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A polynomial chaos method to the analysis of the dynamic behavior of spur gear system

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Abstract. In this paper, we propose a new method for taking into account uncertainties based on the projection on polynomial chaos. The new approach is used to determine the dynamic response of a spur gear system with uncertainty associated to gear system parameters and this uncertainty must be considered in the analysis of the dynamic behavior of this system. The simulation results are obtained by the polynomial chaos approach for dynamic analysis under uncertainty. The proposed method is an efficient probabilistic tool for uncertainty propagation. It was found to be an interesting alternative to the parametric studies. The polynomial chaos results are compared with Monte Carlo simulations.

Keywords: uncertainty; spur gear system; polynomial chaos; random variable; Monte Carlo simulation; gear parameter

1. Introduction

The gearing is the best solution to transmit rotational motions and couple which has been offered numerous advantages (Dalpiaz *et al*. 1996): it ensures a mechanical reliability. Furthermore, its mechanical efficiency is of the order of 0.96 to 0.99. But today, several applications inquire for the gearing transmissions to be more and more reliable, light and having long useful life that requires the control of the acoustic broadcast and the vibratory behavior of these gearings (Begg *et al*. 2000).

Several parametric studies have shown the great sensitivity of the dynamic behavior of gear systems. However, these parameters admit strong dispersions. Therefore, it becomes necessary to take into account these uncertainties to ensure the robustness of the analysis. Also there are several studies in reliability for vibration structures taking into account the uncertainties (Abo Al-kheer *et al*. 2011, Mohsine and El Hami 2010, El Hami *et al*. 2009, Radi and El Hami 2007, El Hami and Radi 1996, El Hami *et al*. 1993).

Several methods are proposed in the literature. Monte Carlo (MC) simulation is a well-known technique in this field (Fishman 1996). It can give the entire probability density function of any system variable, but it is often too costly since a great number of samples are required for

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reasonable accuracy. Parallel simulation (Papadrakakis and Papadopoulos 1999) and proper orthogonal decomposition (Lindsley and Beran 2005) are some solutions proposed to circumvent the computational difficulties of the MC method.

Polynomial chaos expansion (PCE) is presented in the literature as a more efficient probabilistic tool for uncertainty propagation. It was first introduced by Wiener and pioneered by Ghanem and Spanos who used Hermite orthogonal polynomials to model stochastic processes with Gaussian random variables (Wiener 1938, Ghanem and Spanos 1991). The exponential convergence of such expansion has been shown (Cameron and Martin 1947) and generalized to various continuous and discrete distributions using orthogonal polynomials from the so called Askey-scheme (Askey and Wilson 1985, Xiu and Karniadakis 2003).

Polynomial chaos (PC) gives a mathematical framework to separate the stochastic components of a system response from the deterministic ones. The stochastic Galerkin method (Babuska *et al*. 2004, Le Maître *et al*. 2001), collocation and regression methods (Babuska *et al*. 2007, Crestaux *et al*. 2009) are used to compute the deterministic components called stochastic modes in an intrusive and non-intrusive manner while random components are concentrated in the polynomial basis used. Non-intrusive procedures prove to be more advantageous for stochastic dynamic systems since they need no modifications of the system model, contrary to the intrusive method. In the latter, Galerkin techniques are used to generate a set of deterministic coupled equations from the stochastic system model, and then a suitable algorithm is adapted to obtain stochastic modes.

The capabilities of polynomial chaos have been tested in numerous applications, such as treating uncertainties in environmental and biological problems (Isukapalli *et al*. 1998a, Isukapalli *et al*. 1998b), in multibody dynamic systems (Sandu *et al*. 2006a, Sandu *et al*. 2006b), solving ordinary and partial differential equations (Williams 2006, Xiu and Karniadakis 2002), in component mode synthesis techniques (El Hami and Radi 1996, Sarsri *et al*. 2011) and parameter estimation (Saad *et al*. 2007, Blanchard *et al*. 2009, Smith *et al*. 2007).

The main originality of the present paper is that the uncertainty of the gear system parameters in the dynamic behavior study of the one stage gear system is taken into account. The main objective is to investigate of the capabilities of the new method to determine the dynamic response of a spur gear system subject to uncertain gear parameter. So, an eight degree of freedom system modelling the dynamic behavior of a spur gear system is considered. The modelling of a one stage spur gear system is presented in Section 2. In the next section, the theoretical basis of the polynomial chaos is presented. In Section 4, the equations of motion for the eight degrees of freedom are presented. Numerical results are presented in Section 5. Finally in Section 6, to conclude, some comments are made based on the study carried out in this paper.

2. Modelling of a one stage gear system

The global dynamic model of the one stage gear system in 3D is shown in Fig. 1. This model is composed of two blocks (j=1 to 2). Every block (j) is supported by flexible bearing which the bending stiffness is k_j^x and the traction-compression stiffness is k_j^y.

The wheels (11) and (22) characterize respectively the motor side and the receiving side. The shafts (j) admit some torsional stiffness k_j^θ.

Angular displacements of every wheel are noticed by $\theta_{(i,j)}$ with the indices j=1 to 2 designates the number of the block, and i=1 to 2 designate the two wheels of each block. Moreover, the linear displacements of the bearing noted by x_j and y_j are measured in the plan which is orthogonal to the

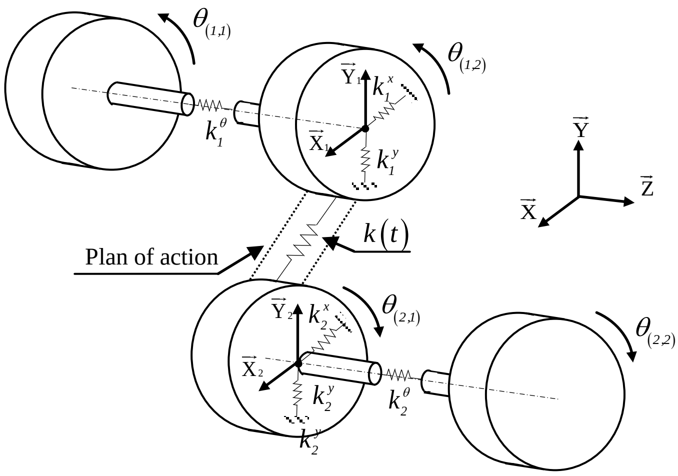

Fig. 1 Global dynamic model of the one stage gear system in 3D

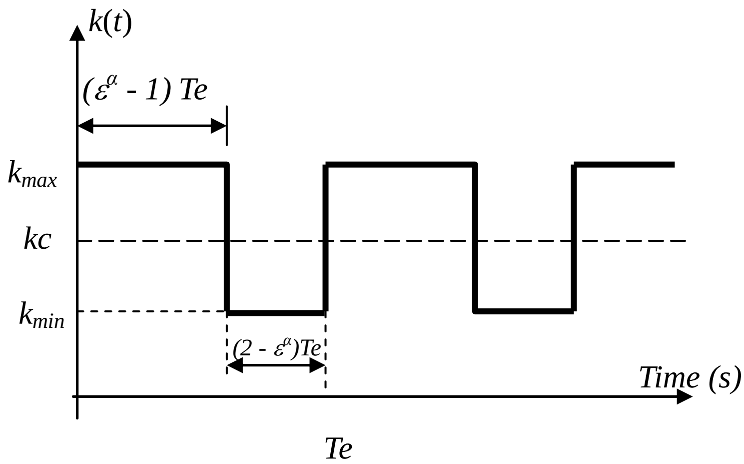

Fig. 2 Modelling of the mesh stiffness variation

wheels axis of rotation.

In this study, we modelled the gear mesh stiffness variation $k(t)$ by a square wave form (Fig. 2). The gear mesh stiffness variation can be decomposed in two components: an average component noted by kc, and a time variant one noted by $kv(t)$ (Walha *et al*. 2009).

The extreme values of the mesh stiffness variation are defined by

$$k_{min} = \frac{kc}{2\varepsilon^{\alpha}} \text{ and } k_{max} = k_{min}\frac{2-\varepsilon^{\alpha}}{\varepsilon^{\alpha}-1} \tag{1}$$

ε^{α} and Te represent respectively the contact ratio and mesh period corresponding to the two gear meshes contacts.

The global dynamic model of the one stage gear system in 2D is presented on Fig. 3.

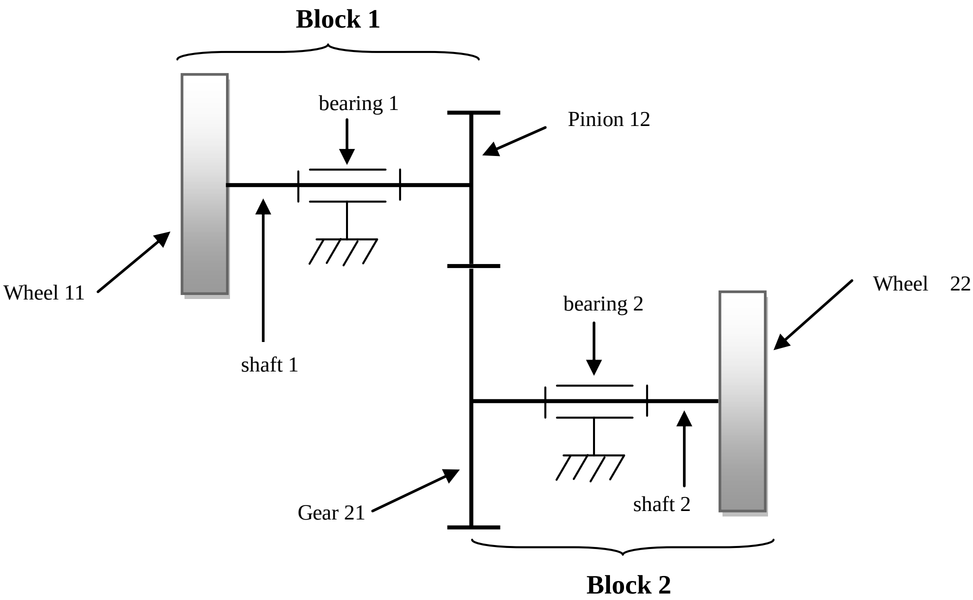

Fig. 3 Global dynamic model of the one stage gear system in 2D

3. Polynomial chaos method

In this section, we propose a new methodological method based on the projection on polynomial chaos. This method consists in projecting the stochastic desired solutions on a basis of orthogonal polynomials in which the variables are Gaussian orthonormal. The properties of the base polynomial are used to generate a linear system of equations by means of projection. The resolution of this system led to an expansion of the solution on the polynomial basis, which can be used to calculate the moments of the random solution. With this method, we can easily calculate the dynamic response of a mechanical system.

Let us consider a multi-degrees of freedom linear system with mass and stiffness matrices $[M_T]$ and $[K_T]$ respectively. The equations of motion describing the forced vibration of a linear system are

$$\left[M_T\right]\left\{\ddot{u}_T\right\}+\left[K_T\right]\left\{u_T\right\}=\left\{f_T\right\} \tag{2}$$

Where $\{u_T\}$ is the nodal displacement vector and $\{f_T\}$is the external excitation.

The chaotic polynomials ψ_m corresponding to the multidimensional Hermite polynomials obtained by the Eq. (3)

$$\psi_m\left(\alpha_1,...,\alpha_P\right)=\left(-1\right)^P e^{\left(\frac{1}{2}{}^T\{\alpha\}\{\alpha\}\right)} \frac{\partial^P e^{\left(-\frac{1}{2}{}^T\{\alpha\}\{\alpha\}\right)}}{\partial\alpha_1...\partial\alpha_P} \tag{3}$$

Where $\{\alpha\}$ is the vector grouping the random variables

$${}^T\{\alpha\}=\left\langle\alpha_1...\alpha_P\right\rangle \tag{4}$$

Where P is the number of random variables.

The random matrices mass and stiffness $[M_T]$ and $[K_T]$ of the mechanical system can be written as

$$\left[M_T\right]=\left[M_T\right]_0+\left[\tilde{M}_T\right] \tag{5}$$

$$\left[K_T\right]=\left[K_T\right]_0+\left[\tilde{K}_T\right] \tag{6}$$

The matrices $[M_T]_0$ and $[K_T]_0$ are deterministic matrices, the matrices $[\tilde{M}_T]$ and $[\tilde{K}_T]$ correspond to the random part of the mass and stiffness matrices.

$[\tilde{M}_T]$ and $[\tilde{K}_T]$ are rewritten from an expression of type Karhunen-Loeve (Ghanem and Spanos 1991) in the following form

$$\left[\tilde{M}_T\right]=\sum_{p=1}^{P}\left[M_T\right]_p\alpha_p \tag{7}$$

$$\left[\tilde{K}_T\right]=\sum_{p=1}^{P}\left[K_T\right]_p\alpha_p \tag{8}$$

Where α_p are independent Gaussian centered reduced which may correspond to the first polynomial ψ_p, while the matrices $[M_T]_p$ and $[K_T]_p$ are deterministic.

We pose α_0=1, we can write then

$$\left[M_T\right]=\sum_{p=0}^{P}\left[M_T\right]_p\alpha_p \tag{9}$$

$$\left[K_T\right]=\sum_{p=0}^{P}\left[K_T\right]_p\alpha_p \tag{10}$$

In the same way, we can write for $\{f_T\}$

$$\left\{f_T\right\}=\sum_{p=0}^{P}\left\{f_T\right\}_p\alpha_p \tag{11}$$

The dynamic response is obtained by solving the following equation knowing that the initial conditions are predefined

$$\left[K_{eq}\right]\left\{u_T\right\}\left(t+\Delta t\right)=\left\{F_{eq}\right\} \tag{12}$$

Where

$$\left[K_{eq}\right]=\left[K_T\right]+a_0\left[M_T\right] \tag{13}$$

$$\left\{F_{eq}\right\}=\left\{f_T\right\}\left(t+\Delta t\right)+\left[M_T\right]\left(a_0\left\{u_T\right\}\left(t\right)+a_1\left\{\dot{u}_T\right\}\left(t\right)+a_2\left\{\ddot{u}_T\right\}\left(t\right)\right) \tag{14}$$

Where

$$a_0=\frac{1}{A\Delta t^2}\,,\quad a_1=\frac{B}{A\Delta t}\quad\text{and}\quad a_2=\frac{1}{A\Delta t} \tag{15}$$

A and B are the parameters of Newmark.

$\{u_T\}(t+\Delta t)$ is decomposed on polynomials to P Gaussian random variables orthnormales

$$\{u_T\}(t+\Delta t)=\sum_{n=0}^{N}\left(\{u_T\}(t+\Delta t)\right)_n \psi_n\left(\{\alpha_i\}_{i=1}^{P}\right) \tag{16}$$

Where N is the polynomial chaos order.

$[K_{eq}]$ and $\{F_{eq}\}$ are written in the following form

$$\left[K_{eq}\right]=\sum_{p=0}^{P}\left[K_T\right]_p \alpha_p + a_0\sum_{p=0}^{P}\left[M_T\right]_p \alpha_p=\sum_{p=0}^{P}\left[K_{eq2}\right]_p \alpha_p \tag{17}$$

$$\begin{aligned}\left\{F_{eq}\right\}&=\sum_{p=0}^{P}\left(\{f_T\}(t+\Delta t)\right)_p \alpha_p+\sum_{p=0}^{P}\left[M_T\right]_p \alpha_p\left(a_0\left(\{u_T\}(t)\right)_0+a_1\left(\{\dot{u}_T\}(t)\right)_0+a_2\left(\{\ddot{u}_T\}(t)\right)_0\right)\\&=\sum_{p=0}^{P}\left\{F_{eq2}\right\}_p \alpha_p\end{aligned} \tag{18}$$

Substituting Eqs. (16), (17) and (18) into Eq. (12) and forcing the residual to be orthogonal to the space spanned by the polynomial chaos ψ_m yield the following system of linear equation

$$\sum_{p=0}^{P}\sum_{n=0}^{N}\left[K_{eq2}\right]_p \{u_T\}_n \left\langle \alpha_p \ \psi_n \ \psi_m\right\rangle=\sum_{p=0}^{P}\left\{F_{eq2}\right\}_p\left\langle \alpha_p \ \psi_m\right\rangle \quad m=0,1,\ldots,N \tag{19}$$

Where N is the order of Polynomial Chaos.

Where $\langle . . \rangle$ denotes the inner product defined by the mathematical expectation operator

This algebraic equation can be rewritten in a more compact matrix form as

$$\begin{bmatrix}[D]^{(00)} & \cdots & [D]^{(0N)}\\ & \ddots & \\ \vdots & [D]^{(ij)} & \vdots\\ & \ddots & \\ [D]^{(N0)} & \cdots & [D]^{(NN)}\end{bmatrix}\begin{Bmatrix}\left(\{u_T\}(t+\Delta t)\right)_0\\ \vdots\\ \left(\{u_T\}(t+\Delta t)\right)_j\\ \vdots\\ \left(\{u_T\}(t+\Delta t)\right)_N\end{Bmatrix}=\begin{Bmatrix}\{f\}^{(0)}\\ \vdots\\ \{f\}^{(j)}\\ \vdots\\ \{f\}^{(N)}\end{Bmatrix}, \tag{20}$$

Where

$$[D]^{(ij)}=\sum_{p=0}^{P}\left[K_{eq2}\right]_p\left\langle \alpha_p \ \psi_i \ \psi_j\right\rangle \tag{21}$$

$$\{f\}^{(j)}=\sum_{p=0}^{P}\left\{F_{eq2}\right\}_p\left\langle \alpha_p \ \psi_j\right\rangle \tag{22}$$

After resolution of the algebraic system (20), the mean values and the variances of the dynamic response are given by the following relationships

$$E\left[\{u_T\}\right]=\left(\{u_T\}(t+\Delta t)\right)_0 \tag{23}$$

$$Var\left[\{u_T\}\right]=\sum_{n=1}^{N}\left(\left(\{u_T\}(t+\Delta t)\right)_n\right)^2\left(\psi_j\right)^2 \tag{24}$$

4. Equations of motion

The equation of motion describing the dynamic behavior of our system (Fig. 1) is obtained by applying Lagrange formulation and is given by

$$m\ddot{x}_1 + k_1^x x_1 + sin(\alpha)\, k(t) \left\langle L^\delta \right\rangle \{Q\} = 0 \tag{25}$$

$$m\ddot{y}_1 + k_1^y y_1 + cos(\alpha)\, k(t) \left\langle L^\delta \right\rangle \{Q\} = 0 \tag{26}$$

$$m\ddot{x}_2 + k_2^x x_2 - sin(\alpha)\, k(t) \left\langle L^\delta \right\rangle \{Q\} = 0 \tag{27}$$

$$m\ddot{y}_2 + k_2^y y_2 - cos(\alpha)\, k(t) \left\langle L^\delta \right\rangle \{Q\} = 0 \tag{28}$$

$$I\ \ddot{\theta}_{(1,1)} + k_1^\theta \left(\theta_{(1,1)} - \theta_{(1,2)}\right) = Cm \tag{29}$$

$$I\ \ddot{\theta}_{(1,2)} - k_1^\theta \left(\theta_{(1,1)} - \theta_{(1,2)}\right) + r_{(1,2)}^b\, k(t) \left\langle L^\delta \right\rangle \{Q\} = 0 \tag{30}$$

$$I\ \ddot{\theta}_{(2,1)} + k_2^\theta \left(\theta_{(2,1)} - \theta_{(2,2)}\right) - r_{(2,1)}^b\, k(t) \left\langle L^\delta \right\rangle \{Q\} = 0 \tag{31}$$

$$I\ \ddot{\theta}_{(2,2)} + k_2^\theta \left(\theta_{(2,1)} - \theta_{(2,2)}\right) = 0 \tag{32}$$

Where I is the moment of inertia of the wheels.
Where $\left\langle L^\delta \right\rangle$ is defined by

$$\left\langle L^\delta \right\rangle = \left[sin(\alpha) \quad -sin(\alpha) \quad cos(\alpha) \quad -cos(\alpha) \quad 0 \quad r_{(1,2)}^b \quad -r_{(2,1)}^b \quad 0 \right] \tag{33}$$

$r_{(1,2)}^b, r_{(2,1)}^b$ represent the base gears radius. α is the pressure angle .
$\{Q(t)\}$ is the vector of the model generalized coordinates, it is in the form

$$\{Q(t)\} = \left[x_1 \quad y_1 \quad x_2 \quad y_2 \quad \theta_{(1,1)} \quad \theta_{(1,2)} \quad \theta_{(2,1)} \quad \theta_{(2,2)} \right]^T \tag{34}$$

5. Numerical simulation

The technological and dimensional features of the one-stage gear system are summarized in the Table 1.

In this section numerical results are presented for the new method formulations derived in the Section 3. The polynomial chaos results are compared with Monte Carlo simulations with 100000 simulations.

The mass and the moment of inertia of gears are defined by

$$m = \rho.\pi.r^2.l \tag{35}$$

$$I = \frac{1}{2}.m.r^2 \tag{36}$$

Table 1 System parameters

Material: 42CrMo4	ρ=7860 Kg/m^3
Motor torque	Cm=200 N.m
Bearing stiffness	$k_j^x=10^7$ N/m $k_j^y=10^7$ N/m
Torsional stiffness of the shaft	$k_j^\theta=10^5$ Nm/rad
Number of teeth	$Z(12)$=40 ; $Z(21)$=50
Module of teeth	module=4.10^{-3} m
Contact ratio	ε^α=1.7341
The pressure angle	α=20°

Where r, ρ and l denote radius, mass density and length.

The radius parameter is supposed random variable and defined as follow:

$$r = r_0 + \sigma_r \xi \qquad (37)$$

Where ξ is a zero mean value Gaussian random variable, r_0 is the mean value and σ_r is the standard deviation of this parameter.

The mean value and standard deviation of the dynamic component of the linear displacement of the first bearing in two directions x and y have been calculated by the polynomial chaos method. The obtained results are compared with those given by the Monte Carlo simulations with 100000 simulations. The results are plotted in Figs. 4 and 6 for σ_r=2% and in Figs. 5 and 7 for σ_r=5%. These figures show that the obtained solutions oscillate around the Monte Carlo simulations reference solution. It can be seen that for small standard deviation σ_r=2%, the polynomial chaos solutions in second order polynomial provides a very good accuracy as compared with the Monte Carlo simulations. When the standard deviation increases the error increases.

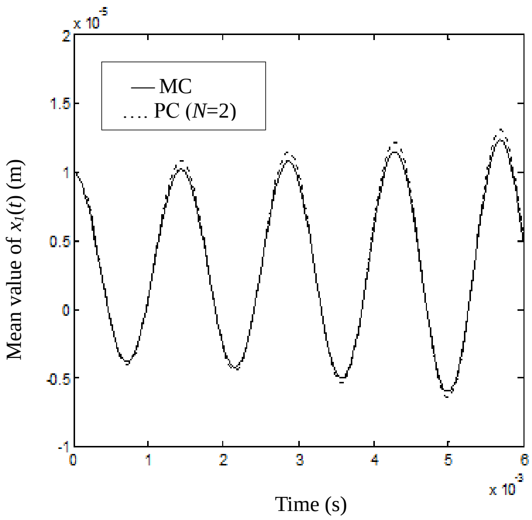

Fig. 4.1 Mean value of $x_1(t)$ σ_r=2%

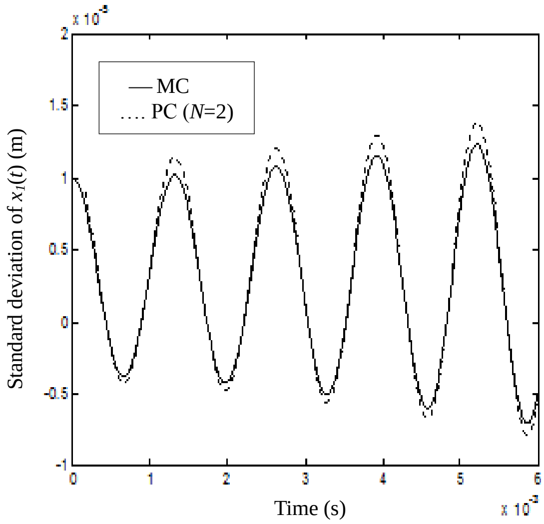

Fig. 5.1 Mean value of $x_1(t)$ σ_r=5%

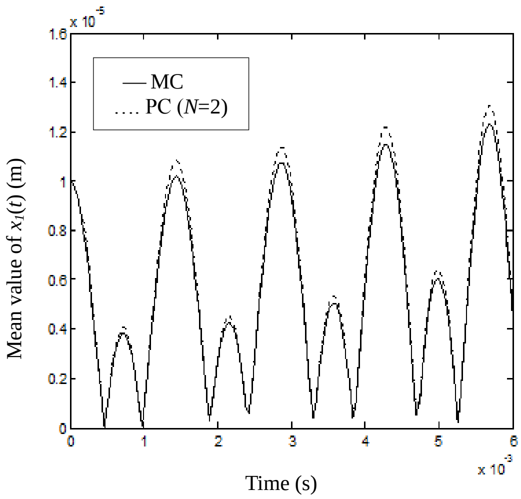

Fig. 4.2 Standard deviation of $x_1(t)$ σ_r=2%

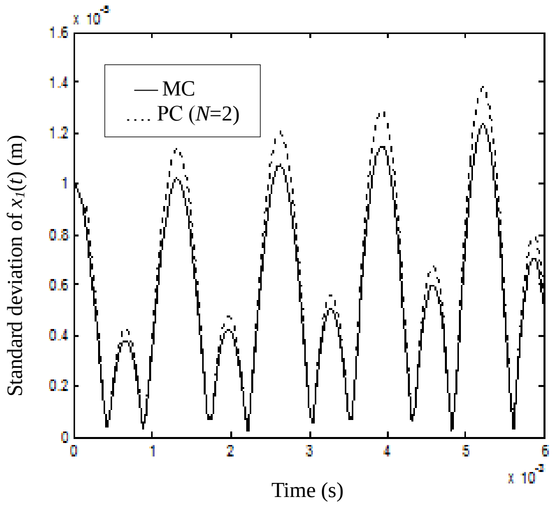

Fig. 5.2 Standard deviation of $x_1(t)$ σ_r=5%

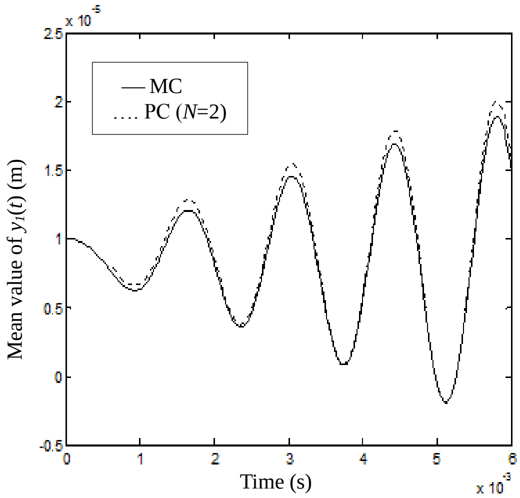

Fig. 6.1 Mean value of $y_1(t)$ σ_r=2%

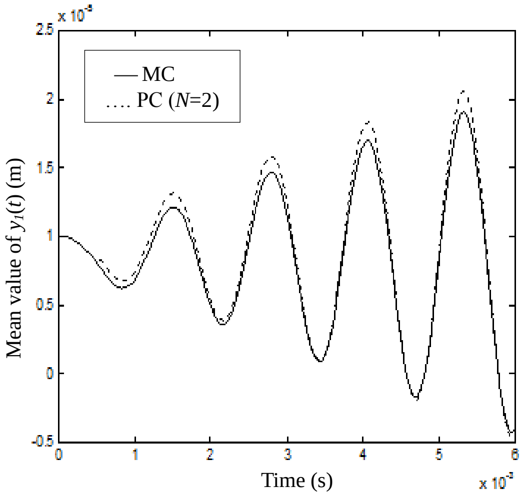

Fig. 7.1 Mean value of $y_1(t)$ σ_r=5%

The mean value of the dynamic component of the angular displacement $\theta_{(1,1)}$ and $\theta_{(2,1)}$ are presented in Figs. 8 and 9 for σ_r=5%.The PC results are compared with Monte Carlo simulations with 100000 simulations. The dynamic response of the angular displacement as predicted by polynomial chaos calculations matches exactly with that of the Monte Carlo analysis. A N=8 has been used for the PC model and is seen to be enough to capture the dynamic response of the angular displacement of our system.

The mean value and standard deviation of the dynamic component of the linear displacement of the second bearing in two directions x and y obtained with different orders of polynomial chaos N=2, N=4 and N=8 are presented in Figs. 10 and 11 for σ_r=10% in order to check the capabilities of the polynomial chaos approach in the analysis of the dynamic behavior of spur gear system.

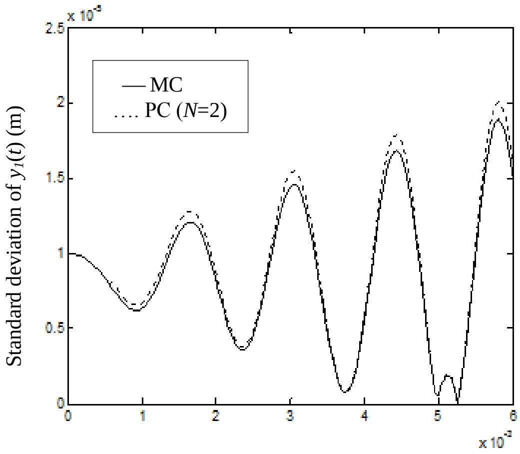

Fig. 6.2 Standard deviation of $y_1(t)$ σ_r=2%

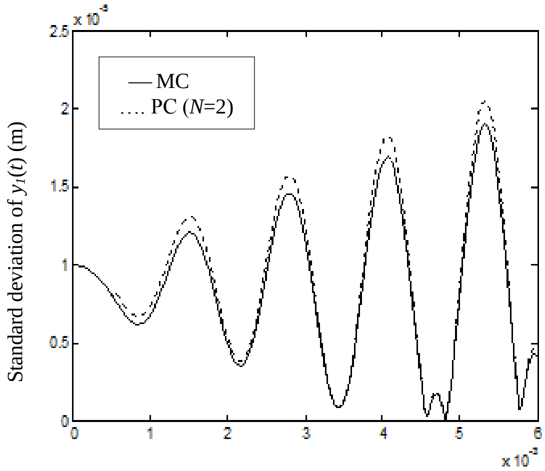

Fig. 7.2 Standard deviation of $y_1(t)$ σ_r=5%

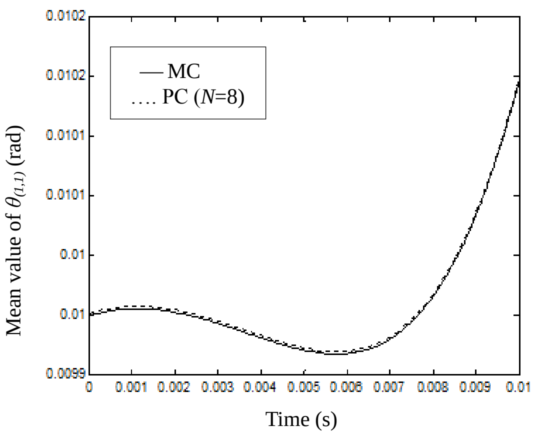

Fig. 8 Mean value of $\theta_{(1,1)}(t)$ σ_r=5%

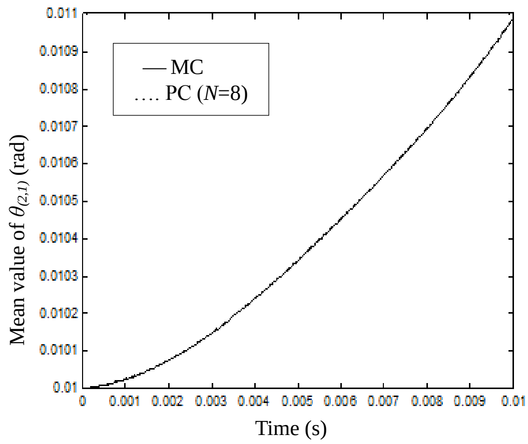

Fig. 9 Mean value of $\theta_{(2,1)}(t)$ σ_r=5%

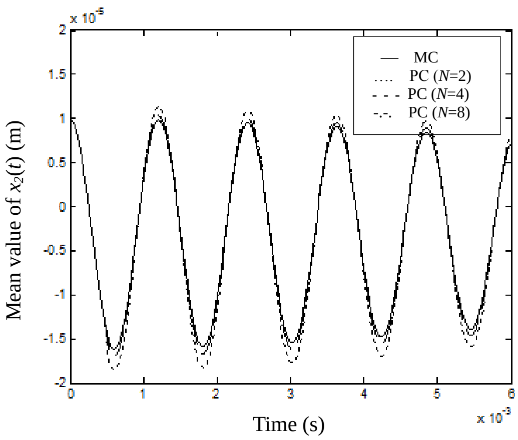

Fig. 10.1 Mean value of $x_2(t)$ σ_r=10%

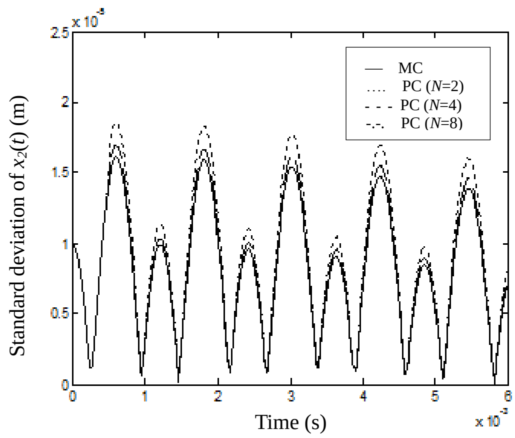

Fig. 10.2 Standard deviation of $x_2(t)$ σ_r=10%

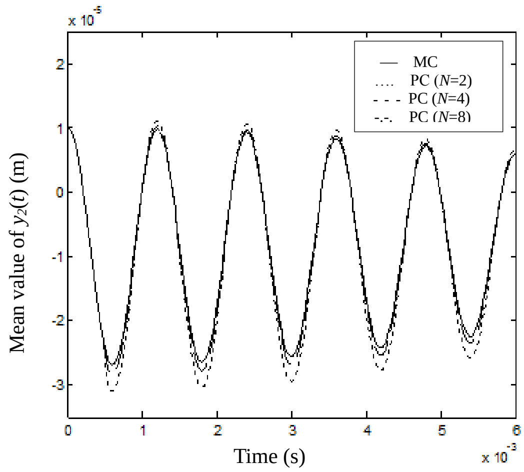

Fig. 11.1 Mean value of $y_2(t)$ σ_r=10%

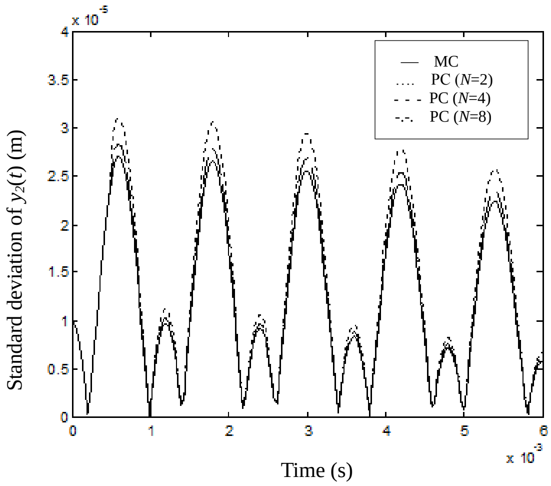

Fig. 11.2 Standard deviation of $y_2(t)$ σ_r=10%

The polynomial chaos results are compared with Monte Carlo simulation with 100000 simulations. It is evident from these figures that *N*=2 case clearly does not have enough chaos terms to represent the output. As *N* increases, the results seem to become better, and with *N*=8, the dynamic response of the linear displacement of the second bearing with polynomial chaos values almost exactly match with the Monte Carlo simulation results. The uncertainty of the radius parameter affects the amplitude of the system responses. It can be noted that the amplitudes of the mean values and the standard deviation are approximated more accurately with *N*=8 than the fourth and the second order polynomial.

6. Conclusions

An approach based on the polynomial chaos method has been proposed to study the dynamic behavior of a spur gear system which is highly sensitive to dispersions of the gear parameters. A complete study of the dynamic behavior including dynamic response analyses has been carried out for an eight degree of freedom model describing a spur gear system characterised by an uncertain gear parameter. The polynomial chaos method has been used to determine the dynamic response of a one stage gear system. The efficiency of the proposed method compared with the Monte Carlo simulation. The main results of the present study show that the polynomial chaos may be an efficient tool to take into account the dispersions of the gear parameter in the dynamic behavior study of a spur gear system. An interesting perspective is to apply this method to a system with higher degree of freedom like epicyclic gear system. Further work in this context is in progress.

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