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Design of isolated footings of circular form using a new model

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Abstract. This paper presents the design of reinforced concrete circular footings subjected to axial load and bending in two directions using a new model. The new model considers the soil real pressure acting on contact surface of the circular footings and these are different, with a linear variation in the contact area, these pressures are presented in terms of the axial load, moments around the axis "X" and the axis "Y". The classical model takes into account only the maximum pressure of the soil for design of footings and it is considered uniform at all points of contact area. Also, a comparison is presented in terms of the materials used (steel and concrete) between the two models shown in table, being greater the classical model with respect the new model. Therefore, the new model is the most appropriate, since it is more economic and also is adjusted to real conditions.

Keywords: circular footings design; ground reaction; moments; bending shear; punching shear

1. Introduction

The foundation is the part of the structure which transmits the loads to the soil. Each building demands the need to solve a problem of foundation. The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems (Das *et al*. 2006).

A superficial foundation is a structural member whose cross section is of large dimensions with respect to height and whose function is to transfer the loads of a building at depths relatively short, less than 4 m approximately with respect the level of the natural ground surface (Bowles 1996).

Superficial foundations, whose constructive systems generally do not present major difficulties, may be of various types according to their function; isolated footing, combined footing, strip footing, or mat foundation (Bowles 1996).

The distribution of soil pressure under a footing is a function of the type of soil, the relative rigidity of the soil and the footing, and the depth of foundation at level of contact between footing and soil. A concrete footing on sand will have a pressure distribution similar to Fig. 1(a). When a rigid footing is resting on sandy soil, the sand near the edges of the footing tends to displace laterally when the footing is loaded. This tends to decrease in soil pressure near the edges, whereas soil away from the edges of footing is relatively confined. On the other hand, the pressure

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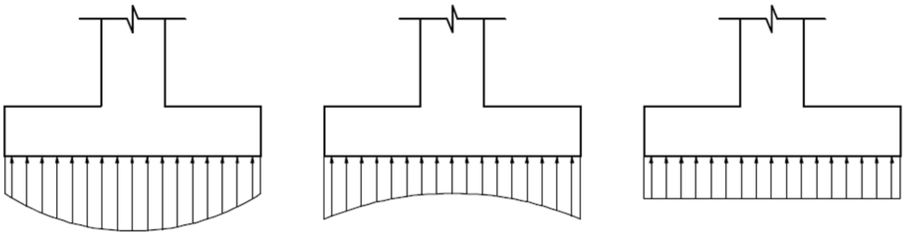

(a) (b) (c)

Fig. 1 Pressure distribution under footing; (a) footing on sand; (b) footing on clay; (c) equivalent uniform distribution

distribution under a footing on clay is similar to Fig. 1(b). As the footing is loaded, the soil under the footing deflects in a bowl-shaped depression, relieving the pressure under the middle of the footing. For design purposes, it is common to assume the soil pressures are linearly distributed. The pressure distribution will be uniform if the centroid of the footing coincides with the resultant of the applied loads, as shown in Fig. 1(c) (Bowles 1996).

In the design of superficial foundations, in the specific case of isolated footings are of three types in terms of the application of loads: 1) The footings subject to concentric axial load, 2) The footings subject to axial load and moment in one direction (unidirectional bending), 3) The footings subject to axial load and moment in two directions (bidirectional bending) (Das *et al.* 2006, Bowles 1996, Calabera-Ruiz 2000, Tomlinson 2008).

The hypothesis used in the classical model is to consider the uniform pressure for the design, i.e., the same pressure at all points of contact in the foundation with the soil, this design pressure is the maximum value that occurs in an isolated footings.

The classical model for dimensioning of circular footings is developed by trial and error, i.e., a dimension is proposed and using the expression of the bidirectional bending for to obtain the stresses acting on the contact surface of the circular footings, which must meet with following conditions: 1) The minimum stress should be equal to or greater than zero, because the soil is not capable of withstand tensile stresses, 2) The maximum stress must be equal or less than the allowable capacity that can withstand the soil (Das *et al.* 2006, Bowles 1996, Calabera-Ruiz 2000, Tomlinson 2008).

Chen *et al.* (2011) proposed the nonlinear partial differential equations of motion for a hybrid composite plate subjected to initial stresses on elastic foundations are established to investigate its nonlinear vibration behavior.

Smith-Pardo (2011) in this study presents a performance-based framework for soil-structure systems using simplified rocking foundation models.

Luévanos-Rojas (2012a) developed a mathematical model to take into account the real pressure of soil acting on the contact surface of the circular footing, when the load is applied to said structural member, this model is presented in function of pressures to obtain solely the moments acting on the circular footings, but in radial shape, i.e., reinforcing steel is available in radial form, which is very difficult for the construction, by the splices of reinforcement that are generated in the center of the circular footings.

Luévanos-Rojas (2012b) presented a mathematical model to obtain solely the most economical diameter, i.e., to find the contact area on the soil for circular footings subjected to axial load and moments in two directions (bidirectional bending), which meets the two conditions mentioned above.

Agrawal and Hora (2012) proposed the building frame and its foundation along with the soil on which it rests, together constitute a complete structural system.

Rad (2012), realized the study on the static behavior of bi-directional functionally graded (FG) non-uniform thickness circular plate resting on quadratically gradient elastic foundations (Winkler-Pasternak type) subjected to axisymmetric transverse and in-plane shear loads is carried out by using state-space and differential quadrature methods.

Luévanos-Rojas (2012c) showed a mathematical model to take into account the real pressure of the soil acting on the contact surface of the circular footings, to obtain the moments and unidirectional shear force in direction of main axes, when applying the load that must support said structural member, to obtain the reinforcement steel in directions "*X*" and "*Y*", also this model is presented in function of pressures.

Orbanich and Ortega (2013) this study aimed to investigate the mechanical behavior of rectangular foundation plates with perimetric beams and internal stiffening beams of the plate is herein analyzed, taking the foundation design into account.

This paper presents a full model for design of reinforced concrete circular footings supporting to a rectangular column subjected to axial load and bending in two directions to obtain: 1) Moment around of an axis *a'-a'* that is parallel to axis "*X-X*" and moment around an axis *b'-b'* that is parallel to axis "*Y-Y*"; 2) Bending shear (unidirectional shear force); 3) Punching shear (bidirectional shear force). The new model considers the real pressure of the soil acting on contact surface of the circular footing with a linear variation in the contact area. The classical model takes into account only the maximum pressure of the soil for design of footings and this is considered uniform at all points of contact area of footings. Also, a comparison is presented in terms of the materials that is used (steel and concrete) between the new model and the traditional model to observe the differences.

2. Methodology

2.1 General conditions

According to Building Code Requirements for Structural Concrete (ACI 318-13) and Commentary the critical sections are: 1) the maximum moment is located in face of column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall; 2) bending shear is presented at a distance "*d*" (distance from extreme compression fiber to centroid of longitudinal tension reinforcement) shall be measured from face of column, pedestal, or wall for footings supporting a column, pedestal, or wall; 3) punching shear is localized so that it perimeter "b_o" is a minimum but need not approach closer than "*d/2*" to: (a) Edges or corners of columns, concentrated loads, or reaction areas; and (b) Changes in slab thickness such as edges of capitals, drop panels, or shear caps.

The general equation for any type of footings subjected to bidirectional bending (Luévanos-Rojas 2012a, b, c, Gere and Goodno 2009)

$$\sigma = \frac{P}{A} \pm \frac{M_x C_y}{I_x} \pm \frac{M_y C_x}{I_y} \tag{1}$$

where: σ is the stress exerted by the soil on the footing (soil pressure), A is the contact area of the

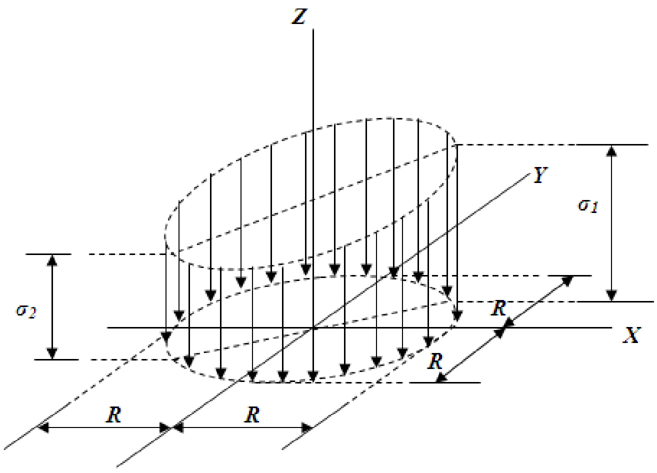

Fig. 2 Pressures soil on the foundation

footing, P is the axial load applied at the center of gravity of the footing, M_x is the moment around the axis "*X*", M_y is the moment around the axis "*Y*", C_x is the distance in the direction "*X*" measured from the axis "*Y*" up the farthest end, C_y is the distance in direction "*Y*" measured from the axis "*X*" up the farthest end, I_y is the moment of inertia around the axis "*Y*" and I_x is the moment of inertia around the axis "*X*".

2.2. New model

Fig. 2 shows the pressures diagram for a circular footing subjected to axial load and moment in two directions (bidirectional bending), where pressures are presented different linearly varying along the entire contact surface.

Fig. 3 presents a typical circular footing to obtain the stresses in any point of the contact surface of said structural member due to pressure exerted by the soil.

The stresses are found by the Eq. (1) at any point on a circular footing subjected bidirectional bending

$$\sigma(x,y) = \frac{P}{\pi R^2} + \frac{4M_x y}{\pi R^4} + \frac{4M_y x}{\pi R^4} \quad (2)$$

where: R is the radius of the footing, $A = \pi R^2$, $I_x = \pi R^4/4$, $I_y = \pi R^4/4$, $C_x = x$, $C_y = y$.

2.2.1 Model to obtain the moments

Critical sections for moments are presented in section *a'- a'* and *b'-b'*, as shown in Fig. 4.

2.2.1.1 Moment around of axis a'-a'

The resultant force "F_{R1}" is obtained through the volume of pressure the area formed of the semicircle that is above the axis *a'-a'* of the footing, it is presented as follows (Piskunov 2004)

$$F_{R1} = \int_{c_1/2}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \sigma(x,y)\,dx\,dy \quad (3)$$

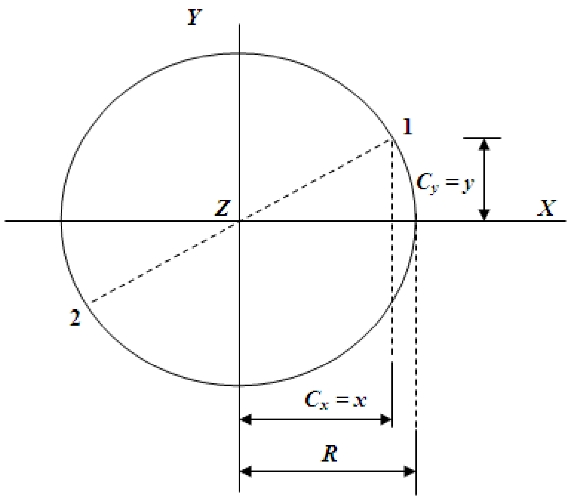

Fig. 3 Typical circular footing

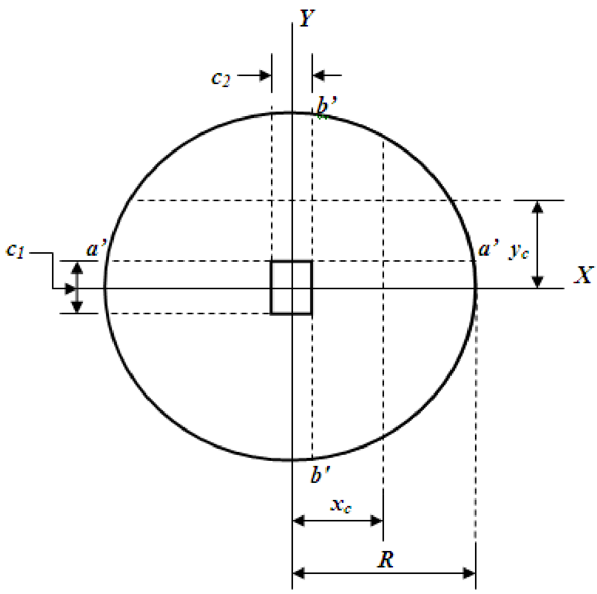

Fig. 4 Critical sections for moments

Eq. (2) is substituted into Eq. (3)

$$F_{R1} = \int_{c_1/2}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \left[\frac{P}{\pi R^2} + \frac{4M_x y}{\pi R^4} + \frac{4M_y x}{\pi R^4}\right] dxdy \tag{4}$$

where: c_1 is the dimension of the column parallel to the axis "*Y*", c_2 is the dimension of the column parallel to the axis "*X*".

The integral double of the Eq. (4) is developed and after the respective boundary conditions are substituted

$$F_{R1}=\left\{\frac{P}{\pi R^2}\left[\frac{\pi R^2}{2}-\frac{c_1\sqrt{4R^2-c_1{}^2}}{4}-R^2\,Asin\left(\frac{c_1}{2R}\right)\right]+\frac{M_x(4R^2-c_1{}^2)^{3/2}}{3\pi R^4}\right\} \tag{5}$$

Now, the following integral is developed to obtain the gravity center "y_c" of soil pressure (Piskunov 2004)

$$y_c=\frac{\int_{c_1/2}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\sigma(x,y)y\,dxdy}{\int_{c_1/2}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\sigma(x,y)dxdy} \tag{6}$$

Eq. (2) is substituted into Eq. (6)

$$y_c=\frac{\int_{c_1/2}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\left[\frac{P}{\pi R^2}+\frac{4M_xy}{\pi R^4}+\frac{4M_yx}{\pi R^4}\right]y\,dxdy}{\int_{c_1/2}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\left[\frac{P}{\pi R^2}+\frac{4M_xy}{\pi R^4}+\frac{4M_yx}{\pi R^4}\right]dxdy} \tag{7}$$

The integral double of the Eq. (7) is developed and after the respective boundary conditions are substituted

$$y_c=\frac{\left\{\frac{P(4R^2-c_1{}^2)^{3/2}}{12\pi R^2}+\frac{M_x}{\pi R^4}\left[\frac{\pi R^4}{2}+\frac{c_1(4R^2-c_1{}^2)^{3/2}}{8}-\frac{R^2c_1\sqrt{4R^2-c_1{}^2}}{4}-R^4\,Asin\left(\frac{c_1}{2R}\right)\right]\right\}}{\left\{\frac{P}{\pi R^2}\left[\frac{\pi R^2}{2}-\frac{c_1\sqrt{4R^2-c_1{}^2}}{4}-R^2\,Asin\left(\frac{c_1}{2R}\right)\right]+\frac{M_x(4R^2-c_1{}^2)^{3/2}}{3\pi R^4}\right\}} \tag{8}$$

Moment around the axis *a'-a'* is found by the following equation

$$M_{a\prime-a\prime}=F_{R1}(y_c-c_1/2) \tag{9}$$

2.2.1.2 Moment around of axis b'-b'

The resultant force "F_{R2}" is obtained through the volume of pressure of the area formed of the circle that is to right side the axis *b*'-*b'* of the footing, it is presented as follows (Piskunov 2004)

$$F_{R2}=\int_{c_2/2}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\sigma(x,y)dxdy \tag{10}$$

Eq. (2) is substituted into Eq. (10)

$$F_{R2}=\int_{c_2/2}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\left[\frac{P}{\pi R^2}+\frac{4M_xy}{\pi R^4}+\frac{4M_yx}{\pi R^4}\right]dxdy \tag{11}$$

The integral double of the Eq. (11) is developed and after the respective boundary conditions are substituted

$$F_{R2}=\left\{\frac{P}{\pi R^2}\left[\frac{\pi R^2}{2}-\frac{c_2\sqrt{4R^2-c_2{}^2}}{4}-R^2\,Asin\left(\frac{c_2}{2R}\right)\right]+\frac{M_y(4R^2-c_2{}^2)^{3/2}}{3\pi R^4}\right\} \tag{12}$$

Now, the following integral is developed to obtain the gravity center "x_c" of soil pressure (Piskunov 2004)

$$x_c=\frac{\int_{c_2/2}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\sigma(x,y)x\,dydx}{\int_{c_2/2}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\sigma(x,y)\,dydx} \tag{13}$$

Eq. (2) is substituted into Eq. (13)

$$x_c=\frac{\int_{c_2/2}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\left[\frac{P}{\pi R^2}+\frac{4M_xy}{\pi R^4}+\frac{4M_yx}{\pi R^4}\right]x\,dydx}{\int_{c_2/2}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-y^2}}\left[\frac{P}{\pi R^2}+\frac{4M_xy}{\pi R^4}+\frac{4M_yx}{\pi R^4}\right]dydx} \tag{14}$$

The integral double of the Eq. (14) is developed and after the respective boundary conditions are substituted

$$x_c=\frac{\left\{\frac{P(4R^2-c_2{}^2)^{3/2}}{12\pi R^2}+\frac{M_y}{\pi R^4}\left[\frac{\pi R^4}{2}+\frac{c_2(4R^2-c_2{}^2)^{3/2}}{8}-\frac{R^2c_2\sqrt{4R^2-c_2{}^2}}{4}-R^4\,Asin\left(\frac{c_2}{2R}\right)\right]\right\}}{\left\{\frac{P}{\pi R^2}\left[\frac{\pi R^2}{2}-\frac{c_2\sqrt{4R^2-c_2{}^2}}{4}-R^2\,Asin\left(\frac{c_2}{2R}\right)\right]+\frac{M_y(4R^2-c_2{}^2)^{3/2}}{3\pi R^4}\right\}} \tag{15}$$

Moment around the axis *b'*-*b'* is found by the following equation

$$M_{b\prime-b\prime}=F_{R2}(x_c-c_2/2) \tag{16}$$

2.2.2 Model to obtain the bending shear (unidirectional shear force)

The critical section for the bending shear is obtained at a distance "*d*" to from the junction of the column with the footing is presented in section *c'*- *c'* as seen in Fig. 5.

The bending shear "V_f" is obtained through the volume of pressure the shaded area of the circle that is above the axis *c'*-*c'* of the footing, it is presented as follows (Piskunov 2004)

$$V_f=\int_{c_1/2+d}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\sigma(x,y)\,dxdy \tag{17}$$

where: "*d*" is distance measured vertically from extreme compression fiber to centroid of longitudinal tension reinforcement of the footing.

Eq. (2) is substituted into Eq. (17)

$$V_f=\int_{c_1/2+d}^{R}\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}}\left[\frac{P}{\pi R^2}+\frac{4M_xy}{\pi R^4}+\frac{4M_yx}{\pi R^4}\right]dxdy \tag{18}$$

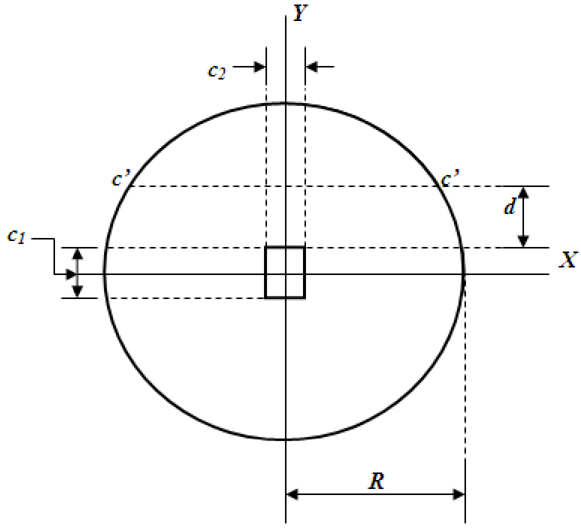

Fig. 5 Critical sections for the bending shear

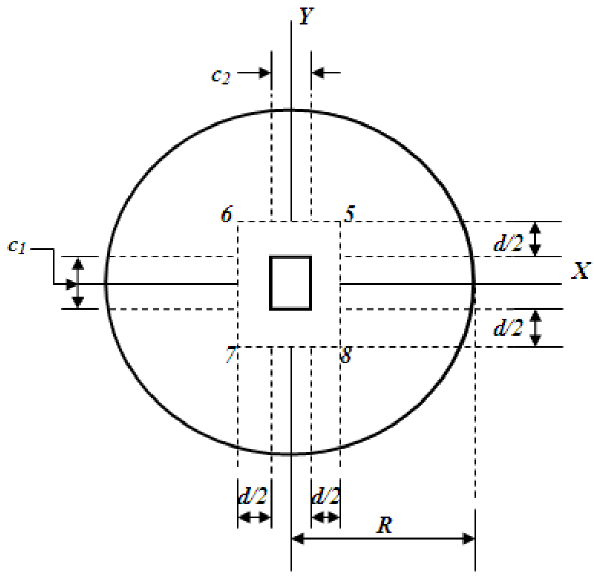

Fig. 6 Critical sections for the punching shear supporting a rectangular column

The integral double of the Eq. (18) is developed and after the respective boundary conditions are substituted

$$V_f = \left\{ \frac{2P}{\pi R^2} \left[\frac{\pi R^2}{4} - \frac{(c_1/2 + d)\sqrt{R^2 - (c_1/2 + d)^2}}{2} - \frac{R^2}{2} Asin\left(\frac{c_1/2 + d}{R} \right) \right] + \frac{8M_x[R^2 - (c_1/2 + d)^2]^{3/2}}{3\pi R^4} \right\} \tag{19}$$

2.2.3 Model to obtain the punching shear (bidirectional shear force)

The critical section for the punching shear appears at a distance "*d/2*" to from the junction of the column with the footing in the two directions occurs in the rectangular section formed by the points 5, 6, 7 and 8, as shown in Fig. 6.

The punching shear acting on the footing "V_p" is obtained through the volume of pressure the total area less the rectangular area formed by points 5, 6, 7 and 8.

The force formed by the total area of the footing "F_T" is as follows (Piskunov 2004)

$$F_T = \int_{-R}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \sigma(x, y) dxdy \tag{20}$$

Eq. (2) is substituted into Eq. (20)

$$F_T = \int_{-R}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \left[\frac{P}{\pi R^2} + \frac{4M_x y}{\pi R^4} + \frac{4M_y x}{\pi R^4}\right] dxdy \tag{21}$$

The integral double of the Eq. (21) is developed and after the respective boundary conditions are substituted

$$F_T = P \tag{22}$$

The force formed by rectangular area of point's 5, 6, 7 and 8, "F_{5678}" of the footing is as follows (Piskunov 2004)

$$F_{5678} = \int_{-c_1/2-d/2}^{c_1/2+d/2} \int_{-c_2/2-d/2}^{c_2/2+d/2} \sigma(x, y) dxdy \tag{23}$$

Eq. (2) is substituted into Eq. (23)

$$F_{5678} = \int_{-c_1/2-d/2}^{c_1/2+d/2} \int_{-c_2/2-d/2}^{c_2/2+d/2} \left[\frac{P}{\pi R^2} + \frac{4M_x y}{\pi R^4} + \frac{4M_y x}{\pi R^4}\right] dxdy \tag{24}$$

The integral double of the Eq. (24) is developed and after the respective boundary conditions are substituted

$$F_{5678} = \frac{P(c_1 + d)(c_2 + d)}{\pi R^2} \tag{25}$$

Now, the punching shear "V_p" is as follows

$$V_P = F_T - F_{5678} \tag{26}$$

Eqs. (22)-(25) is substituted into Eq. (26)

$$V_P = P - \frac{P(c_1 + d)(c_2 + d)}{\pi R^2} \tag{27}$$

2.3 Classical model

This model takes into account only the maximum pressure of the soil for design of footings and it is considered uniform at all points of contact area of footings.

2.3.1 Model to obtain the moments

Critical sections for moments are presented in section *a'-a'* and *b'-b'*, as shown in Fig. 4.

2.3.1.1 Moment around of axis a'-a'

Equation is

$$M_{a'-a'} = \sigma_{umax}\left\{\frac{(4R^2 - {c_1}^2)^{3/2}}{12} - \frac{c_1}{2}\left[\frac{\pi R^2}{2} - \frac{c_1\sqrt{4R^2 - {c_1}^2}}{4} - R^2\,Asin\left(\frac{c_1}{2R}\right)\right]\right\} \quad (28)$$

$$\sigma_{umax} = \frac{P_u}{\pi R^2} + \frac{4\sqrt{{M_{ux}}^2 + {M_{uy}}^2}}{\pi R^3} \quad (29)$$

2.3.1.2 Moment around of axis b'-b'

Analytical expression is

$$M_{b'-b'} = \sigma_{umax}\left\{\frac{(4R^2 - {c_2}^2)^{3/2}}{12} - \frac{c_2}{2}\left[\frac{\pi R^2}{2} - \frac{c_2\sqrt{4R^2 - {c_2}^2}}{4} - R^2\,Asin\left(\frac{c_2}{2R}\right)\right]\right\} \quad (30)$$

2.3.2 Model to obtain the bending shear (unidirectional shear force)

The critical section for the bending shear is obtained at a distance *"d"* to from the junction of the column with the footing is presented in section *c'-c'* as seen in Fig. 5. Equation is

$$V_f = \sigma_{umax}\left[\frac{\pi R^2}{2} - \frac{(c_1 + 2d)}{4}\sqrt{4R^2 - (c_1 + 2d)^2} - R^2\,Asin\left(\frac{c_1 + 2d}{2R}\right)\right] \quad (31)$$

2.3.3 Model to obtain the punching shear (bidirectional shear force)

The critical section for the punching shear appears at a distance "*d*/2" to from the junction of the column with the footing in the two directions as seen in Fig. 6. Analytical expression is

$$V_p = \sigma_{umax}[\pi R^2 - (c_1 + d)(c_2 + d)] \quad (32)$$

2.4 Procedure of design

Step 1: The mechanical elements (P, M_x, M_y) acting on the footing is obtained by the sum of: the dead loads, live loads and accidental loads (wind or earthquake) from each of these effects (Gambhir 2008, González-Cuevas and Robles-Fernández-Villegas 2005, McCormac and Brown 2013, Mosley *et al.* 1999, Parker 1996, Punmia *et al.* 2007).

Step 2: The available load capacity of the soil "σ_{max}" is (Gambhir 2008, González-Cuevas and Robles-Fernández-Villegas 2005, McCormac and Brown 2013, Mosley *et al.* 1999, Parker 1996, Punmia *et al.* 2007)

$$\sigma_{max} = q_a - \gamma_{ppz} - \gamma_{pps} \quad (33)$$

where: q_a is the allowable load capacity of the soil, γ_{ppz} is the self-weight of the footing, γ_{pps} is the self-weight of soil fill.

Step 3: The value of "*R*" is selected according to the following equations (Luévanos-Rojas 2012b)

$$R = \frac{4\sqrt{{M_x}^2 + {M_y}^2}}{P} \tag{34}$$

$$\sigma_{max}\pi R^3 - PR - 4\sqrt{{M_x}^2 + {M_y}^2} = 0 \tag{35}$$

where: the value of "*R*" obtained from the Eq. (34) is when the soil pressure is zero and the value of "*R*" found in Eq. (35) is when the pressure of the soil is load capacity available "$\sigma_{\max}$", of these two values is taken the greater to meet the two conditions, because the pressure generated by footing must greater than zero and less than the load capacity available of the soil. Note: if in the combinations are included the wind and/or the earthquake, the load capacity of the soil can be increased by 33% (ACI 318S-13).

Step 4: The mechanical elements (P, M_x, M_y) acting on the footing is factored (ACI 318S-13).

Step 5: The maximum moment acting on the circular footing is obtained by Eqs. (9)-(16) for new model, and Eqs. (28)-(30) for classical model, such critical section is located in the junction of the column with the footing as shown in Fig. 4.

Step 6: The effective depth "*d*" for the maximum moment is found by means of the following expression (ACI 318S-13)

$$d = \sqrt{\frac{M_u}{\emptyset_f b_w \rho f_y \left[1 - \frac{0.59\rho f_y}{f'_c}\right]}} \tag{36}$$

where: b_w to bending is

$$b_w = \sqrt{4R^2 - {c_1}^2} \tag{37}$$

where: M_u is the factored maximum moment at section acting on the footing, $\emptyset_f$ is the strength reduction factor by bending and its value is 0.90, b_w is width of analysis in structural member, ρ is ratio of "*As*" to "$b_w d$", f_y is the specified yield strength of reinforcement of steel, f'_c is the specified compressive strength of concrete at 28 days.

Step 7: Bending shear (unidirectional shear force) resisted by the concrete "V_{cf}" is given (ACI 318S-13)

$$\emptyset_v V_{cf} = 0.17\emptyset_v\sqrt{f'_c}\, b_w d \tag{38}$$

where: b_w to bending shear is

$$b_w = \sqrt{4R^2 - (c_1 + 2d)^2} \tag{39}$$

To bending shear acting on the footing (V_f) is compared vs. bending shear resisting by concrete (V_{cf}) and must comply with the following expression (ACI 318S-13)

$$V_f \le \emptyset_v V_{cf} \tag{40}$$

where: $\emptyset_v$ is the strength reduction factor by shear is 0.85.

Step 8: Punching shear (shear force bidirectional) resisted by the concrete "V_{cp}" is given (ACI 318S-13)

$$\emptyset_v V_{cp} = 0.17\emptyset_v \left(1 + \frac{2}{\beta_c}\right)\sqrt{f'_c}b_0 d \tag{41a}$$

where: β_c is the ratio of long side to short side of the column and b_0 is the perimeter of the critical section.

$$\emptyset_v V_{cp} = 0.083\emptyset_v \left(\frac{\alpha_s d}{b_0} + 2\right)\sqrt{f'_c}b_0 d \tag{41b}$$

where: α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns.

$$\emptyset_v V_{cp} = 0.33\emptyset_v\sqrt{f'_c}b_0 d \tag{41c}$$

where: $\emptyset_v V_{cp}$ must be the value smallest of Eqs. (41*a*), (41*b*), (41*c*).

To punching shear acting on the footing (V_p) is compared vs. punching shear resisting by concrete (V_{cp}) and must comply with the following expression (ACI 318S-13)

$$V_p \leq \emptyset_v V_{cp} \tag{42}$$

Step 9: The main reinforcement steel (parallel reinforcement steel to the direction of the axis "*Y*" of the footing) "A_{sp}" is calculated with the following expression (ACI 318S-13)

$$A_{sp} = wb_w d - \sqrt{(wb_w d)^2 - \frac{2M_u w b_w}{\emptyset_f f_y}} \tag{43}$$

where: w is $0.85f'_c/f_y$.

Minimum steel "A_{smin}" by rule is (ACI 318S-13)

$$A_{smin} = \rho_{min} b_w d \tag{44}$$

where: ρ_{min} is the minimum percentage of reinforcement steel is obtained (ACI 318S-13)

$$\rho_{min} = \frac{1.4}{f_y} \tag{45}$$

The parallel reinforcement steel in the direction of the axis "*X*" is used same Eq. (43) substituting $M_u = M_{b'-b'}$.

After, the space of the bars "*s*" is obtained

$$s = \frac{b_w a_s}{A_s} \tag{46}$$

where: a_s is the rod area used.

Step 10: The development length for deformed bars "l_d" is expressed (ACI 318S-13)

$$l_d = \frac{f_y \psi_t \psi_e}{6.6\sqrt{f'_c}} d_b \tag{47}$$

where: l_d is the minimum length that should have a deformed bar to prevent slippage, ψ_t is the traditional factor of location of the reinforcing steel which reflects the adverse effects of the position of the bars of the upper part of the section with respect at the height of fresh concrete located beneath them, ψ_e is a coating factor which reflects the effects of the epoxy coating, d_b is the diameter of the bars.

The development length for deformed bars *"l_d"* is compared vs. the available length of the footing *"l_a"* and must comply with the following expression (ACI 318S-13)

$$l_d \leq l_a \tag{48}$$

3. Application

The design of an isolated footing of circular form that supports a square column is presented in Fig. 7, with the basic information following: c_1=40 cm; c_2=40 cm; H=1.5 m; P_D=700 kN; P_L=500 kN; M_{Dx}=40 kN-m; M_{Lx}=100 kN-m; M_{Dy}=120 kN-m; M_{Ly}=80 kN-m; f'_c=21 MPa; f_y=420 MPa; q_a=220 kN/m^2; γ_{ppz}=24 kN/m^3; γ_{pps}=15 kN/m^3.
where: H is the depth of the footing, P_D is the dead load, P_L is the live load, M_{Dx} is the moment around of axis "X-X" of dead load, M_{Lx} is the moment around of axis "X-X" of live load, M_{Dy} is the moment around of axis "Y-Y" of dead load, M_{Ly} is the moment around of axis "Y-Y" of live load.

3.1 New model

Step 1: The loads and moments acting on soil:

$$P = P_D + P_L = 700 + 500 = 1200\ kN$$

$$M_x = M_{Dx} + M_{Lx} = 140 + 100 = 240\ kN\text{-}m$$

$$M_y = M_{Dy} + M_{Ly} = 120 + 80 = 200\ kN\text{-}m$$

Step 2: The available load capacity of the soil:

The thickness "t" of the footing is proposed, the first proposal is the minimum thickness of 25 cm marking regulations, subsequently the thickness is revised to meet the following conditions: moment, shear force by bending and shear force by penetration. If such conditions are not satisfied is proposed a greater thickness until it fulfills the three conditions mentioned.

The thickness of the footing it fulfills the three conditions listed above is *50 cm*.

$$\sigma_{max} = q_a - \gamma_{ppz} - \gamma_{pps} = 220 - 24(0.50) - 15(1.5 - 0.50) = 193.00\ kN/m^2$$

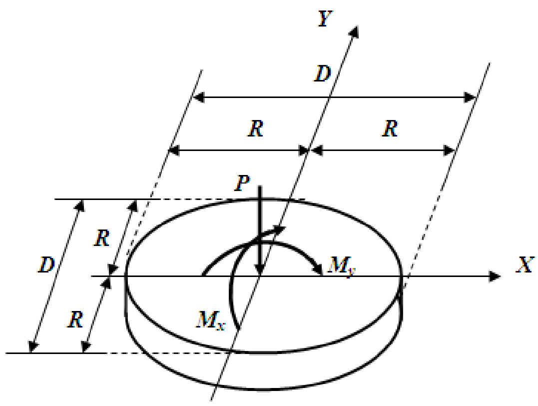

Fig. 7 Isolated footing of circular form

Step 3: The value of “R” is:
First condition is obtained by Eq. (34):

$$R = 1.041\ m$$

Second condition is found by Eq. (35):

$$R = 1.773\ m$$

Then, the greater value of “R” is considered for to meet the two mentioned conditions is 1.773 m.
Therefore the dimension of the footing is:

$$D = 3.60\ m;\ R = 1.80\ m$$

Step 4: The mechanical elements (P, M_x, M_y) acting on the footing is factored:

$$P_u = 1.2P_D + 1.6P_L = 1.2(700) + 1.6(500) = 1640\ kN$$

$$M_{ux} = 1.2M_{Dx} + 1.6M_{Lx} = 1.2(140) + 1.6(100) = 328\ kN\text{-}m$$

$$M_{uy} = 1.2M_{Dy} + 1.6M_{Ly} = 1.2(120) + 1.6(80) = 272\ kN\text{-}m$$

Step 5: The maximum moment acting on the footing is:
The maximum moment around of axis a'-a' acting on the footing according to Fig. 4 is found by Eqs. (5)-(8)-(9):

$$F_{R1} = 856.05\ kN; y_c = 0.91\ m;$$

$$M_{a\prime-a\prime} = 779.01\ kN\text{-}m$$

The maximum moment around of axis b'-b' acting on the footing according to Fig. 4 is found by Eqs. (12)-(15)-(16):

$$F_{R2} = 830.13\ kN;\ x_c = 0.90\ m;$$

$$M_{b\prime-b\prime} = 747.12\ kN\text{-}m$$

Step 6: The effective depth for the maximum moment is found:
Width of analysis “b_w” in structural member is obtained by Eq. (37):

$$b_w = 3.58\ m$$

The effective depth *“d”* for the maximum moment is found by means of the Eq. (36):
where: $M_{a'-a'} = M_u$

$$d = 21.06\ cm$$

Then, we are proposed the final dimensions of footing after performing different proposals:

$$d = 42\ cm; r_1 = 8\ cm; t = 50\ cm$$

where: r_1 is the coating.

Step 7: Bending shear (unidirectional shear force) is:
Width of analysis “b_w” to bending shear is found through Eq. (39):

$$b_w = 3.38\ m$$

Using Eq. (38) is obtained the bending shear resisted by the concrete:

$$\emptyset_v V_{cf} = 940.03\ kN$$

Bending shear acting on footing is found by Eq. (19):

$$V_f = 595.61\ kN$$

$$V_f \leq \emptyset_v V_{cf}, \qquad O.K.$$

Step 8: Punching shear (bidirectional shear force) is:

Using Eqs. (41a)-(41b)-(41c) is obtained the punching shear resisted by the concrete:

$$\emptyset_v V_{cp} = 2736.67\ kN;$$

$$\emptyset_v V_{cp} = 3171.97\ kN;\ \emptyset_v V_{cp} = 1770.78\ kN$$

Punching shear acting on footing is found by Eq. (27):

$$V_p = 1531.66\ kN$$

$$V_p \leq \emptyset_v V_{cp}, \qquad O.K.$$

Step 9: The reinforcement steel is:

* The parallel reinforcement steel in direction of axis "*Y*" is obtained Eq. (43):

$$w = 0.0425;\ A_{sp} = 51.11\ cm^2$$

Minimum percentage of reinforcement steel "ρ_{min}" is obtained steel by Eq. (45) and after, the minimum steel "A_{smin}" is found by Eq. (44):

$$\rho_{min} = 0.00333;\ A_{smin} = 50.07\ cm^2$$

Therefore, main reinforcement steel is proposed "A_{sp}".

Rod "3/4" diameter is used:

The space of the bars "*s*" is obtained by Eq. (46):

$$s = 19.96\ cm \approx 20\ cm$$

* The parallel reinforcement steel in direction of axis "*X*" is:

$$A_{sp} = 48.93\ cm^2$$

Therefore, minimum steel is proposed "A_{smin}".

Rod "3/4" diameter is used:

$$s = 20.85\ cm \approx 20\ cm$$

Step 10: The minimum development length for deformed bars is found by Eq. (47):
where: ψ_t=1 and ψ_e=1.

$$l_d = 83.14\ cm$$

3.2 Classical model

Steps 1 to 4: Those are the same as for the new model**.**

Step 5: The maximum moment acting on the footing is:

The maximum pressure is obtained by Eq. (29):

$$\sigma_{umax} = 254.15\ kN/m^2$$

The maximum moment around of axis *a'*-*a'* and the axis *b'*-*b'* acting on the footing according to Fig. 4 is found by Eqs. (28)-(30) respectively:

$$M_{a'-a'} = 747.72\ kN\text{-}m$$

$$M_{b'-b'} = 747.72\ kN\text{-}m$$

Step 6: The effective depth for the maximum moment is found:

Width of analysis “b_w” in structural member is obtained by Eq. (37):

$$b_w = 3.58\ m$$

The effective depth “*d*” for the maximum moment is found by means of the Eq. (36):
where: $M_{a'-a'}=M_u$

$$d = 20.63\ cm$$

Then, we are proposed the final dimensions of footing after performing different proposals:

$$d = 52\ cm; r_1 = 8\ cm; t = 60\ cm$$

Step 7: Bending shear (unidirectional shear force) is:
Width of analysis “b_w” to bending shear is found through Eq. (39):

$$b_w = 3.30\ m$$

Using Eq. (38) is obtained the bending shear resisted by the concrete:

$$\emptyset_v V_{cf} = 1136.30\ kN$$

Bending shear acting on footing is found by Eq. (31):

$$V_f = 652.78\ kN$$

$$V_f \leq \emptyset_v V_{cf}, \qquad O.K.$$

Step 8: Punching shear (bidirectional shear force) is:
Using Eqs. (41*a*)-(41*b*)-(41*c*) is obtained the punching shear resisted by the concrete:

$$\emptyset_v V_{cp} = 3801.46\ kN;$$

$$\emptyset_v V_{cp} = 4734.16\ kN;\ \emptyset_v V_{cp} = 2459.77\ kN$$

Punching shear acting on footing is found by Eq. (32):

$$V_p = 2371.82\ kN$$

$$V_p \leq \emptyset_v V_{cp}, \qquad O.K.$$

Step 9: The reinforcement steel is:
* The parallel reinforcement steel in direction of axis “*Y*” is obtained Eq. (43):

$$w = 0.0425$$

$$A_{sp} = 39.00\ cm^2$$

Minimum percentage of reinforcement steel “ρ_{min}” is obtained steel by Eq. (45) and after, the minimum steel “A_{smin}” is found by Eq. (44):

$$\rho_{min} = 0.00333;\ A_{smin} = 61.99\ cm^2$$

Therefore, minimum steel is proposed “A_{smin}”.
Rod “3/4” diameter is used:
The space of the bars “*s*” is obtained by Eq. (46):

$$s = 16.46\ cm \approx 16\ cm$$

* The parallel reinforcement steel in direction of axis “*X*” is:
Then, if the moment is the same as in the direction the axis “*Y*”, will be the same reinforcement steel.

Step 10: This step is the same as for the new model.

Table 1 Comparison of results

Concept	Classical model CM	New model NM	CM/NM
Maximum moment acting $M_{a'-a'}$ (kN-m)	747.72	779.01	0.96
Maximum moment acting $M_{b'-b'}$ (kN-m)	747.72	747.12	1.00
Effective depth d (cm)	52	42	1.24
Total thickness t (cm)	60	50	1.20
Volume of concrete (m^3)	6.11	5.09	1.20
Bending shear acting V_f (kN)	652.78	595.61	1.10
Punching shear acting V_p (kN)	2371.82	1531.66	1.55
Parallel reinforcement steel in direction of axis "Y" of the footing A_s (cm^2)	61.99	51.11	1.21
Parallel reinforcement steel in direction of axis "X" of the footing A_s (cm^2)	61.99	50.07	1.24

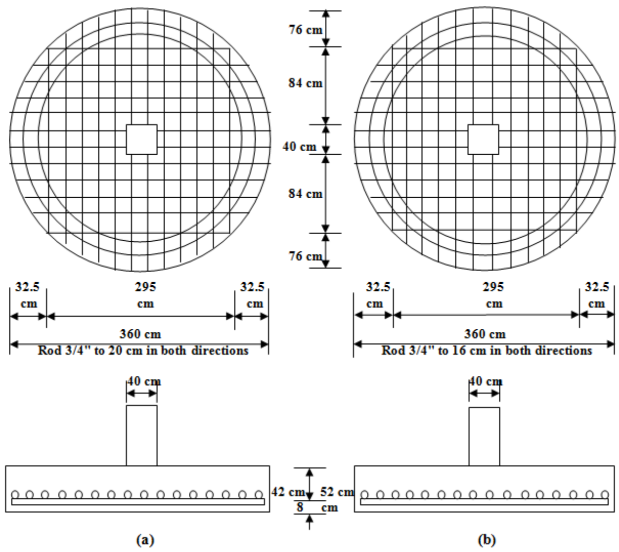

Fig. 8 Isolated footing in plant and elevation: (a) New model, (b) Classical model

4. Results

Table 1 shows the differences between the two models and Fig. 8 presents the concrete dimensions and reinforcement steel of the two footings. Minimum length of the rods is 84 cm to comply with the length of development presented in step 10, and the length of 295 cm is the width where these rods are distributed, length is obtained as follows:

$$\sqrt{(3.60)^2 - (0.84 + 0.40 + 0.84)^2} = 2.94\, m \ \approx 2.95\, m.$$

Reinforcing steel in edge is placed to meet the development length in annular distance of 32.5 cm, as show Fig. 8.

Effects that govern the design for isolated footings are: moments, bending shear, and punching shear:

a) With regarding the maximum moments acting on the footing in direction of axis "Y", there is an increase in a 4% of the new model with respect to classical model, and in direction of axis "X" there is not increase between the two models.

b) In terms bending shear acting on the footing there is an increase of 10% in classical model with respect to new model.

c) According to punching shear acting on the footing, in this concept is presented the greater increase of a 55% in classical model with respect to new model.

Materials used for the construction of an isolated footing are: concrete and reinforcement steel:

a) In terms of concrete, there is a saving of 20% in the new model with respect to the classical model.

b) For reinforcement steel in direction of axis "Y" of the footing, there is a saving of 21% in the new model with respect to the classical model, and in direction of axis "X" of the footing, there is a saving of 24 % in the new model with respect to the classical model.

5. Conclusions

The maximum moments acting on the isolated footing are greater in the traditional model in comparison to proposed model. This is a logical situation, because in traditional model, the design pressure is same in all the contact area of the footing on soil, being this the maximum pressure that is presented in said structural member, But the proposed model reduces the pressure, due to the fact it counts real pressure, which is a linear variation of contact total area and it goes from a maximum pressure up to the minimum pressure. Consequently the effective depth and the thickness of the footing are reduced.

In terms of the dimensions of the isolated footing it is showed that diameter "D" is equal in the two models, but the thickness of the footing "t" is different, being less in the proposed model with respect to traditional model.

With respect the parallel reinforcement steel in direction of axis "X" and "Y" of the footing, the traditional model is greater with respect to proposed model, but in direction of axis "Y" the difference is less, because the traditional model governs the minimum steel for design and in proposed model governs the reinforcement steel main that is the maximum moment acting on the isolated footing.

This means that can have great savings in terms of materials used (reinforcing steel and concrete) for the fabrication of footings isolated under conditions mentioned above. Since that the principle in civil engineering, in terms of structural conditions is that be safe and economical, and the last is not satisfied in traditional model.

Therefore, the practice of using the traditional model is not a recommended solution, because are very exceeded the materials in some cases, with regard to design of these structural members.

Then, we propose using the model developed in this paper for the structural design of isolated footings of circular form subject to axial load and moment in two directions (bidirectional bending), also, it can be applied to others cases: 1) The footings subjected to a concentric axial load, 2) The footings subjected to a axial load and moment in one direction (unidirectional bending). Moreover, the proposed model is the most appropriate, since it is more economic and also is adjusted to real conditions.

The model presented in this paper applies only for the foundations design, footings are assumed to be rigid and the supporting soil layers elastic, which meet expression of the bidirectional bending, i.e., the variation of pressure is linear. The suggestions for future research, when is presented another type of soil, by example in pure cohesive soils and pure granular soils, the pressures diagram is not linear and should be treated differently.

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