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Design for shear strength of concrete beams longitudinally reinforced with GFRP bars

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Abstract. In this paper, a model for the evaluation of shear strength of fibre reinforced polymer (FRP)-reinforced concrete beams is given. The survey of literature indicates that the FRP reinforced beams tested with shear span to depth ratio less than or equal to 1.0 is limited. In this study, eight concrete beams reinforced with GFRP rebars without stirrups are cast and tested over shear span to depth ratio of 0.5 and 1.75. The concrete compressive strength is varied from 40.6 to 65.3 MPa. The longitudinal reinforcement ratio is varied from 1.16 to 1.75. The experimental shear strength and load-deflection response of the beams are determined and reported in this paper. A model is proposed for the prediction of shear strength of beams reinforced with FRP bars. The proposed model accounts for compressive strength of concrete, modulus of FRP rebar, longitudinal reinforcement ratio, shear span to depth ratio and size effect of beams. The shear strength of FRP reinforced concrete beams predicted using the proposed model is found to be in better agreement with the corresponding test data when compared with the shear strength predicted using the eleven models published in the literature. Design example of FRP reinforced concrete beam is also given in the appendix.

Keywords: shear strength; GFRP; concrete deep beams; shear span to depth ratio; size effect

1. Introduction

The corrosion of reinforcement is a major factor affecting the deterioration of steel reinforced concrete structures. Non-corroding reinforcement such as fibre reinforced polymers (FRP) bars is one of the solutions to mitigate this problem. FRP materials exhibit a high level of durability and strength together with high strength to weight ratio. The cost of the FRP materials is now competitive with that of the steel. It is expected that due to the increased durability, lifecycle cost of the FRP structures will be less than the steel reinforced concrete structures. The mechanical properties namely, the tensile strength and the young's modulus of elasticity of FRP bars are different from that of steel bars. The stress strain behaviour of FRP bars is linear and brittle, whereas behaviour of steel bars is linear, yielding, strain hardening and failure. Soric *et al*. (2010) suggested that behavior of FRP reinforced concrete members is significantly different from that of

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members reinforced with steel due to the difference in mechanical properties of FRP and steel. However, in most of the approaches for the design of FRP reinforced members, a modification of models originally proposed for the steel reinforced beams is suggested. It is found from the literature by Bank (2006) that the shear behavior of concrete members reinforced longitudinally with FRP bars has not yet been fully explored.

In concrete beams, the shear resistance is mobilized through five mechanisms, namely, shear stresses in uncracked concrete, interlocking action of aggregate, dowel action of the longitudinal reinforcing bars, arch action and residual tensile stresses transmitted directly across the cracks and is given in ASCE-ACI Committee 445 (1988) report. The shear resistance of uncracked concrete depends on the strength of concrete and the depth of uncracked zone in the beam. The shear sustenance through aggregate interlock is mobilised due to relative slip at crack interface. Duthinh (1997) suggested that the shear resistance offered through aggregate interlock depends on the width of the crack. The longitudinal bars crossing the crack act as dowels and they are effective in resisting the shear displacement at crack interface. The arch action in beam is effected by the transfer of load in beam to the support by direct compression. The arch action is significant when the loading regime is nearer to the support. The crack initiate in beams when the principal stresses exceeds the residual tensile strength of concrete. The special concretes having greater tensile strength contribute greater shear sustenance in concrete beams.

In this study, the shear strength of concrete beams reinforced with FRP bars is evaluated. The shear strength of FRP reinforced concrete beams is predicted using the models proposed by Tottori and Wakui (1993), Michaluk *et al.* (1998), Deitz *et al.* (1999), Wegain and Abdalla (2005), Razaqpur and Isgor (2006), Bentz and Collins (2006), El-Sayed *et al.(2006)*, Nehdi *et al.* (2007), El-Sayed and Benmokrane (2008), Hoult *et al.* (2008), Razaqpur and Spadea (2010). The predicted strength is compared with the corresponding experimental results.

2. Research significance

The behavior and shear strength of concrete members reinforced with FRP bars as main tensile reinforcement is not fully explored. The models for the prediction of shear strength of FRP reinforced concrete beams by Tottori and Wakui (1993), Michaluk *et al.* (1998), Deitz *et al.* (1999), Wegain and Abdalla (2005), Razaqpur and Isgor (2006), Bentz and Collins (2006), El-Sayed *et al.(2006)*, Nehdi *et al.* (2007), El-Sayed and Benmokrane (2008), Hoult *et al.* (2008), Razaqpur and Spadea (2010) are developed by modifying the models originally proposed for steel reinforced concrete beams. This can be attributed to the fact that the mechanism of load transfer in steel reinforced beams and FRP reinforced beam are similar (Guadagagni 2006).

It is found from the literature by Tottori and Wakui (1993), Michaluk *et al.* (1998), Deitz *et al.* (1999), Wegain and Abdalla (2005), Razaqpur and Isgor (2006), Bentz and Collins (2006), El-Sayed *et al.(2006)*, Nehdi *et al.* (2007), El-Sayed and Benmokrane (2008), Hoult *et al.* (2008), Razaqpur and Spadea (2010) that the test data corresponding to shear span to depth ratio less than or equal to 1.0 is limited. Hence, in this study, eight concrete beams longitudinally reinforced with GFRP bars are cast and tested over a shear span to depth ratio between 0.5 and 1.75. The prediction model originally proposed by IS 456 (2000) for steel rebar is modified to account for the influence of GFRP rebar. The model for predicting the shear strength of FRP reinforced concrete beams accounting for the influence of effect of the reinforcement ratio, shear span to depth ratio and the modulus of elasticity of the reinforcing bars is proposed. It is expected that the model proposed in this study would help the designer to arrive at economical sections for the design of concrete beams reinforced with FRP rebars.

3. Review of current design provisions

The details of the model proposed by Tottori and Wakui (1993), Michaluk *et al.* (1998), Deitz

Table 1 Models given by various researchers for FRP reinforced concrete beams

Sl. No.	Reference	Shear strength of concrete beam without stirrups (V_c)
1	Tottori and Wakui (1993)	$V_c = 0.2\left(100\ \rho_f\ f_c^{'}\frac{E_f}{E_s}\right)^{\frac{1}{3}}\left(\frac{d}{1000}\right)^{-\left(\frac{1}{4}\right)}\left(0.75+\frac{1.4}{a/d}\right)bd$
2	Michaluk et al. (1998)	$V_c = \frac{E_f}{E_s}\left(\frac{1}{6}\sqrt{f_c^{'}}bd\right)$
3	Deitz et al. (1999)	$V_c = 3\frac{E_f}{E_s}\left(\frac{1}{6}\sqrt{f_c^{'}}bd\right)$
4	Wegian and Abdalla (2005)	$V_c = 2\left(f_c^{'}\rho_f\ \frac{E_f}{E_s}\frac{d}{a}\right)^{\frac{1}{3}}bd$
5	Razaqpur and Isgor(2006)	$V_c = 0.035\,k_m k_s k_a\left(1+k_r\right)\sqrt{f_c^{'}}bd \le 0.2k_s\sqrt{f_c^{'}}bd$ $k_m = \left(\frac{V_F d}{M_F}\right)^{2/3}$ $k_r = \left(E_f\rho_f\right)^{1/3}$ $k_a = \begin{cases} 1.0 & \text{for } \left(\frac{M_F}{V_F d}\right) \ge 2.5 \\ \frac{2.5}{\left(\frac{M_F}{V_F d}\right)} & \text{for } \left(\frac{M_F}{V_F d}\right) < 2.5 \end{cases}$ $k_s = \begin{cases} 1.0 & \text{for beams with } d \le 300\text{mm} \\ \frac{750}{450+d} & \text{for beams with } d > 300\text{mm} \end{cases}$
6	Bentz and Collins (2006)	$V_c = \left(\frac{0.4}{1+1500\,\varepsilon_x}\right)\left(\frac{1300}{1000+S_{xe}}\right)\sqrt{f_c^{'}}\,b\,d_v$ $d_v = \text{shear depth} = 0.9d$ $\varepsilon_x = \frac{M_f/d_v + V_f}{2E_r A_r}$ $S_{xe} = \frac{31.5d}{16+a_g} \ge 0.77d$

Table 1 Continued

7	El-sayed, El-Salakawy, Benmokrane (2006)	$V_c = \left(\frac{\rho_f E_f}{90\beta_1 f_c^{'}}\right)^{\frac{1}{3}}\left(\frac{\sqrt{f_c^{'}}}{6} bd\right) \le \frac{\sqrt{f_c^{'}}}{6} bd$ $\beta_1 = 0.85 \quad \text{for} \quad f_c^{'} = 28\text{MPa}$ $\beta_1 = 0.85 - \frac{0.05(f_c^{'} - 28)}{7} \ge 0.65 \quad \text{for} \quad f_c^{'} > 28\text{MPa}$
8	Nehdi, Chabib and Said (2007)	$V_c = 2.1\left(\frac{f_c^{'}\rho_f d}{a}\ \frac{E_f}{E_s}\right)^{0.3} bd \ \text{ for } \frac{a}{d} > 2.5$ For $\frac{a}{d} < 2.5$, V_c is multiplied by $2.5\frac{d}{a}$
9	El-Sayed and Benmokrane (2008)	$V_c = 2.5\left(\frac{0.40}{1 + 1500\,\varepsilon_x}\right)\left(\frac{1300}{1000 + S_{xe}}\right)\varphi_c f_{cr} b d_v$ φ_c = resistance factor for concrete made equal to 1.0 d_v = effective shear depth = 0.9d $\varepsilon_x = \frac{M_f/d_v + V_f}{2E_r A_r}$ $S_{xe} = \frac{31.5d}{16 + a_g} \ge 0.77d$ $f_{cr} = 0.4\sqrt{f_c^{'}}$
10	Hoult et al.(2008)	$V_c = \left(\frac{0.3}{0.5 + (1000\,\varepsilon_x + 0.15)^{0.7}}\right)\left(\frac{1300}{1000 + S_{xe}}\right)\sqrt{f_c^{'}}\, b d_v$ d_v = effective shear depth = 0.9d $\varepsilon_x = \frac{M_f/d_v + V_f}{2E_r A_r}$ $S_{xe} = \frac{31.5d}{16 + a_g} \ge 0.77d$
11	Razaqpur and Spadea (2010)	$V_c = 0.045 k_m^{*} k_s k_a k_r^{*}\left(f_c^{'}\right)^{1/3} bd$ $k_m^{*} = \left(\frac{Vd}{M}\right)^{1/2}$ $k_s = 1.0 \text{ for } d \le 300\text{mm}$ $= \frac{750}{450 + d} \text{ for } d > 300\text{mm}$ $k_a = 1.0 \text{ for } \left(\frac{M}{Vd}\right) \ge 2.5$ $= \left(\frac{2.5}{M/Vd}\right) \text{ for } \left(\frac{M}{Vd}\right) < 2.5$ $k_r^{*} = 1 + (E_f \rho_f)^{1/3}$

et al. (1999), Wegain and Abdalla (2005), Razaqpur and Isgor (2006), Bentz and Collins (2006), El-Sayed *et al*.(2006), Nehdi *et al.* (2007), El-Sayed and Benmokrane (2008), Hoult *et al.* (2008), Razaqpur and Spadea (2010) are reviewed and given in Table 1.

The total shear resistance V_u of concrete beam is computed by

$$V_u = V_c + V_s \tag{1}$$

where V_c is the shear resistance of concrete and V_s is the shear resistance due to stirrups.

The model accounting for the shear enhancement due to load transfer through arch action is limited Nehdi *et al*. (2007). El-Sayed and Soudki (2011) reported that the models proposed by various researchers account for the influence of different characteristic parameters of the FRP reinforcement.

The models proposed by Tottori and Wakui (1993), Wegain and Abdalla (2005), Razaqpur and Isgor (2006), Nehdi *et al.(*2007), Razaqpur and Spadea (2010) account for the influence of shear span to depth ratio and is given in Table 1. The model proposed by Bentz and Collins (2006), El-Sayed and Benmokrane (2008), Hoult *et al.* (2008) utilizes iterative procedure to compute the shear strength of beams. The concept of equivalent reinforcement ratio is accounted for in the model proposed by Tottori and Wakui (1993), Wegian and Abdalla (2005) and Nehdi *et al*. (2007).

The concrete compressive strength is accounted in the form of $(\sqrt{f_c'})$ in the models proposed by Michaluk *et al*. (1998), Deitz *et al*. (1999), Razaqpur and Isgor (2006), Bentz and Collins(2006), El-Sayed *et al*. (2006), El-sayed and Benmokrane (2008), Hoult *et al*. (2008). Tottori and Wakui (1993), Wegian and Abdalla (2005), Nehdi *et al*. (2007), Razaqpur and Spadea (2010) account for the influence of concrete compressive strength in the form of $(f_c')^{1/3}$.

The axial stiffness factor $(1+(E_f)^{1/3})$ is accounted for in the model proposed by Razaqpur and Isgor (2006) and Razqpur and Spadea (2010).

In this paper, the model originally proposed for the prediction of shear strength of concrete beams longitudinally reinforced with steel reinforcement by IS:456 (2000) is modified to account for the arch action, size effect of beams, axial stiffness factor and the influence of FRP longitudinal reinforcement. The proposed model for the prediction of shear strength of concrete V_c for beams reinforced with FRP bars is given by

$$V_c = k_1 k_2 \tau_c bd \tag{2}$$

where k_1 is the shear enhancement factor accounting for the arch action in beams and k_2 is the size effect factor. τ_c is the average permissible shear stress of concrete beam. b and d are the width and depth of the beam cross section. k_1 in Eq. (2) is modified and given by

$$k_1 = \begin{cases} 2.5\, d/a & ;\text{when } a/d \le 2.5 \\ 1.0 & ;\text{when } a/d \ge 2.5 \end{cases} \tag{3}$$

The factor k_1 accounts for the load sustenance by arch action. The maximum value of shear enhancement factor k_1 given by IS:456 (2000) is 2.0 and k_1 is modified based on regression analysis. The size effect factor k_2 proposed by Razaqpur and Spadea (2010) is used in this study and is given by

$$k_2 = \begin{cases} 1.0 & ;\text{when } d \le 300 \text{ mm} \\ \dfrac{750}{450 + d} & ;\text{when } d > 300 \text{ mm} \end{cases} \tag{4}$$

τ_c proposed by SP:24(1983) is used and is given by

$$\tau_c = \frac{0.85\sqrt{f_c'}\left(\sqrt{1+5\beta}-1\right)}{6\beta} \qquad (5)$$

where, f_c' is the cylinder compressive strength of concrete and is computed by

$$f_c' = 0.8\, f_{ck} \qquad (6)$$

where f_{ck} is the characteristic compressive strength of concrete. β in Eq. (5) is the factor accounting for the influence of the longitudinal reinforcement and is given by

$$\beta = \frac{0.8 f_{ck}}{k_0\ p_t} > 1.0 \qquad (7)$$

where p_r is the longitudinal reinforcement ratio and is given by

$$p_t = \frac{100 A_e}{bd} \qquad (8)$$

where A_e is the equivalent area of reinforcement to be provided when steel is used in place of FRP and is proposed by Guadagagni *et al*. (2006). A_e is given by

$$A_e = A_f\,\frac{E_f}{E_s} \qquad (9)$$

where A_f is the area of FRP longitudinal reinforcement. E_f and E_s are Young's modulus of FRP rebar and steel reinforcement respectively. The magnitude of E_s is assumed as 200GPa.

The magnitude of numerical constant k_0 in the denominator in Eq. (7) is computed by statistical regression analysis of test data and its magnitude is found to be 48.2. Based on the proposed model, the shear strength of eight concrete test beams reinforced with FRP bars is predicted and by the models proposed by various researchers and the results is compared.

4. Experimental programme

A total of eight concrete beams reinforced with GFRP bars are cast and tested under four point loading by varying the shear span to depth (*a/d*) ratio, longitudinal reinforcement ratio, cross section, span of testing and are given in Table 2. The test beams are longitudinally reinforced with glass fibre reinforced polymer (GFRP) bars. The schematic of cross section and reinforcement detailing of beams and the test set-up are given in Fig. 1. The designation of test beams is given in Table 2. The Young's modulus of GFRP bars is observed as 40.8 GPa.

The details of the end-zone and longitudinal reinforcements are given in Fig. 1(a). The smooth and straight GFRP bars available in market are used. The length of smooth GFRP bars beyond the supports is limited to 300 mm. The longitudinal bars are anchored at the ends to avoid bond failure of the bars. The stainless steel reinforcement cages in the form of rectangular stirrups are provided at the end zone to resist the high stresses developed during loading. The reinforcement in the cross section of the beam at the mid-span is shown in Fig. 1(b). In the test beams containing six longitudinal bars, the bars are provided in three layers. The bars are provided in two layers in test beams containing four longitudinal bars. Two GFRP bars are provided at top and number of bars

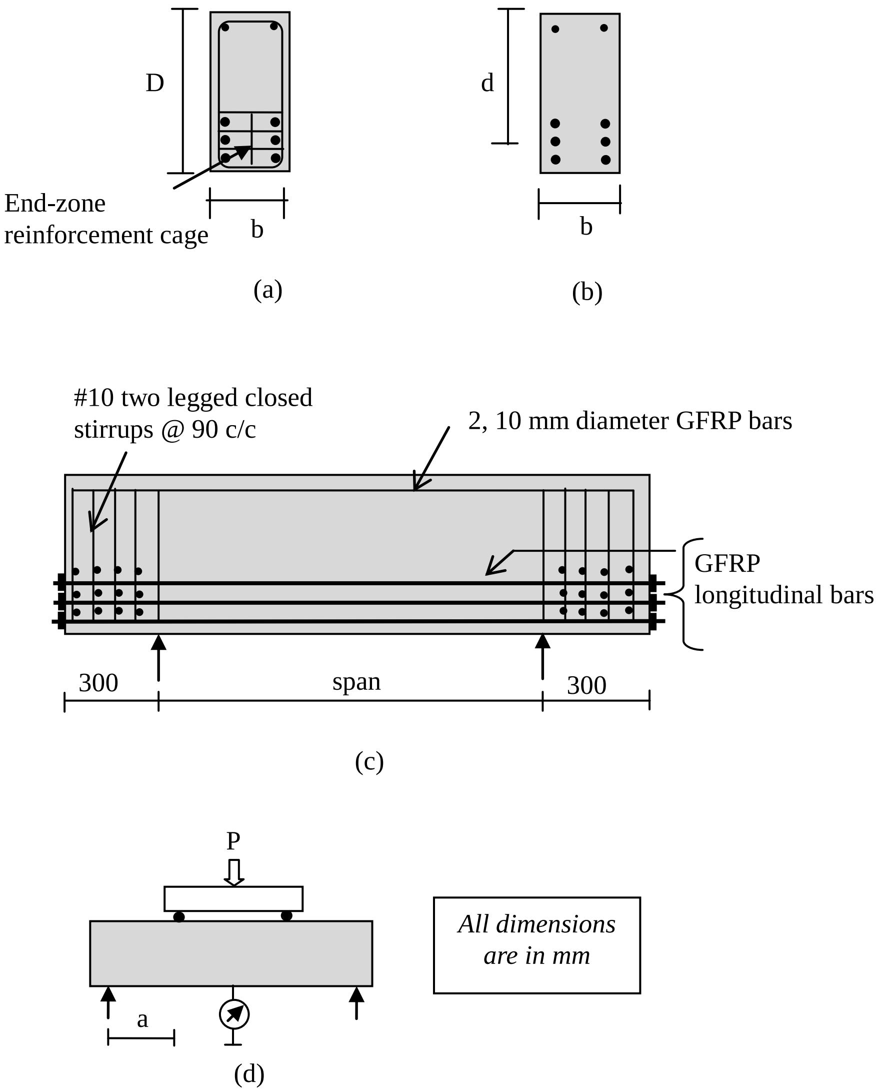


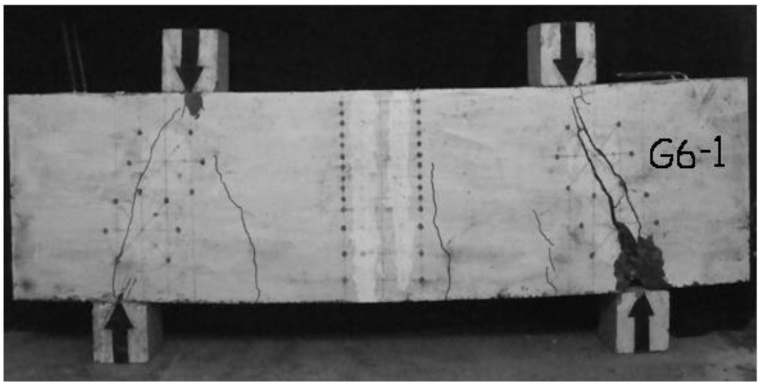
Fig. 1 Details of the test beam and test –Setup. (a) cross-section at end, (b) cross-section at mid span, (c) longitudinal elevation showing the reinforcement details (d) test set-up

at bottom is varied. The arrangement of longitudinal reinforcement is given in Fig. 1(c). The bottom longitudinal bars are externally bolted. In the field, it is recommended to provide dummy concrete block to mask the projecting lead of the reinforcing bars. The beams are cast and cured for 28days using moist burlap. The test set-up is shown in Fig. 1(d). The bearing plates at the loading and support points were of 100×170 mm. The beams are tested using 1000 kN digital beam testing machine. The load is applied gradually. The load and mid-span deflection in beams are recorded using a multichannel data logging system. The loads at first crack and ultimate stage are recorded.

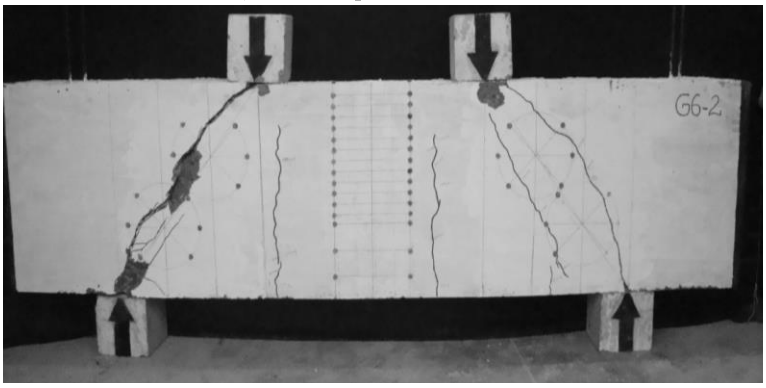
Table 2 Details of test beams

Designation	f_{ck} (MPa)	b (mm)	D (mm)	d (mm)	span (mm)	a/d	Longitudinal GFRP reinforcement			
							Bar details*	Tensile strength (MPa)	p_t (%)	E_f (GPa)
G6-a	65.3	170	500	416	990	0.50	6#16	655	0.35	40.8
G6-b	65.3	170	500	416	990	1.00	6#16	655	0.35	40.8
G6-1	59.5	170	500	405	1400	0.50	6#16	655	0.36	40.8
G6-2	59.5	170	500	405	1400	1.00	6#16	655	0.36	40.8
G6-3	45.3	100	350	270	1100	1.25	6#10	760	0.36	40.8
G6-4	45.3	100	350	270	1100	1.75	6#10	760	0.36	40.8
G4-1	40.6	100	350	270	1100	1.25	4#10	760	0.24	40.8
G4-2	40.6	100	350	270	1100	1.75	4#10	760	0.24	40.8

*total number of bars# diameter of bar in mm



(a) Crack pattern in test beam G6-1



(b) Crack pattern in test beam G6-2

Fig. 2 Crack pattern indicating the shear failure of deep concrete beams reinforced with FRP bars

5. Results and discussions

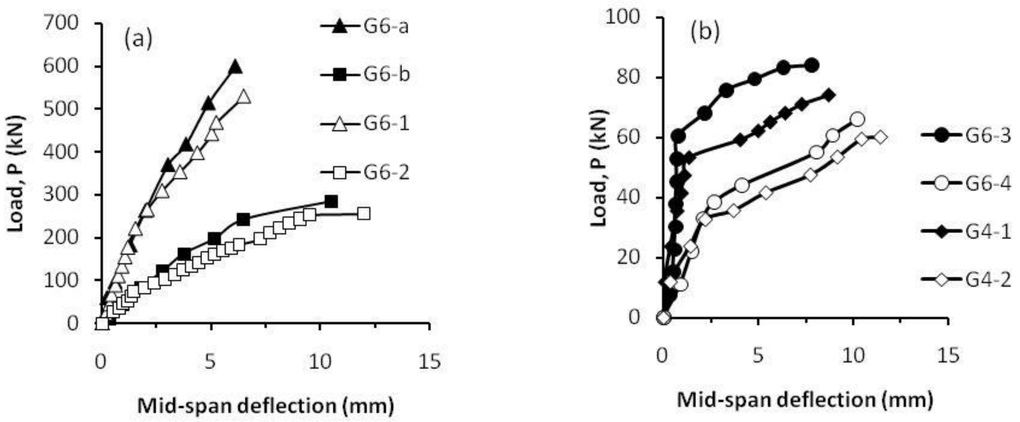
The eight concrete beams longitudinally reinforced with glass fiber reinforced polymer (GFRP) bars are cast and tested under four-point loading. The variables of the study are concrete compressive strength, shear span to depth (*a/d*) ratio, longitudinal reinforcement ratio, cross section and span of testing. In the initial stages of load, the deformations are found to be gradual and linear. Cracks initiated in the shear span at mid-height of the beam, in the subsequent stages of loading. With the increase of load, formation of new cracks is observed at the tension zone in shear and flexure span of the beam. The tension zone cracks are progressed towards the compression zone. The crack near the support extended towards the loading point and lead to the formation of diagonal crack. In later stages of loading, an increase in the width of the diagonal crack is observed. The test beams failed in diagonal shear. The cracks in test beams at failure are shown in Fig. 2.

The load at first crack and failure in the test beams is recorded and is given in Table 3. The load carrying capacity of the GFRP beam is found to increase significantly with the decrease in shear span to depth (*a/d*) ratio. The increase in the strength is attributed to the load transfer by arch action. This indicates that the shear enhancement due to arch action is important and shall be accounted for in the model for the prediction of shear strength of concrete beams reinforced with GFRP bars. The first crack load in test beam is found to be fifty percent of the ultimate load. This indicates that the cracking and failure are sufficiently apart to provide warning prior to the failure of the structure. Localized crushing and spalling of concrete at post peak regime are also observed.

Table 3 Load corresponding to different stages in test beams

Test beams	Load at first crack (kN)	Failure load (kN)	Shear strength (kN)*	Mode of failure
G6-a	298.0	600.0	300.0	Diagonal shear
G6-b	122.4	285.0	142.5	Diagonal shear
G6-1	259.7	530.0	265.0	Diagonal shear
G6-2	109.4	255.0	127.5	Diagonal shear
G6-3	41.3	84.0	42.0	Diagonal shear
G6-4	29.0	66.0	33.0	Diagonal shear
G4-1	34.1	74.0	37.0	Diagonal shear
G4-2	20.9	60.0	30.0	Diagonal shear

*Shear strength = Failure load/2



(a) Beams with *a/d* ≤ 1.0 (b) Beams with *a/d* > 1.0

Fig. 3 Load-deflection response of concrete beam reinforced with GFRP bars

Table 4 Comparison of shear strength of test beams with corresponding prediction using the models proposed by various researchers and proposed model

References	Ratio of experimental shear strength to predicted shear strength of test beams ($V_{c,exp}/V_{c,pred}$)									
	G6-a	G6-b	G6-1	G6-2	G6-3	G6-4	G4-1	G4-2	Mean	S.D.*
Tottori and Wakui (1993)	1.83	1.43	1.68	1.34	1.28	1.21	1.34	1.31	1.43	0.22
Michaluk *et al.* (1998)	17.24	8.19	16.36	7.87	7.64	6.00	7.12	5.77	9.52	4.58
Deitz *et al*. (1999)	5.75	2.73	5.45	2.62	2.55	2.00	2.37	1.92	3.17	1.53
Wegian and Abdalla (2005)	2.98	1.78	2.76	1.67	1.66	1.45	1.74	1.57	1.95	0.58
Razaqpur and Isgor (2006)	3.39	1.61	3.18	1.53	1.29	1.02	1.2	0.97	1.77	0.96
Bentz and Collins (2006)	4.44	2.38	4.11	2.22	2.00	1.72	2.15	1.92	2.62	1.05
El-Sayed *et al*. (2006)	5.85	2.78	5.43	2.61	2.37	1.86	2.48	2.01	3.17	1.55
Nehdi *et al*. (2007)	0.55	0.64	0.51	0.6	0.73	0.89	0.75	0.95	0.70	0.16
El-Sayed and Benmokrane (2008)	4.44	2.38	4.11	2.22	2.00	1.72	2.15	1.92	2.62	1.05
Hoult *et al*. (2008)	3.96	2.07	3.65	1.93	1.74	1.47	1.82	1.59	2.28	0.97
Razaqpur and Spadea (2010)	0.42	0.56	0.38	0.52	0.59	0.77	0.61	0.81	0.58	0.15
Proposed model	0.97	0.92	0.87	0.84	0.80	0.88	0.83	0.94	0.88	0.05

*S.D. = Standard deviation

5.1 Load-deflection response of test beams

The load deflection response of test beams is given in Fig. 3. The mid-span deflection in beam with lower shear span to depth ratio is found to be lower than the beams with higher shear span to depth ratio. The major factor that influences the deflection is the gross stiffness of the beam in pre-cracking regime. The mid-span deflection in beam in the post cracking regime is influenced by axial stiffness of the reinforcement, longitudinal reinforcement ratio and crack depth. The rotation of concrete blocks at critical diagonal crack interfaces and elongation of the reinforcement significantly influences in the non-linear deflection of the shear dominant beam. The lower axial stiffness of the GFRP rebar is attributed to the increased concrete block rotation.

6. Comparison of prediction with experimental data of present study

The shear strength of eight concrete test beams reinforced with GFRP rebars predicted using the models given by Tottori and Wakui (1993), Michaluk *et al.* (1998), Deitz *et al.* (1999), Wegain and Abdalla (2005), Razaqpur and Isgor (2006), Bentz and Collins (2006), El-Sayed *et al*. (2006), Nehdi *et al.* (2007), El-Sayed and Benmokrane (2008), Hoult *et al.* (2008), Razaqpur and Spadea (2010) and by the model proposed in this study is compared and are given in Table 4. The mean of $V_{c,exp}/V_{c,pred}$ for the eight test beams of the present study for the model proposed by Michaluk *et al.* (1998) is highly conservative and is found to be 9.52. For the models proposed by Deitz *et al*. (1999), El-Sayed and Benmokrane (2006), the mean of the ratio of experimental to predicted shear strength is found be 3.17 and standard deviation of 1.53 and 1.55 respectively. The the mean of $V_{c,exp}/V_{c,pred}$ proposed by Tottori and Wakui (1993), Nehdi *et al*. (2007) is found to be 1.43 and 0.707 respectively. The model proposed by Wegian and Abdalla (2005), Bentz and Collins (2006), Razaqpur and Isgor (2006), El-sayed and Benmokrane (2008) are conservative.

The prediction based on the model proposed in this study is in better agreement with the experimental results and the mean of $V_{c,exp}/V_{c,pred}$ for the eight test beams is found to be 0.88. A design example for the evaluation of shear strength of FRP reinforced concrete beam is given in appendix.

7. Conclusions

The strength and behavior of concrete beams reinforced with FRP bars were investigated experimentally and analytically. It is found that the shear strength of FRP bar reinforced concrete beams is considerably influenced by the shear span to depth ratio and size effect of the beam. The degradation of stiffness of FRP bar reinforced concrete beam is significant in the post cracking regime when shear span to depth ratio is greater than 1.0. The splitting failure of web is also observed for concrete beam reinforced with GFRP bars when tested over a shear span to depth ratio between 0.5 and 1.0. The first cracking load is found to be less than 50 percent of ultimate load. Hence, the experiment demonstrated that there is scope for providing sufficient warning in the post cracking regime in concrete beams reinforced with GFRP bars.

The shear strength of eight concrete test beams reinforced with GFRP bars is evaluated using various models proposed in the literature by Tottori and Wakui (1993), Michaluk *et al.* (1998), Deitz *et al.* (1999), Wegain and Abdalla (2005), Razaqpur and Isgor (2006), Bentz and Collins (2006), El-Sayed *et al.(2006)*, Nehdi *et al.* (2007), El-Sayed and Benmokrane (2008), Hoult *et al.* (2008), Razaqpur and Spadea (2010). The predicted shear strength is compared with corresponding experimental data. The shear span to depth ratio is accounted for in the models proposed by Tottori and Wakui (1993), Wegian and Abdalla (2005), Razaqpur and Isgor (2006), Nehdi *et al.* (2007), Razaqpur and Spadea (2010). The shear strength of FRP reinforced concrete beam predicted using Tottori and Wakui (1993), Razaqpur and Isgor (2006), Bentz and Collins (2006), Nehdi *et al*.(2007), El-Sayed and Benmokrane (2008), Razaqpur and Spadea (2010) was found to be comparable with the experimental results.

The model proposed in this study accounts for the enhancement of shear capacity due to arch action and is considered when shear span to depth ratio is less than 2.5 and size effect of beams. The prediction of shear strength of eight concrete test beams reinforced with GFRP rebars using the proposed model is found to be in better agreement with the corresponding experimental data. The proposed model can be used for predicting the shear strength for a wide range of shear span to depth ratio accurately. A design example for the prediction of shear strength of concrete beam reinforced with FRP bars using the model proposed in this study is illustrated in the Appendix.

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Abbreviations

A_f = area of FRP reinforcement, mm^2
A_r = area of tension reinforcement, mm^2
a = shear span, mm
a/d = shear span to depth ratio
a_g = nominal maximum size of coarse aggregate, mm
b = width of beam, mm
b_w = width of the web, mm
c = depth of neutral axis, mm
d = effective depth of beam, mm
d_v = effective shear depth in mm =0.9d
E_c = modulus of elasticity of concrete, N/mm^2
E_f = modulus of elasticity of FRP longitudinal bars, N/mm^2
E_r = modulus of elasticity of reinforcing bars, N/mm^2
E_s = modulus of elasticity of steel reinforcement, N/mm^2
f_{cr} = cracking strength of concrete, N/mm^2
f_{cu} = cube compressive strength of concrete, N/mm^2
f'_c = compressive strength of concrete, N/mm^2
k = factor which accounts for the load transfer to the support due to strut action
E_r = moment at the section of interest, N-mm
M_d = design bending moment, N-mm
M_f = factored bending moment, N-mm
M_0 = decompression moment, N-mm
N'_d = design axial compressive force, N
Sxe = effective crack spacing for members without stirrups, mm
s_v = spacing of stirrups, mm
V = shear at the section of interest, N
V_c = concrete shear resistance, N
$V_{c,exp}$ = experimental shear strength, N
$V_{c,p}$ = predicted shear strength, N
V_f = factored shear force, N
V_u = total shear resistance of concrete beam, N
λ = a factor that accounts for concrete density, made equal to 1.0
β = longitudinal reinforcement factor
γ_b = member factor (taken equal to 1.0)
γ_c = strength safety factor for concrete (taken equal to 1.0)
ρ_f = FRP longitudinal reinforcement ratio
τ_c = design shear strength of concrete in beams, N/mm^2
η_f = ratio of modulus of elasticity of reinforcing bars to modulus of elasticity of concrete
φ_c = Resistance factor for concrete, made equal to 1.0

Appendix: Design example

Test beam G6-a given in Table 4 is considered.

Compressive strength of concrete, f_{ck} = 65.3 N/mm^2.

Width of beam, b = 170 mm

Overall depth of beam, D = 500 mm

Effective depth of beam, d = 416 mm

Shear span of beam, a = 208 mm

Quantity of tension reinforcement = 6 numbers of 16 mm diameter GFRP bars

Young's modulus of GFRP bar, E_f = 40.8 GPa

Young's modulus of steel bar, E_s = 200 GPa

Shear span to effective depth ratio, a/d = 208/416

= 0.5 < 2.5

Shear enhancement factor, $k_1 = \begin{cases} 2.5/(a/d) & ;\text{when } a/d < 2.5 \\ 1.0 & ;\text{when } a/d \geq 2.5 \end{cases}$

= 2.5/0.5

= 5.0

Size effect factor, $k_2 = \begin{cases} 1.0 & ;\text{when } d \leq 300 \text{ mm} \\ \dfrac{750}{450+d} & ;\text{when } d > 300 \text{ mm} \end{cases}$

= 0.87 (corresponding to d = 416 mm)

Area of GFRP bars, $A_f = n \times \dfrac{\pi \times d_b^2}{4}$

$= 6 \times \dfrac{\pi \times 16^2}{4}$

= 1206.37 mm^2

Area of equivalent steel reinforcement, $A_e = A_f \dfrac{E_f}{E_s}$

$= 1206.37 \times \dfrac{40.8}{200}$

= 246.09 mm^2

Tension reinforcement ratio, $p_t = \dfrac{100A_e}{bd}$

$= \dfrac{100 \times 246.09}{170 \times 416}$

= 0.35

Longitudinal reinforcement factor, $\beta = \dfrac{0.8 f_{ck}}{48.2\ p_t} > 1.0$

$= \dfrac{0.8 \times 65.3}{48.2 \times 0.35}$

= 3.09

Cylinder compressive strength $f_c' = 0.8 * f_{ck}$

$= 0.8 * 65.3$

		= 52.24 MPa
Shear stress of concrete	τ_c	$= \dfrac{0.85\sqrt{f_c^{'}}\left(\sqrt{1+5\beta}-1\right)}{6\beta}$
		$= \dfrac{0.85\sqrt{52.24}\left(\sqrt{1+(5\times 3.09)}-1\right)}{6\times 3.09}$
		= 1.01 N/mm^2
Shear strength predicted,	$V_{c,p}$	$= k_1 k_2 \tau_c bd$
		$= 5.0\times 0.87\times 1.01\times 170\times 416$
		= 310708 N
		= 310.7 kN
Failure load from the experiment		= 600 kN
$V_{c,\exp}$		= 600/2
		= 300 kN
$V_{c,\exp}/V_{c,pred}$		= 300/310.7
		= 0.97