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Structural damage and force identification under moving load

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Abstract. Structural damage and moving load identification are the two aspects of structural system identification. However, they universally coexist in the damaged structures subject to unknown moving load. This paper proposed a dynamic response sensitivity-based model updating method to simultaneously identify the structural damage and moving force. The moving force which is equivalent as the nodal force of the structure can be expressed as a series of orthogonal polynomial. Based on the system Markov parameters by the state space method, the dynamic response and the dynamic response derivatives with respect to the force parameters and elemental variations are analytically derived. Afterwards, the damage and force parameters are obtained by minimizing the difference between measured and analytical response in the sensitivity-based updating procedure. A numerical example for a simply supported beam under the moving load is employed to verify the accuracy of the proposed method.

Keywords: structural damage identification; response sensitivity; system Markov parameters; moving load

1. Introduction

Structural damage identification under the moving load has been widely used in structural health monitoring and damage assessment of the bridge structures, such as the damage identification for the bridge structure under the vehicle load. Many researchers have studied the approaches of structural damage identification under the known moving load based on the structural dynamic responses.

Gonzalez and Hester (2013) used the acceleration response of a beam-type structure which is composed of static, damage and dynamic responses to identify the location and severity of the damage when the structure is traversed by a moving load. This method first established the nature of the damage singularity in an acceleration response and discussed how the singularity changes with damage location and severity and with the properties of the beam and moving force. Zhan *et*

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al. (2011) proposed a damage identification approach using train-induced responses and sensitivity analysis for the nondestructive evaluation of railway bridges. In this method, the forward problem for train-induced bridge response is calculated considering the interaction between the train subsystem and bridge subsystem. Afterwards, the damage is located and quantified with the finite element model updating technique based on the response sensitivity analysis. Lu and Liu (2011) also used a dynamic response sensitivity-based finite element model updating approach to identify the damages in the bridge. Furthermore, the vehicular parameters are identified from the structural dynamic responses simultaneously. Sieniawska *et al*. (2009) proposed a structural identification approach under a moving load to transform the dynamical problem into a static one by integrating the input and output signals. Furthermore, the stochastic disturbances following the movement of vehicles through the pavement roughness are taken into account in this method. Hester and Gonzalez (2012) proposed a novel wavelet-based approach using wavelet coefficient versus scale plots at different points in time to identify the damage from the bridge acceleration signal. Khorram *et al*. (2013) combined Continuous Wavelet Transform (CWT) and factorial design method to detect the multiple cracks in a simply supported beam subjected to a moving load. All the above approaches are based on the assumption that the information of the moving loads or input signals is known.

However, the moving load applied on the structure is usually unknown or difficult to be precisely measured in practice. Therefore, structural damage and moving load identification universally coexist in the damaged structures subject to unknown moving load.

In the field of the moving load identification, Yu and Chan (2007) comprehensively introduced four methods (interpretive method I, interpretive method II, time domain method, and frequency-time domain method) in determining the dynamic axle loads from bridge responses. The moving force identification is an inverse problem, which is based on simulating the structural response caused by a set of time-varying forces running across a bridge. Law *et al*. (2007) studied the problems of moving loads identification with a three-dimensional bridge deck. In this method, the equation of motion is formulated in state space and the resulting damped least-square identification problem is solved using the dynamic programming method with regularization on the solution. Zhu and Law (2006) presented a time domain method based on regularization technique and modal superposition to identify moving loads more accurately. The relationship between the response and the moving loads on an Euler-Bernoulli beam is formulated in this method. Jiang *et al* (2004) proposed the parameter identification of vehicles moving on the multi-span continuous bridges based on the genetic algorithms, taking into account the random road surface roughness. The above moving loads identification approaches assume that the structure is known initially.

As far as the combination of the structural damage and moving force identification, some researchers further developed the structural damage identification methods under moving loads without knowledge of the time-histories of the moving forces. Zhu and Law (2007) proposed a time-domain method based on the measured displacement response to identify the moving load and damage in the bridge without prior knowledge of the loads. The unknown moving loads and local damages are indentified by using two-step approach that separately adjusts the loads and the damage factors in each iteration of the optimization procedure, when the number of measurements is equal to the number of beam elements minus one. Li *et al*. (2013) presented a dynamic response sensitivity-based identification procedure with the unknown moving loads based on the dynamic response reconstruction technique in wavelet domain. In this damage identification process, the sensitivity matrix is obtained using numerical finite difference method which may be computational time-consuming. Zhang *et al.* (2010) proposed a technique for simultaneous identification of moving masses and damages based on virtual distortion method, which treats the masses and damage extents as the optimization variables. The numerical cost in this study is significantly reduced by introducing the moving dynamic influence matrix and the smaller number of optimization variables. However, the location of the damage has to be known a priori. Zhu *et al.* (2013) proposed a substructural method in state space domain to indentify the local damages of the large systems. In this method, the external moving forces combining with the interface forces of adjacent substructures are considered as the unknown parameters to be identified.

In this paper, a structural damage identification method with unknown moving force is proposed to simultaneously identify the structural damage and moving force. The moving force is represented as a sum of a series of orthogonal polynomial, and the dynamic response derivatives with respect to the force orthogonal parameters and elemental variations are derived based on the system Markov parameters in state space domain. Different from the intelligent optimization algorithms to tackle the damage identification problems (Tang *et al*. 2013, Hakim and Razak 2013), the damage and moving force identification is performed using sensitivity-based model updating method, by minimizing the difference between the measured and calculated dynamic response. Dynamic response sensitivities are calculated to indicate a search direction in the optimization process of model updating. Comparing the conventional method which calculates the sensitivity matrix by the numerical finite difference method, the proposed method derives sensitivity matrix of the dynamic response explicitly in state space domain, which accelerates the process of the damage identification. A numerical example for a simply supported beam under the moving load is employed to verify the accuracy of the proposed method.

2. Motion equations under moving load in state space domain

2.1 State space method for Markov parameters H_k

The motion equation of a linear structure subjected to a moving force at the speed of v can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}\boldsymbol{\delta}(l - vt) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ represent the mass, damping and stiffness matrices, respectively. The structure is assumed to exhibit Rayleigh damping as $\mathbf{C}=a\mathbf{M}+b\mathbf{K}$, where a and b are the Rayleigh damping coefficients. $\ddot{\mathbf{x}}$**,** $\dot{\mathbf{x}}$ and $\mathbf{x}$ are respectively the dynamic acceleration, velocity and displacement of the structure, and $\mathbf{F}\boldsymbol{\delta}(l-vt)$ is the external moving force. In Eq. (1), $\mathbf{F}$ is the time-varying force at a constant speed of v, l is the location point of the structure, vt donates the location of moving force at time t, and $\boldsymbol{\delta}(\cdot)$ is Dirac delta function.

The moving force can be decomposed by Chebyshev polynomial (Rivlin 1990) as follows

$$\mathbf{F}\boldsymbol{\delta}(l - vt) = \sum_{k=1}^{M} c_k \mathbf{T}_k \boldsymbol{\delta}(l - vt) \tag{2}$$

In Eq. (2), c and $\mathbf{T}$ are respectively the orthogonal factors and orthogonal items of the moving force. The subscript k indicates the number of orthogonal items, and the Chebyshev orthogonal items can be expressed as

$$\mathbf{T}_1=\frac{1}{\sqrt{\pi}}\,,\qquad \mathbf{T}_2=\sqrt{\frac{2}{\pi}}\left(\frac{2t}{D_t}-1\right),\qquad \mathbf{T}_{k+1}=2\left(\frac{2t}{D_t}-1\right)\mathbf{T}_k-\mathbf{T}_{k-1} \tag{3}$$

where D_t is the duration time of the moving force. c_k is considered as the unknown parameter to be identified together with the structural damage in the identification process in the following sections.

Combining Eqs. (1) and (2), the motion equation of the structure can be rewritten by using state space formulation as following

$$\dot{\mathbf{X}}=\mathbf{K}^*\mathbf{X}+\mathbf{B}^*\sum_{k=1}^{M}c_k\mathbf{T}_k\boldsymbol{\delta}(l-vt) \tag{4}$$

$$\mathbf{Y}=\mathbf{R}\mathbf{X}+\mathbf{D}\sum_{k=1}^{M}c_k\mathbf{T}_k\boldsymbol{\delta}(l-vt) \tag{5}$$

where $\mathbf{X}=\begin{bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{bmatrix}$, $\mathbf{K}^*=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}$, $\mathbf{B}^*=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\end{bmatrix}$. $\mathbf{K}^*$ is the system matrix, $\mathbf{B}^*$ is the input matrix, and the column vector $\mathbf{X}$ is the state vector of the system. In Eq. (5), $\mathbf{R}$=[$\mathbf{R_d}$−$\mathbf{R_a}\mathbf{M}^{-1}\mathbf{K}$ $\mathbf{R_v}$−$\mathbf{R_a}\mathbf{M}^{-1}\mathbf{C}$] and $\mathbf{D}$=$\mathbf{R_a}\mathbf{M}^{-1}$. $\mathbf{Y}$ represents the output matrix. $\mathbf{R_a}$, $\mathbf{R_v}$, $\mathbf{R_d}$ are respectively the mapping matrices associated with the measured acceleration, velocity and displacement.

Eqs. (4)-(5) can be discretized as the following equations by using the exponential matrix algorithm

$$\mathbf{X}(j+1)=\mathbf{A}\mathbf{X}(j)+\mathbf{B}\sum_{k=1}^{M}c_k\mathbf{T}_k(j)\boldsymbol{\delta}(l-vt) \tag{6}$$

$$\mathbf{Y}(j)=\mathbf{R}\mathbf{X}(j)+\mathbf{D}\sum_{k=1}^{M}c_k\mathbf{T}_k(j)\boldsymbol{\delta}(l-vt)\qquad (j=1,2,\dots,N) \tag{7}$$

where $\mathbf{A}=\exp(\mathbf{K}^*h)$ and $\mathbf{B}=\mathbf{K}^{*-1}(\mathbf{A}-\mathbf{I})\mathbf{B}^*$. N is total number of the sampling points, and h is the time step between the state vectors $\mathbf{X}(j)$ and $\mathbf{X}(j+1)$.

From Eqs. (6)-(7), the output $\mathbf{Y}(j)$ can be calculated with zero initial conditions, and it can be written as

$$\mathbf{Y}(j)=\sum_{k=1}^{M}\sum_{m=0}^{j}c_k\mathbf{H}_m\mathbf{T}_k(j-m)\boldsymbol{\delta}(l-vt)\qquad (m=0,1,\dots,j) \tag{8}$$

where $\mathbf{H}_0=\mathbf{D}$ and $\mathbf{H}_m=\mathbf{R}\mathbf{A}^{(m-1)}\mathbf{B}$. The matrices $\mathbf{H}_m$ are called the system Markov parameters which represent the response of the discrete system to unit impulses. The Markov parameters represent the inherent system response characteristics and are unique for a given linear system. Eq. (8) can be expressed as the Toeplitz matrix forms

$$\begin{Bmatrix}\mathbf{Y}(0)\\ \mathbf{Y}(1)\\ \vdots\\ \mathbf{Y}(N-1)\end{Bmatrix}=\sum_{k=1}^{M}c_k\begin{bmatrix}\mathbf{H}_0 & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{H}_1 & \mathbf{H}_0 & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{H}_{N-1} & \mathbf{H}_{N-2} & \cdots & \mathbf{H}_0\end{bmatrix}\begin{Bmatrix}\mathbf{T}_k(0)\\ \mathbf{T}_k(1)\\ \vdots\\ \mathbf{T}_k(N-1)\end{Bmatrix}\boldsymbol{\delta}(l-vt) \tag{9}$$

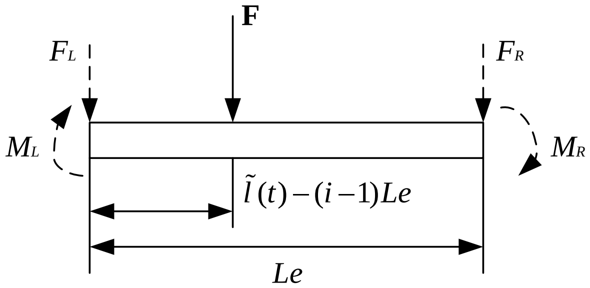


Fig. 1 The equivalent nodal loads of the moving force for the *i*th beam element

In consequence, Eq. (9) can be rewritten as the simple form

$$\mathbf{Y}=\sum_{k=1}^{M} c_k \mathbf{H_L}\mathbf{T}_k \boldsymbol{\delta}(l-vt) \tag{10}$$

2.2 Equivalence of moving load

In this paper, the continuous moving force is equivalent as the nodal force at any time step by the shape function of the element (Zhu and Law 2007). The equivalent nodal loads by a moving force **F** for the *i*th element is shown in Fig. 1. Based on the shape function of the beam element, the equivalent nodal loads (F_L, M_L, F_R and M_R) of the *i*th element can be expressed as

$$\begin{Bmatrix} F_L \\ M_L \\ F_R \\ M_R \end{Bmatrix}=\begin{Bmatrix} 1-3\left(\dfrac{\tilde{l}(t)-(i-1)Le}{Le}\right)^2+2\left(\dfrac{\tilde{l}(t)-(i-1)Le}{Le}\right)^3 \\ (\tilde{l}(t)-(i-1)Le)\left(\dfrac{\tilde{l}(t)-(i-1)Le}{Le}-1\right)^2 \\ 3\left(\dfrac{\tilde{l}(t)-(i-1)Le}{Le}\right)^2-2\left(\dfrac{\tilde{l}(t)-(i-1)Le}{Le}\right)^3 \\ (\tilde{l}(t)-(i-1)Le)\left(\left(\dfrac{\tilde{l}(t)-(i-1)Le}{Le}\right)^2-\left(\dfrac{\tilde{l}(t)-(i-1)Le}{Le}\right)\right) \end{Bmatrix}\mathbf{F}\,,\quad ((i-1)\cdot Le\le \tilde{l}(t)<i\cdot Le) \tag{11}$$

where $\tilde{l}(t)=vt$ is the location of the moving force, and *Le* is the length of the element.

3. Sensitivity analysis for structural damage and force identification

3.1 Dynamic response sensitivity of the structure

The local damage in the structure is assumed as a change in the elemental stiffness factors $\Delta\alpha$, and the perturbation of the structural stiffness matrix is described as $\Delta K = \sum_{i=1}^{Ncal} \Delta\alpha_i k_i^e$ (0<Δα<1). The dynamic response sensitivity of the damaged structure with respect to the structural parameters and the orthogonal factors of the moving force are calculated by differentiating Eq. (10) as

$$\frac{\partial \mathbf{Y}}{\partial \alpha_i} = \frac{\partial(\sum_{k=1}^{M} c_k \mathbf{H_L T}_k \boldsymbol{\delta}(l - vt))}{\partial \alpha_i} = \frac{\partial \mathbf{H_L}}{\partial \alpha_i} \sum_{k=1}^{M} c_k \mathbf{T}_k \boldsymbol{\delta}(l - vt) \tag{12}$$

$$\frac{\partial \mathbf{Y}}{\partial c_k} = \frac{\partial(\sum_{k=1}^{M} c_k \mathbf{H_L T}_k \boldsymbol{\delta}(l - vt))}{\partial c_k} = \mathbf{H_L T}_k \boldsymbol{\delta}(l - vt) \tag{13}$$

In Eq. (12)

$$\frac{\partial \mathbf{H_L}}{\partial \alpha_i} = \begin{bmatrix} \frac{\partial \mathbf{H}_0}{\partial \alpha_i} & \mathbf{0} & \cdots & \mathbf{0} \\ \frac{\partial \mathbf{H}_1}{\partial \alpha_i} & \frac{\partial \mathbf{H}_0}{\partial \alpha_i} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{H}_{N-1}}{\partial \alpha_i} & \frac{\partial \mathbf{H}_{N-2}}{\partial \alpha_i} & \cdots & \frac{\partial \mathbf{H}_0}{\partial \alpha_i} \end{bmatrix} \tag{14}$$

Differentiating $\mathbf{H}_0 = \mathbf{D}$ and $\mathbf{H}_m = \mathbf{RA}^{(m-1)}\mathbf{B}$ with respect to α_i, the derivative matrices $\frac{\partial \mathbf{H}_m}{\partial \alpha_i}$ (k=0,…, N-1) can be written as

$$\begin{cases} \frac{\partial \mathbf{H}_0}{\partial \alpha_i} = 0 \qquad (m = 0) \\ \frac{\partial \mathbf{H}_m}{\partial \alpha_i} = \frac{\partial(\mathbf{RA}^{(m-1)}\mathbf{B})}{\partial \alpha_i} = \frac{\partial \mathbf{R}}{\partial \alpha_i}\mathbf{A}^{(m-1)}\mathbf{B} + \mathbf{R}\frac{\partial(\mathbf{A}^{(m-1)})}{\partial \alpha_i}\mathbf{B} + \mathbf{RA}^{(m-1)}\frac{\partial \mathbf{B}}{\partial \alpha_i} \qquad (m = 1, 2, \cdots, N-1) \end{cases} \tag{15}$$

where

$$\frac{\partial \mathbf{R}}{\partial \alpha_i} = \left[\mathbf{R_a M}^{-1}\frac{\partial \mathbf{K}}{\partial \alpha_i} \quad \mathbf{R_a M}^{-1}\frac{\partial \mathbf{C}}{\partial \alpha_i} \right] = \left[\mathbf{R_a M}^{-1}\frac{\partial \mathbf{K}}{\partial \alpha_i} \quad b\mathbf{R_a M}^{-1}\frac{\partial \mathbf{K}}{\partial \alpha_i} \right] \tag{16}$$

In the second equation of Eq. (15), $\frac{\partial(\mathbf{A}^{m-1})}{\partial \alpha_i}$ is calculated by

$$\frac{\partial(\mathbf{A}^2)}{\partial \alpha_i} = \frac{\partial \mathbf{A}}{\partial \alpha_i}\mathbf{A} + \mathbf{A}\frac{\partial \mathbf{A}}{\partial \alpha_i}$$

$$\frac{\partial(\mathbf{A}^3)}{\partial\alpha_i}=\frac{\partial\mathbf{A}}{\partial\alpha_i}\mathbf{A}^2+\mathbf{A}\frac{\partial\mathbf{A}}{\partial\alpha_i}\mathbf{A}+\mathbf{A}^2\frac{\partial\mathbf{A}}{\partial\alpha_i}=\frac{\partial(\mathbf{A}^2)}{\partial\alpha_i}\mathbf{A}+\mathbf{A}^2\frac{\partial\mathbf{A}}{\partial\alpha_i}$$

$$\frac{\partial(\mathbf{A}^4)}{\partial\alpha_i}=\frac{\partial\mathbf{A}}{\partial\alpha_i}\mathbf{A}^3+\mathbf{A}\frac{\partial\mathbf{A}}{\partial\alpha_i}\mathbf{A}^2+\mathbf{A}^2\frac{\partial\mathbf{A}}{\partial\alpha_i}\mathbf{A}+\mathbf{A}^3\frac{\partial\mathbf{A}}{\partial\alpha_i}=\frac{\partial(\mathbf{A}^3)}{\partial\alpha_i}\mathbf{A}+\mathbf{A}^3\frac{\partial\mathbf{A}}{\partial\alpha_i}$$

$$\cdots\frac{\partial(\mathbf{A}^{m-1})}{\partial\alpha_i}=\frac{\partial(\mathbf{A}^{m-2})}{\partial\alpha_i}\mathbf{A}+\mathbf{A}^{m-2}\frac{\partial\mathbf{A}}{\partial\alpha_i} \tag{17}$$

Differentiating $\mathbf{A}=\exp(\mathbf{K}^*h)$ and $\mathbf{B}=\mathbf{K}^{*-1}(\mathbf{A}-\mathbf{I})\mathbf{B}^*$ with respect to α_i, $\frac{\partial\mathbf{A}}{\partial\alpha_i}$ and $\frac{\partial\mathbf{B}}{\partial\alpha_i}$ are calculated as

$$\frac{\partial\mathbf{A}}{\partial\alpha_i}=\mathbf{A}h\frac{\partial\mathbf{K}^*}{\partial\alpha_i} \tag{18}$$

$$\begin{aligned}\frac{\partial\mathbf{B}}{\partial\alpha_i}&=\frac{\partial}{\partial\alpha_i}[\mathbf{K}^{*-1}(\mathbf{A}-\mathbf{I})\mathbf{B}^*]=\frac{\partial\mathbf{K}^{*-1}}{\partial\alpha_i}(\mathbf{A}-\mathbf{I})\mathbf{B}^*+\mathbf{K}^{*-1}\frac{\partial\mathbf{A}}{\partial\alpha_i}\mathbf{B}^*\\&=(-\mathbf{K}^{*-1}\frac{\partial\mathbf{K}^*}{\partial\alpha_i}\mathbf{K}^{*-1})(\mathbf{A}-\mathbf{I})\mathbf{B}^*+\mathbf{K}^{*-1}\mathbf{A}h\frac{\partial\mathbf{K}^*}{\partial\alpha_i}\mathbf{B}^*\end{aligned} \tag{19}$$

where

$$\frac{\partial\mathbf{K}^*}{\partial\alpha_i}=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ -\mathbf{M}^{-1}\frac{\partial\mathbf{K}}{\partial\alpha_i} & -\mathbf{M}^{-1}\frac{\partial\mathbf{C}}{\partial\alpha_i}\end{bmatrix}=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ -\mathbf{M}^{-1}\frac{\partial\mathbf{K}}{\partial\alpha_i} & -b\mathbf{M}^{-1}\frac{\partial\mathbf{K}}{\partial\alpha_i}\end{bmatrix} \tag{20}$$

Finally, the dynamic response derivatives with respect to the elemental stiffness variation and orthogonal force parameters $\frac{\partial\mathbf{Y}}{\partial\alpha_i}$ and $\frac{\partial\mathbf{Y}}{\partial c_k}$ can be calculated from Eqs. (12)-(13).

3.2 Structural damage and force identification based on sensitivity analysis

The sensitivity-based model updating method (Brownjohn *et al*. 2001) is employed for structural damage and moving force identification. In the sensitivity-based updating procedure, the unknown parameters are repeatedly adjusted to minimize the discrepancy between the analytical responses from FE model and the practical measurement counterparts in an optimal way. The sensitivities of dynamic responses with respect to unknown parameters are used to indicating the search direction of the optimization process. As a result, based on the sensitivities of dynamic response with respect to the stiffness and force parameters, the unknown damage and force are identified using the sensitivity-based updating method. The objective function of the dynamic response is expressed as

$$\mathbf{J}(\beta)=\sum(\mathbf{Y}^A(\beta)-\mathbf{Y}^M)^2 \tag{21}$$

where $\mathbf{Y}^M$ represents the measured response, and $\mathbf{Y}^A$ represents the dynamic response from the analytical FE model which is expressed as the function of the uncertain physical parameters $\{\beta\}$. The objective function is minimized by adjusting continuously the parameters $\{\beta\}$ of the analytical model through a sensitivity-based iterative procedure. The sensitivity matrix of the dynamic response with respect to the parameter β is expressed as

$$\left[\mathbf{S}(\beta)\right] = \frac{\partial \mathbf{Y}}{\partial \beta} \tag{22}$$

Given that $\beta=\{\alpha\ c\}^T$, Eq. (22) can be rewritten as

$$[\mathbf{S}(\beta)] = \left[\mathbf{S_K}(\alpha)\quad \mathbf{S_F}(c)\right] = \begin{bmatrix} \dfrac{\partial \mathbf{Y}}{\partial \alpha} & \dfrac{\partial \mathbf{Y}}{\partial c} \end{bmatrix} \tag{23}$$

where $\mathbf{S_K}(\alpha)$ and $\mathbf{S_F}(c)$ are the sensitivity matrices of the dynamic response with respect to the damage parameters α and the orthogonal factors of the moving force c, respectively. During the updating process, the dynamic sensitivity matrices are recalculated based on the iteratively updated structural matrices and the force vector.

Thus the dynamic response sensitivity-based identification equation by a first-order Taylor series is donated as

$$\Delta \mathbf{Y} = [\mathbf{S}]\{\Delta\beta\} = \left[\mathbf{S_K}\quad \mathbf{S_F}\right]\begin{Bmatrix} \Delta\alpha \\ \Delta c \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial \mathbf{Y}}{\partial \alpha} & \dfrac{\partial \mathbf{Y}}{\partial c} \end{bmatrix}\begin{Bmatrix} \Delta\alpha \\ \Delta c \end{Bmatrix} \tag{24}$$

where $\Delta\mathbf{Y}$ is the difference between the measured response and the response from the finite element model. In consequence, the unknown elemental stiffness parameters and orthogonal factors of the equivalent external force are obtained through an optimization process.

4. Numerical example

A simply supported beam is taken as the example to demonstrate the proposed method in identifying the structural damages and the moving load simultaneously. The finite element model of the simply supported beam is shown in Fig. 2. The beam is numerically modeled by 12 elements with each 1m long. The beam has 13 nodes and 36 DOFs in total. The Young's modulus of each

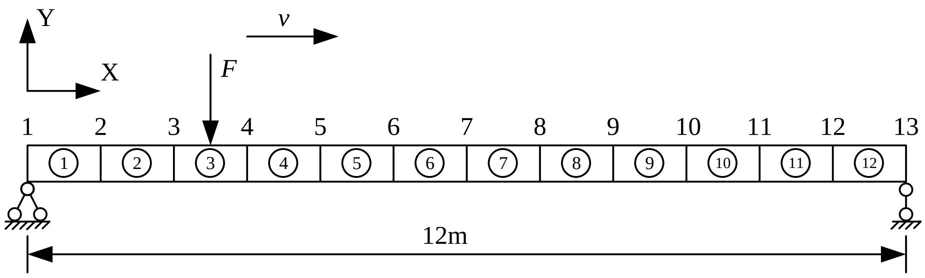


Fig. 2 Finite element model of a simply supported beam

Table 1 The comparison of the first eight frequencies between intact and damaged state

No.	Intact state *f* (Hz)	Damaged state *f* (Hz)	Relative difference *e* (%)
1	8.142	8.028	1.400
2	32.572	32.531	0.126
3	73.301	72.455	1.154
4	107.817	106.929	0.824
5	130.385	129.846	0.413
6	203.957	202.272	0.826
7	294.285	292.262	0.687
8	325.301	323.834	0.451

element is 210 GPa and the mass density is 7800 kg/m^3. The Poisson's ratio is 0.3. The cross-section area of all members is 0.5 m×0.5 m. The Rayleigh damping coefficients a and b are 0.8180 and 7.823×10^{-5}, which give about 1% damping ratio for the first two modes. The force $F=-12000(\sin(24t)+0.5\sin(12t))N$ moves from Node 1 to Node 13 at the speed of v=12 m/s.

The procedure of identifying structural damages with the unknown moving force consists of the following steps:

(1) Select the initial damage parameter α_0 and Chebysheve orthogonal polynomial coefficients c_0 as zeros in this study.

(2) Afterwards, the parameter for elemental stiffness variation α and orthogonal polynomial coefficients c are identified through an iterative scheme. In the r^{th} iteration, the matrix $\mathbf{H}_\mathbf{L}$, which consists of the system Markov parameters $\mathbf{H}_m$ in state space domain, is calculated from Eq. (9).

(3) The first derivative of the system Markov parameters with respect to elemental stiffness factors $\frac{\partial \mathbf{H}_m}{\partial \alpha_i}$ is computed from Eq. (15). Then, $\frac{\partial \mathbf{H}_\mathbf{L}}{\partial \alpha_i}$ can be obtained after substituting $\frac{\partial \mathbf{H}_\mathbf{m}}{\partial \alpha_i}$ into Eq. (14).

(4) The dynamic response sensitivity with respect to the elemental stiffness variation and the orthogonal coefficients of the moving force ($\frac{\partial \mathbf{Y}}{\partial \alpha_i}, \frac{\partial \mathbf{Y}}{\partial c_k}$) are calculated from Eqs. (12)-(13) to form the dynamic response sensitivity matrix **S**.

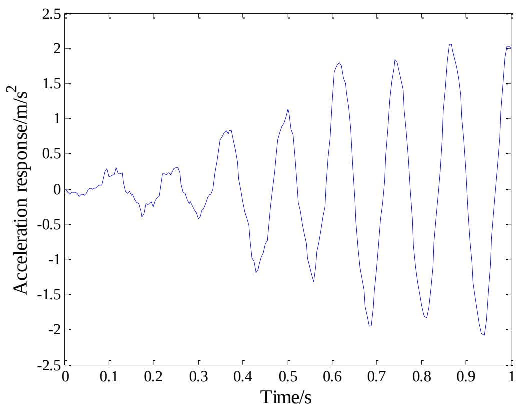
(5) The parameters for elemental stiffness variation α and orthogonal coefficients of the moving force c in the r^{th} iteration are identified from Eq. (24) by the sensitivity-based model updating technique.

(6) The vector of identified parameters α and c in the r^{th} iteration is used as the initial value in the $(r+1)^{th}$ iteration. Repeat steps 2-5 until the convergence criterion is satisfied. The tolerance is taken as 1×10^{-12} in this study.

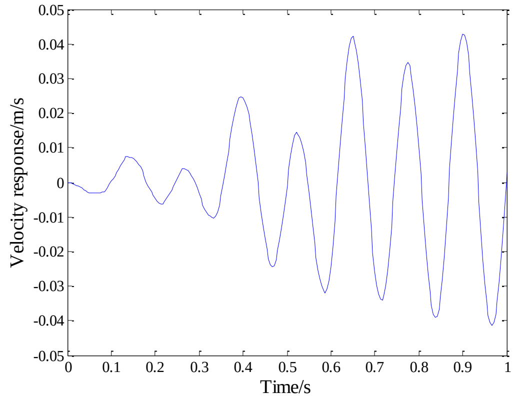
It is assumed that the bending rigidity of Element 6 is reduced by 15%, and Table 1 shows the first eight frequencies of the simply supported beam before and after the damage occurred. It is shown in Fig. 3 the simulated measured dynamic acceleration, velocity and displacement responses on Node 6 of the damaged beam. In this study, the dynamic acceleration response and acceleration response sensitivities are employed to identify the damage and moving force.

4.1 Dynamic acceleration response sensitivity results

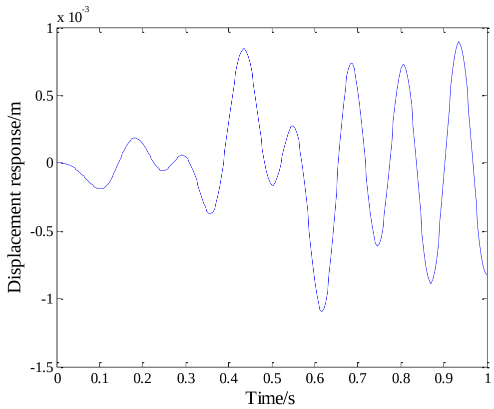
In order to validate the accuracy of the dynamic acceleration response sensitivities calculated by the system Markov parameters in state space domain, Newmark method (Lu and Law 2007) is taken as a reference. Fig. 4 shows that the sensitivity of the acceleration responses (UY(6) and UY



(a) The measured dynamic acceleration response on Node 6

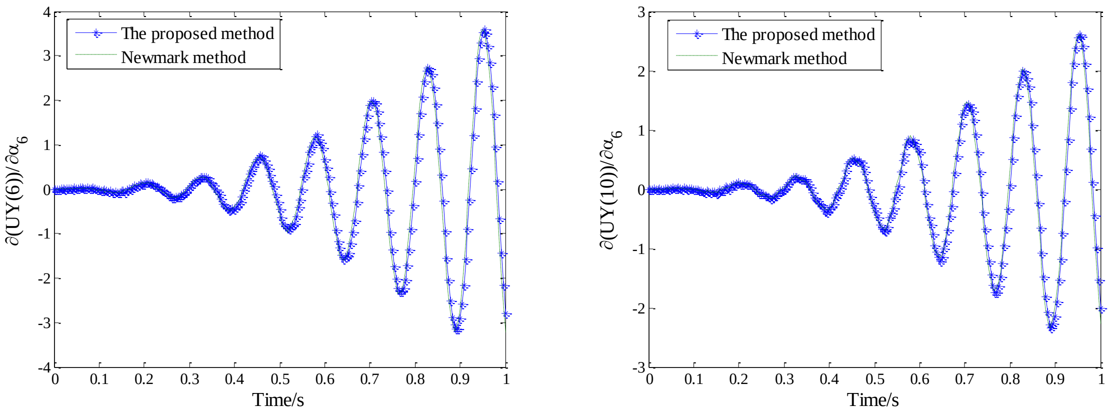


(b) The measured dynamic velocity response on Node 6



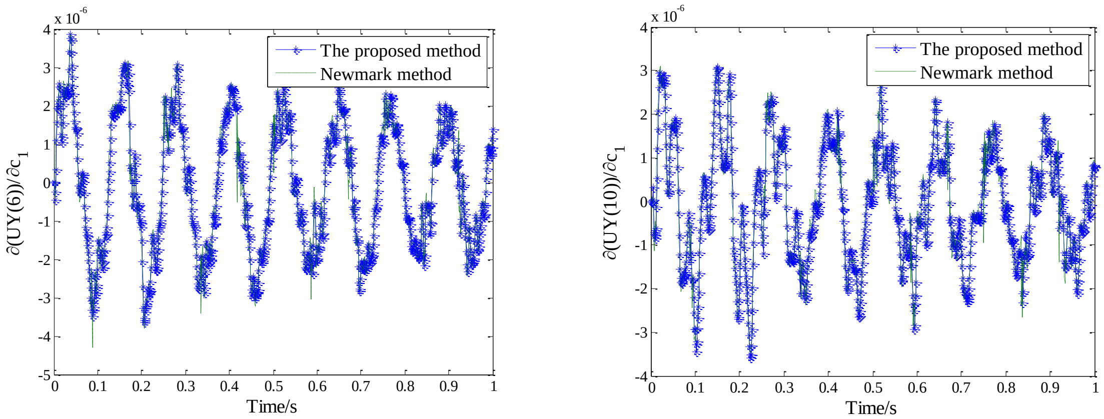
(c) The measured dynamic displacement response on Node 6

Fig. 3 The measured dynamic responses on Node 6 of the simply supported beam



(a) $\dfrac{\partial(UY(6))}{\partial\alpha_6}$ (b) $\dfrac{\partial(UY(10))}{\partial\alpha_6}$

Fig. 4 The first derivative of acceleration response with respect to α_6

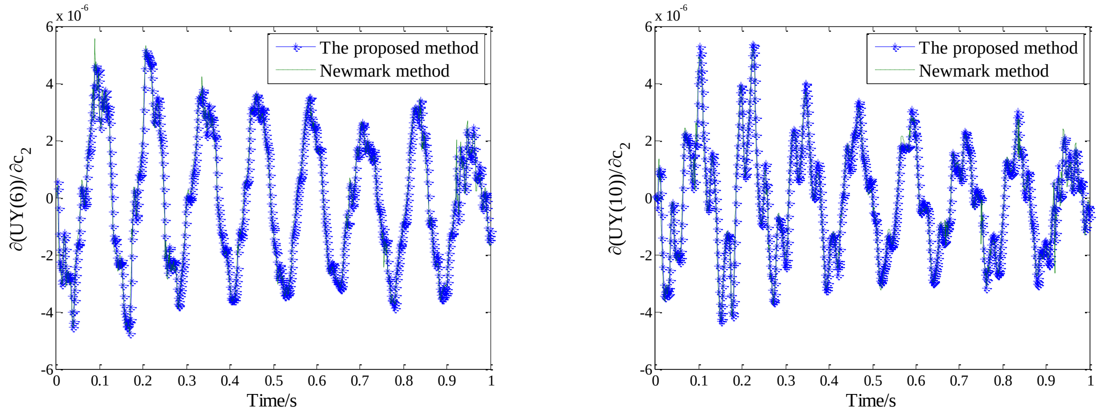


(a) $\dfrac{\partial(UY(6))}{\partial c_1}$ (b) $\dfrac{\partial(UY(10))}{\partial c_1}$

Fig. 5 The first derivative of acceleration response with respect to c_1

(10)) with respect to the sixth elemental parameter α_6. It is seen from Fig. 4(a) that the first derivative of UY(6) with respect to α_6 obtained from Newmark method and the proposed method are matched closely each other, and the similar observation for the sensitivities of UY(10) is shown in Fig. 4(b). It shows that the calculation of the dynamic acceleration response sensitivity by the proposed method is accurate.

Fig. 5 and Fig. 6 show the sensitivities of the acceleration responses (UY(6) and UY(10)) with respect to the first two force orthogonal factors c_1 and c_2, respectively. From Figs. 5-6, the first derivatives of acceleration response with respect to the force orthogonal factors by the proposed method are consistent with the results from Newmark method, which shows the accuracy of the acceleration response sensitivity with respect to the force orthogonal factors.



(a) $\dfrac{\partial(UY(6))}{\partial c_2}$ (b) $\dfrac{\partial(UY(10))}{\partial c_2}$

Fig. 6 The first derivative of acceleration response with respect to c_2

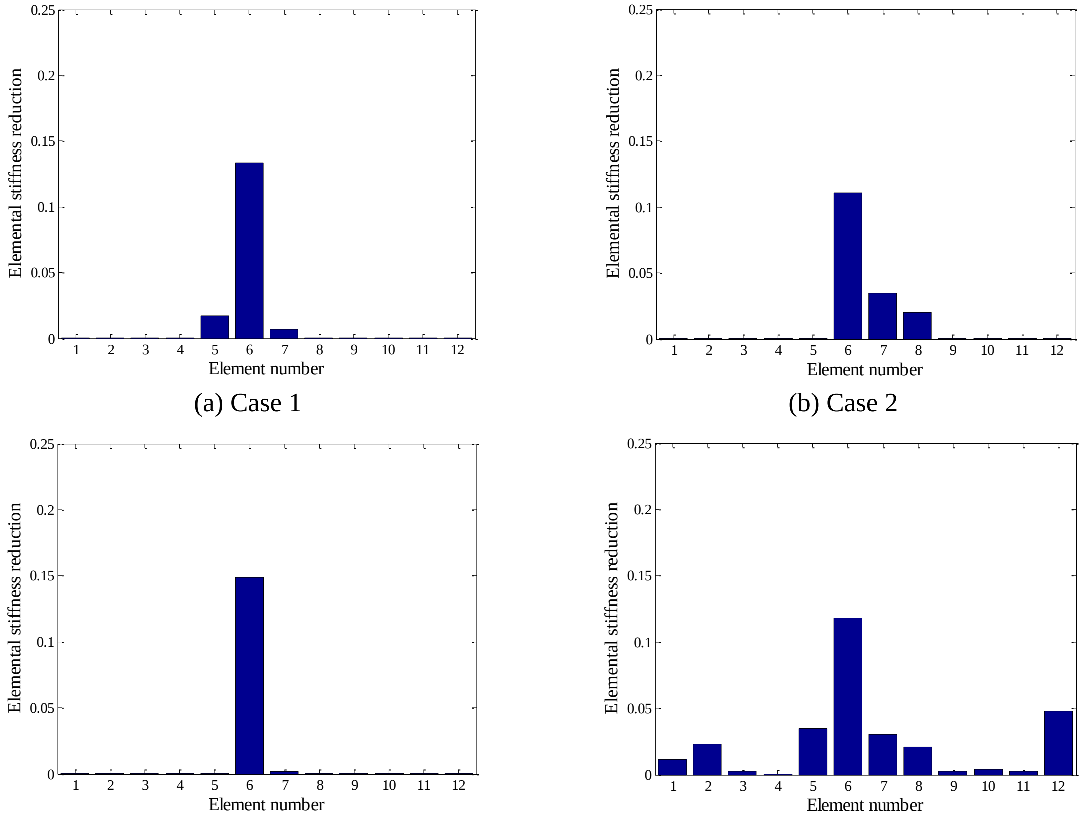
Table 2 Four different cases for damage and force identification

No.	Measurement response	Sampling frequency (Hz)	Noise level
Case 1	UY(6), UY(10)	120	No noise
Case 2	UY(2), UY(3)	120	No noise
Case 3	UY(6), UY(10)	480	No noise
Case 4	UY(6), UY(10)	120	5% noise

4.2 Structural damage identification results

In order to study the factors for affecting the accuracy of the proposed method, Table 2 shows the four different cases for the damage and force identification. The number of orthogonal items of the Chebyshev polynomials is determined by the complexity of the moving force (Qin. 2007). The orthogonal items are chosen to be 15 in all four cases of this study, which is sufficient for the numerically simulated sine-wave moving force. If a real moving vehicle force with surface roughness considered, a larger number of orthogonal items are definitely required. The selection of polynomials items undoubtedly influences the accuracy and efficiency in the damage and force identification process. In Case1, the beam is measured at UY(6) and UY(10) , which represent acceleration measurements in Y direction on Node 6 and Node 10, respectively. The sampling frequency is 120 Hz in Case 1. Case 2 has the same sampling frequency with Case 1, but UY(2) and UY(3) which are located far from the damaged element are selected as the measurement responses in Case 2. In Case 3, the sampling frequency is selected as 240 Hz, and the measurement responses are located UY(6) and UY(10). In Case1, Case 2 and Case 3, the measurement responses with no noise are considered, and 5% measurement noise is considered in Case 4.

Fig. 7 shows the identified damage results in different four cases. It is shown that the identified damage is consistent with the real damage in four cases From Fig. 7(a)-(d). It is shown in Fig. 7(a) that the measurement responses near the damaged element give better accuracy damage identification result with relative difference 1.64% in Case 1. In Case 2 (Fig. 7(b)), the



(a) Case 1 (b) Case 2

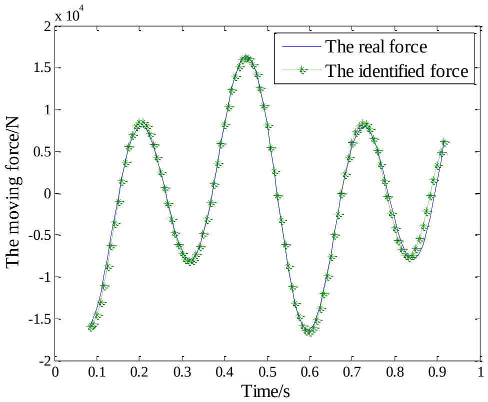
(c) Case 3 (d) Case 4

Fig. 7 The identified structural damage results under different cases

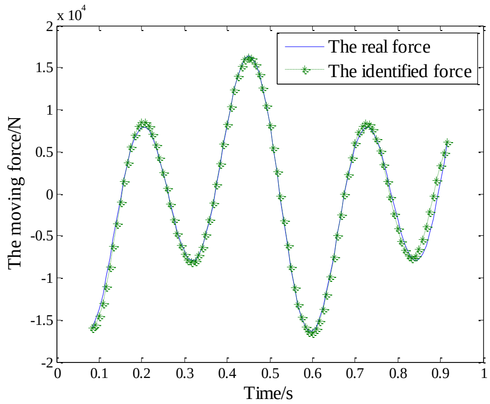
measurement responses (UY(2), UY(3)) which are far from the damaged element can also identified the location and extent of the damage with the relative difference 4.47%. The sampling frequency of 480Hz employed in Case 3 gives better identification result for the exact damage location and extent than the sampling frequency of 120Hz employed in Case 1. The reason is that the lower sampling frequency 120Hz only covers fewer low modes of the structure. Fig. 7(d) shows that the stiffness of Element 6 is reduced by 11.80% with 5% measurement noise considered in Case 4. In Case 4, when the measurement noise is considered, the relative differences are larger than those without measurement noise. The identified damage result is sensitive to the measurement noise.

4.3 Moving force identification results

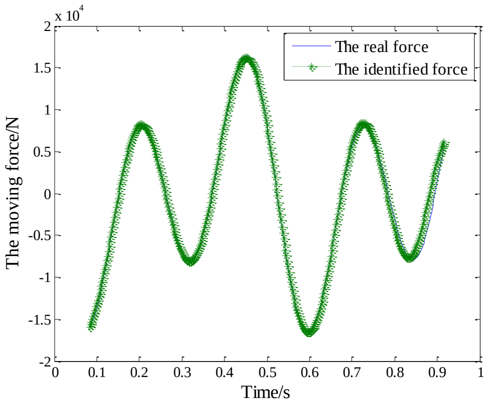
Fig. 8 shows the time history of the identified moving force in different four cases. It is shown in Fig. 8 that the moving load is identified accurately with the real force in four cases. The relative difference of the identified moving force in different cases is listed in Table 3, which is estimated by $e = \frac{norm(F_{id} - F_{real})}{norm(F_{real})} \times 100\%$. From Table 3, the relative differences of the identified force are



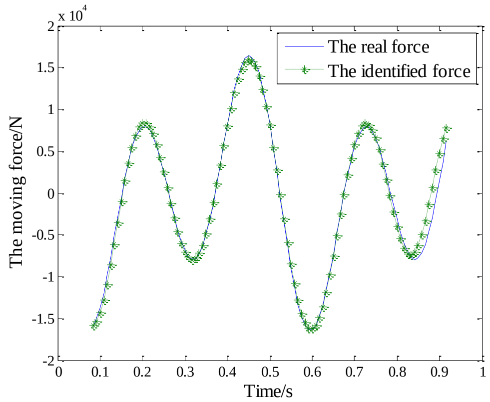
(a) Case 1



(b) Case 2



(c) Case 3



(d) Case 4

Fig. 8 The identified force results under different cases

Table 3 The relative difference of the identified moving force in different cases

	Case 1	Case 2	Case 3	Case 4
Relative difference *e* (%)	8.33	8.76	8.27	10.97

about 8% in Case 1, Case 2 and Case 3, and the relative difference of the identified force is increased to 10.97% in Case 4 when 5% measurement noise considered. As usual, the error is introduced by a large number of sources, for example, the order of Chebyshev polynomial, the sampling time, sampling rate and so on.

5. Conclusions

This paper presents a structural damage identification method under the unknown moving load. Combining the finite element method and the shape functions of the element, the unknown moving force is equivalent as the nodal force of the structure which is expressed as a series of orthogonal polynomial. Afterward, the sensitivities of the dynamic response with respect to elemental stiffness factors and the orthogonal parameters of the moving force are derived analytically using the system Markov parameters in state space domain. A sensitivity-based model updating approach is employed to identify the local damages and the moving force simultaneously by minimizing the difference between the measured response and the analytical response.

A numerical simply supported beam subjected to the unknown moving force is employed to validate the accuracy of the proposed method. It is shown that the dynamic acceleration response sensitivities with respect to the elemental parameters and force orthogonal parameters can be calculated based on the system Markov parameters in state space domain accurately. The numerical results have shown that the local damages and moving force can be identified simultaneously without the knowledge of the time-history of the moving force even with 5% measurement noise considered. In addition, the sampling rate, the sensor location and the noise level affect the accuracy of the identified results. Furthermore, the moving force which is expressed as the equivalent nodal force can be also identified accurately by the proposed method when 5% noise is included in the measured responses.

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