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A new damage detection indicator for beams based on mode shape data

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Abstract. In this paper, a new damage indicator based on mode shape data is introduced to identify damage in beam structures. In order to construct the indicator proposed, the mode shape, mode shape slope and mode shape curvature of a beam before and after damage are utilized. Mode shape data of the beam are first obtained here using a finite element modeling and then the slope and curvature of mode shape are evaluated via the central finite difference method. In order to assess the robustness of the proposed indicator, two test examples including a simply supported beam and a two-span beam are considered. Numerical results demonstrate that using the proposed indicator, the location of single and multiple damage cases having different characteristics can be accurately determined. Moreover, the indicator shows a better performance when compared with a well-known indicator found in the literature.

Keywords: damage detection; damage indicator; beam structure; mode shape data

1. Introduction

Structural damage detection is of a great importance, because early detection and repair of damage in a structure can increase its life and prevent it from an overall failure. Structural damage detection consists of four different levels (Rytter 1993). The first level determines the presence of damage in the structure. The second level includes locating the damage, while the third level quantifies the severity of the damage. The final level uses the information from the first three steps to predict the remaining service life of the damaged structure. After discovering the damage occurrence, damage localization is more important than damage quantification. In the last years, numerous methods have been proposed to accurately locate structural damage (Cawley and Adams 1979, Rizos *et al.* 1990, Doebling *et al.* 1992, Doebling *et al.* 1998, Abdo and Hori 2002, Koh and Dyke 2007, Sinou 2007). The damage location assurance criterion (DLAC) and multiple damage location assurance criterion (MDLAC) based on the changes in natural frequencies have been proposed by Messina *et al*. (1992, 1998). The DLAC and MDLAC criteria are used to locate single and multiple damages, respectively. The modal assurance criterion (MAC) was used by West

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(1984) to determine the level of correlation between modes from the test of an undamaged Space Shuttle Orbiter body flap and the modes from the test of the flap after it has been exposed to an acoustic loading. The MAC criterion can't show explicitly where the source of the damage in the structure lies. Therefore, the coordinate modal assurance criterion (COMAC) has been proposed by Lieven and Ewins (1988). Damage detection from changes in mode shape curvature has been proposed by Pandey *et al*. (1991). The curvature of mode shapes was investigated as a possible candidate for identifying and locating damage in a beam structure. The results showed that the absolute changes in the mode shape curvature are localized in the region of damage and hence can be used to detect damage in the structure. Damage detection in structures using changes in flexibility has been presented by Pandey and Biswas (1994). Appraisal of changes in the flexibility matrix of a structure was presented as a candidate method for recognizing the presence and locating the damage. Some other related criteria based on the mode shape curvature as damage indicators including mode shape amplitude comparison (MSAC), flexibility index (FI), mode shape slope (MSS), and mode shape curvature square (MSCS) have been proposed by Ho and Ewins (2000). The changes of modal flexibility matrix and modal strain energy of flexural members before and after damage have been used by Shih *et al.* (2009) as a basis for locating the structural damage. Results showed that the proposed method is effective in multiple damage assessment in beam and plate structures. Locating structural damage through an adaptive neuro-fuzzy inference system (ANFIS) has been introduced by Fallahian and Seyedpoor (2010). Damage detection using an efficient correlation based index and a modified genetic algorithm has been proposed by Nobahari and Seyedpoor (2011). A two-stage method for determining structural damage sites and extent using a modal strain energy based index (MSEBI) and particle swarm optimization (PSO) has been proposed by Seyedpoor (2012). An efficient method for structural damage localization based on the concepts of flexibility matrix and strain energy of a structure has been suggested by Nobahari and Seyedpoor (2013). An efficient indicator for structural damage localization using the change of strain energy based on static noisy data (SSEBI) has been proposed by Seyedpoor and Yazdanpanah (2014). The acquired results clearly showed that the proposed indicator can precisely locate the damaged elements.

The main purpose of this study is to introduce an efficient mode shape data based indicator for estimating the damage location in beam structures. Two test examples with and without considering measurement noise are considered to assess the efficiency of the proposed indicator for accurately locating the damage. Numerical results demonstrate that the proposed indicator can well determine the location of single and multiple damage cases having different characteristics.

2. Vibration based damage detection techniques

There are various vibration based damage detection techniques that can be utilized to identify the damage in a beam structure. In this section, three more common methods including natural frequency based methods, mode shape based methods and mode shape curvature based methods are described briefly.

2.1 Natural frequency based methods

The change of natural frequencies of a structure before and after damage can be considered as an index for identifying structural damage. One of the most advantages of this technique is that frequency measurements can be quickly and easily conducted (Gholizadeh and Barzegar 2013). However, there are quite a few limitations for using natural frequencies. The practical frequencies are a global parameter and do not provide any spatial information about a structure. Also, the sensitivity of frequencies to damage is relatively low, especially for large structures. Numerous researchers used the changes in natural frequencies for damage detection. For example, Messina *et al*. (1992) proposed the damage location assurance criterion (DLAC) as follows

$$DLAC(i) = \frac{\left|\Delta\omega_A^{\mathrm{T}}\,\Delta\omega_B(i)\right|^2}{(\Delta\omega_A^{\mathrm{T}}\,\Delta\omega_A)\,(\Delta\omega_B(i)^{\mathrm{T}}\,\Delta\omega_B(i))} \tag{1}$$

where $\Delta\omega_A$ is the experimental frequency shift vector and $\Delta\omega_B$ is the theoretical frequency change vector for damage that is situated at the i^{th} position.

2.2 Mode shape based methods

The mode shapes are another basic dynamic characteristic of a structure. Mode shape vectors are a spatial characteristic and can provide information regarding both the existence and location of damage. Mode shape changes before and after damage can be considered as one of prevalent damage detection indicator in structures. For example, the coordinate modal assurance criterion (COMAC) is a mode shape based damage indicator proposed by Lieven and Ewins (1988). The COMAC factor at a point *i* between two sets of mode shape is given by

$$COMAC_i = \frac{\left(\sum_{j=1}^{nm}\left|(\Phi_A)_{ij}\,(\Phi_B)_{ij}\right|\right)^2}{\sum_{j=i}^{nm}(\Phi_A)_{ij}^2 \cdot \sum_{j=i}^{nm}(\Phi_B)_{ij}^2} \tag{2}$$

where *nm* defines the number of correlated mode shapes. Also, $(\Phi_A)_{ij}$ and $(\Phi_B)_{ij}$ indicate the value of the *j*th mode shape at a point *i* of the structure for the states A and B, respectively.

2.3 Mode shape curvature based methods

Mode shape derivatives such as curvature are widely used as an alternative to mode shape in order to obtain spatial information about vibration characteristic changes for damage identification. It is first noted that for beams, plates and shells there is a direct relationship between curvature and bending strain. The value of curvature at a point in the structure is equal to M/EI, so if the stiffness at a point reduces by damage, the curvature at that point will increase. This can be used to detect and locate damage in the structure. Pandey *et al*. (1991) demonstrated that the absolute change in mode shape curvature can be used as an efficient indicator of damage. They estimated the mode shape curvature using the central finite difference approximation (the second derivative of mode shape) as

$$\Phi''_{q,i} = \frac{\Phi_{q-1,i} - 2\Phi_{q,i} + \Phi_{q+1,i}}{l_e^{\,2}} \tag{3}$$

where l_e is the distance between the measurement co-ordinates or it can be the element length.

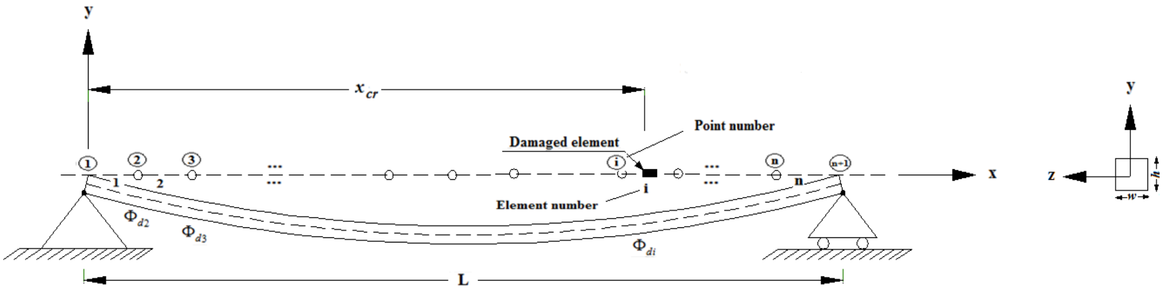


Fig. 1 A simply supported beam having a crack located at x_{cr} from the left end

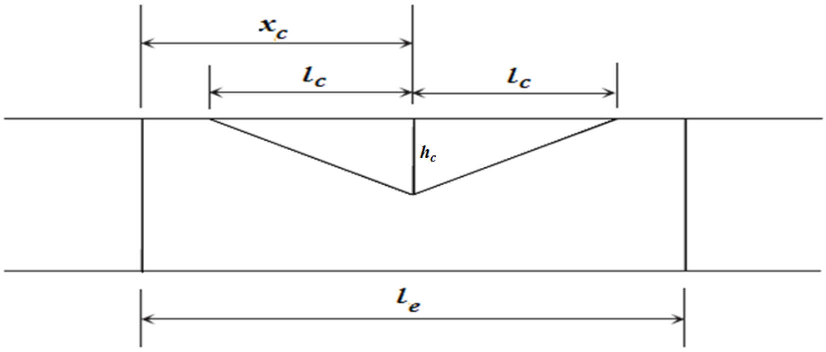


Fig. 2 Variation of EI due to the crack in an element with length l_e

Also, $\Phi_{q,i}$ represents the modal displacement for the i^{th} mode shape at the measurement co-ordinate q. Then, they proposed a damage indicator named mode shape curvature (MSC) as

$$MSC_q = \sum_{i=1}^{nm} \left| (\Phi_{q,i}^{\text{damaged}})'' - (\Phi_{q,i}^{\text{undamaged}})'' \right| \tag{4}$$

where nm is the number of modes to be considered.

Moreover, for considering the effect of higher modes of a structure, Abdel Wahab and De Roeck (1999) proposed the curvature damage factor (CDF) as

$$CDF_q = \frac{1}{nm} \sum_{i=1}^{nm} \left| (\Phi_{q,i}^{\text{damaged}})'' - (\Phi_{q,i}^{\text{undamaged}})'' \right| \tag{5}$$

3. Damage (crack) modeling in beams

In this study, it is assumed that damage occurs by a transverse surface crack located at x_{cr} from the left end of a beam as shown in Fig. 1. For crack modeling, a fully open transverse surface crack model, illuminated by Sinha *et al.* (2002), is adopted. The effect of the crack on the mass is small and can be neglected. The crack only leads to local stiffness reduction in a specified length adjacent to the crack. It is assumed that the reduction of stiffness due to the crack is inside one element. Considering one cracked element as shown in Fig. 2, the flexural rigidity *EI* of the cracked element varies linearly from the cracked position towards both sides in an effective length l_c. The stiffness matrix of the damaged element can be represented as

$$K_c^e = K_u^e - \Delta K_c^e \tag{6}$$

where K_u^e represents the element stiffness matrix of the intact element; ΔK_c^e is the stiffness reduction on the intact element stiffness matrix due to the crack. According to Euler–Bernoulli beam element, the element stiffness matrix of the intact beam is expressed as

$$K_u^e = \frac{2EI_o}{l_e^3}\begin{bmatrix} 6 & 3l_e & -6 & 3l_e \\ & 2l_e^2 & -3l_e & l_e^2 \\ & \text{symmetric} & 6 & -3l_e \\ & & & 2l_e^2 \end{bmatrix} \tag{7}$$

By using the linear variation of *EI* as proposed by Sinha *et al*. (2002), the reduction on the element stiffness matrix can be obtained as

$$\Delta K_c^e = \begin{bmatrix} \Delta K_{11} & \Delta K_{12} & -\Delta K_{11} & \Delta K_{14} \\ & \Delta K_{22} & -\Delta K_{12} & \Delta K_{24} \\ & \text{symmetric} & \Delta K_{11} & -\Delta K_{14} \\ & & & \Delta K_{44} \end{bmatrix} \tag{8}$$

where the stiffness factors are given by

$$\begin{aligned}
\Delta K_{11} &= \frac{12E(I_o - I_c)}{I_e^4}\left[\frac{2l_c^3}{l_e^2} + 3l_c\left(\frac{2x_c}{l_e} - 1\right)^2\right], \\
\Delta K_{12} &= \frac{12E(I_o - I_c)}{I_e^3}\left[\frac{l_c^3}{l_e^2} + l_c\left(2 - \frac{7x_c}{l_e} + \frac{6x_c^2}{l_e^2}\right)\right], \\
\Delta K_{14} &= \frac{12E(I_o - I_c)}{I_e^3}\left[\frac{l_c^3}{l_e^2} + l_c\left(1 - \frac{5x_c}{l_e} + \frac{6x_c^2}{l_e^2}\right)\right], \\
\Delta K_{22} &= \frac{2E(I_o - I_c)}{I_e^2}\left[\frac{3l_c^3}{l_e^2} + 2l_c\left(\frac{3x_c}{l_e} - 2\right)^2\right], \\
\Delta K_{24} &= \frac{2E(I_o - I_c)}{I_e^2}\left[\frac{3l_c^3}{l_e^2} + 2l_c\left(2 - \frac{9x_c}{l_e} + \frac{9x_c^2}{l_e^2}\right)\right], \\
\Delta K_{44} &= \frac{2E(I_o - I_c)}{I_e^2}\left[\frac{3l_c^3}{l_e^2} + 2l_c\left(\frac{3x_c}{l_e} - 1\right)^2\right],
\end{aligned} \tag{9}$$

where x_c is the crack location in the local coordinate, l_e is the length of the element and l_c is the effective length of the stiffness reduction. The value of l_c is assumed to be 1.5 times the beam height. Also, E is the Young's modulus, $I_o=wh^3/12$ and $I_c=w(h-h_c)^3/12$ are the moment of inertia of the intact and cracked cross sections, respectively; w and h are the width and height of the intact beam and h_c is the crack depth. For cases of more than one cracked elements, the same procedure can be followed. The global stiffness matrix of the beam K_c is obtained by assembling the element stiffness matrices including those of cracked elements.

4. The proposed damage detection indicator

In this paper, damage detection of a prismatic beam with a specified length is studied. First, the beam is divided into a number of finite elements. Then, mode shapes of the healthy beam in measurement points are evaluated using the finite element method. A MATLAB (R2010b) code is prepared here for this purpose. Henceforward, consider the nodal coordinates (x_q, q=1,2,...,n+1) and ith mode shape ($\Phi_{h(q,i)}$, q=1,2,...,n+1) obtained for the healthy beam as follows

$$\lfloor x_q, \Phi_{h(q,i)} \rfloor = \lfloor (x_1, \Phi_{h(1,i)}), (x_2, \Phi_{h(2,i)}), \ldots, (x_{n+1}, \Phi_{h(n+1,i)}) \rfloor$$

Now by having the mode shapes, the slope of mode shapes (the first derivative of mode shapes, $\Phi'=d\Phi/dx$) of healthy beam can be achieved using the central finite difference approximation as

$$\Phi'_{h(q,i)} = \frac{\Phi_{h(q+1,i)} - \Phi_{h(q-1,i)}}{2l_e} \tag{10}$$

where i, q and l_e are defined in Eq.(3).

Also, the mode shape curvature (the second derivative of mode shapes) of healthy beam can now be determined using the central finite difference approximation as

$$\Phi''_{h(q,i)} = \frac{\Phi_{h(q-1,i)} - 2\Phi_{h(q,i)} + \Phi_{h(q+1,i)}}{l_e^{\,2}} \tag{11}$$

This process can also be repeated for damaged beam. It should be noted, it is assumed that the damage decreases the stiffness and therefore can be simulated by a reduction in the moment of inertia (I) at the location of damage. In this paper, it is supposed the damage occurs in the center of an element. So, consider the nodal coordinates and ith mode shape ($\Phi_{d(q,i)}, q = 1, 2, ..., n+1$) obtained for the damaged beam as follows:

$$\lfloor x_q, \Phi_{d(q,i)} \rfloor = \lfloor (x_1, \Phi_{d(1,i)}), (x_2, \Phi_{d(2,i)}), \ldots, (x_{n+1}, \Phi_{d(n+1,i)}) \rfloor$$

Now by having the damaged mode shapes, the mode shape slope of damaged beam can be achieved using the central finite difference method as

$$\Phi'_{d(q,i)} = \frac{\Phi_{d(q+1,i)} - \Phi_{d(q-1,i)}}{2l_e} \tag{12}$$

Also, the mode shape curvature of damaged beam can now be approximated as

$$\Phi''_{d(q,i)} = \frac{\Phi_{d(q-1,i)} - 2\Phi_{d(q,i)} + \Phi_{d(q+1,i)}}{l_e^{\,2}} \tag{13}$$

Finally, using the dynamic responses (mode shapes, slope and curvature of mode shapes) obtained for two above states, a new mode shape data based indicator (MSDBI) is introduced here as

$$MSDBI_q = \frac{\sum_{i=1}^{nm} \left\| \left[\left| \Phi''_{d(q,i)} - \Phi''_{h(q,i)} \right| \times \left(\Phi_{d(q,i)} \right)^2 \right] - \left[\left(\left| \Phi'_{d(q,i)} \right| - \left| \Phi'_{h(q,i)} \right| \right)^2 \times \Phi_{h(q,i)} \right] \right\|}{nm} \tag{14}$$

where *nm* is the number of mode shapes considered.

Assuming that the set of the *MSDBI* of all points ($MSDBI_q$, q=1,2,...,n+1) represents a sample population of a normally distributed variable, a normalized form of *MSDBI* can be defined as follows

$$nMSDBI_q = \max\left[0, \left(\frac{MSDBI_q - \mathrm{mean}(MSDBI)}{\mathrm{std}(MSDBI)}\right)\right], \; q = 1,2,...,n+1 \tag{15}$$

where $MSDBI_q$ is defined by Eq. (14). Also, mean (*MSDBI*) and std (*MSDBI*) represent the mean and standard deviation of ($MSDBI_q$, q=1,2,...,n+1), respectively.

In this paper, the result of proposed indicator given by Eq. (15) is compared with that proposed by Abdel Wahab and De Roeck (1999) given by Eq. (5). Therefore, it is required the *CDF* to be normalized in a similar way as

$$nCDF_q = \max\left[0, \left(\frac{CDF_q - \mathrm{mean}(CDF)}{\mathrm{std}(CDF)}\right)\right], \; q = 1,2,...,n+1 \tag{16}$$

5. Numerical examples

In order to assess the efficiency of the proposed indicator for damage detection of beams, the result of nMSDBI is compared with that of nCDF. For this, two test examples including a simply supported beam and a two-span beam are considered. Various parameters that may affect the performance of the method are studied.

5.1 Example 1: a simply supported beam

A simply supported beam with span L=6 m shown in Fig. 3 is selected as the first example. Parameters of the beam consist of width w=0.2 m, height h=0.2 m, elasticity modulus E=200 GPa and the mass density ρ=7850 kg/m^3. The beam is discretized by twenty 2D-beam elements leading to 40 DOFs. In order to assess the efficiency of the proposed indicator, four different damage cases listed in Table 1 are considered. It should be noted that damage in the cracked beam is simulated here by reducing the height and thereinafter the inertia moment (I) at the crack location.

5.1.1 Damage identification without considering noise

Damage identification charts of the simply supported beam for cases 1 to 4 are shown in Figs. 4-7, respectively. As shown in the figures, the value of nMSDBI is further in vicinity of some elements that this indicates, damage occurs in these elements. As shown in the damage identification charts of cases 1 to 4, for precisely locating the damage, 1, 2, 3 and 4 vibration

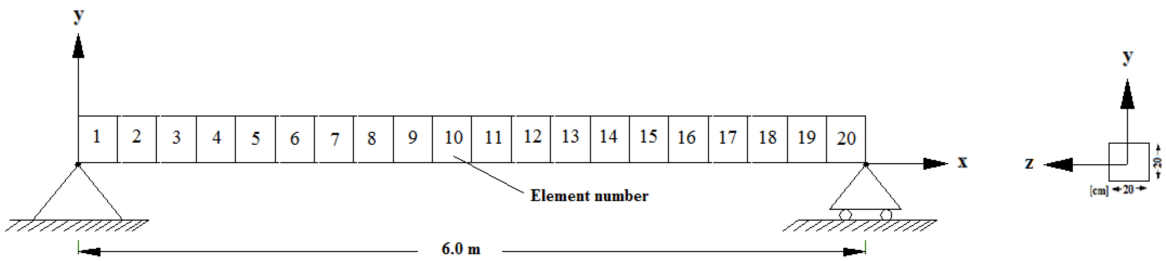


Fig. 3 (a) Geometry of the simply supported beam (b) Cross-section of the beam

Table 1 Four different damage cases induced in simply supported beam

Case 1		Case 2		Case 3		Case 4	
Element number	Damage ratio*	Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
2	0.10	5	0.10	3	0.20	4	0.15
-	-	17	0.15	10	0.15	9	0.10
-	-	-	-	16	0.10	13	0.20
-	-	-	-	-	-	18	0.10

*Damage ratio is $\frac{h_c}{h}$ where h_c is the crack depth

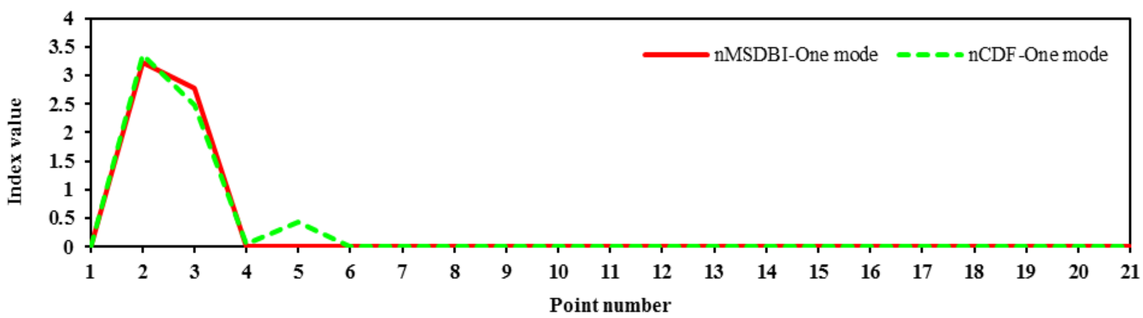


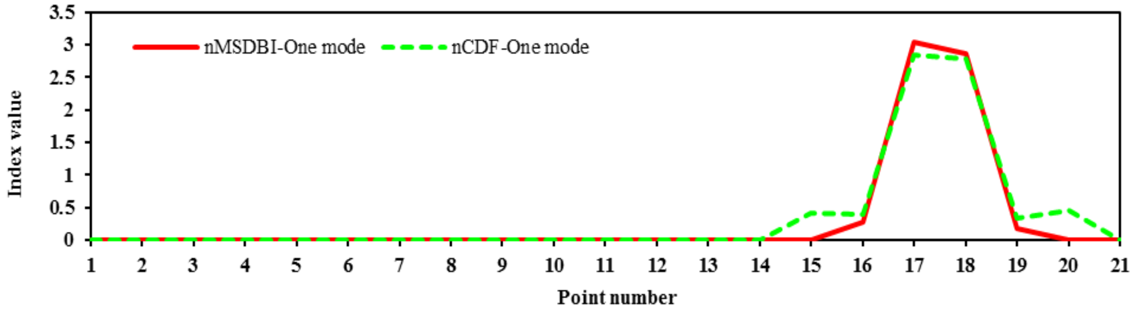
Fig. 4 Damage identification chart of 20-element beam for damage case 1 considering one mode

modes of the beam are necessitated to be considered, respectively. It should be noted that with considering higher vibration modes, the results of nMSDBI and nCDF do not change significantly. Hence, the study of considering the higher vibration modes after exact identification has been neglected.

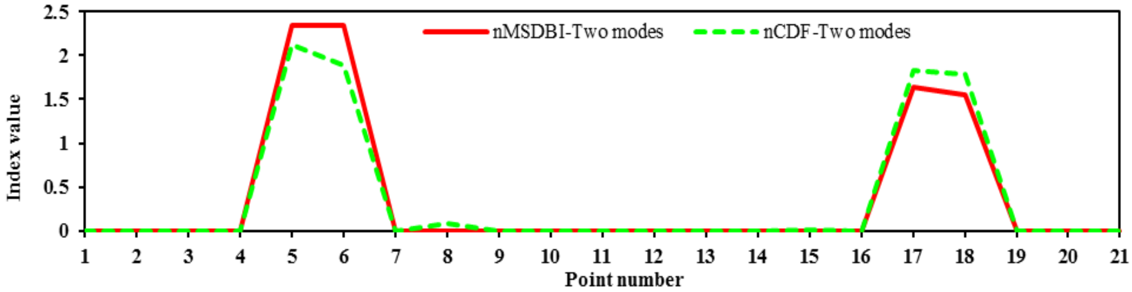
In addition, for verifying the proposed indicator, the result of nMSDBI can be compared with that of nCDF. As can be observed in the figures, the efficiency of the proposed indicator for damage localization is high when comparing with the available damage indicator.

5.1.2 The effect of measurement noise

Measurement noise can’t be avoided practically. Hence, the effect of noise is considered here by perturbing the responses of damaged structure. In this example, 3% noise is assumed in case 1.

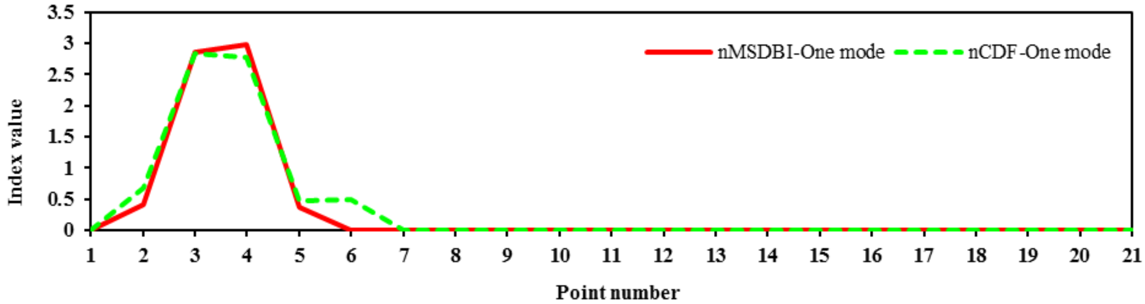


(a)

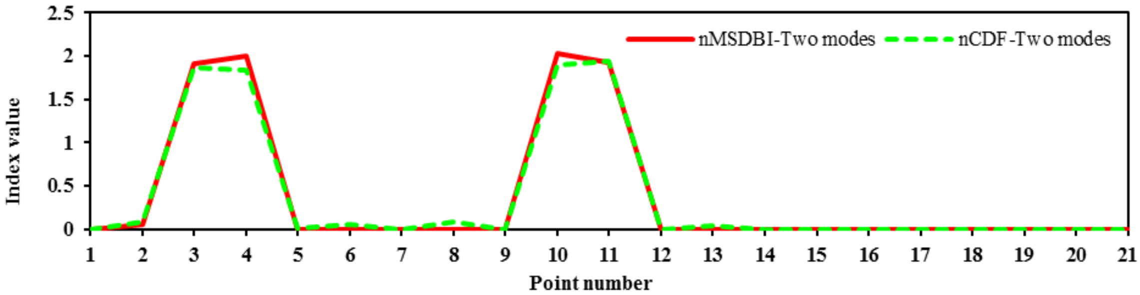


(b)

Fig. 5 Damage identification chart of 20-element beam for damage case 2 considering: (a) one mode, (b) two modes

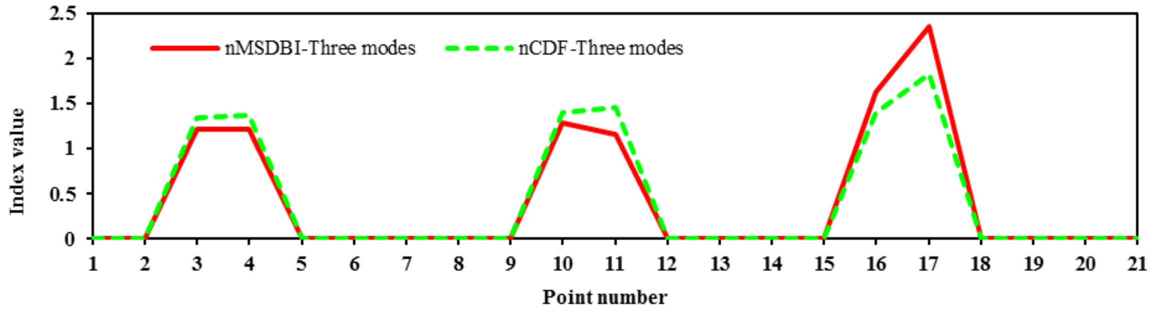


(a)



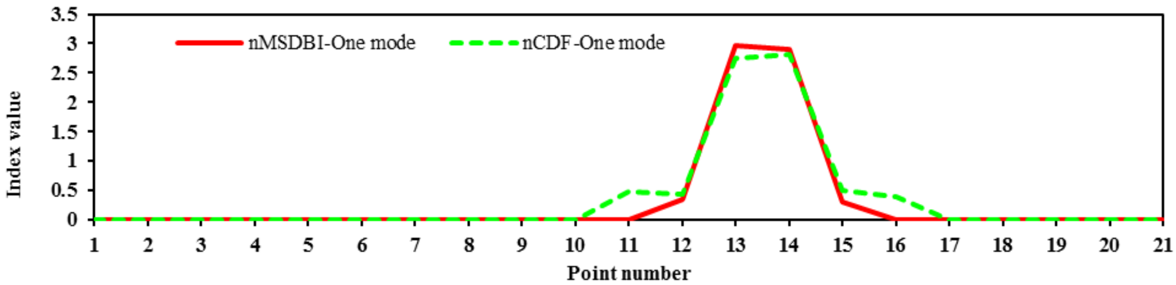
(b)

Fig. 6 Damage identification chart of 20-element beam for damage case 3 considering: (a) one mode, (b) two modes and (c) three modes

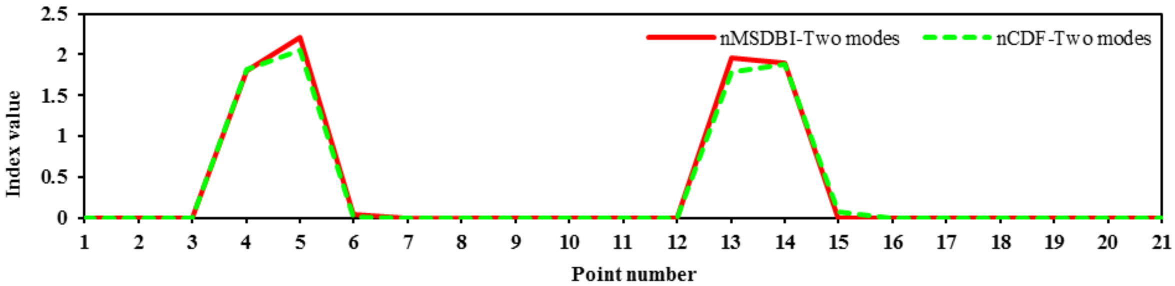


(c)

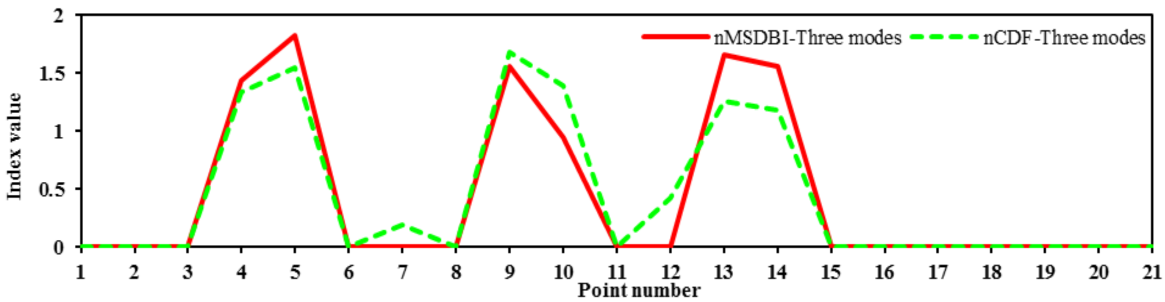
Fig. 6 Continued



(a)

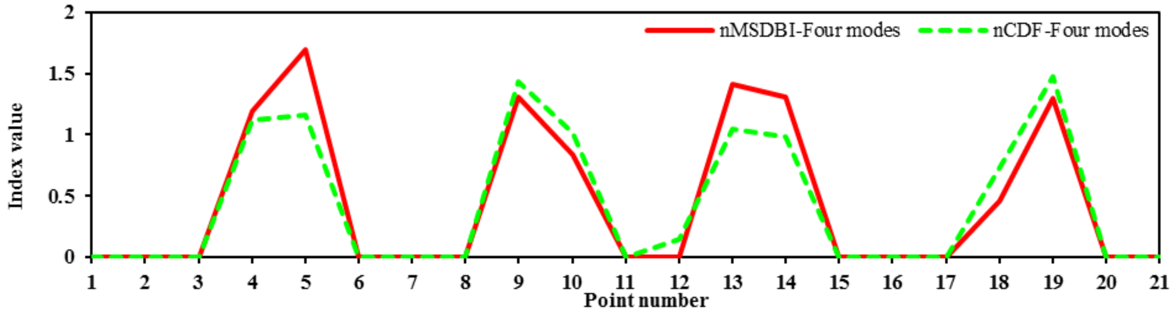


(b)



(c)

Fig. 7 Damage identification chart of 20-element beam for damage case 4 considering: (a) one mode, (b) two modes, (c) three modes and (d) four modes



(d)

Fig. 7 Continued

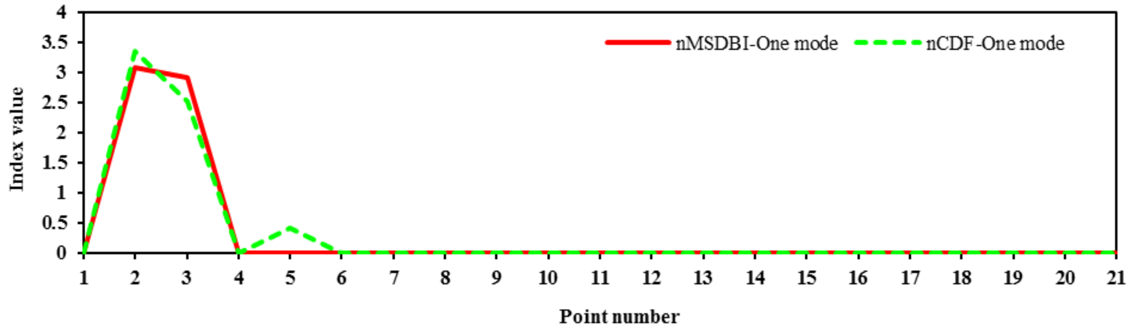


Fig. 8 Damage identification chart of 20-element beam for damage case 1 considering: one mode and 3% noise

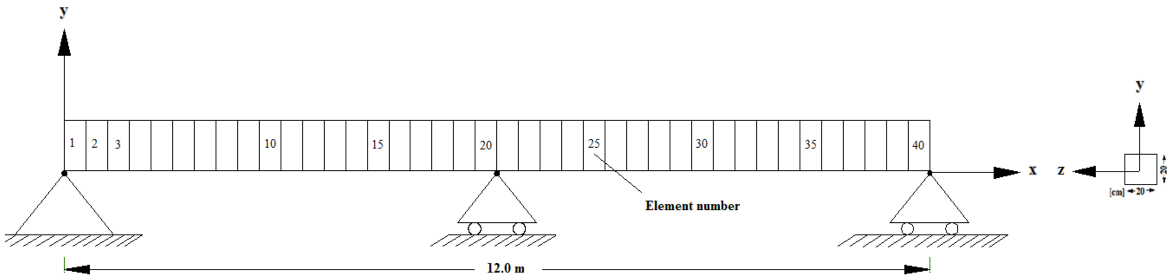


Fig. 9 (a) Geometry of the two-span continuous beam (b) Cross-section of the beam

Fig. 8 shows damage identification chart for the damage scenario 1 considering 3% noise. When comparing it with the one shown in Fig. 4 for scenario 1 (state without noise), it can be indicated that there is a good compatibility between them. In other words, the noise has a negligible effect on the performance of nMSDBI.

5.2 Example 2: a two-span beam

An indeterminate beam with two spans shown in Fig. 9 is considered as the second example. Parameters of the beam are length L=12 m, width w=0.2 m, height h=0.2 m, elasticity modulus

Table 2 Four different damage cases induced in two-span continuous beam

Case 1		Case 2		Case 3		Case 4	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
21	0.20	8	0.10	2	0.10	3	0.15
-	-	34	0.10	21	0.15	12	0.20
-	-	-	-	37	0.20	20	0.15
-	-	-	-	-	-	39	0.10

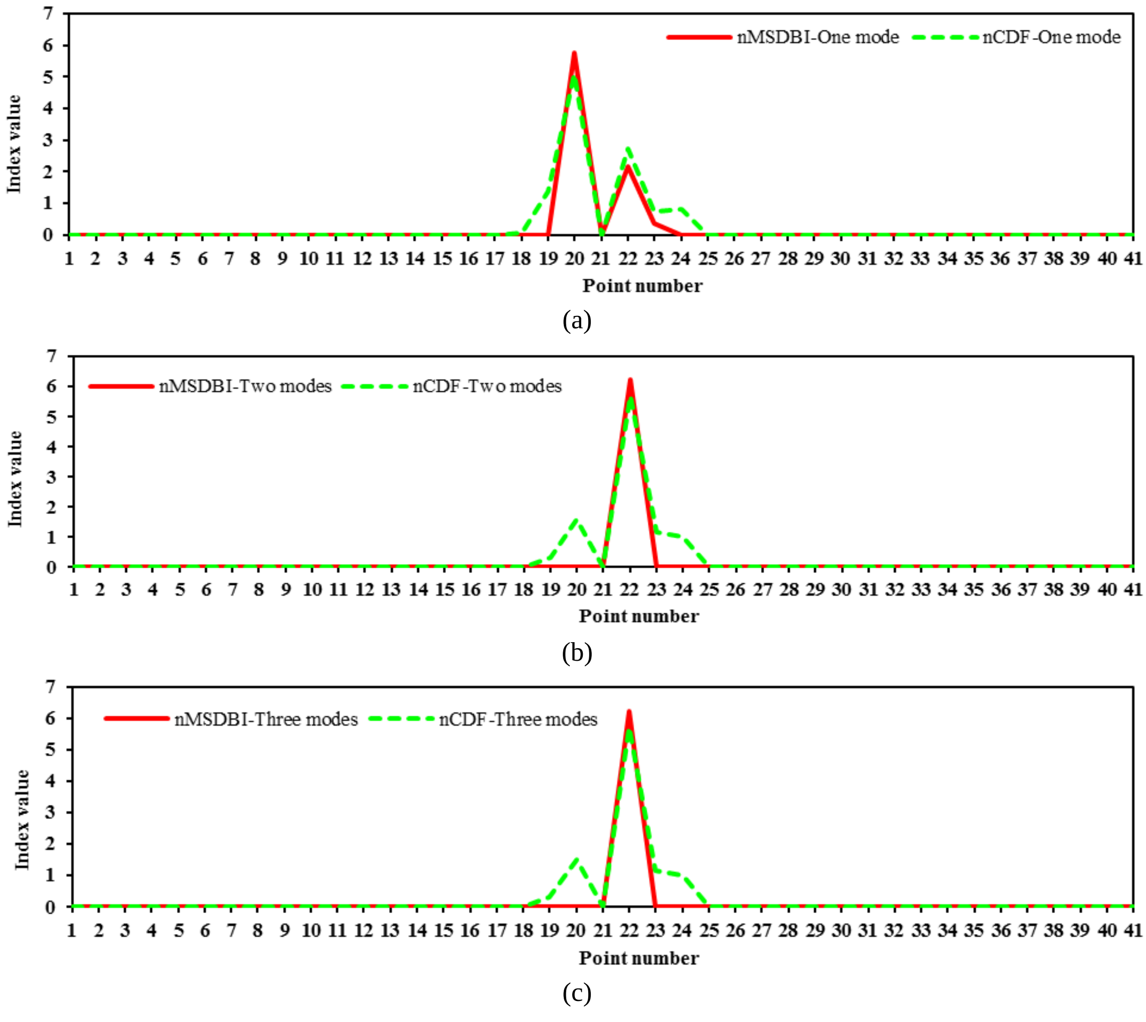


Fig. 10 Damage identification chart of 40-element beam for damage case 1 considering: (a) one mode, (b) two modes and (c) three modes

E=200 GPa and the mass density ρ=7850 kg/m^3. The beam is discretized by forty 2D-beam elements leading to 79 DOFs. For assessing the robustness of the proposed indicator, four different damage cases, including single and multiple damage cases, listed in Table 2 are considered.

5.2.1 Damage identification without considering noise

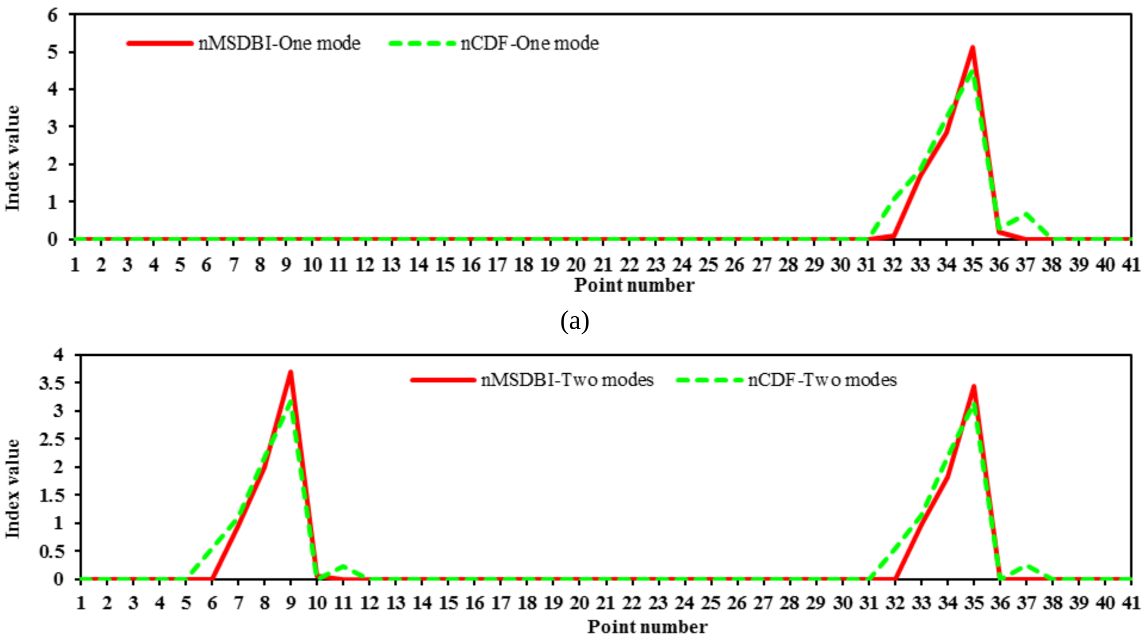
Damage identification charts of the beam for cases 1 to 4 are shown in Figs. 10-13, respectively. As shown in the figures, the value of nMSDBI is further in vicinity of some elements that this indicates there is damage in these elements. As shown in the damage identification charts of cases 1 to 4, for locating the damage precisely, 3, 5, 6 and 8 vibration modes of the beam are required to be considered, respectively. It should be noted that with considering the higher vibration modes, the results of nMSDBI and nCDF do not change. Hence, the effect of higher vibration modes after exact identification has been neglected.

In addition, for verifying the proposed indicator, the performance of nMSDBI is compared with that of nCDF. As can be observed in the figures, the proposed indicator is capable of identifying all damage cases correctly, but when damage occurs in the elements that have placed in the vicinity of the middle support, the nCDF indicates other elements which damage don't happen in them. It seems that the use of nMSDBI for these elements can be better than nCDF. In addition, as can be observed in Fig. 10 (c), even with considering three modes of the beam, the nCDF can not indicate the crack location correctly. In other word, three damaged elements are identified by this figure. It reveals that the results obtained by the proposed indicator are more reliable.

As can be observed in Fig. 11 (e), even with considering five vibration modes for the structure, the result of nCDF can not show the crack location properly. In other word, six elements are introduced by this figure as damaged ones.

5.2.2 The effect of measurement noise

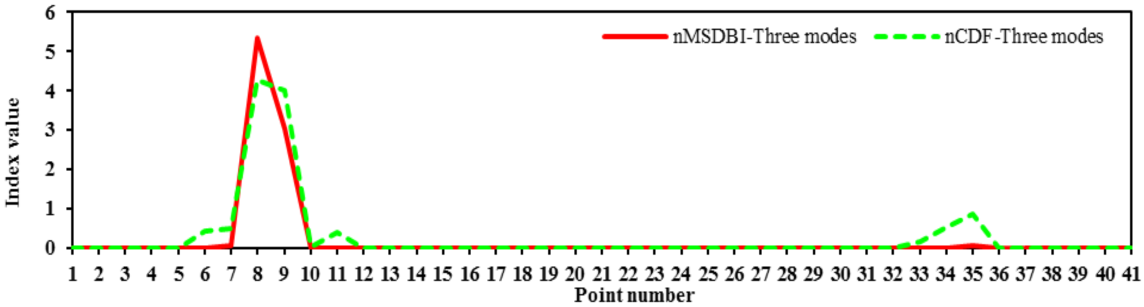
Measurement noise can't be avoided. Hence, the effect of noise is considered here by



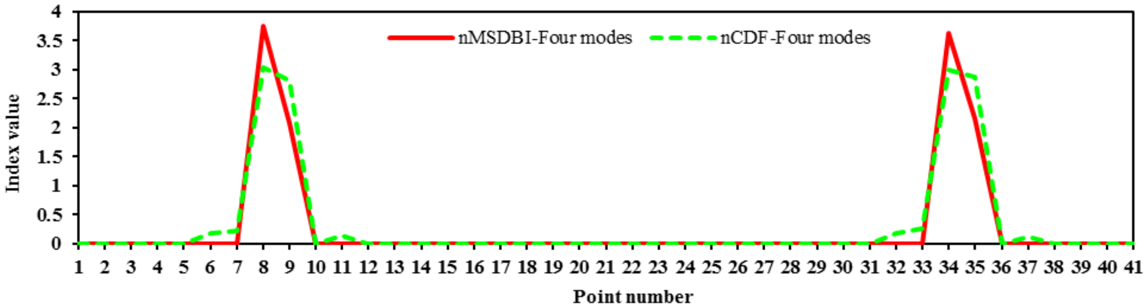
(a)

(b)

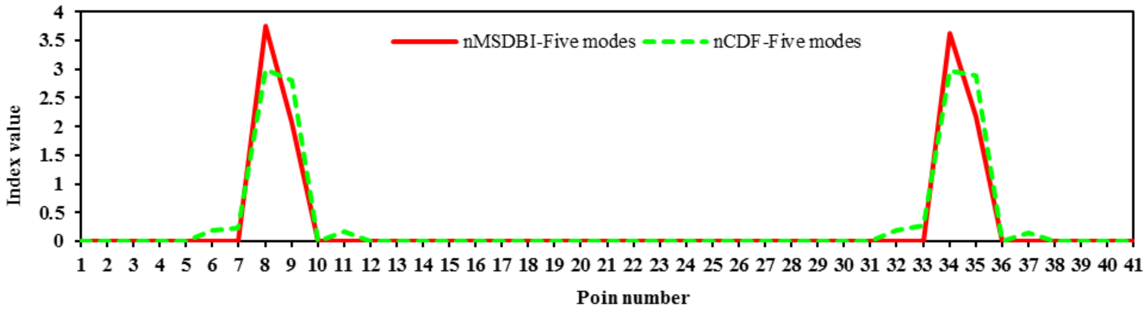
Fig. 11 Damage identification chart of 40-element beam for damage case 2 considering: (a) one mode, (b) two modes, (c) three modes, (d) four modes and (e) five modes



(c)

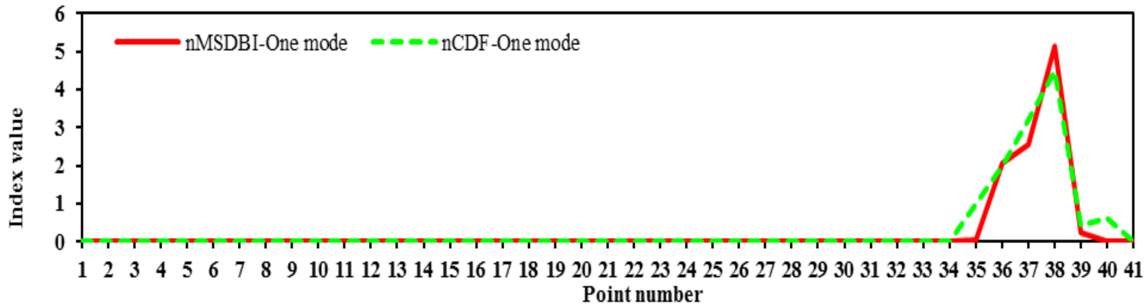


(d)



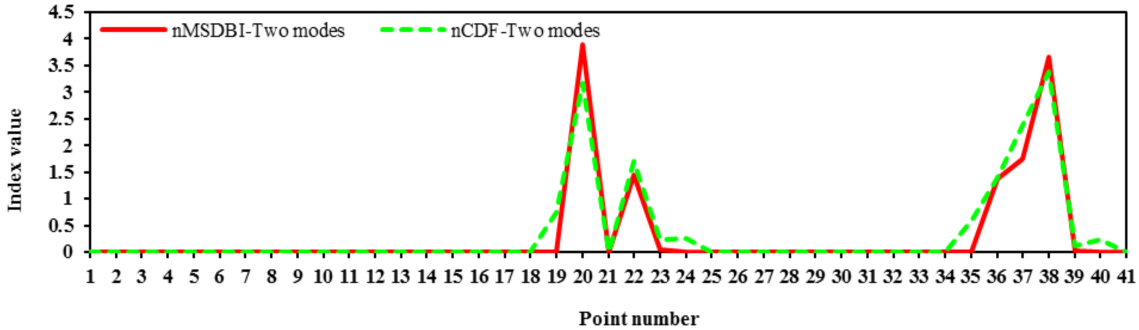
(e)

Fig. 11 Continued

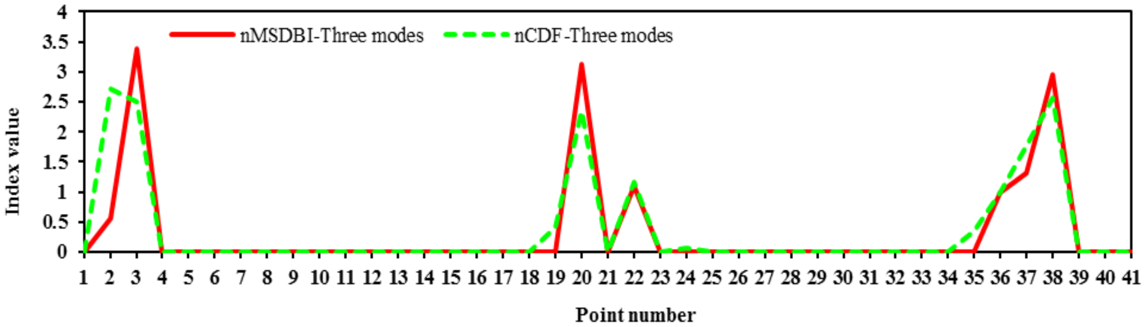


(a)

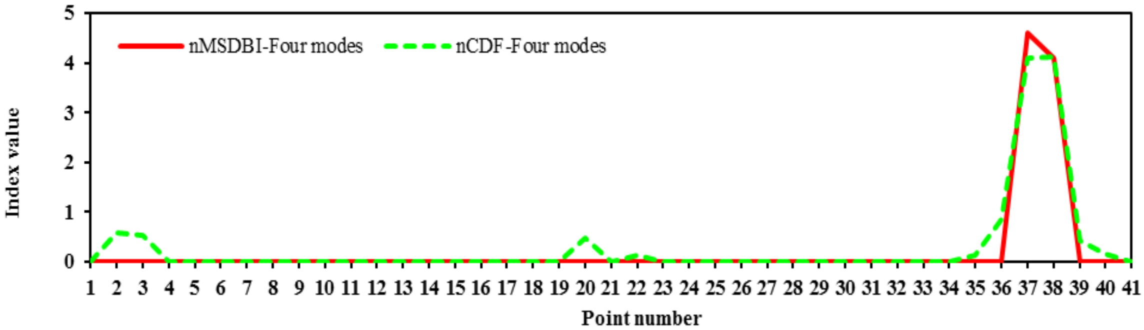
Fig. 12 Damage identification chart of 40-element beam for damage case 3 considering: (a) one mode, (b) two modes, (c) three modes, (d) four modes, (e) five modes and (f) six modes



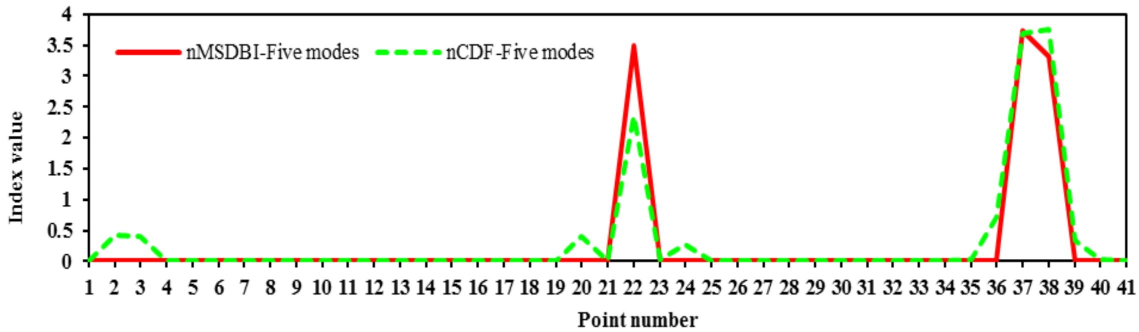
(b)



(c)

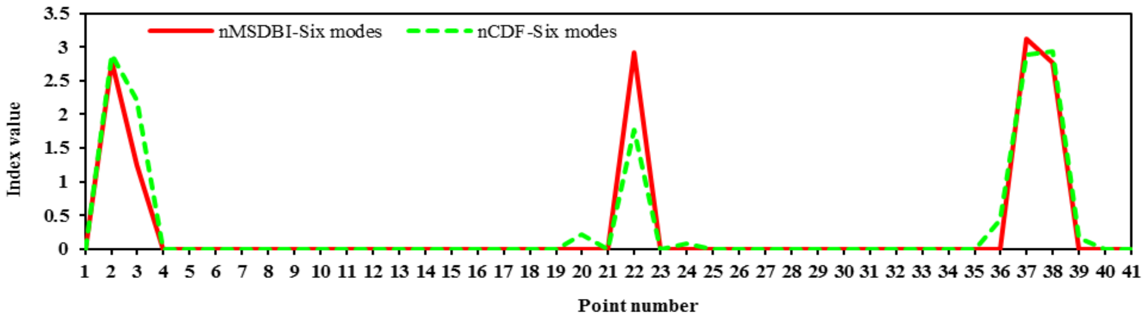


(d)



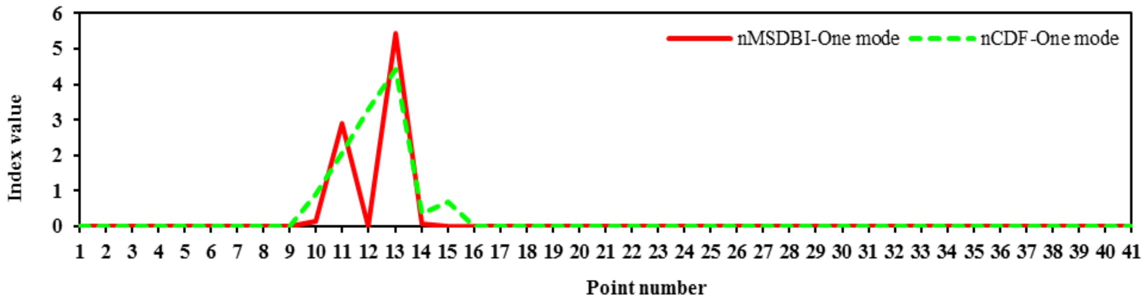
(e)

Fig. 12 Continued

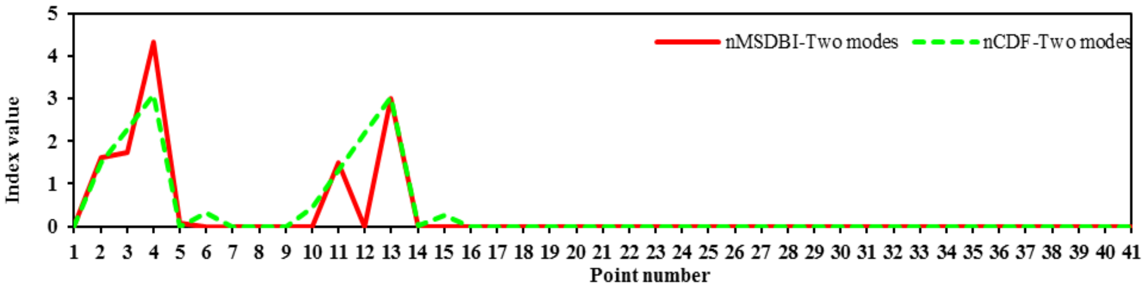


(f)

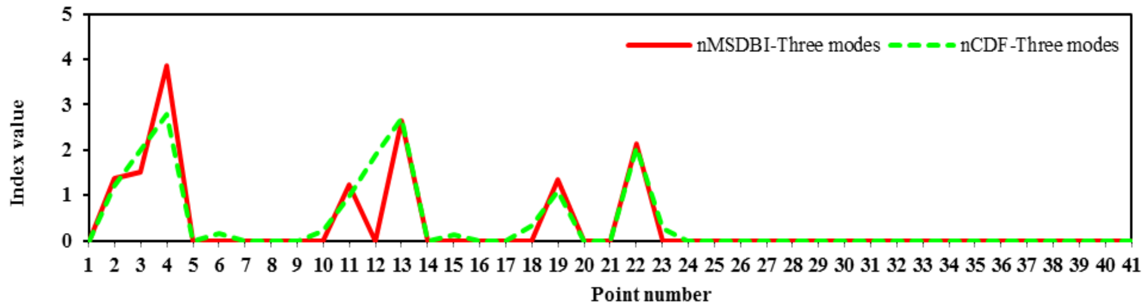
Fig. 12 Continued



(a)

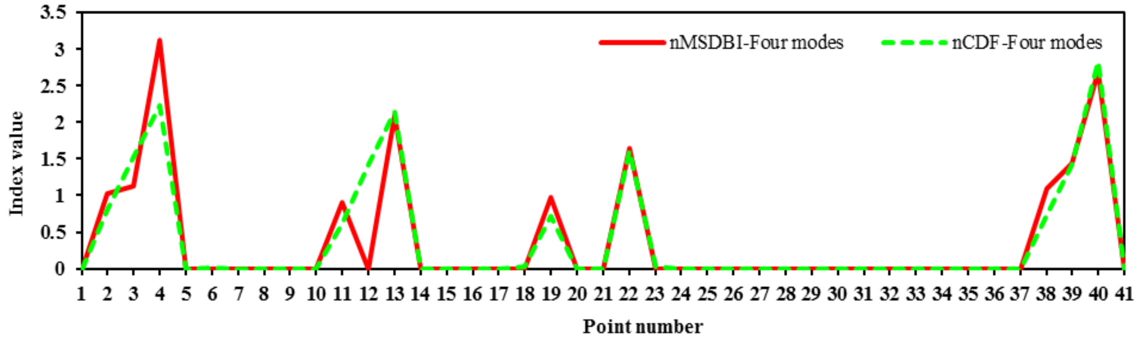


(b)

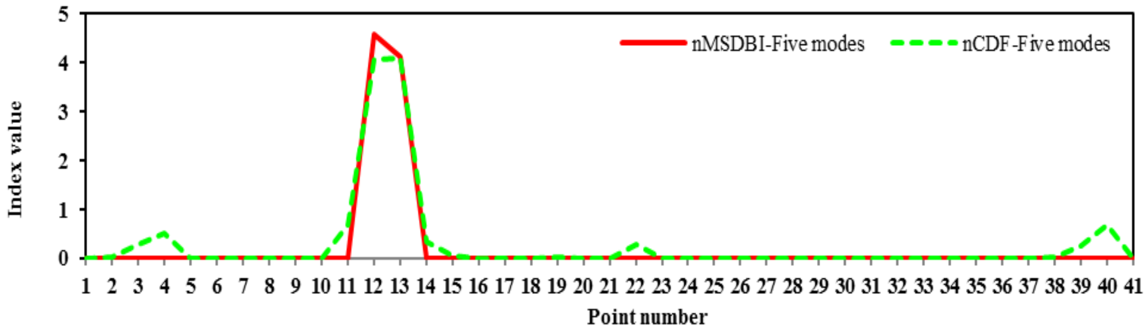


(c)

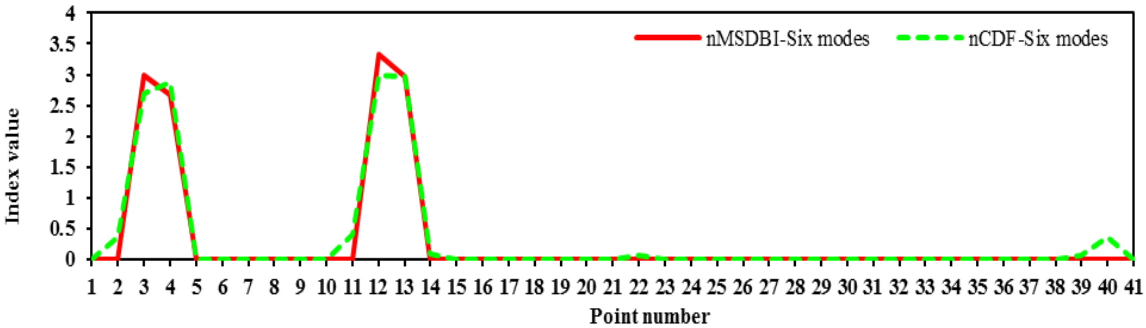
Fig. 13 Damage identification chart of 40-element beam for damage case 4 considering: (a) one mode, (b) two modes, (c) three modes, (d) four modes, (e) five modes, (f) six modes, (g) seven modes and (h) eight modes



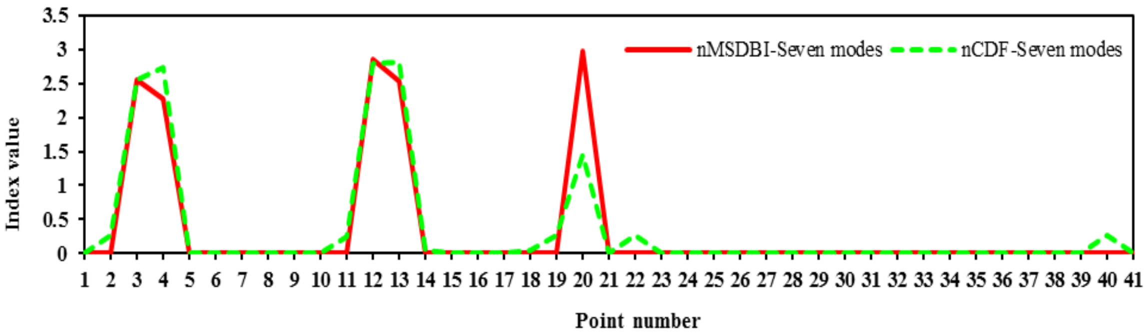
(d)



(e)

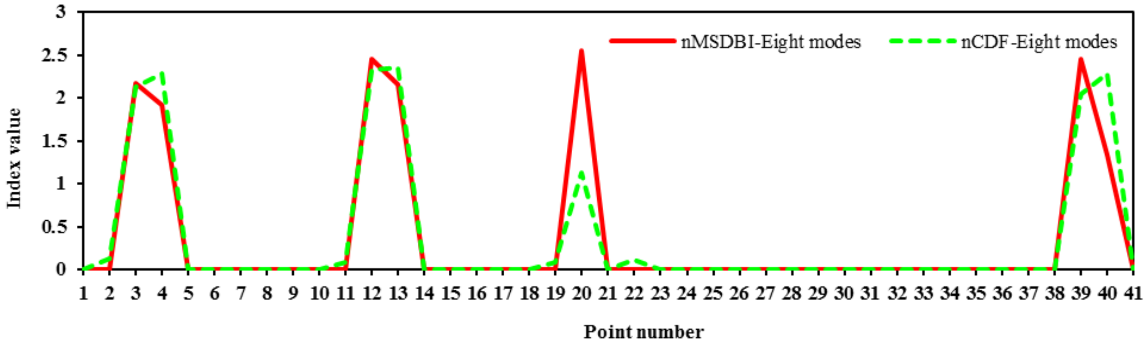


(f)



(g)

Fig. 13 Continued



(h)

Fig. 13 Continued

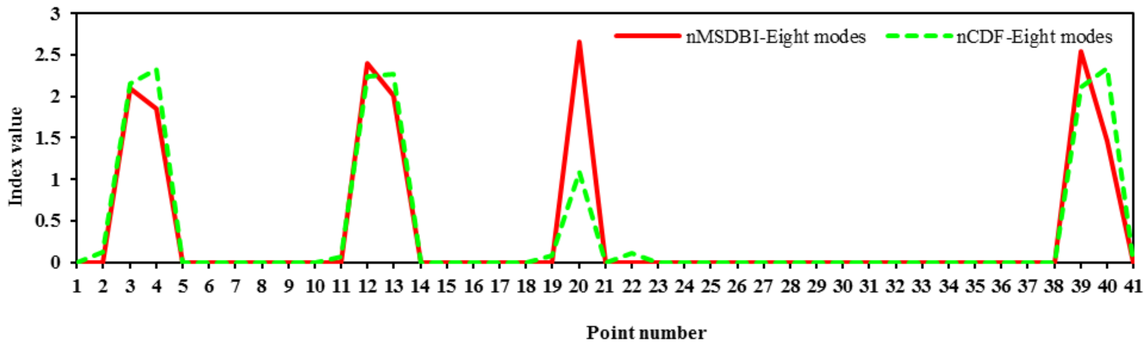


Fig. 14 Damage identification chart of 40-element beam for damage case 4 considering 8 modes and 3% noise

perturbing the responses of damaged structure. In this example, 3% noise is assumed in case 4 of Table 2. Fig. 14 shows damage identification chart for the damage scenario 4 considering 3% noise. When comparing it with the one shown in Fig. 13 for scenario 4 (state without noise), it can be indicated that there is a good compatibility between them. In other words, the measurement noise has a negligible effect on nMSDBI.

6. Conclusions

In this paper, damage identification of beams using a mode shape data based indicator (MSDBI) proposed here has been investigated. The effects of many parameters, may affect the efficiency of the damage indicator, have been assessed with considering a simply supported and an indeterminate beam as test examples. Based on the numerical studies, the following results can be concluded:

• The MSDBI is sensitive to the stiffness reduction (moments of inertia), in other words, it has characteristics from damaged area and can be used as a good indicator for damage detection. As the identification charts revealed, the proposed indicator could identify all damage scenarios properly.

• In order to verify the proposed indictor, the results of MSDBI have been compared with those of a well-known indicator, CDF. Results have shown a better performance for the proposed indicator when compared to that of CDF.

• One of important characteristics of the proposed indicator is that it can precisely detect damaged elements adjacent to middle supports in continues beams. Whereas the CDF identifies other elements which damage don’t occur in them. It seems that the use of MSDBI for these cases is better than CDF.

• As can be observed in identification charts, with increasing damaged elements, more vibration modes were required to be considered to exactly identify the elements. Of course, after identification of all damaged elements by a few modes, by considering the higher vibration modes, the results of MSDBI do not change significantly. In other words, the indicator has a high convergence in terms of number of modes.

•Measurement noise has a negligible effect on the efficiency of the proposed indicator for damage detection.

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