG plate made from a mixture of two material phases, for example, a metal and a ceramic as shown in Fig. 1. The material properties of FG plates are assumed to vary continuously through the thickness of the plate. In this investigation, the imperfect plate is assumed to have porosities spreading within the thickness due to defect during production. Consider an imperfect FGM with a porosity volume fraction, α (α<<1), distributed evenly among the metal and ceramic, the modified rule of mixture proposed by Wattanasakulpong and Ungbhakorn (2014) is used as

$$P = P_m\left(V_m - \frac{\alpha}{2}\right) + P_c\left(V_c - \frac{\alpha}{2}\right) \tag{1}$$

Now, the total volume fraction of the metal and ceramic is: $V_m+V_c=1$, and the power law of volume fraction of the ceramic is described as

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{2}$$

Hence, all properties of the imperfect FGM can be written as

$$P = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + P_m - (P_c + P_m)\frac{\alpha}{2} \tag{3}$$

It is noted that the positive real number n ($0 \le n < \infty$) is the power law or volume fraction index, and z is the distance from the mid-plane of the FG plate. The FG plate becomes a fully ceramic plate when n is set to zero and fully metal for large value of n.

Thus, the Young's modulus (E) and material density (ρ) equations of the imperfect FGM plate can be expressed as

$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + E_m - (E_c + E_m)\frac{\alpha}{2} \tag{4}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + \rho_m - (\rho_c + \rho_m)\frac{\alpha}{2} \tag{5}$$

However, Poisson's ratio (v) is assumed to be constant. The material properties of a perfect FG plate can be obtained when α is set to zero.

3. Fundamental equations

3.1 Basic assumptions and constitutive equations

The displacement fields of various shear deformation plate theories are chosen based on following assumptions: (1) the axial and transverse displacements are partitioned into bending and shear components; (2) the bending component of axial displacement is similar to that given by the classical plate theory (CPT); and (3) the shear component of axial displacement gives rise to the higher-order variation of shear strain and hence to shear stress through the thickness of the plate in such a way that shear stress vanishes on the top and bottom surfaces. Based on these assumptions, the displacement fields of various higher-order shear deformation plate theories are given in a general form as

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{6a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \tag{6b}$$

$$w(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t) \tag{6c}$$

where u_0 and v_0 are the mid-plane displacements of the plate in the x and y direction, respectively; w_b and w_s are the bending and shear components of transverse displacement, respectively; and $f(z)$ $f(z)$ is a shape function determining the distribution of the transverse shear strain and shear stress through the thickness of the plate. The shape functions $f(z)$ are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the plate, thus a shear correction factor is not required. In this study, these shape functions are chosen based on the third-order shear deformation theory (TSDT) of Reddy (2000), sinusoidal shear deformation theory (SSDT) of Touratier (1991), hyperbolic shear deformation theory (HSDT) of Soldatos (1992), and exponential shear deformation theory (ESDT) of Karama *et al*. (2003), as presented in Table 1.
The nonzero linear strains associated with the displacement field in Eq. (6) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{7}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} \\ \dfrac{\partial w_s}{\partial x} \end{Bmatrix} \tag{8a}$$

and

$$g(z) = 1 - \frac{df(z)}{dz} \tag{8b}$$

Table 1 Shape functions

model	$f(z)$	$g(z)=1-f'(z)$
Third plate theory (TSDT)	$\dfrac{4z^3}{3h^2}$	$1-\dfrac{4z^2}{h^2}$
Sinusoidal plate theory (SSDT)	$z-\dfrac{h}{\pi}\sin\left(\dfrac{\pi z}{h}\right)$	$\cos\left(\dfrac{\pi z}{h}\right)$
Hyperbolic plate theory (HSDT)	$z-h\sinh\left(\dfrac{z}{h}\right)+z\cosh\dfrac{1}{2}$	$\cosh\left(\dfrac{z}{h}\right)-\cosh\dfrac{1}{2}$
Exponential plate theory (ESDT)	$z-ze^{-2(z/h)^2}$	$\left(1-\dfrac{4z^2}{h^2}\right)e^{-2(z/h)^2}$

For elastic and isotropic FGMs, the constitutive relations can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{9}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eq. (4), stiffness coefficients, C_{ij}, can be expressed as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \tag{10a}$$

$$C_{12} = \frac{\nu\, E(z)}{1-\nu^2}, \tag{10b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{10c}$$

3.2 Governing equations

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as

$$0 = \int_0^t (\delta U - \delta K)\, dt \tag{11}$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.

The variation of strain energy of the plate stated as

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x\, \delta\varepsilon_x + \sigma_y\, \delta\varepsilon_y + \tau_{xy}\, \delta\gamma_{xy} + \tau_{yz}\, \delta\gamma_{yz} + \tau_{zx}\, \delta\gamma_{zx}\right] dAdz \\ &= \int_A \left[N_x\, \delta\varepsilon_x^0 + N_y\, \delta\varepsilon_y^0 + N_{xy}\, \delta\varepsilon_{xy}^0 + M_x^b\, \delta k_x^b + M_y^b\, \delta k_y^b + M_{xy}^b\, \delta k_{xy}^b + M_x^s\, \delta k_x^s \right. \\ &\quad \left. + M_y^s\, \delta k_y^s + M_{xy}^s\, \delta k_{xy}^s + S_{yz}^s\, \delta\gamma_{yz}^s + S_{xz}^s\, \delta\gamma_{xz}^s\right] dA \end{aligned} \tag{12}$$

where the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i\, dz, \quad (i = x, y, xy) \text{ and } \quad S_i = \int_{-h/2}^{h/2} g\sigma_i\, dz, \quad (i = xz, yz) \tag{13}$$

The variation of kinetic energy is expressed as

$$
\begin{aligned}
\delta K &= \int_V \left[\dot{u}\delta\,\dot{u} + \dot{v}\delta\,\dot{v} + \dot{w}\delta\,\dot{w}\right]\rho(z)\,dV \\
&= \int_A \Big\{ I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \left(\dot{w}_b + \dot{w}_s\right)\left(\delta\dot{w}_b + \delta\dot{w}_s\right)\right] \\
&- I_1\left(\dot{u}_0\,\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial x}\,\delta\,\dot{u}_0 + \dot{v}_0\,\frac{\partial\delta\dot{w}_b}{\partial y} + \frac{\partial\dot{w}_b}{\partial y}\,\delta\,\dot{v}_0\right) \\
&- J_1\left(\dot{u}_0\,\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\,\delta\,\dot{u}_0 + \dot{v}_0\,\frac{\partial\delta\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\,\delta\,\dot{v}_0\right) \\
&+ I_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\,\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\,\dot{w}_b}{\partial y}\right) + K_2\left(\frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\,\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\,\dot{w}_s}{\partial y}\right) \\
&+ J_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\,\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\,\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\,\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\,\dot{w}_b}{\partial y}\right)\Big\} dA
\end{aligned} \tag{14}
$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$
\left(I_0, I_1, J_1, I_2, J_2, K_2\right) = \int_{-h/2}^{h/2} \left(1, z, f, z^2, z\,f, f^2\right)\rho(z)dz \tag{15}
$$

Substituting the expressions for δU and δK from Eqs. (12) and (14) into Eq. (11) and integrating by parts, and collecting the coefficients of δu_0, δv_0, δw_b and δw_s, the following equations of motion of the plate are obtained

$$
\begin{aligned}
&\delta\,u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_b}{\partial x} - J_1\frac{\partial\ddot{w}_s}{\partial x} \\
&\delta\,v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v}_0 - I_1\frac{\partial\ddot{w}_b}{\partial y} - J_1\frac{\partial\ddot{w}_s}{\partial y} \\
&\delta\,w_b: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0\left(\ddot{w}_b + \ddot{w}_s\right) + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_b - J_2\nabla^2\ddot{w}_s \\
&\delta\,w_s: \ \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = I_0\left(\ddot{w}_b + \ddot{w}_s\right) + J_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - J_2\nabla^2\ddot{w}_b - K_2\nabla^2\ddot{w}_s
\end{aligned} \tag{16}
$$

By substituting Eq. (7) into Eq. (9) and the subsequent results into Eq. (13), the stress resultants are obtained as

$$
\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \quad S = A^s\gamma\,, \tag{17}
$$

where

$$N = \{N_x, N_y, N_{xy}\}^t, \quad M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t, \quad M^s = \{M_x^s, M_y^s, M_{xy}^s\}^t, \tag{18a}$$

$$\varepsilon = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}^t, \quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}^t, \quad k^s = \{k_x^s, k_y^s, k_{xy}^s\}^t, \tag{18b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{18c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{18d}$$

$$S = \{S_{xz}^s, S_{yz}^s\}^t, \quad \gamma = \{\gamma_{xz}, \gamma_{yz}\}^t, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{18e}$$

where A_{ij}, B_{ij}, D_{ij}, etc., are the plate stiffness, defined by

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} Q_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{19a}$$

and

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{22}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) \tag{19b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} \frac{E(z)}{2(1+\nu)} [g(z)]^2 \, dz, \tag{19c}$$

By substituting Eq. (17) into Eq. (16), the governing equations can be expressed in terms of displacements (u_0, v_0, w_b and w_s) as

$$\begin{aligned} & A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y} - B_{11}\frac{\partial^3 w_b}{\partial x^3} - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_b}{\partial x \partial^2 y} \\ & \quad - \left(B_{12}^s + 2B_{66}^s\right)\frac{\partial^3 w_s}{\partial x \partial y^2} - B_{11}^s\frac{\partial^3 w_s}{\partial x^3} = I_0 \ddot{u}_0 - I_1\frac{\partial \ddot{w}_b}{\partial x} - J_1\frac{\partial \ddot{w}_s}{\partial x}, \end{aligned} \tag{20a}$$

$$\begin{aligned} & A_{22}\frac{\partial^2 v_0}{\partial y^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y} - B_{22}\frac{\partial^3 w_b}{\partial y^3} - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_b}{\partial^2 x \partial y} \\ & \quad - \left(B_{12}^s + 2B_{66}^s\right)\frac{\partial^3 w_s}{\partial x^2 \partial y} - B_{22}^s\frac{\partial^3 w_s}{\partial y^3} = I_0 \ddot{v}_0 - I_1\frac{\partial \ddot{w}_b}{\partial y} - J_1\frac{\partial \ddot{w}_s}{\partial y}, \end{aligned} \tag{20b}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x\partial^2 y}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial^2 x\partial y}+B_{22}\frac{\partial^3 v_0}{\partial^3 y}d_{222}v_0-D_{11}\frac{\partial^4 w_b}{\partial x^4}$$
$$-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_b}{\partial y^4}-D_{11}^s\frac{\partial^4 w_s}{\partial x^4}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2} \tag{20c}$$
$$-D_{22}^s\frac{\partial^4 w_s}{\partial y^4}=I_0\left(\ddot{w}_b+\ddot{w}_s\right)+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-I_2\nabla^2\ddot{w}_b-J_2\nabla^2\ddot{w}_s,$$

$$B_{11}^s\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 u_0}{\partial x\partial y^2}+\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 v_0}{\partial x^2\partial y}+B_{22}^s\frac{\partial^3 v_0}{\partial y^3}-D_{11}^s\frac{\partial^4 w_b}{\partial x^4}$$
$$-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-D_{22}^s\frac{\partial^4 w_b}{\partial y^4}-H_{11}^s\frac{\partial^4 w_s}{\partial x^4}-2\left(H_{12}^s+2H_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}-H_{22}^s\frac{\partial^4 w_s}{\partial y^4} \tag{20d}$$
$$+A_{55}^s\frac{\partial^2 w_s}{\partial x^2}+A_{44}^s\frac{\partial^2 w_s}{\partial y^2}=I_0\left(\ddot{w}_b+\ddot{w}_s\right)+J_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-J_2\nabla^2\ddot{w}_b-K_2\nabla^2\ddot{w}_s$$

4. Dispersion relations

We assume solutions for u_0, v_0, w_b and w_s representing propagating waves in the x-y plane with the form

$$\begin{Bmatrix} u_0(x,y,t) \\ v_0(x,y,t) \\ w_b(x,y,t) \\ w_s(x,y,t) \end{Bmatrix}=\begin{Bmatrix} U\exp\left[i(k_1x+k_2y-\omega t\right] \\ V\exp\left[i(k_1x+k_2y-\omega t\right] \\ W_b\exp\left[i(k_1x+k_2y-\omega t\right] \\ W_s\exp\left[i(k_1x+k_2y-\omega t\right] \end{Bmatrix} \tag{21}$$

where U; V; W_b and W_s are the coefficients of the wave amplitude, k_1 and k_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively, ω is the frequency.

Substituting Eq. (21) into Eqs. (20), we obtain

$$\left(\left[K\right]-\omega^2\left[M\right]\right)\left\{\Delta\right\}=\left\{0\right\} \tag{22}$$

where

$$\left\{\Delta\right\}=\left\{U,V,W_b,W_s\right\}^T, \tag{23a}$$

$$\left[K\right]=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix},\quad \left[M\right]=\begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \tag{23b}$$

in which

$$a_{11} = -\left(A_{11}\, k_1^2 + A_{66}\, k_2^2\right)$$
$$a_{12} = -k_1\; k_2\left(A_{12} + A_{66}\right)$$
$$a_{13} = i\; k_1^3\, B_{11} + i\,(B_{12} + 2\,B_{66})k_1\; k_2^2$$
$$a_{31} = -i\; k_1^3\, B_{11} - i\,(B_{12} + 2\,B_{66})k_1\; k_2^2$$
$$a_{14} = i\,k_1\; k_2^2\; B_{12}^s + 2\,i\,k_1\; k_2^2\; B_{66}^s + i\,B_{11}^s\,k_1^3$$
$$a_{41} = -i\,k_1\; k_2^2\; B_{12}^s - 2\,i\,k_1\; k_2^2\; B_{66}^s - i\,B_{11}^s\,k_1^3$$
$$a_{22} = -\left(A_{22}\, k_2^2 + A_{66}\, k_1^2\right)$$
$$a_{23} = i\; k_2^3\, B_{22} + i\,(B_{12} + 2\,B_{66})k_1^2\, k_2$$
$$a_{32} = -i\; k_2^3\, B_{22} - i\,(B_{12} + 2\,B_{66})k_1^2\, k_2$$
$$a_{24} = i\,k_1^2\; k_2\; B_{12}^s + 2\,i\,k_1^2\; k_2\; B_{66}^s + i\,B_{22}^s\,k_2^3$$
$$a_{33} = -\left(2\,k_1^2\; k_2^2\; D_{12} + 4\,k_1^2\; k_2^2\; D_{66} + D_{11}\,k_1^4 + D_{22}\,k_2^4\right)$$
$$a_{34} = -\left(2\,k_1^2\; k_2^2\; D_{12}^s + 4\,k_1^2\; k_2^2\; D_{66}^s + D_{11}^s\,k_1^4 + D_{22}^s\,k_2^4\right)$$
$$a_{44} = -\left(H_{11}^s k_1^4 + 2(H_{12}^s + 2H_{66}^s)k_1^2 k_2^2 + H_{22}^s k_2^4 + A_{55}^s k_1^2 + A_{44}^s k_2^2\right)$$
$$m_{11} = m_{22} = -I_0$$
$$m_{13} = i\,I_1\,k_1\;,\quad m_{31} = -i\,I_1\,k_1$$
$$m_{14} = i\,J_1\,k_1\;,\quad m_{41} = -i\,J_1\,k_1$$
$$m_{23} = i\,I_1\,k_2\;,\quad m_{32} = -i\,I_1\,k_2$$
$$m_{24} = i\,J_1\,k_2\;,\quad m_{42} = -i\,J_1\,k_2$$
$$m_{34} = -I_0 - J_2\left(k_1^2 + k_2^2\right) = m_{43}$$
$$m_{44} = -I_0 - K_2\left(k_1^2 + k_2^2\right) \tag{23c}$$

The dispersion relations of wave propagation in the functionally graded plate are given by

$$\left|\,[K] - \omega^2 [M]\,\right| = 0 \tag{24}$$

Assuming $k_1=k_2=k$, the roots of Eq. (24) can be expressed as

$$\omega_1 = W_1(k)\,,\quad \omega_2 = W_2(k)\,,\quad \omega_3 = W_3(k)\ \text{ and }\ \omega_4 = W_4(k) \tag{25}$$

They correspond with the wave modes M_0, M_1, M_2 and M_3, respectively. The wave modes M_0 and M_3 correspond to the flexural wave, the wave modes M_1 and M_2 correspond to the extensional wave.

The phase velocity of wave propagation in the functionally graded plate can be expressed as

$$C_i = \frac{W_i(k)}{k}\,,\quad (i = 1,2,3,4) \tag{26}$$

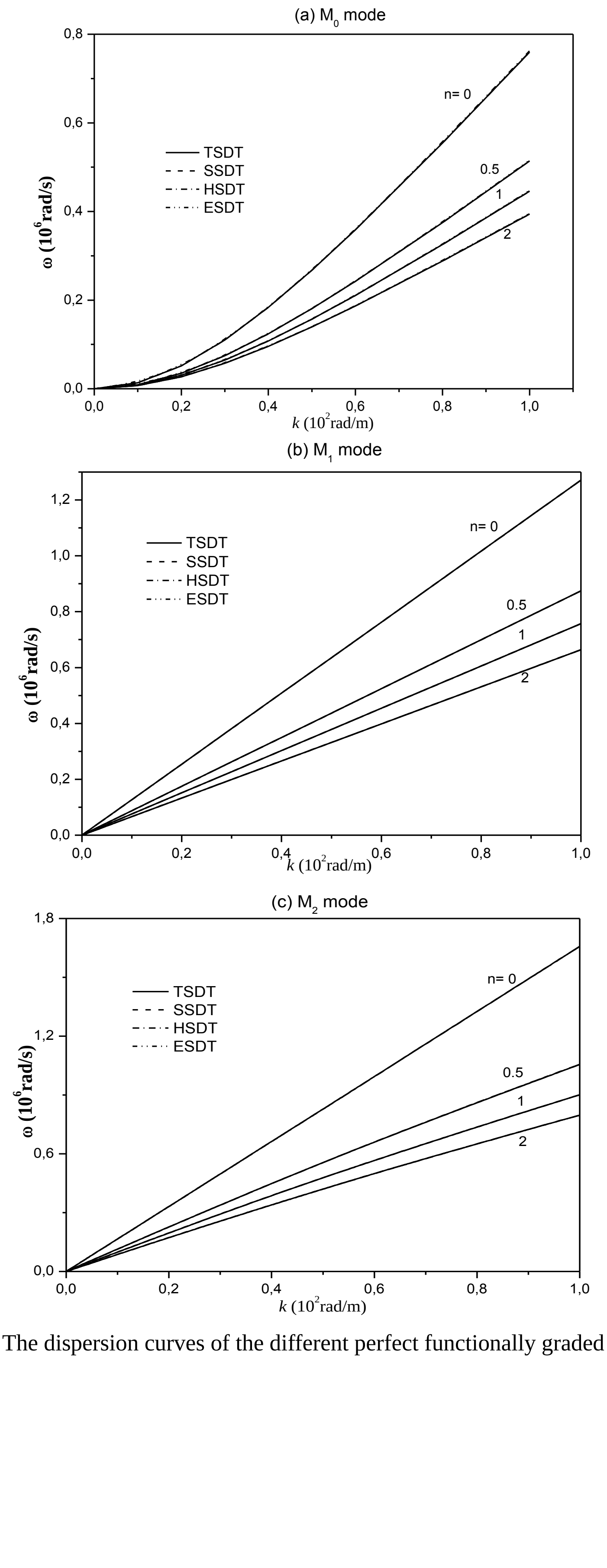


Fig. 2 The dispersion curves of the different perfect functionally graded plates

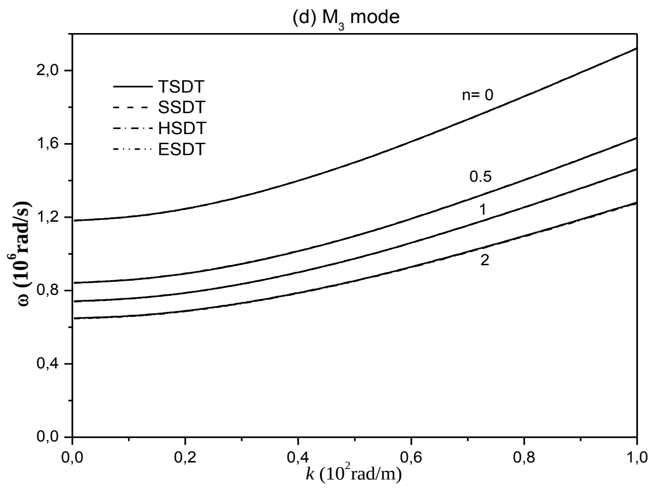


Fig. 2 Continued

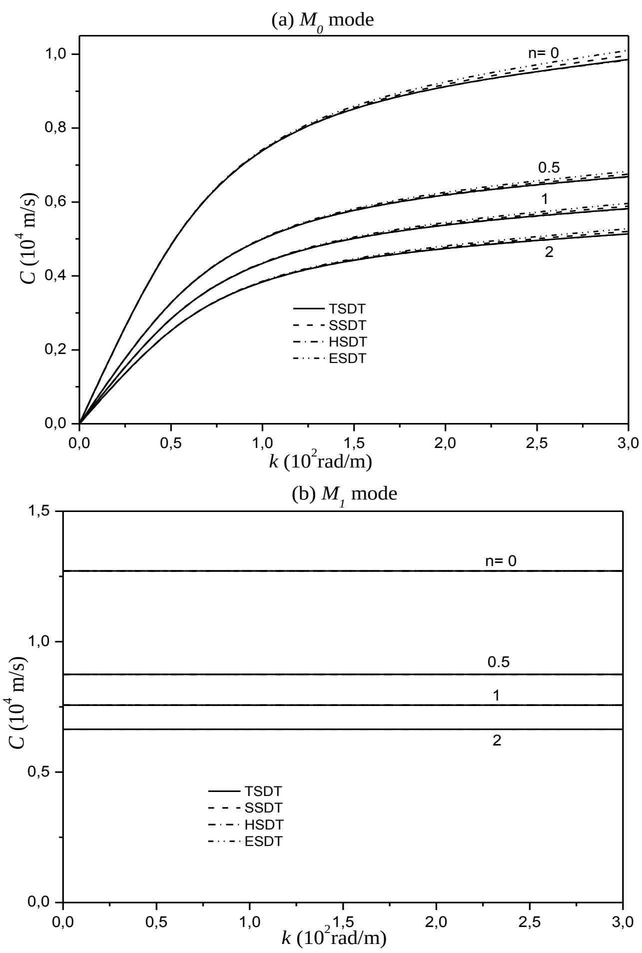


Fig. 3 The phase velocity curves of the different perfect functionally graded plates

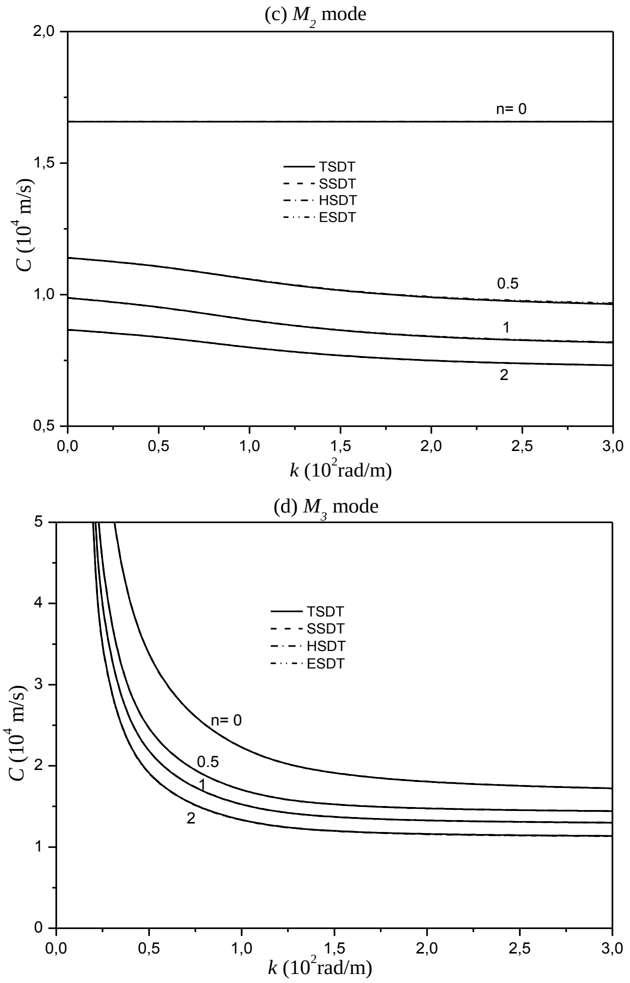


Fig. 3 Continued

5. Numerical results and discussion

In this section, a FG plate made from Si_3N_4/SUS304; whose material properties are: E=348.43 GPa, ρ=2370 kg/m^3, v=0.3 for Si_3N_4 and E= 201.04 GPa, ρ= 8166 kg/m^3, v=0.3 for SUS304; are chosen for this work. The thickness of the FG plate is 0.02 m. The analysis based on the present TSDT, SSDT, HSDT, and ESDT are carried out using MAPLE.

Fig. 2 plots the dispersion curves of the different perfect FG plates using various shear deformation plate theories. It can be seen that the dispersion curves predicted by all proposed plate theories are almost identical to each other and this regardless the power law index n and wave modes (M_0, M_1, M_2 and M_3). For the same k, the frequency of the wave propagation in the perfect FG plate decreases with the increase of the power law index n whatever the wave modes. Also, the frequency of the wave propagation becomes maximum in the homogeneous plate (n=0).

Fig. 3 shows the phase velocity curves of the different perfect FG plates predicted using various shear deformation plate theories. It can be seen that the phase velocity of the wave propagation in

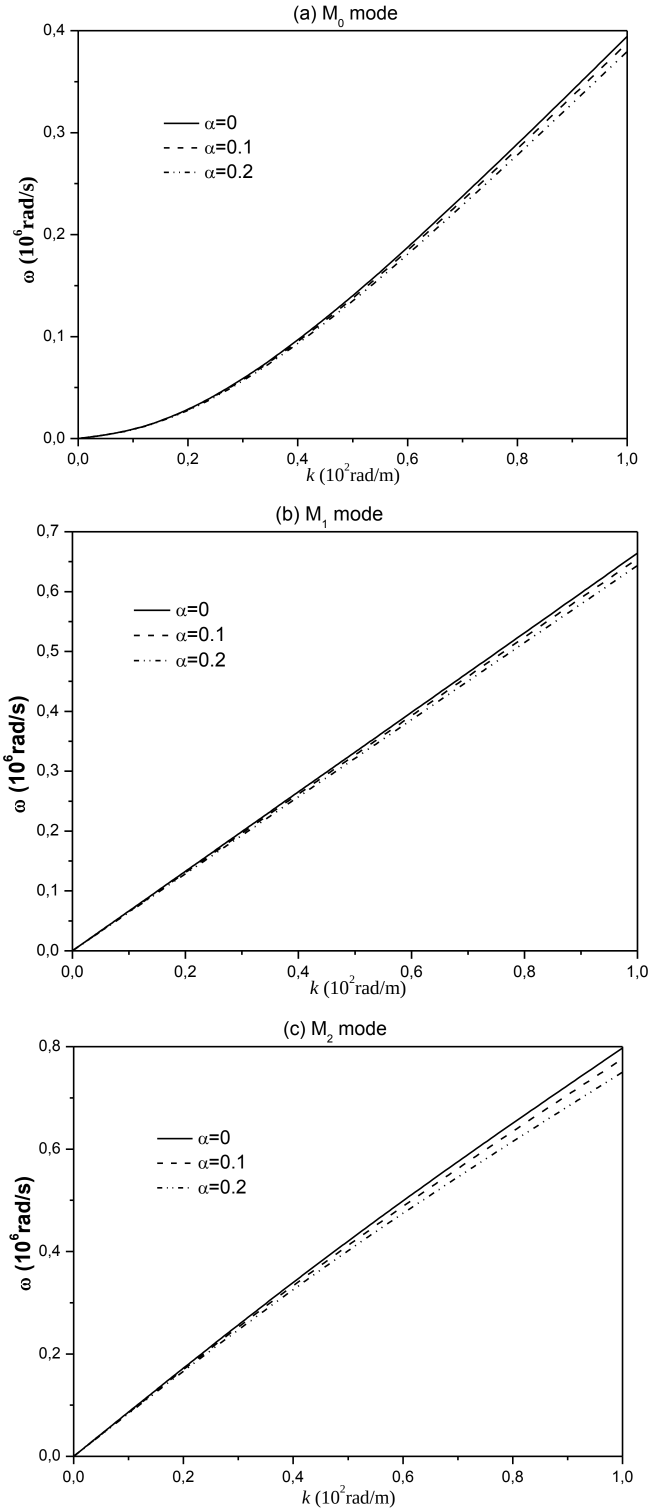


Fig. 4 The dispersion curves of the different imperfect functionally graded plates using TSDT

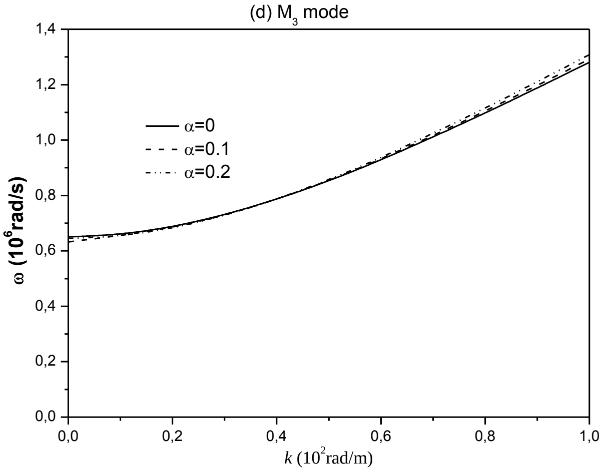


Fig. 4 Continued

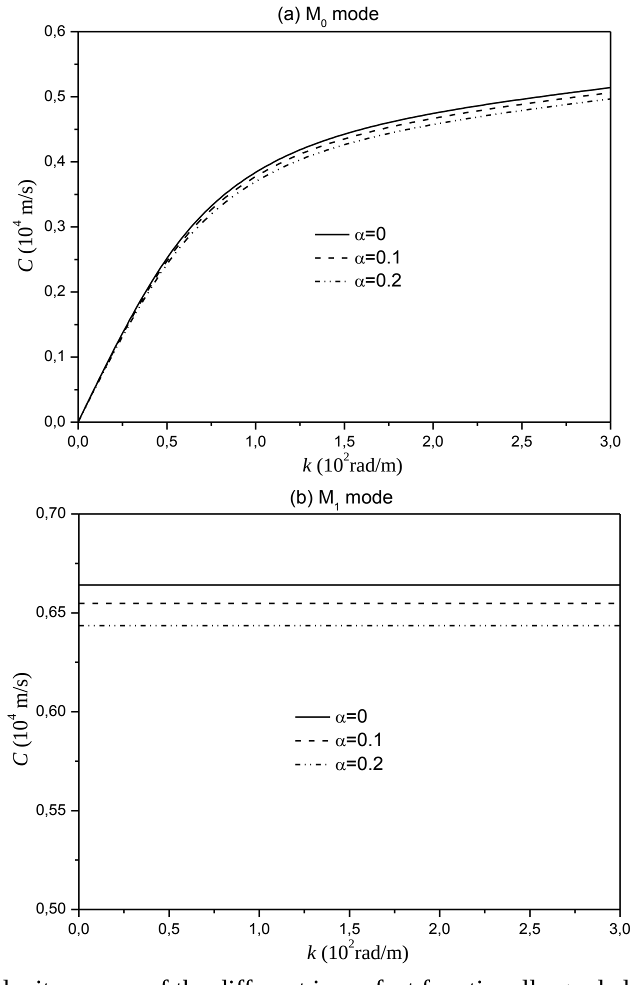


Fig. 5 The phase velocity curves of the different imperfect functionally graded plates using TSDT

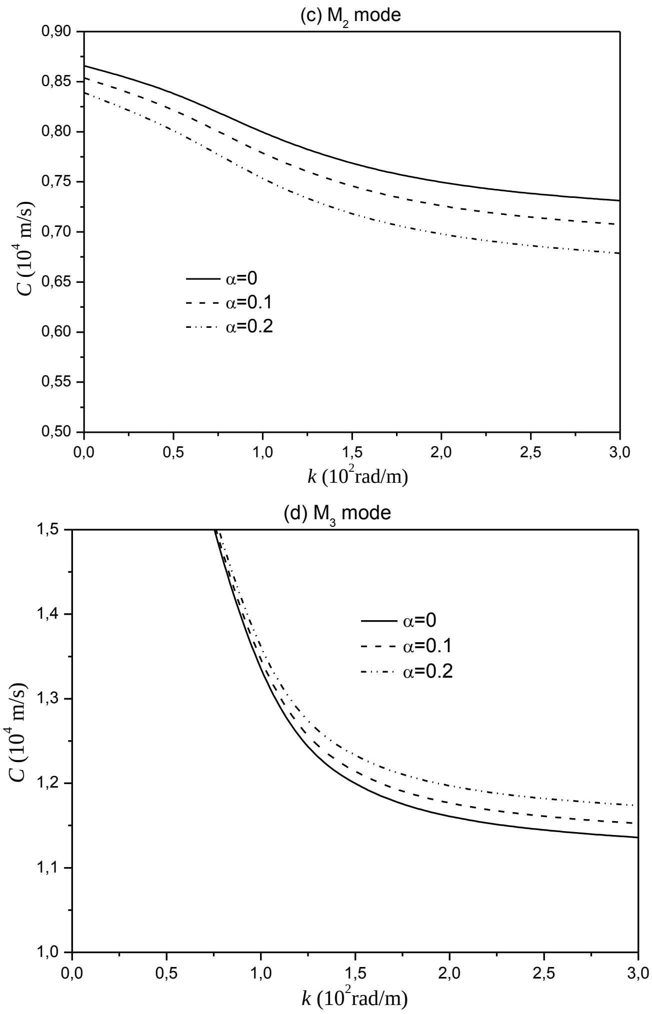


Fig. 5 Continued

the perfect FG plate decreases as the power law index *n* increases for the same wave number *k*. The phase velocity for the extensional wave modes M_1 and M_2 of the plate (n=0) is a constant, but it is not a constant for the plate (n≠0). In the case of the homogeneous plate (n=0), the phase velocity takes the maximum among those of all FG plates. Also, it can be seen that the phase velocity curves predicted by all proposed plate theories are almost identical to each other.

Fig. 4 shows the dispersion curves of different imperfect FG plate with n=2. It can be seen that the porosity has effect on the frequency of the wave propagation in FG plate for the large wave numbers (*k*) and especially for the extensional wave mode M_2. Indeed, the frequencies are reduced when the porosity increases.

Fig. 5 shows, the phase velocity curves of different imperfect FG plate with n=2. It can be seen from Fig. 5 that the phase velocity of the FG plate decreases as the porosity increases, except for flexural wave mode M_3, where an opposite behaviour is observed. Furthermore, it is seen that the influence of porosity on the phase velocity for M_1 and M_2 modes is also obvious at lower wave

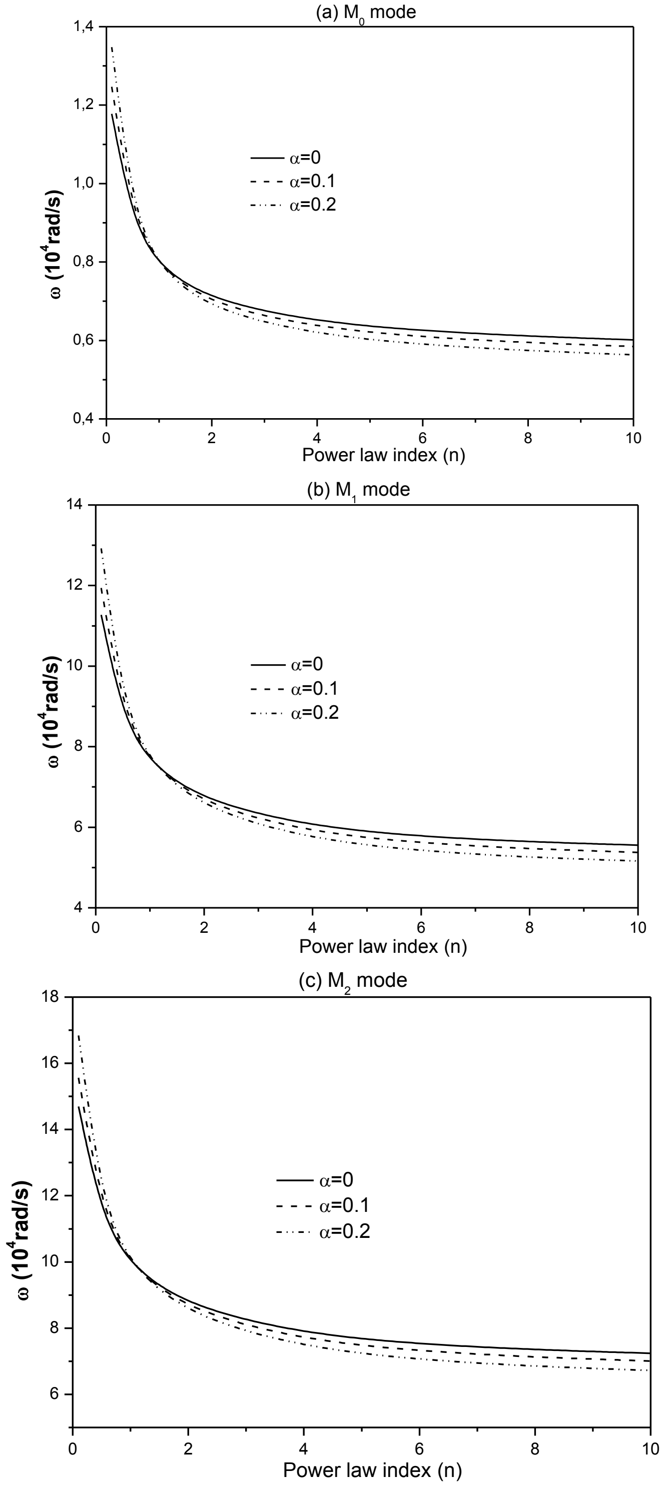


Fig. 6 The effects of power law index and the porosity on the frequency of the wave propagation in the perfect and imperfect FG plates using TSDT for the wave number k=10 rad/m

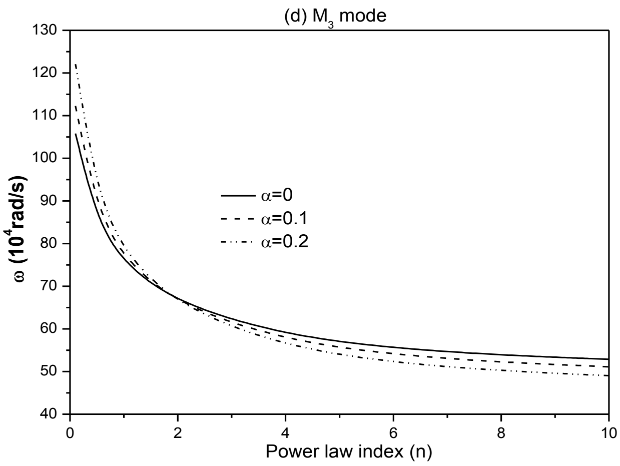
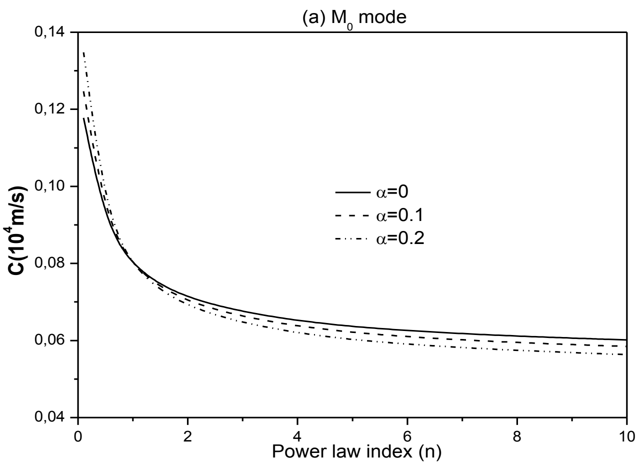


Fig. 6 Continued



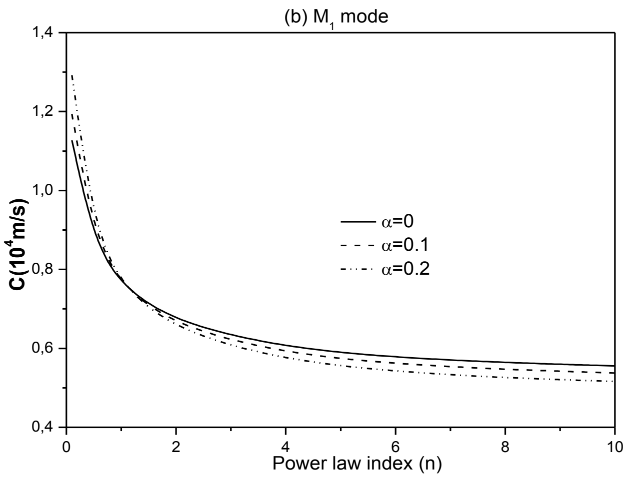


Fig. 7 The effects of power law index and the porosity on the phase velocity of the perfect and imperfect FG plates using TSDT for the wave number k=10 rad/m

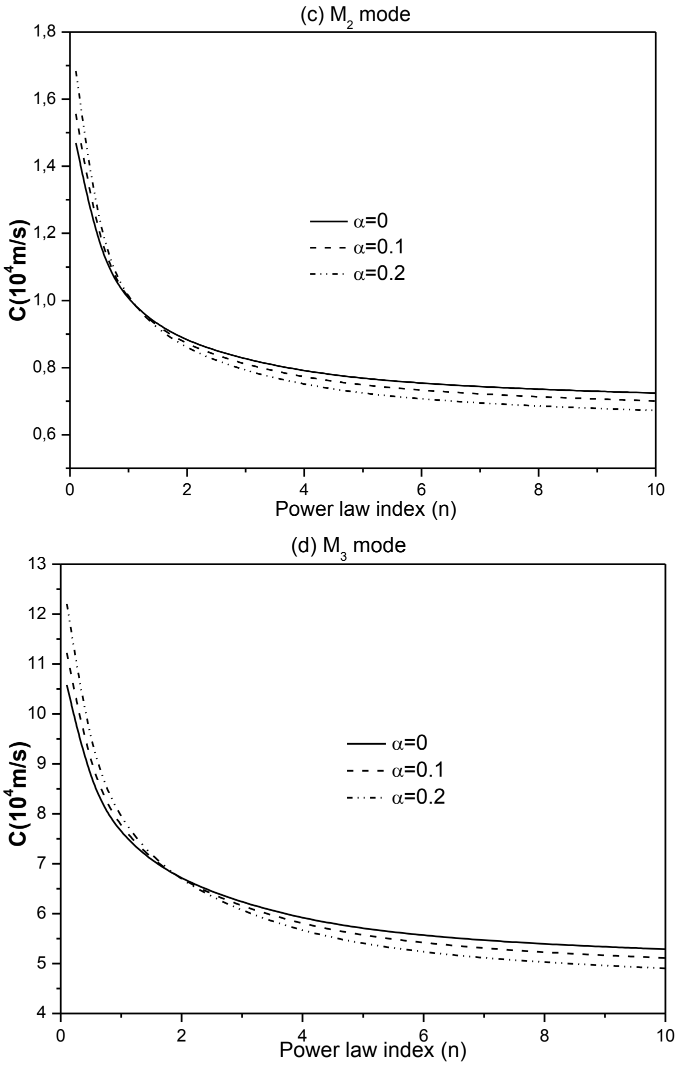


Fig. 7 Continued

number. The influence of porosity on the phase velocity for the flexural wave modes M_0 and M_3 is very little at lower wave number, but the influence is obvious as wave numbers increases.

To investigate the influences of power law index of material constituents (n) and porosity volume index (α) on the frequency and the phase velocity, the results of perfect and imperfect FG plates are shown in Figs. 6 and 7, respectively, using TSDT for the wave number k=10 rad/m. It is seen that when the power law index n>1, both the frequency and the phase velocity decrease with increasing the porosity contrary to the case where the power law index is less to 1. However, it is observed that the increase of the power law index leads to reducing the frequency and the phase velocity and this regardless the value of the porosity.

6. Conclusions

The wave propagation of an infinite perfect and imperfect functionally graded plate is analyzed using various higher-order shear deformation plate theories. The main advantage of the proposed theories over the existing higher-order shear deformation theories is that the present ones involve fewer unknowns as well as the dispersion relations of wave propagation in the FG plate. The computational cost can therefore be reduced. The modified rule of mixture covering porosity phases is employed to describe and approximate material properties of the imperfect FG plates. The analytic dispersion relation of the functionally graded plate is obtained by solving an eigenvalue problem. From the present work, it can be concluded that the influence of the volume fraction distributions and porosity volume index on wave propagation in the FG plate is significant. An improvement of present formulation will be considered in the future work to account for the thickness stretching effect by employing quasi-3D shear deformation models (Bessaim *et al.* 2013, Saidi *et al.* 2013, Bousahla *et al.* 2014, Bourada *et al.* 2015, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Houari *et al.* 2014, Larbi Chaht *et al.* 2014, Meradjah *et al.* 20154, Hamidi *et al.* 2015, Swaminathan and Naveenkumar 2014, Sayyad and Ghugal 2014).

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