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Disturbance analysis of hydropower station vertical vibration dynamic characteristics: the effect of dual disturbances

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Abstract. The purpose of this work is to analyze the effect of structure parameter disturbance on the dynamic characteristics of a hydropower station powerhouse. A vibration model with a head-cover system is established, and then the general disturbance problem analysis methods are discussed. Two new formulae based on two types of disturbances are developed from existing methods. The correctness and feasibility of these two formulae are validated by analyzing the hydropower station powerhouse vibration model. The appropriate calculation method for disturbance of the hydropower station powerhouse vibration dynamic characteristics is derived.

Keywords: hydropower station; transfer path; stochastic perturbation method; Hadamard product; linear stochastic structure; time-invariable parameters; dual disturbances; dynamic characteristics

1. Introduction

Field and model tests reveal three main paths of vibration transfer from the water turbine to the powerhouse (Ma and Dong 2003): (1) runner→shafting→bearing→fixed components (machine frame, head cover) →powerhouse; (2) flow pressure→spiral case→powerhouse; and (3) runner→runner negative pressure region→head cover→powerhouse.Ma conducted vertical vibration analyses of the low machine pier, the high machine pier, and the mutual coupling between the powerhouse and the units of the circle beam column hydropower station from the perspective of the dynamic balance equation. The scope of these studies included the effect of path (1), whereas paths (2) and (3) were ignored. However, the continually increasing scale and capacity of hydropower stations corresponds to an increase in the flow passage area, which comprises the head-cover system and the bearing control components. This process, in turn, leads to an increase in head-cover vibration. Therefore, the effect of the head-cover system has become more important in the process of hydraulic vibration transfer, such that ignoring path (3) will result in significant error. Here, a head-cover system is introduced to augment Ma's research (Ma and

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Dong 2003). With this amendment, the vibration source is the same as that in path (1), the elastic foundation restraint is then selected, and the new vibration model is established. Next, our study introduces the treatment of additive disturbance, which is provided by Zhang, and the treatment of multiplicative disturbance, which is probed by Gao. Combining the mathematical traits of these two disturbance types, the above two treatments are extended in this work. Two new solution formulae are proposed to represent dual disturbances using the variation algebra synthesis method (Gao *et al*. 2009), Kronecker algebra (Vetter 1973) and Hadamard product (Benjamin 1979). Finally, the study employs computational research and model experiments to verify the two new treatments and attempts to explore and analyze the hydropower station stochastic parameter structure.

2. Literature review

In analyses of engineering structures, structural parameters are disturbed due to materials, manufacturing and other reasons. When these types of disturbances occur within structural calculations, it is termed a multiplicative disturbance. In addition, some parameters will change due to various noises caused by measurement. This type of disturbance is an additive disturbance. Prior research has indicated that structural parameter disturbance may cause large changes in the structural dynamic characteristics and dynamic response. This lends itself to mechanical parameters becoming a dominant factor under certain conditions. Li (1993) suggested that the C.V. (coefficient of variation) of the structure maximum response is 2-4 times that of the C.V. of structure parameters. This ratio has been reported to be as high as 7 times in some publications. Zhang (2001) demonstrated that the contribution of the structure parameters’ randomness is more than the contribution of the external excitation with regard to the dynamic response. Therefore, it is necessary to clearly and accurately elucidate the contribution of each vibration path to the structure vibration when considering the parameters’ randomness. This is not a simple problem.

The analysis of the vibration transfer characteristics with disturbance should be assigned to stochastic structure system fields. At present, the main methods used to quantify additive disturbances include the Monte-Carlo simulation method (MCSM) (Singh *et al*. 2001, Popescu 2011) and the perturbation method (Kaplunov *et al*. 2005, Kamiński 2011, Madani *et al*. 2011). MCSM is limited to small structure analysis and verification analysis; consequently, the perturbation method is widely used. Colliins and Thompson (1969) initially explored using the perturbation method to analyze the stochastic dynamic systematic characteristic values, and then subsequent studies mainly focused on how to employ the perturbation method in static (Hisada and Nakagiri 1982) and dynamic analyses (Liu *et al*. 1988). Vetter (1973) introduced Kronecker algebra to expand the perturbation method. Perturbation theory was widely used and experienced multiple developments over the next several decades. Major developments include the L-P method (Poincare 1960), the multiple scale method (Friemann 1963), the average method (Krylov and Bogoliubov 1947), the KBM (Krylov-Bogoliubov Mitropolsky) method (Krylov and Bogoliubov 1947, Bogoliubov and Mitropolsky 1961) and the singular perturbation method (Kaplun 1967, Tsien 1956). The homotopy perturbation method (Abbasbandy 2006) is a recent development. Zhang built a theoretical model describing the vibration transfer path by employing the perturbation method with Kronecker algebra into the static (Zhang *et al*. 1996), dynamic (Zhang 2007) and reliability analyses (Zhang *et al*. 2003). This method is used for mechanical component analysis and design. The results are used for isolation vibration analysis. In the multiplicative

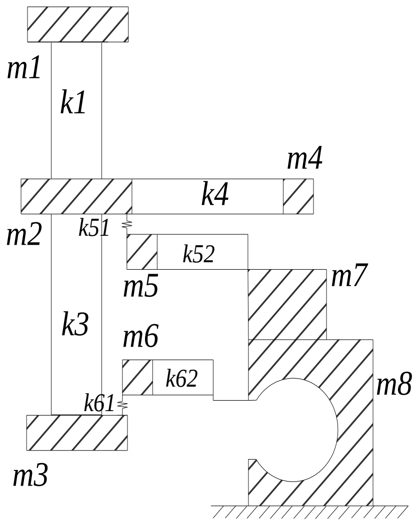

Fig. 1 Simplified model of the coupling system between umbrella unit and powerhouse

disturbance analysis area, Gao analyzed dynamic characteristics of truss structures using the interval factor method (Gao 2006a), the random factor method (Gao 2006b) and the non-stationary random excitation method (Gao *et al*. 2004). Ma J. performed a dynamic characteristics analysis on linear (Ma *et al*. 2006a) and non-linear (Ma *et al*. 2006b) truss structures using fuzzy variables. In 2010, the two-factor method was proposed considering two multiplicative disturbances by Ma *et al*. (2010).

The above analyses mainly focus on singular disturbances, which illustrate the shortage of research on stochastic structure that accounts for two types of disturbances. It is therefore imperative to analyze this type of contribution (3).

2. Analysis model with introduction of the head-cover system

For both suspension and umbrella hydroelectric generating units, the weight of the rotating parts is transferred into the reinforced concrete machine foundation successively through thrust bearing, the frame (suspension units contain the stator frame), and the sole screw (Ueda 1981). The head-cover system is always fixed on the base ring strengthening plate, and it becomes one entity with the strengthening plate. For example, the model for the umbrella vertical vibration characteristics’ contains the shafting system, thrust bearing and lower bracket (see Fig. 1). The heave shaft can be simplified as a massless elastic continuous beam, and its mass can be regarded as an attached mass added to three nodes: $m1$, $m2$ and $m3$. $m1$ can be defined as the mass of the exciter rotor zone axis, with half of the shafting mass as measured from the heavy shaft top to the rotor frame and the other half of the mass added on top of the heavy shaft; $m2$ can be defined as the mass of the central body of the rotor frame, with half the mass of the whole gate arm and half the mass of the whole shaft; and $m3$ can be defined as the mass of the water turbine runner, the additional water mass, and the half shafting mass as measured from the rotor frame to the hydraulic turbine. The rotor gate arm can be simplified as a massless elastic continuous rod, with its mass assigned to the runner margin and the central body of the rotor frame. $k4$ can be defined as the sum of the vertical stiffness of the whole gate arm; $m4$ can be defined as the lumped mass of the runner margin. The outer end of the lower bracket is fixed on the concrete foundation. Then, the lower bracket gate arm can be simplified as a gravity-free beam, ignoring the coupling effect of the foundation. $k52$ can be defined as the vertical stiffness of the lower bracket gate arm. $m5$ can be defined as the lumped mass of one end of the lower bracket that is close to the heavy shaft and half the mass of the gate arm; meanwhile, $m5$ can be connected with $m2$ by the thrust bearing, which is simplified as an equivalent stiffness, $k51$.

Introducing the head-cover system establishes the vibration transfer path as follows: the hydraulic vertical vibration is transferred to the head cover by the heavy shaft seal and the guide bearing in the water turbine runner chamber; next, the vibration is transferred to the spiral case base ring strengthening plate, which is connected to the outer end of the head cover by the head cover; finally, the vibration is transferred to the machine foundation by the wrapped concrete outside the spiral case. By temporarily ignoring the coupling effect on the foundation and assuming the foundation is a rigid body, the control parts and other additional parts on the head cover can be regarded as the attached mass of the head cover system. Consequently, $m6$, as the lumped mass close to the heavy shaft, can be defined as the mass of the central body and half the mass of the whole head-cover system. The head cover system can be simplified as a gravity free beam, so $k62$ can be defined as the vertical stiffness. $m6$ can be connected with $m3$ via the sealing spring, which lies between the head-cover structure and the water turbine runner. The vertical stiffness of the connection can be simplified as an equivalent stiffness $k61$. The head-cover is simplified as a singular free degree node.

$k51$ and $k61$ are the parallel stiffnesses of the elastic oil tank and oil film. However, the elastic oil tank stiffness is unstable because the unit's axial water thrust varies with the unit's conditions; thus, $k51$ and $k61$ are simplified as one stochastic variable in this study.

Assuming the system is linear, the vibration differential equation can be defined as

$$M\ddot{u} + Ku = 0 \tag{1}$$

Next, transforming the frequency domain yields

$$(-\omega^2 M + K)U = 0 \tag{2}$$

By merging the stiffness matrix of the shafting, rotor, lower bracket, head-cover system and machine foundation pier, an equation with 7 degrees of freedom and 21 parameters is obtained such that the total stiffness matrix can be expressed as

$$K = \begin{pmatrix} k_1 & -k_1 & 0 & 0 & 0 & 0 & 0 \\ -k_1 & k_1 + k_3 + k_4 + k_{51} & -k_3 & -k_4 & -k_{51} & 0 & 0 \\ 0 & -k_3 & k_3 + k_{61} & 0 & 0 & -k_{61} & 0 \\ 0 & -k_4 & 0 & k_4 & 0 & 0 & 0 \\ 0 & -k_{51} & 0 & 0 & k_{51} + k_{52} & 0 & -k_{52} \\ 0 & 0 & -k_{61} & 0 & 0 & k_{61} + k_{62} & -k_{62} \\ 0 & 0 & 0 & 0 & -k_{52} & -k_{62} & k_{52} + k_{62} + k_7 \end{pmatrix} \tag{3}$$

The dampness matrix is similar to the stiffness matrix. The total mass matrix is obtained via the lumped mass

$$M = diag\{m_1, m_2, m_3, m_4, m_5, m_6, m_7\} \tag{4}$$

The structure dynamic characteristics can thus be determined from Eqs. (2)-(4).

4. The treatments of disturbances

4.1 Additive random factor

Zhang proposed applying the stochastic perturbation theory (Zhang *et al.*1996) to decompose the stochastic variables containing additive disturbance into deterministic and perturbation portions, that is

$$x = x^d + \varepsilon x^p; \quad \omega_i\left(x\right) = \omega_i^{\ d}(x) + \varepsilon\omega_i^{\ p}(x) \tag{5}$$

In Eq. (5), assume that x is the stochastic parameter vector $\mathbf{x}=(x_1,x_2,...,x_m)^T$ and follows the normal distribution; $\omega_i(x)$ represents the natural frequency obtained by using general structural dynamic characteristic analysis. According to the central limit theorem, $\omega_i(a)$ can also be assumed to fit the normal distribution. ε is a small parameter; superscript d denotes its deterministic portion and is defined as the mean value after multi-sampling in practical applications; superscript p denotes its stochastic portion, which has the zero mean characteristics. The mathematical expectation in terms of Eq. (5) can then be expressed as

$$\begin{cases} E_{\mathbf{x}} = E\left[\mathbf{x}^d + \varepsilon\mathbf{x}^p\right] = E(\mathbf{x}^d) + \varepsilon E(\mathbf{x}^p) = \mathbf{x}^d \\ E_{\omega_i} = E\left[\omega_i^{\ d}(x) + \varepsilon\omega_i^{\ p}(x)\right] = E\left[\omega_i^{\ d}(x)\right] + \varepsilon E\left[\omega_i^{\ p}(x)\right] = \omega_i^{\ d}(x) \end{cases} \tag{6}$$

According to the Taylor expansion, ω_i is expanded in the first-order on ω_i^d, yielding

$$\omega_i = \omega_i^{\ d} + \frac{\partial\omega_i^{\ d}}{\partial x}(x - x^d) + O(x - x^d) = \omega_i^{\ d} + \frac{\partial\omega_i^{\ d}}{\partial x}\varepsilon x^p + O(\varepsilon x^p) \tag{7}$$

$\dfrac{\partial\omega_i^{\ d}}{\partial x}$ is the derivative of the deterministic term ω_i over the vector x. Ignoring the higher order terms, the stochastic components of ω_i can be expressed as

$$\omega_i^p = \sum_{k=1}^{m}\frac{\partial\omega_i^{\ d}}{\partial x_k}\varepsilon x_k^p \tag{8}$$

According to the Kronecker algebra method and the stochastic perturbation theorem (Tsien 1956), the variance of the stochastic vector and natural frequency can be expressed as

$$\begin{cases} Var\left(x\right) = \sigma_x^2 = E\left\{\left[x - E_x\right]^{[2]}\right\} = E\left[\varepsilon\left(x^p\right)^{[2]}\right] = \varepsilon^2\left(x^p\right)^{[2]} \\ Var\left(\omega_i\right) = \sigma_{\omega_i}^2 = E\left\{\left[\omega_i - E_{\omega_i}\right]^2\right\} = E\left[\left(\varepsilon\sum_{j}^{n}\frac{\partial\omega_i^{\ d}}{\partial x_j}x_j^p\right)^2\right] = \left|\frac{\partial\omega_i^{\ d}}{\partial x}\right|^{[2]} Var(x^T) \end{cases} \tag{9}$$

Superscript [2] denotes the Krnoecker power, that is, $x^{[2]}=x\otimes x$. Herein, x contains m stochastic variables, that is: x is of the $m\times$order; $x^{[2]}$ is of the $m^2\times 1$order; $Var(x)$ is of the $m^2\times 1$ order; $Var(x)$ contains the variance and covariance of each vector of x; and $Var(\omega_i)$ is the overall variance including the partial derivative of each vector and the covariance among vectors, which is of the 1×1 order.

Zhang conducted these transfer path analysis and reliability computations via the above-described method in 1996 (Zhang *et al.*1996) and 2003 (Zhang *et al.* 2003). The results indicate that the system function disturbance could have been calculated based on calculations of the first order partial derivative.

4.2 Multiplicative random factor

Gao (2006b) introduced the random factor method for multiplicative disturbance. By using the finite element method, the element stiffness is defined as

$$\left[K_e\right]=K_e^p\left[K_e^d\right] \tag{10}$$

In Eq. (10), $[K_e]$ is the stiffness matrix of element e; K_e^p is the perturbation factor of the stiffness matrix for element e; K_e^d is the deterministic portion of the stiffness matrix for element e. Assuming there are n elements in the structure, the overall stiffness matrix $[K]$ can be expressed as

$$\left[K\right]=\sum_{e=1}^{n}\left\{\left[G_e\right]^T\left[K_e\right]\left[G_e\right]\right\}=\sum_{e=1}^{n}\left\{K_e^p\left[G_e\right]^T\left[K_e^d\right]\left[G_e\right]\right\}=\sum_{e=1}^{n}\left\{K_e^p\left[K_e^{\#}\right]\right\} \tag{11}$$

In Eq. (11), $[G_e]$ is the coordinate transformation matrix for element e, and $[K_e^{\#}]$ is the deterministic portion of the overall stiffness matrix for element e.

Similarly, the overall mass matrix of the structure of the lumped mass matrix can be represented as

$$\left[M\right]=\sum_{e=1}^{n}\left\{M_e^p\left[M_e^{\#}\right]\right\} \tag{12}$$

In Eq. (12), M_e^p is the mass matrix disturbance factor for element e; $[M_e^{\#}]$ is the deterministic portion of the overall mass matrix for element e. Analysis of the structural dynamic characteristics primarily focuses on each order natural frequencies and on the structural vibration mode. Due to the mutual correspondence between the natural frequency and the modal vibration shape, that is, once the natural frequency of a given order is identified, the corresponding modal vibration shape can be determined. Therefore, the structural vibration perturbation can be determined based on analyzing the structural natural frequency perturbation. The natural frequency can be computed via the Rayleigh quotient, as follows

$$\omega_i^2=\frac{\{\phi\}_i^T\left[K\right]\{\phi\}_i}{\{\phi\}_i^T\left[M\right]\{\phi\}_i}=\frac{\{\phi\}_i^T\sum_{e=1}^{n}\left\{K_e^p\left[K_e^{\#}\right]\right\}\{\phi\}_i}{\{\phi\}_i^T\sum_{e=1}^{n}\left\{M_e^p\left[M_e^{\#}\right]\right\}\{\phi\}_i}=\frac{\sum_{e=1}^{n}\left\{K_e^p\{\phi\}_i^T\left[K_e^{\#}\right]\{\phi\}_i\right\}}{\sum_{e=1}^{n}\left\{M_e^p\{\phi\}_i^T\left[M_e^{\#}\right]\{\phi\}_i\right\}} \tag{13}$$

If the stochastic variable probability characteristic is consistent for every element parameter, thus following the hypotheses of Gao (2006a, b), Gao *et al.* (2004), Ma *et al.* (2006a, b, 2010), then

$$\omega_i^2 = \frac{K^p \sum_{e=1}^{n} \left\{ \{\phi\}_i^T \left[K_e^{\#} \right] \{\phi\}_i \right\}}{M^p \sum_{e=1}^{n} \left\{ \{\phi\}_i^T \left[M_e^{\#} \right] \{\phi\}_i \right\}} = \frac{K^p}{M^p} \frac{K_i^d}{M_i^d} = \omega^p \left(\omega_i^d \right)^2 \tag{14}$$

In Eqs. (13) and (14), ω_i is the natural frequency of the structure in the i^{th} order; ω_i^d is the deterministic portion of the natural frequency in the i^{th} order; ω^p is the disturbance portion of $(\omega_i^d)^2$; $\{\phi\}_i$ is the correspondent modal vibration mode; K_i^d is the deterministic portion of the main stiffness matrix of the i^{th} order modal matrix structure; and M_i^d is the deterministic portion of the main mass matrix of the i^{th} order modal matrix structure. If the parameters are mutually independent in accordance with Eq. (14), probability characteristics of the natural frequency can be calculated via the stochastic variable algebra synthetic method

$$E(\omega_i) = (1 + v_{M_i}^2)\omega_i^d \tag{15}$$

$$Var(\omega_i) = (v_{K_i}^2 + v_{M_i}^2)\left(\omega_i^d\right)^2 \tag{16}$$

V is the coefficient of variation; C_{KiMi} is the covariance function for K_i and M_i, $\omega_i^d = \dfrac{E(K_i)}{E(M_i)}$.

To treat the multiplicative noise, Gao (2006a) used the internal factor method, and Ma *et al.* (2006a, b) used the fuzzy factor method. Their research results illustrate that the natural frequency disturbance may be calculated out in the case of parameter consistency.

5. The dynamic characteristic analysis of the hydropower station regarding the effect of dual disturbances

In the two, above-mentioned classical models for treatment disturbances with time-invariable parameters, the methods are completely independent, which requires different methods for different disturbance analysis. In fact, these two disturbances always co-exist, yet no methods exist for conjoint analysis or study. This research seeks to deduce the dynamic characteristic formula via expansion of the above two treatments in view of dual disturbances.

5.1 Dual disturbances form

The time-invariable parameters, including multiplicative and additive disturbances, can be expressed as below

$$x = x^1 x^d + x^2 \tag{17}$$

In Eq. (17), x is obtained by direct measurement and represents basic parameters such as materials, loads, and geometric dimensions. The time-invariable x^d is the mean value of x. x incorporates two types of disturbances vectors: x^1 is the multiplicative disturbance factor. Direct measurement of x^1 is difficult due to limitations in techniques and practical conditions that would significantly influence the structural analysis. x^2 is the additive disturbance factor and mainly represents phenomena such as environmental noise, etc. When the SNR (Signal Noise Ratio) is small, the noise outweighs the effective signals. The additive noise can be obtained by measurement.

The normal distribution is the most common stochastic variable distribution. Assuming the disturbance is stochastic then, the multiplicative factor x^1 and the additive factor x^2 represent the stochastic variables, which respectively obey $N(1,\sigma_1^2)$ and $N(0,\sigma_2^2)$ normal distributions. The result of adding or subtracting two normal distribution stochastic variables also obeys the normal distribution. Therefore, x would also obey the normal distribution. Thus, x is represented by $N(x^d,\sigma_1^2,\sigma_2^2)$.

5.2 Perturbation method

The treatment for dual disturbance is represented below as an expansion of Zhang's method.

The numerical characteristics of Eq. (17) are

$$E(x) = E(x^1x^d + x^2) = E(x^1x^d) + E(x^2) = x^d \tag{18}$$

$E(x)$ is the mathematical expectation for x. If the multiplicative factor and the additive factor are independent and irrelevant, then according to the Krnoecker algebra, Hadamard product and perturbation theory

$$\begin{aligned} Var\left(x\right) &= E\left\{\left[x - E_x\right]^{[2]}\right\} = E\left\{\left[(x^1-1)x^d + x^2\right]^{[2]}\right\} \\ &= Var\left(x^1\right)\circ\left(x^d\right)^{[2]} + Var\left(x^2\right) + 2\times\left[\sigma_{x^1}\circ\left(x^d\right)\right]\otimes\sigma_{x^2} \end{aligned} \tag{19}$$

According to Eq. (9), the natural frequency variance would be

$$\begin{aligned} Var\left(\omega\right) &= E\left\{\left[\omega - E_\omega\right]^2\right\} = E\left[\left\{\sum_j^n \frac{\partial \omega^d}{\partial x_j}\left[(x_k^1-1)x_k^d + x_k^2\right]\right\}^2\right] \\ &= \left|\frac{\partial \omega^d}{\partial x}\right|^{[2]} \left\{Var\left(x^1\right)\circ\left(x^d\right)^{[2]} + Var\left(x^2\right) + 2\times\left[\sigma_{x^1}\circ\left(x^d\right)\right]\otimes\sigma_{x^2}\right\} \end{aligned} \tag{20}$$

The symbol $\circ$ denotes the Hadamard product (Benjamin 1979). According to Eqs. (18) and (20), the dynamic analysis of dual disturbances may be performed based on the perturbation theory. Comparing Eq. (9) and Eq. (20), the nature of the algorithm is shown not to change after introducing the dual disturbances. Consequently, this treatment obtains the probabilistic characteristic of natural frequency via the first order sensitivity of random parameters and the probabilistic characteristic of random variables. This simplifies the calculations because the sample itself does not participate in the calculations; this treatment is also suitable for other function forms (e.g. transmissibility solution).

5.3 Random factor method

The analyses of Gao and Ma have illustrated that the random factor method requires unification between parameter disturbances: that is, the $k1$, $k2$..., $k62$'s disturbances are consistent when

$$K = K^1 K^d + K^2; M = M^1 M^d + M^2 \tag{21}$$

Substituting Eq. (10) yields

$$K^p = K^1 + \frac{K^2}{K^d}; M^p = M^1 + \frac{M^2}{M^d} \tag{22}$$

Substituting Eq. (14) yields

$$\omega_i^2 = \frac{K^p}{M^p}\frac{K_i^d}{M_i^d} = \frac{\left(K^1 + \alpha K^2\right)}{\left(M^1 + \beta M^2\right)}\left(\omega_i^d\right)^2 \tag{23}$$

In Eq. (23), $\alpha=1/K^d$, $\beta=1/M^d$. In Eq. (22), the disturbance includes item K^d. Only when the stiffness of each element is consistent can the total stiffness matrix disturbance be represented by a singular value, and then Eq. (23) will be tenable. When the stiffness is independent of the mass, the probabilistic characteristic of the natural frequency can be obtained by the C.V. algebra comprehensive method

$$E(\omega_i) = \left[1 + \frac{\sigma_{M^1}^2 + \beta^2 \sigma_{M^2}^2}{\left(\mu_{M^1} + \beta \mu_{M^2}\right)^2}\right] \frac{\mu_{K^1} + \alpha \mu_{K^2}}{\mu_{M^1} + \beta \mu_{M^2}} \omega_i^d \tag{24}$$

$$Var(\omega_i) = \left[\frac{\sigma_{K^1}^2 + \alpha^2 \sigma_{K^2}^2}{\left(\mu_{K^1} + \alpha \mu_{K^2}\right)^2} + \frac{\sigma_{M^1}^2 + \beta^2 \sigma_{M^2}^2}{\left(\mu_{M^1} + \beta \mu_{M^2}\right)^2}\right] \left(\frac{\mu_{K^1} + \alpha \mu_{K^2}}{\mu_{M^1} + \beta \mu_{M^2}}\right)^2 \left(\omega_i^d\right)^2 \tag{25}$$

In the above two Eqs., μ denotes the mathematical expectation, and σ denotes the standard deviation. The structure dynamic characteristic problem considers dual disturbances in accordance with Eqs. (24) and (25) and can be solved by the random factor method. It should be noted that the random factor method, fuzzy factor method and interval factor method all require that the parameter disturbances be consistent. Furthermore, in view of dual disturbances, the requirement may be improved by insistence that the mean value of each element is consistent. Thus, objects for this analysis are limited. The method is only used for regular structures, and it is not suitable for a complicated hydropower station powerhouse structure. However, it should not be overlooked that the random factor method avoids solving the first order partial derivative. This superior method should not be neglected for a structure with plenty of elements when compared with other methods.

6. Numerical simulation

Due to the complexity of the hydropower station structure, the multiplicative treatment could not satisfy the usage condition of the stochastic method. This stochastic method requires that the parameters are consistent and that the parameters' mean values are consistent when considering dual disturbances. Therefore, only the feasible analysis of the hydropower station structure was conducted by the dual perturbation method in this algorithm. Fig. 1 shows the structure simplification schematic diagram for this instance of modeling the coupling vertical vibration

Table 1 Frequency orders and their probability characteristic values in umbrella units, simplified by the model structure

Theory(rad/s)		w_1	w_2	w_3	w_4	w_5	w_6	w_7
		41.31	134.90	264.30	394.94	501.93	936.39	4531.03
Exp.	Direct	40.87	133.85	263.15	387.54	498.16	924.45	4419.69
	Sole	41.31	134.90	264.30	394.51	501.93	936.38	4515.90
	Dual	41.31	134.90	264.30	394.51	501.93	936.38	4515.90
S.D.	Direct	6.02	16.85	24.69	76.10	61.69	149.01	998.26
	Sole	0.066	0.015	0.064	18.42	0.065	0.015	369.89
	Dual	0.063	0.011	0.025	17.89	0.13	1.79	364.47
C.V.	Direct	0.15	0.13	0.094	0.20	0.12	0.16	0.23
	Sole	1.59e-3	1.11e-4	2.42e-4	0.047	1.29e-4	1.59e-5	0.082
	Dual	1.52e-3	8.36e-5	9.61e-5	0.045	2.64e-4	1.91e-3	0.081

Exp.: Expectation
S.D.: Standard deviation
C.V.: Coefficient of variation
Direct: values are calculated according to the statistical characteristics of every disturbance.
Sole: to solve based on the singular form, see Chapter 4.1.
Dual: to solve based on the dual form, see Chapter 5.1.

between the axis system and powerhouse in the background of one giant hydropower station umbrella units' hydraulic vibration source. Ignoring the impact from the spiral case and its inferior structure, the mean value for the stochastic parameters are as follows: $m1=8.28\times10^4$, $m2=1.042\times10^6$, $m3=3.29\times10^5$, $m4=9\times10^5$, $m5=1.2\times10^5$, $m6=1.15\times10^5$, and $m7=4.79\times10^6$, where the unit of mass m is kg, and $k1=7.26\times10^{10}$, $k3=5.72\times10^{10}$, $k4=2.32\times10^{10}$, $k51=2.20\times10^{12}$, $k52=9.41\times10^9$, $k61=1.73\times10^8$, and $k62=1.73\times10^{10}$, where the unit of stiffness k is N/m. Stochastic parameters obey the normal distribution. It is difficult to obtain the stiffness for the oil film and water seal. In addition, because the stiffness changes with time, the error will increase. Therefore, the multiplicative stochastic quantities' variances coefficients are all set to 0.15 about the vertical stiffness $k51$, which includes the thrust bearing and the seal equivalent vertical stiffness $k61$ between the head cover and the runner. The control parts and other additional parts, which are carried on the head cover, are too heavy. This trend leads to stiffness and dampness, with more uncertainty. The multiplicative stochastic quantity's variance coefficient is set to 0.1 about the equivalent bending stiffness $k62$. For the other parameters, the multiplicative stochastic quantity's variances coefficients are set to 0.05. Additive stochastic quantities include environmental noise and measurement noise. Additionally, the range of the stochastic quantity amplitude is relevant to the measurement size. Thus, according to each parameter's mean value, the additive stochastic variables variances of mass and stiffness are separately set to 10^4 and 10^8, respectively.

Table 1 displays the theoretical solutions. The expectation (Exp.), standard deviation (S.D.) and coefficient of variation (C.V.) are listed for each characteristic frequency as obtained by the Direct, Sole and Dual methods, respectively. Theoretical solutions are calculated using the general method, by which the structure dynamics are used to compute the characteristics under the assumption of a disturbance-free system. With the often-employed Direct method, the characteristics are computed using sample values. The characteristic frequency vector is represented by $w_{1,}$, with the ($1\times n$) order, where n is the number of sampling points; this vector is the result of statistical analysis when neglecting the covariance between parameters. The Sole method calculates characteristics according to Eq. (9), treating the two noise types as additive noises and not distinguishing between them. The Dual method calculates characteristics according to Eq. (20), strictly distinguishing these two disturbance types based on the disturbance-parameter relationship.

Table 1 indicates that (1) the mathematical expectations calculated via the Sole and Dual methods are very close to theoretical solutions. This difference is because in the theoretical solution process, the natural frequency is a directly extracted root, whereas in the process of the comprehensive algebra method, the resulting value can be adjusted in accordance with the stochastic variables' probabilistic characteristics. The difference between the Sole method and the Dual methods is small because there is no coefficient correlation between these two disturbance types in the process of calculating the disturbance variance. (2) The statistical values of Exp, S.D. and C.V. show that the results obtained via the Direct method are inferior to the other methods. (3) Comparing the results of the Sole and Dual methods, the Dual method is superior; the difference between the two is not evident due to the lack of coefficient correlation.

The vertical vibration model for coupling between hydropower station units and powerhouses verifies that using a dual method to address the dual disturbances problem in a linear stochastic structure is feasible and that this method is accurate.

7. Conclusions

In practice, the disturbance factor is complicated, and the disturbance source is almost always uncertain. Analyses that only consider the sole disturbance are therefore limited. The dual disturbance treatment provided in this study completely describes the practical parameter disturbance.

• This study builds on the research of Zhang and Gao; by considering two types of disturbances, it elucidates two new formulae for the stochastic structure dynamic characteristics. Moreover, the work includes computations of multiple noises and a sensitivity analysis of Eqs. (20) and (25) for each disturbance. This inclusion widens the potential applications of this approach.

• The stochastic factor method was used for the simple stochastic structure problem with dual disturbances; however, the complicated structure does not satisfy the computation conditions. The perturbation method can be used for calculating the hydropower station structure dynamic characteristics, and it ensures the computations' accuracy when the first order sensitivity is considered. The method does not strictly limit the structure and function forms of Eq. (20), which suggests it has wide applicability.

• As shown in Table 1, some individual items did not perform better than the results of sole disturbance forms. The computation of the dynamic characteristic disturbance is merely one step in this treatment. Thus, future efforts should focus on improving this approach.

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