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The stress analysis of a shear wall with matrix displacement method

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Abstract. Finite element method (FEM) is an effective quantitative method to solve complex engineering problems. The basic idea of FEM for a complex problem is to be able to find a solution by reducing the problem made simple. If mathematical tools are inadequate to obtain precise result, even approximate result, FEM is the only method that can be used for structural analyses. In FEM, the domain is divided into a large number of simple, small and interconnected sub-regions called finite elements. FEM has been used commonly for linear and nonlinear analyses of different types of structures to give us accurate results of plane stress and plane strain problems in civil engineering area. In this paper, FEM is used to investigate stress analysis of a shear wall which is subjected to concentrated loads and fundamental principles of stress analysis of the shear wall are presented by using matrix displacement method in this paper. This study is consisting of two parts. In the first part, the shear wall is discretized with constant strain triangular finite elements and stiffness matrix and load vector which is attained from external effects are calculated for each of finite elements using matrix displacement method. As to second part of the study, finite element analysis of the shear wall is made by ANSYS software program. Results obtained in the second part are presented with tables and graphics, also results of each part is compared with each other, so the performance of the matrix displacement method is demonstrated. The solutions obtained by using the proposed method show excellent agreements with the results of ANSYS. The results show that this method is effective and preferable for the stress analysis of shell structures. Further studies should be carried out to be able to prove the efficiency of the matrix displacement method on the solution of plane stress problems using different types of structures.

Keywords: finite element method; plane stress problem; constant strain triangle element; ANSYS

1. Introduction

When the effects of massive earthquakes on buildings are investigated, the resistance of shear wall buildings against earthquake forces is much better than framed systems have been identified. Shear walls are consider as the most suitable structural member in terms of displacement constraints when horizontal direction rigidity is taken into considered. In seismic zones, shear walls are used together with frame structures to provide resistance and ductility. Vecchio (1998) performed a three-dimensional static nonlinear finite element analysis of shear walls. Also, shear

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walls are the most convenient and inexpensive constriction element to repair of earthquake-damaged buildings. Shear walls are effective elements to carry lateral loads and these elements service as vertical structural elements. Bozdogan (2013) proposed modified finite element – transfer matrix for free vibration analysis of asymmetric structures. The bearing systems of structures consist of shear-wall frames. The purpose of this paper is to analyze a shear wall as plane stress problem using matrix displacement method. The matrix displacement method is a structural analysis method used in many applications in civil engineering Martin (1966). In the past, the studies on the behaviors of the shear walls with openings were carried out using shell and brick elements. Solid 65 element was used by Musmar (2013) for analysis of shear walls. Also, he investigated the effects of the size of the openings on the behaviors of the reinforced concrete shear walls. Masood *et al*. (2012) studied on behavior of shear wall with base opening. Analytical methods were used to perform studies of shell structures over a century ago; however studies about behavior of shell structures have majorly increased since the developments of finite element methods. Finite element methods have been used commonly for linear and nonlinear analyses of different types of engineering structures Ed Akin (1984), Hutton (2004) and Zienkiewicz and Taylor (2005). The analysis of shear walls was considered as an example by (Ghorbani *et al*. 2009) to show the effect of nonlinearity. They obtained nonlinear behavior of shear walls using a finite element code developed by using Galerkin weighted residual formulation. Studies have been conducted by researchers for years to develop new finite elements presenting properly behaviors of shell structures. Rebiai and Belounar (2014) developed a new simple and efficient four-node quadrilateral membrane finite element with drilling rotation. A new three-node triangular shell element was developed using discrete Kirchhoff theory and mixed method by Yagawa and Miyamura (2005). The goal of this study is to make stress analyses of shell structures using the free mesh method. Lee and Bathe (2004) developed a simple method to design isotropic triangular shell finite elements based on the Mixed Interpolation of Tensorial Components (MITC) approach. The proposed method is mechanically clear as well as simple and effective. Numerical tests are carried out by using selected MITC elements. Proposed elements show good performance for test elements having different thickness. Saritas and Filippou (2013) struck a balance between the computational efficiency of frame type models and the accuracy of solid finite element models by proposing a frame finite element. This element explains interaction between shear and normal stress at material level.

Complicated problems are divided into sub-problems to make them more understable and easily solvable problems. Main problem can be solved by combining the solutions of created sub-problems. An approximate solution is preferred in an acceptable level rather than full solution due to the complexity of the solution of problems in engineering applications. There are some problems that their complete solutions are considered impossible, so approximate solutions are adopted as the only way. Finite element method used to solve sensitively complex engineering problems is an effective quantitive method. In 1950s, this method was used commonly for stress analysis of aircraft bodies, within the next ten years finite element method could be used accomplishedly in the solution of problems in applied sciences and engineering area. In later years, finite element method has been one of the best methods for solving practical problems. Finite element method has been used commonly in various engineering fields for years. One of the main reasons of this popularity is that this method can be used to solve any particular problem by changing input data of a general computer program. Also, this method is very appropriate to create computer software; so many studies have been conducted for years to develop computer software for analyses of shear walls. For example, an alternate formulation was developed using optimal membrane triangle elements by (Paknahad *et al*. 2007). This formulation was employed to implement a computational algorithm. The implemented code is applied to the analyses of shear wall structures with and without openings. (Oztorun *et al*. 1998) created a finite element computer program named TUNAL to carry out elastic analysis of shear wall building structures based on finite element technical. This program automatically evaluates the statically equivalent earthquake loads and when necessary modifies these loads together with the boundary conditions. Also, a semi- automatic algorithm for finite element analysis was presented by Alyavuz (2007) to obtain the stress and strain distribution in shear wall-frame structures. The proposed algorithm was developed in MATLAB using a constant strain triangle with six degrees of freedom and mesh refinement-coarsening algorithms. The basic idea of finite element method for a complex problem is to be able to find a solution by reducing the problem made simple. If mathematical tools are inadequate to obtain precise result, even approximate result, finite element method is the only method that can be used. In finite element method, the domain is divided into a large number of simple, small and interconnected sub-regions called finite elements such as triangular and rectangular elements. Mousa and Tayeh (2004) developed a new triangular finite element named SBTREIR for the general plane elasticity. This element has three degrees of freedom at each of the three corner nodes. The performance of the new element was compared with well-known constant strain triangle CST element. The new finite element shows good behavior in the elasticity theory. Also, it has fewer discontinuities in the corner stresses than the CST.

Components of a building such as columns and beams can be individually called finite elements. The overall property of the structure depends on properties of individual finite elements. Behavior of the structure in the global coordinates can be specified by assembling the properties of the individual finite elements in this own local coordinates. In finite element method, individual properties of elements are presented by the help of numerical equations. The numerical equations of the individual finite elements are gathered together to specify behavior of the entire structure. Using of finite elements to obtain more accurate solutions leads to complex calculations. However, finite element method has been used commonly for analysis of different types of domain with advances in computer technology.

Some different methods for analyses of shear walls are enhanced by researchers based on finite element method. For example, Clough (1960) used finite element method in stress analysis. Corradi and Panzeri (2004) proposed a method based on sequential limit analyses. This method is regarded as an effective tool to estimate the behavior of the past-collapse response of some shell structures. Also, an analytical method used to carry out analyses of reinforced shear walls using discrete element method was presented by Xinzheng and Jianjing (2001). More than 13000 combination elements are used for analyzing of a two-limb shear wall in this method, so powerful computer software should be developed to use this method. Apart from above studies, Lashgari (2009) carried out finite element analysis of low yield point of thin steel plate shear walls. Severn (1966) solved foundation mat problems by using finite-element methods. In his study, he was interested in plate bending problems in which the plate was resting on an elastic foundation. Also, fundamentally considerations are presented by Chapelle and Bathe (1997) for the finite element analyses of shell structures. Minaine *et al*. (2014) developed nonlinear finite element model of reinforced masonry shear walls for bidirectional loading response. The objective of their study is to analytically establish the effects of bidirectional loading on the response of reinforced masonry shear walls.

This study is consisting of two parts. The first part of the paper is concerned with the plane stress elasticity problem of a shear wall. Fundamental principles of stress analysis of the shear wall

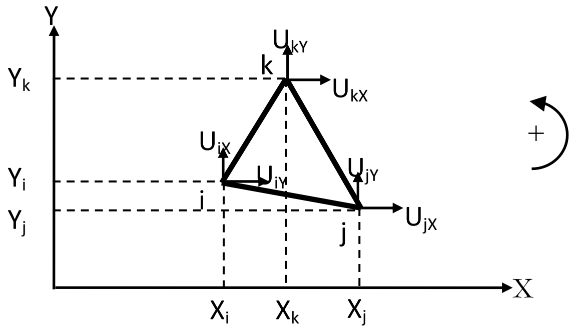


Fig. 1 Triangle element and its coordinates displacement and force vectors

Holand and Bell (1969) are applied by using matrix method in this paper. The matrix analysis of this structure is carried out in three phases; idealization of the system with constant strain triangular finite elements, developing of the structural and loading characteristics of the structure in matrix form and the matrix algebra analysis for displacements and stresses of the structure. Each of these topics is discussed in numerical example in the following section of this paper. In the second part, finite element analysis of the shear wall is made by ANSYS software program. Results obtained from both the first and the second parts of the paper are compared with each other, so the performance of the matrix displacement method is demonstrated.

2. Finite element formulation

2.1 Constant strain triangle

2.1.1 Discretization of the domain into finite elements

In this study, Constant strain triangle (CST) element is used to create finite element model of a plane stress problem. Firstly, domain has to be discretized into finite elements. Triangular are connected at the nodes and each element has three straight sides and three nodes. Smaller elements should be used on the regions that are under the influence of high stress and strain gradients. After discretization of the domain, total number of nodes, total number of elements, coordinates of each node, equivalent nodal forces and boundary conditions are specified along with material properties of the domain such as modulus of elasticity and poisson's ratio. Elements and nodes are usually listed in an element counterclockwise for consistency. For plane strain problem, thickness is taken as equal to 1 and thickness is taken as equal to t for plane stress problem.

2.2 Solution of a shell structure with Matrix Displacement Method

Development of the stiffness matrixes for each element using finite element methodology is the most important task in the matrix displacement method. Essential steps for the developments of the element stiffness matrix using matrix displacement method are explained as below.

2.2.1 Choosing convenient coordinate system and numbering

There are two displacement parameters at both *x* and *y* directions of each node of the CST element as shown in Fig. 1, so the degree of freedom is equal to two for CST element. In brief, three nodes for a triangle and two displacement parameters at each node, so six degrees of foredoom in total.

$$\{U\}=\begin{Bmatrix} U_i \\ U_j \\ U_k \end{Bmatrix}=\begin{Bmatrix} U_{iX} \\ U_{iY} \\ U_{jX} \\ U_{jY} \\ U_{kX} \\ U_{kY} \end{Bmatrix} \qquad \{F\}=\begin{Bmatrix} F_i \\ F_j \\ F_k \end{Bmatrix}=\begin{Bmatrix} F_{iX} \\ F_{iY} \\ F_{jX} \\ F_{jY} \\ F_{kX} \\ F_{kY} \end{Bmatrix} \tag{1}$$

Relation between force and displacement is known as below

$$\{F\}=[K]\{U\} \tag{2}$$

where [K] denotes the stiffness matrix of triangle element.

2.2.2 Choosing displacement function {N(X,Y)} which defines displacements {U(X,Y)} at every point of element

Pascal Triangle

$$\begin{array}{c} 1 \\ XY \\ X^2XYY^2 \\ X^3X^2YY^2XY^3 \end{array}$$

$$U_X=\alpha_1+\alpha_2X+\alpha_3Y \tag{3a}$$

$$U_Y=\alpha_4+\alpha_5X+\alpha_6Y \tag{3b}$$

where α_1, α_2, α_3, α_4, α_5 and α_6 are adjustable parameters. It is considered that displacements of the element change linearly. A state of constant strain within the element is achieved by the selection of the displacement function.

Displacement functions can be written in matrix form as below

$$\{U(X,Y)\}=\begin{Bmatrix} U_X \\ U_Y \end{Bmatrix}=\begin{bmatrix} 1 & X & Y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & X & Y \end{bmatrix}\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix} \tag{4}$$

or briefly

$$\{U(X,Y)\} = \begin{Bmatrix} U_X \\ \\ U_Y \end{Bmatrix} = [N(X,Y)]\{\alpha\} \tag{5}$$

2.2.3 Expressing displacements {U(X,Y)} at triangle element by the help of Displacements {U}

From the nodal coordinates, the nodal displacement parameters can be written as

$$\{U_i\} = \{U(X_i, Y_i)\} = [N(X_i, Y_i)]\{\alpha\} = \begin{bmatrix} 1 & X_i & Y_i & 0 & 0 & 0 \\ \\ 0 & 0 & 0 & 1 & X_i & Y_i \end{bmatrix}\{\alpha\} \tag{6}$$

$$\{U_j\} = \{U(X_j, Y_j)\} = [N(X_j, Y_j)]\{\alpha\} = \begin{bmatrix} 1 & X_j & Y_j & 0 & 0 & 0 \\ \\ 0 & 0 & 0 & 1 & X_j & Y_j \end{bmatrix}\{\alpha\} \tag{7}$$

$$\{U_k\} = \{U(X_k, Y_k)\} = [N(X_k, Y_k)]\{\alpha\} = \begin{bmatrix} 1 & X_k & Y_k & 0 & 0 & 0 \\ \\ 0 & 0 & 0 & 1 & X_k & Y_k \end{bmatrix}\{\alpha\} \tag{8}$$

2.2.4 Strain {ε(X,Y)}- Displacement {U(X,Y)} relations at any point of the element

The deformed shape of a domain under the external loads and temperature distribution can be completely described by the three components of displacement *u,v* and *w* in the *x,y* and *z* directions, respectively. In general, each of these components *u,v* and *w* is a function of coordinates *x,y* and *z*. The strains induced in the domain can be expressed in terms of the displacement components.

For plane stress problem the strain vector as

$$\{\varepsilon(X,Y)\} = \begin{Bmatrix} \varepsilon_X \\ \\ \varepsilon_Y \\ \\ \gamma_{XY} \end{Bmatrix} \tag{9}$$

Strain and displacement relations

$$\varepsilon_X = \frac{\partial U_X}{\partial X}, \ \varepsilon_Y = \frac{\partial U_Y}{\partial Y} \ \text{and} \ \gamma_{XY} = \frac{\partial U_X}{\partial Y} + \frac{\partial U_Y}{\partial X} \ ;$$

$$\varepsilon_X = \frac{\partial U_X}{\partial X} = \frac{\partial}{\partial X}\left(\alpha_1 + \alpha_2 X + \alpha_3 Y\right) = \alpha_2 \tag{10}$$

$$\varepsilon_Y = \frac{\partial U_Y}{\partial Y} = \frac{\partial}{\partial y}\left(\alpha_4 + \alpha_5 X + \alpha_6 Y\right) = \alpha_6 \tag{11}$$

$$\gamma_{XY} = \frac{\partial U_X}{\partial Y} + \frac{\partial U_Y}{\partial X} = \frac{\partial}{\partial Y}\left(\alpha_1 + \alpha_2 X + \alpha_3 Y\right) + \frac{\partial}{\partial X}\left(\alpha_4 + \alpha_5 X + \alpha_6 Y\right) = \alpha_3 + \alpha_5 \tag{12}$$

If Eqs. (10)-(12) are written in their own places at Eq. (9), we obtain Eq. (13) as below

$$\{\varepsilon(X,Y)\}=\begin{Bmatrix}\varepsilon_X\\ \varepsilon_Y\\ \gamma_{XY}\end{Bmatrix}=\begin{Bmatrix}\alpha_2\\ \alpha_6\\ \alpha_3+\alpha_5\end{Bmatrix} \tag{13}$$

In matrix form

$$\{\varepsilon(X,Y)\}=\begin{Bmatrix}\varepsilon_X\\ \varepsilon_Y\\ \gamma_{XY}\end{Bmatrix}=\begin{bmatrix}0&1&0&0&0&0\\0&0&0&0&0&1\\0&0&1&0&1&0\end{bmatrix}\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\\ \alpha_5\\ \alpha_6\end{Bmatrix} \tag{14}$$

briefly

$$\{\varepsilon(X,Y)\}=[C]\{\alpha\} \tag{15}$$

$$\{\alpha\}=[A]^{-1}\{U\} \tag{16}$$

where

$$[A]=\begin{pmatrix}1&X_i&Y_i&0&0&0\\0&0&0&1&X_i&Y_i\\1&X_j&Y_j&0&0&0\\0&0&0&1&X_j&Y_j\\1&X_k&Y_k&0&0&0\\0&0&0&1&X_k&Y_k\end{pmatrix}$$

If Eq. (16) is put in its own place at Eq. (15), we obtain Eq. (17) as below

$$\{\varepsilon(X,Y)\}=[C][A]^{-1}\{U\} \tag{17}$$

When $[C][A]^{-1}$ at Eq. (17) is described as $[B]=[C][A]^{-1}$, Eq. (18) can be written as below

$$\{\varepsilon(X,Y)\}=[B]\{U\} \tag{18}$$

where

$$[B]=\frac{1}{2\Delta}\begin{bmatrix}Y_j\text{-}Y_k&0&Y_k\text{-}Y_i&0&Y_i\text{-}Y_j&0\\0&X_k\text{-}X_j&0&X_i\text{-}X_k&0&X_j\text{-}X_i\\X_k\text{-}X_j&Y_j\text{-}Y_k&X_i\text{-}X_k&Y_k\text{-}Y_i&X_j\text{-}X_i&Y_i\text{-}Y_j\end{bmatrix} \tag{19}$$

where Δ is area of triangle element and given by

$$2\Delta = X_j Y_k - Y_i X_j - X_i Y_k - X_k Y_j + X_k Y_i + X_i Y_j \tag{20}$$

2.2.5 Stress {σ(X,Y)}- Strain {ε(X,Y)}- Displacement {U(X,Y)} relations

In general, equations in continuum mechanics involve 81 independent material constants. If material is considered as homogeneous, isotropic and linearly elastic material, only two independent material constants are required to specify the relations. These constants are modulus of elasticity (*E*) and poisson's ratio (*ν*). For plane stress problem the stress vector as below

$$\{\sigma(X,Y)\} = \begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{Bmatrix} \tag{21}$$

The general stress-strain relations for a homogeneous, isotropic, linearly elastic material subjected to a general two dimensional deformation are as follows

$$\varepsilon_X = \frac{\sigma_X}{E} - \nu \frac{\sigma_Y}{E} \tag{22}$$

$$\varepsilon_Y = -\nu \frac{\sigma_X}{E} + \frac{\sigma_Y}{E} \tag{23}$$

$$\gamma_{XY} = \frac{\tau_{XY}}{G} = \frac{2(1+\nu)}{E} \tau_{XY} \tag{24}$$

where shear modulus or modulus of rigidity, defined by

$$G = \frac{E}{2(1+\nu)} \tag{25}$$

Strains in terms of stresses

$$\{\varepsilon(X,Y)\} = \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{Bmatrix} \tag{26}$$

Stresses in terms of strains

$$\{\sigma(X,Y)\} = \begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} \tag{27}$$

or briefly

$$\{\sigma(X,Y)\}=[D]\{\varepsilon(X,Y)\} \tag{28}$$

When Eq. (18) is written in its place at Eq. (28), we obtain Eq. (29) as below

$$\{\sigma(X,Y)\}=[D][B]\{U\} \tag{29}$$

where [D] is material property matrix. For plane strain problem, constitutive equations

$$\varepsilon_X=\frac{1}{E}[\sigma_X-\nu(\sigma_Y+\sigma_Z)] \tag{30}$$

$$\varepsilon_Y=\frac{1}{E}[\sigma_Y-\nu(\sigma_X+\sigma_Z)] \tag{31}$$

$$\gamma_{XY}=\frac{\tau_{XY}}{G}=\frac{2(1+\nu)}{E}\tau_{XY} \tag{32}$$

$$\begin{Bmatrix}\sigma_X\\ \sigma_Y\\ \tau_{XY}\end{Bmatrix}=\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix}1 & \frac{\nu}{1-\nu} & 0\\ \frac{\nu}{1-\nu} & 1 & 0\\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)}\end{bmatrix}\begin{Bmatrix}\varepsilon_X\\ \varepsilon_Y\\ \gamma_{XY}\end{Bmatrix} \tag{33}$$

For plane elasticity

$$[D]=\begin{bmatrix}d_{11} & d_{12} & 0\\ d_{21} & d_{22} & 0\\ 0 & 0 & d_{33}\end{bmatrix}=\begin{bmatrix}C_1 & C_1C_2 & 0\\ C_1C_2 & C_1 & 0\\ \hline 0 & 0 & C_{12}\end{bmatrix} \tag{34}$$

where C_1, C_2 and C_{12} are constants used to create element stiffness matrixes and their values are given in Table 1 for plane stress and plane strain problems.

Table 1 C_1, C_2 and C_{12} constants for plane stress and plane strain problems

Variable	Plane stress	Plane strain
C1	$\frac{E}{1-\nu^2}$	$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$
C2	ν	$\frac{\nu}{1-\nu}$
C12	$\frac{C_1(1-C_2)}{2}$	$\frac{C_1(1-C_2)}{2}$

2.2.6 Displacement-force relations

According to virtual work principle; strain work done by internal forces equal to strain work done by external forces. If triangular element is subjected to virtual displacement, work done by external forces that affect nodes equals to work done by stresses.

{F}: force at nodes and {U} displacement at nodes. Work which was done by external point forces

$$W_{ext}=\{U^*\}^T\{F\} \tag{35}$$

An arbitrary virtual displacement causes strains $\{\varepsilon(X,Y)\}$ at any point in element. $\{\sigma(X,Y)\}$ are real stresses in triangular element. Strains done by internal forces

$$W_{i?}=\int_0^V \{\varepsilon(X,Y)^*\}^T\{\sigma(X,Y)\}dv \tag{36}$$

If Eqs. (18) and (29) are written in their own places at Eq. (36), we obtain Eq. (37) as below

$$W_{int}=\int_V \left[[B]\{U^*\}\right]^T[D][B]\{U\}dv \tag{37}$$

where {U} is displacement vector. If $(AB)^T=B^TA^T$ is applied to Eq. (37)

$$W_{int}=\int_V \{U^*\}^T[B]^T[D][B]\{U\}dv \tag{38}$$

is obtained. If strain work done by internal forces at Eq. (38) is equalized to work done by external forces given at Eq. (35), Eq. (39) can be written as below

$$\{U^*\}^T\{F\}=\int_V \{U^*\}^T[B]^T[D][B]\{U\}dv \tag{39}$$

Providing that both sides of Eq. (39) are divided with $\{U^*\}^T$

$$\{F\}=\int_V [B]^T[D][B]\{U\}dv \tag{40}$$

is attained. When (40) and (2) Eqs. are compared with each other, stiffness of triangular element is calculated as below

$$[K]=\int_V [B]^T[D][B]dv \tag{41}$$

dv=tdxdy, In this case, stiffness of triangular element can be written again as below

$$[K]=t\iint_{dA}[B]^T[D][B]dxdy=2\Delta t[B]^T[D][B] \tag{42}$$

where *t* is thickness of element.

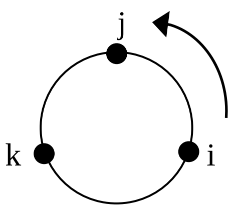
Finally, providing Eqs. (19) and (34) are written in their own places at Eq. (42) and integral is calculated, stiffness matrix (43) is obtained as below

$$[K]=\frac{t}{4\Delta}\left[\begin{array}{c:c:c:c:c:c}
C_1b_i^{\,2}+C_{12}c_i^{\,2} & & & & & \\ \hdashline
C_1C_2b_ic_i+C_{12}b_ic_i & C_1c_i^{\,2}+C_{12}b_i^{\,2} & & & & \\ \hdashline
C_1b_ib_j+C_{12}c_ic_j & C_1C_2b_jc_j+C_{12}b_ic_j & C_1b_j^{\,2}+C_{12}c_j^{\,2} & & & \\ \hdashline
C_1C_2b_ic_j+C_{12}b_jc_i & C_1c_ic_j+C_1b_ib_j & C_1C_2b_jc_j+C_{12}b_jc_j & C_1c_j^{\,2}+C_{12}b_j^{\,2} & & \\ \hdashline
C_1b_ib_k+C_{12}c_ic_k & C_1C_2b_kc_i+C_{12}b_ic_k & C_1b_jb_k+C_{12}c_jc_k & C_1C_2b_kc_j+C_{12}b_jc_k & C_1b_k^{\,2}+C_{12}c_k^{\,2} & \\ \hdashline
C_1C_2b_ic_k+C_{12}b_kc_i & C_1c_ic_k+C_{12}b_ib_k & C_1C_2b_jc_k+C_{12}b_kc_j & C_1c_jc_k+C_{12}b_jb_k & C_1C_2b_kc_k+C_{12}b_kc_k & C_1c_k^{\,2}+C_{12}b_k^{\,2}
\end{array}\right] \tag{43}$$

where

$$\begin{array}{lll}
a_i=X_jY_k-X_kY_j & a_j=X_kY_i-X_iY_k & a_k=X_iY_j-X_jY_i \\
b_i=Y_j-Y_k=Y_{jk} & b_j=Y_k-Y_i=Y_{ki} & b_k=Y_i-Y_j=Y_{ij} \\
c_i=X_k-X_j=X_{kj} & c_j=X_i-X_k=X_{ik} & c_k=X_j-X_i=X_{ji}
\end{array}$$

$$2\Delta=\begin{vmatrix} 1 & X_i & Y_j \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix}=2(\text{area of ijk triangle})\rightarrow 2\Delta=X_jY_k-Y_iX_j-X_iY_k-X_kY_j+X_kY_i+X_iY_j$$



t= thickness of triangular element

3. Numerical example

In this example, stresses at shear wall under external loads are calculated by the help of matrix displacement method. Mechanic and material properties of the structure are given as below;

3.1 Creating stiffness matrix of element-1

The area of triangle element-1 (Δ) is equal to 2 m^2

C_1, C_2, C_{12} and b_{ijk}, c_{ijk} parameters are calculated by depending on problem type and coordinates of nodes respectively as shown below

E=2.1×10^7 KN/m^2

v=0.2

t=20 cm

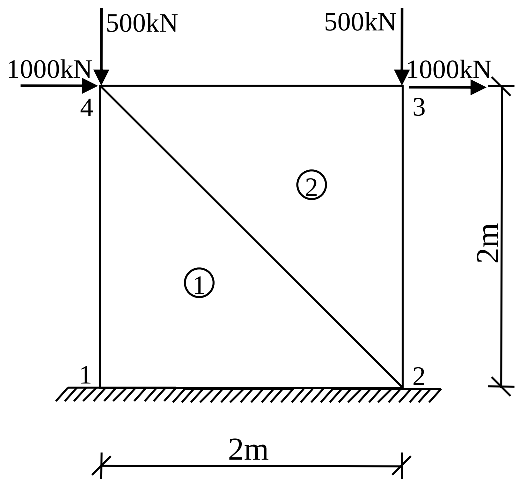


Fig. 2 Shear wall

$$C_1=\frac{E}{1-v^2}=\frac{2.1\text{x}10^7}{1-0.2^2}=2.1875\text{x}10^7$$

$$C_2=0.2$$

$$C_{12}=\frac{C_1(1\text{-}C_2)}{2}=\frac{2.1875\text{x}10^7(1\text{-}0.2)}{2}=0.875\text{x}10^7$$

$$b_1=Y_2-Y_3=0-2=-2 \qquad c_1=X_2-X_3=0-2=-2$$

$$b_2=Y_3-Y_1=2-0=2 \qquad c_2=X_3-X_1=0-0=0$$

$$b_3=Y_1-Y_2=0-0=0 \qquad c_3=X_1-X_2=2-0=2$$

Stiffness matrix of element-1 shown below is obtained by using these parameters

$$[K]_1=2500\begin{bmatrix} U_{1X} & U_{1Y} & U_{2X} & U_{2Y} & U_{4X} & U_{4Y} & \\ 1225 & 525 & -875 & -350 & -350 & -175 & U_{1X} \\ 525 & 1225 & -175 & -875 & -350 & -875 & U_{1Y} \\ -875 & -175 & 875 & 0 & 0 & 175 & U_{2X} \\ -350 & -875 & 0 & 350 & 350 & 0 & U_{2Y} \\ -350 & -350 & 0 & 350 & 350 & 0 & U_{4X} \\ -175 & -875 & 175 & 0 & 0 & 875 & U_{4Y} \end{bmatrix}$$

3.2 Creating stiffness matrix of element-2

The area of triangle element-2 (Δ) is equal to 2 m^2

C_1, C_2, C_{12} and b_{ijk}, c_{ijk} parameters are calculated by depending on problem type and coordinates of nodes respectively as shown below;

$$C_1 = \frac{E}{1-\nu^2} = \frac{2.1x10^7}{1-0.2^2} = 2.1875x10^7$$

$$C_2 = 0.2$$

$$C_{12} = \frac{C_1(1-C_2)}{2} = \frac{2.1875x10^7(1-0.2)}{2} = 0.875x10^7$$

$$b_2 = Y_4 - Y_3 = 2-2=0 \qquad c_2 = X_3 - X_4 = 0-2=-2$$
$$b_4 = Y_3 - Y_2 = 2-0=2 \qquad c_4 = X_2 - X_3 = 2-0=2$$
$$b_3 = Y_2 - Y_4 = 0-2=-2 \qquad c_3 = X_4 - X_2 = 2-2=0$$

Stiffness matrix of element-2 shown below is obtained by using these parameters;

$$[K]_2 = 2500 \begin{array}{c} \begin{matrix} U_{2X} & U_{2Y} & U_{3X} & U_{3Y} & U_{4X} & U_{4Y} \end{matrix} \\ \left[\begin{array}{ccc|ccc} 350 & 0 & -350 & -350 & 0 & 350 \\ 0 & 875 & -175 & -875 & 175 & 0 \\ -350 & -175 & 1225 & 525 & -875 & -350 \\ \hline -350 & -875 & 525 & 1225 & -175 & -350 \\ 0 & 175 & -875 & -175 & 875 & 0 \\ 350 & 0 & -350 & -350 & 0 & 350 \end{array}\right] \end{array} \begin{matrix} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \\ U_{4X} \\ U_{4Y} \end{matrix}$$

3.3 Creating system stiffness matrix

$$[K]^* = 2500 \left[\begin{array}{cc|cc} 1225 & 525 & -875 & -350 \\ 525 & 1225 & -175 & -350 \\ \hline -875 & -175 & 875+350 & 0+0 \\ -350 & -350 & 0+0 & 350+875 \end{array}\right] = 2500 \left[\begin{array}{cc|cc} 1225 & 525 & -875 & -350 \\ 525 & 1225 & -175 & -350 \\ \hline -875 & -175 & 1225 & 0 \\ -350 & -350 & 0 & 1225 \end{array}\right]$$

3.4 Creating system displacement and force vectors

$$\{U\}=\begin{Bmatrix} U_{1X} \\ U_{1Y} \\ \hline U_{2X} \\ U_{2Y} \\ \hline U_{3X} \\ U_{3Y} \\ \hline U_{4X} \\ U_{4Y} \end{Bmatrix} \rightarrow \{U\}^{*}=\begin{Bmatrix} U_{3X} \\ U_{3Y} \\ U_{4X} \\ U_{4Y} \end{Bmatrix} \qquad \{F\}=\begin{Bmatrix} F_{1X} \\ F_{1Y} \\ \hline F_{2X} \\ F_{2Y} \\ \hline F_{3X} \\ F_{3Y} \\ \hline F_{4X} \\ F_{4Y} \end{Bmatrix} \rightarrow \{F\}^{*}=\begin{Bmatrix} F_{3X} \\ F_{3Y} \\ \hline F_{4X} \\ F_{4Y} \end{Bmatrix}=\begin{Bmatrix} 1000 \\ -500 \\ \hline 1000 \\ -500 \end{Bmatrix}$$

3.5 Solution

$$\{F\}^{*}=[K]^{*}\{U\}^{*}$$

$$\begin{Bmatrix} 1000 \\ -500 \\ \hline 1000 \\ -500 \end{Bmatrix}=2500\left[\begin{array}{cc:cc} 1225 & 525 & -875 & -350 \\ 525 & 1225 & -175 & -350 \\ \hdashline -875 & -175 & 1225 & 0 \\ -350 & -350 & 0 & 1225 \end{array}\right]\begin{Bmatrix} U_{3X} \\ U_{3Y} \\ U_{4X} \\ U_{4Y} \end{Bmatrix} \rightarrow \begin{Bmatrix} U_{3X} \\ U_{3Y} \\ U_{4X} \\ U_{4Y} \end{Bmatrix}=\begin{Bmatrix} 1.63\text{x}10^{-3} \\ -6.27\text{x}10^{-4} \\ \hline 1.40\text{x}10^{-3} \\ 1.24\text{x}10^{-4} \end{Bmatrix}\text{m}$$

3.6 Calculation of stresses

$\{\sigma\}=\{D\}\{\varepsilon\}$

$\{\sigma\} \rightarrow$ Stress vector

$\{D\} \rightarrow$ Material matrix

$\{\varepsilon\} \rightarrow$ Strain vector

$$\begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{Bmatrix}=\left[\begin{array}{ccc} C_1 & C_1C_2 & 0 \\ C_1C_2 & C_1 & 0 \\ \hline 0 & 0 & C_{12} \end{array}\right]\begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix}$$

3.7 Calculating stresses of element-1 (i=1, j=2, k=4)

$$\begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \frac{1}{2\Delta} \left[\begin{array}{ccc:ccc} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{array} \right] \begin{Bmatrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{4X} \\ U_{4Y} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \left[\begin{array}{ccc:ccc} 0.5 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & 0 & 0 & 0.5 \\ -0.5 & -0.5 & 0 & 0.5 & 0.5 & 0 \end{array} \right] \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1.40\text{x}10^{-3} \\ 1.24\text{x}10^{-4} \end{Bmatrix} \rightarrow \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 6.221\text{x}10^{-5} \\ 7.019\text{x}10^{-4} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{Bmatrix} = \left[\begin{array}{ccc} C_1 & C_1C_2 & 0 \\ C_1C_2 & C_1 & 0 \\ \hdashline 0 & 0 & C_{12} \end{array} \right] \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = 10^3 \begin{bmatrix} 21875 & 4375 & 0 \\ 4375 & 21875 & 0 \\ 0 & 0 & 8750 \end{bmatrix} \begin{Bmatrix} 0 \\ 6.221\text{x}10^{-5} \\ 7.019\text{x}10^{-4} \end{Bmatrix} \rightarrow \begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{Bmatrix} = \begin{Bmatrix} 272 \\ 1359 \\ 6141 \end{Bmatrix} \text{kN/m}^2$$

3.8 Calculating stresses of element-2 (i=2, j=3, k=4)

$$\begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \frac{1}{2\Delta} \left[\begin{array}{ccc:ccc} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{array} \right] \begin{Bmatrix} U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \\ U_{4X} \\ U_{4Y} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \left[\begin{array}{ccc|ccc} 0 & 0.5 & 0.5 & 0 & -0.5 & 0 \\ 0 & -0.5 & 0 & 0.5 & 0 & 0 \\ -0.5 & 0 & 0.5 & 0.5 & 0 & -0.5 \end{array}\right] \left\{\begin{array}{c} 0 \\ 0 \\ 1.63\text{x}10^{-3} \\ -6.27\text{x}10^{-4} \\ \hline 1.40\text{x}10^{-3} \\ 1.24\text{x}10^{-4} \end{array}\right\} \rightarrow \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \begin{Bmatrix} 1.149\text{x}10^{-4} \\ -3.137\text{x}10^{-4} \\ 4.410\text{x}10^{-4} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{Bmatrix} = \left[\begin{array}{ccc} C_1 & C_1C_2 & 0 \\ C_1C_2 & C_1 & 0 \\ \hline 0 & 0 & C_{12} \end{array}\right] \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = 10^3 \begin{bmatrix} 21875 & 4375 & 0 \\ 4375 & 21875 & 0 \\ 0 & 0 & 8750 \end{bmatrix} \begin{Bmatrix} 1.149\text{x}10^{-4} \\ -3.137\text{x}10^{-4} \\ 4.410\text{x}10^{-4} \end{Bmatrix} \rightarrow \begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{Bmatrix} = \begin{Bmatrix} 1141 \\ -6359 \\ 3859 \end{Bmatrix} \text{kN/m}^2$$

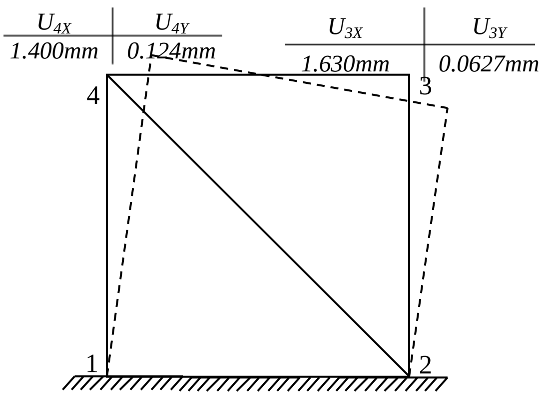


Fig. 3 Deformed shape and nodal displacements

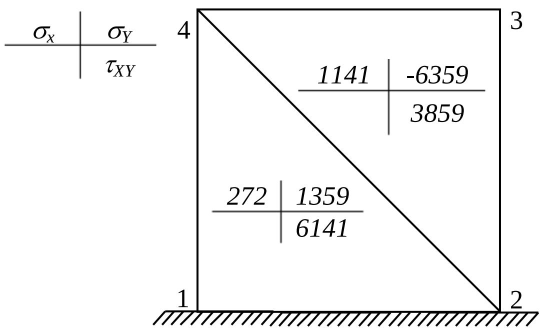


Fig. 4 Stress distribution of shear wall

4. Stress analysis of shear wall with ANSYS software program

ANSYS is used commonly for numerically solving a wide variety of mechanical problems. These problems are static/dynamic structural analysis (both linear and non-linear), heat transfer and fluid problems, as well as acoustic and electro-magnetic problems. Analyses of structures are carried out by ANSYS with the following three stages; preprocessing (defining the problem), solution (assigning loads, constraints and solving) and post processing (further processing and viewing of the results). In this part of the paper, finite element model shown in Fig. 5 is created and stress analysis of the shear wall is carried out by ANSYS software.

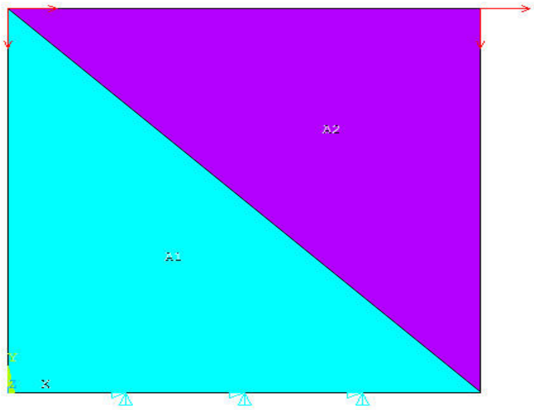


Fig. 5 Solid model of the shear wall under concentrated loads

4.1 Nodal displacements of the shear wall

Table 2 Nodal displacements values of the shear wall under concentrated loads

Node	U_X (m)	U_Y (m)	U_Z (m)	U_{SUM} (m)
1	0.14037E-02	0.12422E-03	0.0000	0.14092E-02
2	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000
4	0.16335E-02	0.62733E-03	0.0000	0.17499E-02

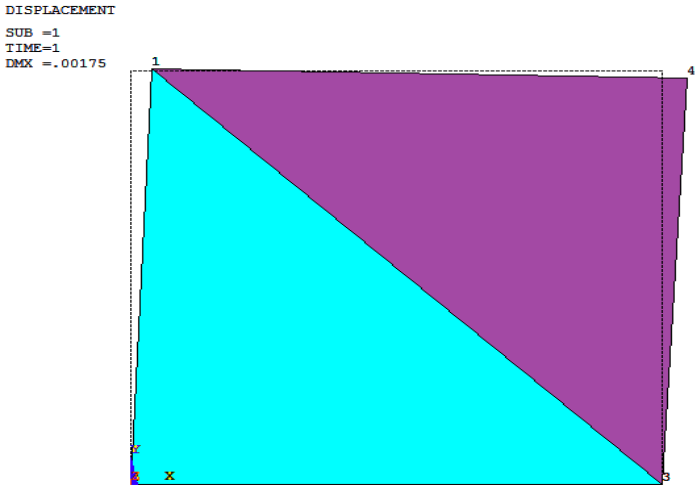


Fig. 6 Deformed shape and maximum nodal displacement of the shear wall

4.2 Elastic strain components of the shear wall

After static analysis of the shear wall with ANSYS, elastic strain components of the each element are obtained as Tables 3-4. The following *X*, *Y*, *Z* values are in global coordinates.

Table 3 Elastic strain values of the Element-1 under concentrated loads

Node	ε_X	ε_Y	ε_Z	γ_{XY}	γ_{YZ}	γ_{XZ}
2	0.0000	0.62112E-04	0.15528E-04	0.70186E-03	0.0000	0.0000
3	0.0000	0.62112E-04	0.15528E-04	0.70186E-03	0.0000	0.0000
1	0.0000	0.62112E-04	0.15528E-04	0.70186E-03	0.0000	0.0000
1	0.0000	0.62112E-04	0.15528E-04	0.70186E-03	0.0000	0.0000

Table 4 Elastic strain values of the Element-2 under concentrated loads

Node	ε_X	ε_Y	ε_Z	γ_{XY}	γ_{YZ}	γ_{XZ}
4	0.11491E-03	0.31366E-03	0.49689E-04	0.44099E-03	0.0000	0.0000
1	0.11491E-03	0.31366E-03	0.49689E-04	0.44099E-03	0.0000	0.0000
3	0.11491E-03	0.31366E-03	0.49689E-04	0.44099E-03	0.0000	0.0000
3	0.11491E-03	0.31366E-03	0.49689E-04	0.44099E-03	0.0000	0.0000

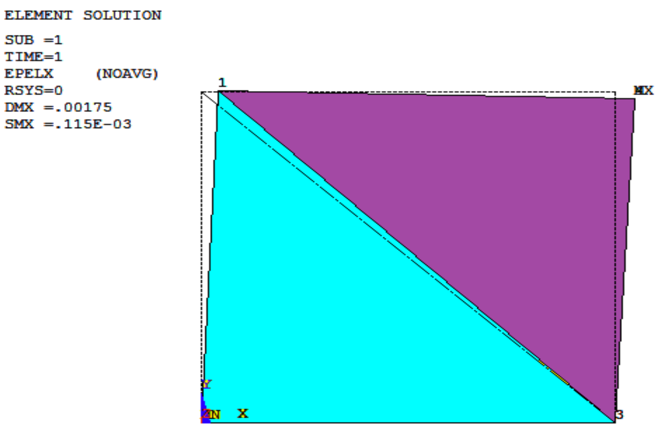


Fig. 7 Deformed shape and *X*-component of elastic strain of the shear wall

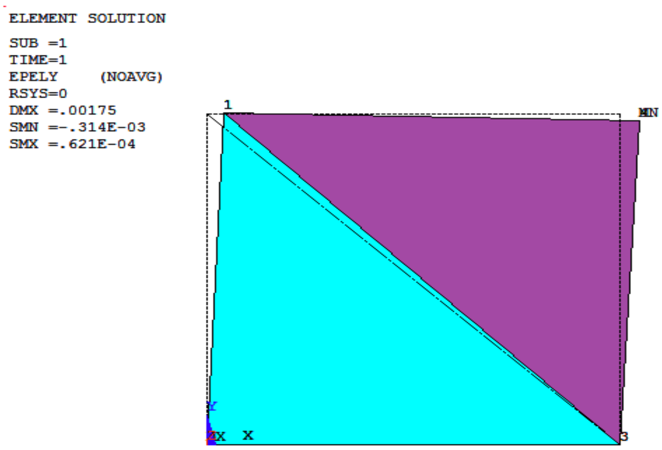


Fig. 8 Deformed shape and *Y*-components of elastic strain of the shear wall

4.3 Stress components of the shear wall

After static analysis of the shear wall with ANSYS, stress components of the each element are obtained as Tables 5-6. The following *X*, *Y*, *Z* values are in global coordinates.

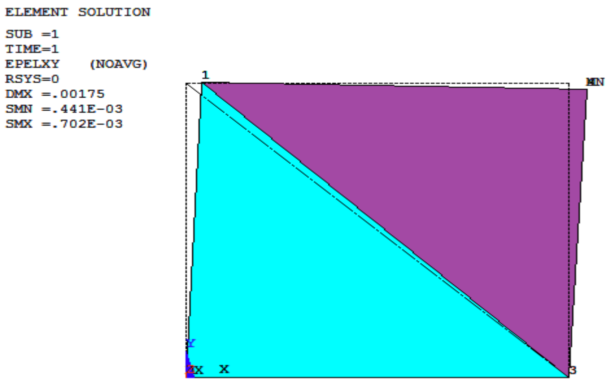


Fig. 9 Deformed shape and *XY*-components of elastic strain of the shear wall

Table 5 Stress values of the Element-1 under concentrated loads

Node	σ_X	σ_Y	σ_Z	τ_{XY}	τ_{YZ}	τ_{XZ}
2	271.74	1358.7	0.0000	6141.3	0.0000	0.0000
3	271.74	1358.7	0.0000	6141.3	0.0000	0.0000
1	271.74	1358.7	0.0000	6141.3	0.0000	0.0000
1	271.74	1358.7	0.0000	6141.3	0.0000	0.0000

Table 6 Stress values of the Element-2 under concentrated loads

Node	σ_X	σ_Y	σ_Z	τ_{XY}	τ_{YZ}	τ_{XZ}
4	1141.3	-6358.7	0.0000	3858.7	0.0000	0.0000
1	1141.3	-6358.7	0.0000	3858.7	0.0000	0.0000
3	1141.3	-6358.7	0.0000	3858.7	0.0000	0.0000
3	1141.3	-6358.7	0.0000	3858.7	0.0000	0.0000

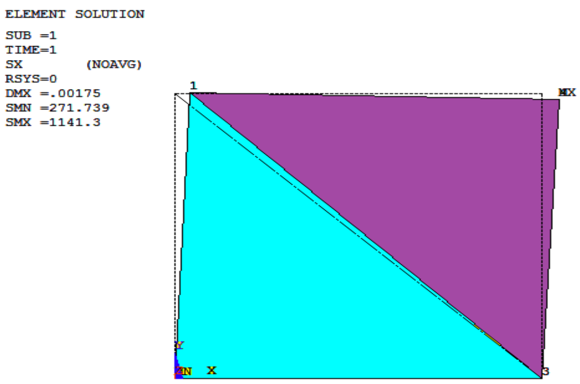


Fig. 10 Deformed shape and *X* normal stresses of the shear wall

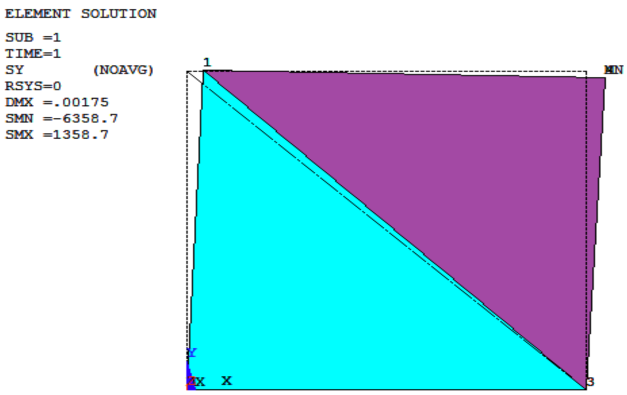


Fig. 11 Deformed shape and *Y* normal stresses of the shear wall

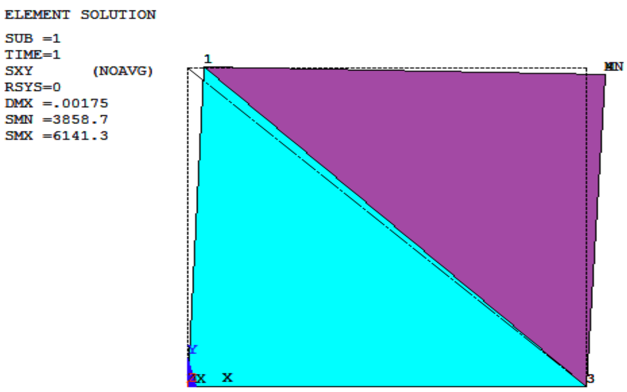


Fig. 12 Deformed shape and *XY* shear stresses of the shear wall

5. Conclusions

In this paper, the efficiency of matrix displacement method on the solution of plane stress problems is investigated by using a shear wall as an example. Proposed shear wall is discretized into constant strain triangle finite elements. The stress values of the shear wall are obtained by using matrix displacement method. Then, the stress analysis of the structure is carried out by ANSYS software.

From the results of this study, the following observations can be made:

• Elastic strain values of the element-1 and element-2 under concentrated loads obtained by using proposed method show excellent agreements with the results of ANSYS.

• Nodal displacement values of the shear wall obtained from both proposed method and ANSYS are almost the same.

• Stress values of element-1 and element-2 obtained from proposed method overlap ones obtained from ANSYS.

The solutions obtained by using the proposed method show excellent agreements with the results of ANSYS software. According to results obtained from this study, the matrix displacement method can be used effectively for stress analyses of shear wall structures. Further studies should be carried out to be able to prove the efficiency of the matrix displacement method on the solution of plane stress problems using different types of structures.

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