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Buckling load optimization of laminated plates via artificial bee colony algorithm

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Abstract. In this present work, Artificial Bee Colony Algorithm (ABCA) is used to optimize the stacking sequences of simply supported antisymmetric laminated composite plates with criticial buckling load as the objective functions. The fibre orientations of the layers are selected as the optimization design variables with the aim to find the optimal laminated plates. In order to perform the optimization based on the ABCA, a special code is written in MATLAB software environment. Several numerical examples are presented to illustrate this optimization algorithm for different plate aspect ratios, number of layers and load ratios.

Keywords: laminated plates; artificial bee colony algorithm; buckling; optimization

1. Introduction

As part of the massive introduction of the composite materials within aircraft manufacturing, a particular attention is turned on the buckling behaviour and energy dissipation capacities of basic structures. In industry, composite laminated materials have proved their efficiency in the manufacture of primary structures parts, due to their performance, lightness and form versatility. The design of this type of structures requires more and more sophisticated mechanical modeling tools for taking into account the particularities of these materials. Numerical methods and notably the finite element method are necessary for dimensioning complex composite structures. Due to their behaviour complexity, the analysis of laminated plates remains an open research problem.

Structural instability becomes an important challenge regarding reliable and feasible design of composite plates. Buckling load optimization of the laminated composite plates has been extensively studied in the literature. Karakaya and Soykasap (2009) used genetic algorithm and generalized pattern search algorithm for optimal stacking sequence of a simply supported composite panel subjected to biaxial in-plane compressive loads. Sebaey *et al.* (2011) maximized critical buckling load of laminated composite panels under biaxial compression loads using ant colony optimization technique. Raju *et al.* (2012) presented differential evolution algorithm technique to design a rectangular composite plate by optimising the laminate stacking sequence whose aim was to maximise the buckling load capability without any failure at ply level. Rao and Arvind (2007) investigated optimal stacking sequence design of laminated composite plates for

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maximizing buckling load using tabu embedded simulated annealing algorithm. Ahmadian *et al.* (2011) presented a totally general approach to optimal design of composite laminates using plar-genetic method. Wu *et al.* (2012) investigated buckling load optimization of variable angle tow composite plates using genetic algorithm. Narita and Turvey (2004) determined the optimum lay-ups and maximum buckling loads of symmetrically laminated rectangular plates using layerwise optimization. Honda *et al.* (2007) compared three optimum design approaches to clarify the advantages and disadvantages for optimizing the buckling performance of laminated composite plates. Iyengar and Vyas (2011) carried out optimization of composite laminates for maximizing the buckling load with and without cut-out using genetic algorithm. Honda and Narita (2006) proposed an optimum design approach to optimize buckling performance of laminated composite plates by using the lamination parameters. Lingaard and Lund (2011) focused on criterion functions for gradient based optimization of the buckling load of laminated composite structures considering different types of buckling behaviour. Correia *et al.* (2003) investigated optimal design of laminated composite plates with integrated piezoelectric actuators. Refined finite element models based on equivalent single layer high-order shear deformation theories were used. These models were combined with simulated annealing, a stochastic global optimization technique, in order to find the optimal location of piezoelectric actuators and also to find the optimal fiber reinforcement angles in both cases having the objective of maximizing the buckling load of the composite adaptive plate structure. De Faria (2002) considered optimal design for elastic buckling loads of composite plates under uncertain loading conditions. More results can be found in the literature.

This study investigates the applicability of artificial bee colony algorithm to optimize the stacking sequences of simply supported antisymmetric laminated composite plates with criticial buckling load as the objective functions. The fibre orientations of the layers are selected as the optimization design variables with the aim to find the optimal laminated plates. In order to perform the optimization based on the ABCA, a special code is written in MATLAB software environment. Several numerical examples are presented to illustrate this optimization algorithm for different plate aspect ratios, number of layers and load ratios.

2. General formulations

A typical laminated composite plate subjected to biaxial loading is shown in Fig. 1. In this paper, first order shear deformation theory (FSDT) has been applied in the analytical formulation to account for the displacement fields in each plate. The displacement field of this theory is as follows

$$u = u_o + z\Psi_x$$

$$v = v_o + z\Psi_y$$

$$w = w_o \tag{1}$$

where u, v and w are components of displacement at a general point, whilst u_o, v_o and w_o are similar components at the middle surface (z=0), Ψ_x is the rotation of a transverse normal about the axis y and Ψ_y is the rotation of a transverse normal about the axis x.

The strain vector ε can be written in terms of the mid-plane deformation

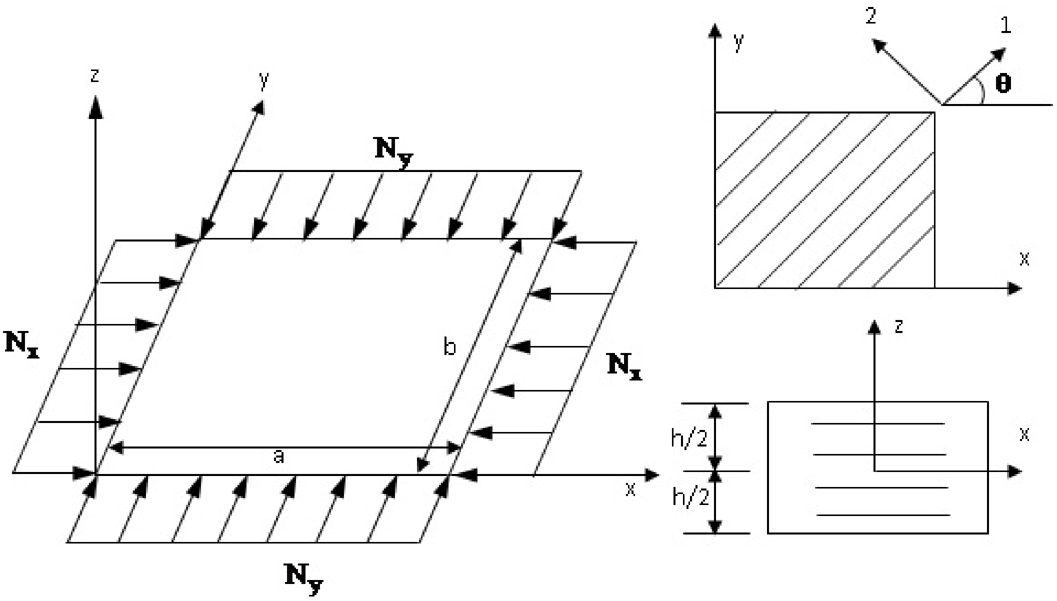

Fig. 1 Geometry and loading of a laminated plate

$$\varepsilon = \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^T = \begin{Bmatrix} \varepsilon_m \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \varepsilon_s \end{Bmatrix} \tag{2}$$

where ε_m is the membrane strain, ε_b is the bending strain and ε_s is the shear strain, which can be given by

$$\varepsilon_m = \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{\partial v_o}{\partial y} \\ \dfrac{\partial u_o}{\partial y} + \dfrac{\partial v_o}{\partial x} \end{Bmatrix}, \ \varepsilon_m = \begin{Bmatrix} \dfrac{\partial \Psi_x}{\partial x} \\ \dfrac{\partial \Psi_y}{\partial y} \\ \dfrac{\partial \Psi_x}{\partial y} + \dfrac{\partial \Psi_y}{\partial x} \end{Bmatrix}, \ \varepsilon_s = \begin{Bmatrix} \dfrac{\partial w_o}{\partial x} - \Psi_x \\ \dfrac{\partial w_o}{\partial y} - \Psi_y \end{Bmatrix} \tag{3}$$

The constitutive relationship of laminated composite plates can be expressed as

$$\hat{\sigma} = \hat{D}\hat{\varepsilon} \tag{4}$$

where

$$\hat{\sigma} = \{\hat{\sigma}_m^T \quad \hat{\sigma}_B^T \quad \hat{\sigma}_s^T\}^T, \ \hat{\varepsilon} = \{\hat{\varepsilon}_m^T \quad \hat{\varepsilon}_B^T \quad \hat{\varepsilon}_s^T\}^T, \ \hat{D} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & S \end{bmatrix} \tag{5}$$

where $\hat{\sigma}_m = \{N_x \quad N_y \quad N_{xy}\}^T$ is the membrane force vector, $\hat{\sigma}_b = \{M_x \quad M_y \quad M_{xy}\}^T$ is the bending moment vector and $\hat{\sigma}_s = \{Q_x \quad Q_y\}^T$ is the transverse shear force vector. The membrane stiffness (A), the bending stiffness (D), the bending-extensional coupling stiffness (B) and the transverse shear stiffness (S) are defined as

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix},$$

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}, \quad S = \begin{bmatrix} \kappa_1 S_{44} & \sqrt{\kappa_1 \kappa_2} S_{45} \\ \sqrt{\kappa_1 \kappa_2} S_{45} & \kappa_2 S_{55} \end{bmatrix} \tag{6}$$

in which κ_1 and κ_2 are the shear correction factors. The stiffness in Eq. (6) can be defined as

$$A_{ij} = \sum_{k=1}^{N_i} \overline{Q}_{ij}^{(k)} \left(h_k - h_{k+1}\right), \quad B_{ij} = \frac{1}{2} \sum_{k=1}^{N_i} \overline{Q}_{ij}^{(k)} \left(h_k^2 - h_{k+1}^2\right),$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N_i} \overline{Q}_{ij}^{(k)} \left(h_k^3 - h_{k+1}^3\right), \quad S_{ij} = \sum_{k=1}^{N_i} \overline{Q}_{ij}^{(k)} \left(h_k - h_{k+1}\right) \tag{7}$$

where A_{ij}, B_{ij} and D_{ij} are defined for i, j=1, 2, 6, whereas S_{ij} is defined for i, j=4, 5. h_k and h_{k+1} denote the distances from the plate reference mid-plane to the outer and inner surfaces of the kth layer, respectively. N_i is the total number of layers in the laminated plate and $\overline{Q}_{ij}^{(k)}$ (i, j=1,2,4,5,6) are related to the engineering constans for the kth layer.

The basic equation of buckling analysis in the form of an eigenproblem is

$$K^e \phi = \lambda K_G \phi \tag{8}$$

where K^e and K_G are the elastic and geometric stiffness matrices of the structure, respectively, ϕ is the generalized global displacementvector. This eigenproblem is solved by the subspace iteration procedure that is an effective method widely used in engineering practice for the solution of eigenvalues and eigenvectors of finite element equations. This technique is particularly suited for the calculation of a few eigenvalues and eigenvectors of large finite element system. The smallest eigenvalue λ_1 among eigenvalues obtained by the subspace iteration method is the buckling load N_{cr}.

3. Artificial bee colony algorithm (ABCA)

ABCA is a new population based metaheuristic approach proposed by Karaboğa and Baştürk (2007). The algorithm, simulates the intelligent foraging behaviour of honey bee swarms, consists of three essential components such as; food sources, employed and unemployed bees. Employed bees employed at a specific food source which is discovered before. They carry information about distance, the direction and profitability of the source and share it with the other bees in the hive. Unemployed bees are divided into two groups. One of the groups is called scout bees who search

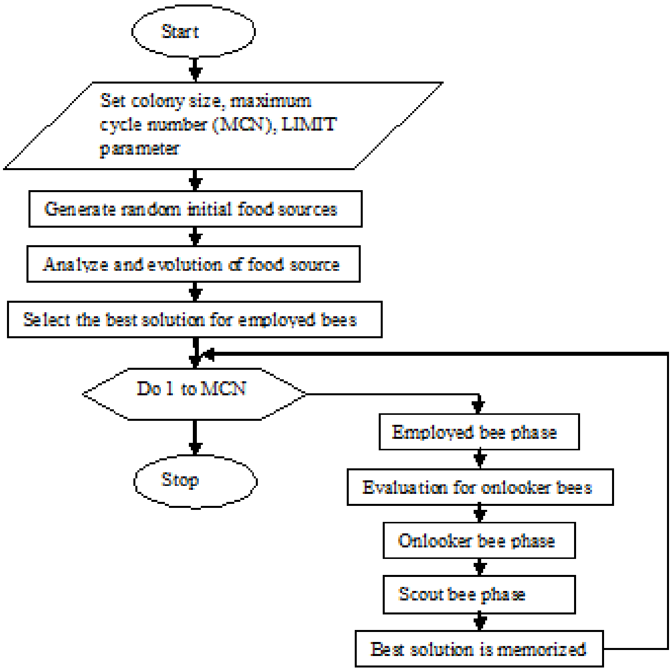

Fig. 2 Flow chart of artificial bee colony algorithm

the environment randomly and the other group called onlookers who try to find a food source by means of the information given by the employed bees. In the algorithm, the position of a food source represents a possible solution to the optimization problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution (Karaboğa and Akay 2006). The flow chart of this algorithm is shown in Fig. 2. At the first step, algorithm generates random solutions for all bees. This operation can be defined as

$$x_{ij} = x_j^{\min} + r\text{ and }(0,1)(x_j^{\max} - x_j^{\min}) \tag{9}$$

where i=1...SN, j=1…D, SN denotes the number of food sources or employed bees, D is the number of design variables. $x_j^{\min}$ and $x_j^{\max}$ are lower and upper bounds of the jth parameter, respectively. In employed bee phase, each employed bee searches for candidate food source having more nectar in the neighborhood of its current food source by following expression

$$\upsilon_{ij} = x_{ij} + \varphi_{ij}(x_{kj} - x_{ij}) \tag{10}$$

where $k \in \{1,2,\dots,SN\}$ and $j \in \{1,2,\dots,D\}$are randomly chosen index and $k \neq i$. φ_{ij} is a random real number between [-1,1]. After producing the candidate food source, its fitness is calculated as

$$fitness_i = \begin{cases} \dfrac{1}{1+f_i} & if \quad f_i \geq 0 \\ 1+abs(f_i) & if \quad f_i < 0 \end{cases} \tag{11}$$

where, f_i is the value of cost function for ith solution. After then greedy selection is applied between old and candidate food source. If nectar amount (fitness) of candidate food source is higher than that of the old food source, the bee memorizes the new source and forgets the old one. Otherwise it keeps the position of the old one in its memory. After all employed bees complete the search process; they share the nectar information of the food sources and their position information with the onlooker bees on the dance area. Onlooker bees produce candidate food sources, according to probability value of the old sources, in the neighbourhood of the food source chosen by them. In other words onlooker bees select a food source according to a probability proportional to the amount of nectar (Öztürk and Durmuş (2013)). After producing the candidate food source, a greedy selection process is applied between old and candidate food source again. Probability value calculated by

$$p_i=(0.9\, fitness_i + \max(fitness))+0.1 \tag{12}$$

In scout bee phase, when a food source cannot be improved by a predetermined number of trials which is called limit, then that food source is abandoned by its employed bee and the employed bee is converted to a scout to find a new source. Then, a new food source positions is generated randomly by Eq. (9) and replaced with the abandoned ones by scouts. The best food source is determined and position of that food source is memorized. This cycle is repeated until requirements are met.

4. Optimization problem

The main objective in this study is to determine the optimal layer sequences which maximize the critical buckling load of the simply supported antisymmetric laminated composite plates. The optimization problem can be defined as

Find: $\theta_{opt} = \left(\theta_1 / -\theta_2 / \ldots. / -\theta_N\right)_{a,s}$

which maximizes: $N_{cr} = N_{cr}\left(\theta_{opt}\right)$

subjected to the constraints: $-90^\circ \le \theta_k \le 90^\circ$

where N is the number of layers.

5. Numerical results and discussion

In order to demonstrate the accuracy and applicability of the proposed method, firstly the buckling problems of simply supported cross ply and angle ply square laminated plates are investigated and compared with the corresponding results for different b/h ratios. The mechanical properties of each layer are taken to be E_1=25E_2, G_{12}=G_{13}=0.5E_2, G_{23}=0.2E_2, v_{12}=0.25. The non-dimensional buckling load is defined by

$$\overline{N}_{cr} = N_{cr}(b^2 / E_2 h^3) \tag{13}$$

From Table 1, it is obvious that the results of the present study give an accurate prediction of buckling loads in comparison with the literature results. The results presented in the Table 1 shows that the magnitude of buckling load is higher for angle-ply composite plate than corresponding

Table 1 Non-dimensional buckling loads of simply supported cross ply and angle ply square laminated plates

b/h	(0/90/0/90/0)		(-45/45)$_4$	
	Uniaxial compression (N_y/N_x=0)			
	Present study	Reddy (2004)	Present study	Reddy (2004)
10	16.140	16.309	21.082	21.082
20	21.045	21.125	34.990	34.990
100	23.495	23.389	41.163	40.875
	Biaxial compression (N_y/N_x =1)			
	Present study	Reddy (2004)	Present study	Reddy (2004)
10	8.070	8.154	12.067	12.067
20	10.523	10.562	17.495	17.495
100	11.747	11.695	20.581	20.437

Table 2 Optimal results for simply supported symmetric 8-layered composite plates ($\Delta\theta$=5°)

a/b				
	Uniaxial compression			
	Optimal stacking (Present study)	$\bar{\lambda}$ (Present study)	Optimal stacking (Honda and Narita 2006)	$\bar{\lambda}$ (Honda and Narita 2006)
1	(45/-45/-45/-45)$_s$	323.7	(45/-45/-45/-45)$_s$	321.0
2	(45/-45/-45/45)$_s$	1295.0	(45/-45/-45/-45)$_s$	1282.0
	Biaxial compression			
	Optimal stacking (Present study)	$\bar{\lambda}$ (Present study)	Optimal stacking (Honda and Narita 2006)	$\bar{\lambda}$ (Honda and Narita 2006)
1	(45/-45/45/45)$_s$	161.9	(45/-45/-45/-45)$_s$	160.5
2	(70/-75/70/60)$_s$	489.7	(85/-50/75/85)$_s$	473.7

cross-ply composite plate. This is due to the fact that in cross-ply laminated plate every alternate fiber is oriented perpendicular to the direction of the applied load and fibers have very small strength in transverse direction.

Second, the optimal solutions for critical buckling load of simply supported symmetric 8-layered composite plates $(\theta_1/\theta_2/\theta_3/\theta_4)_s$ are searched using ABCA and the results are compared with the literature results for different a/b ratios. The fiber angle of each ply in the composite plate is changed with a step of $\Delta\theta$=5° between (−90°≤θ≤90°). The laminate material properties are E_1=138 GPa, E_2=138 GPa, G_{12}=7.1 GPa, v_{12}=0.30. The non-dimensional buckling load is defined by:

$$\bar{\lambda} = -\frac{N_x a^2}{D_o} \tag{14}$$

where $D_o=E_2h^3/12(1-v_{12}v_{21})$. From Table 2, it is evident that ABCA is successful in the determination of the optimal layer sequences maximizing the critical buckling load of the laminated plates. In this study, optimum results are investigated for simply supported antisymmetric composite plates for different plate aspect ratios, number of layers and load ratios (*b/h*=25). The composite material properties are given as E_1=181 GPa, E_2=10.3 GPa, G_{12}=7.17 GPa and v_{12}=0.28. The non-dimensional buckling load is defined as Eq. (13). The fiber angle of each ply in the composite plate is changed with a step of $\Delta\theta$=1° between (−90°≤θ_k≤90°).

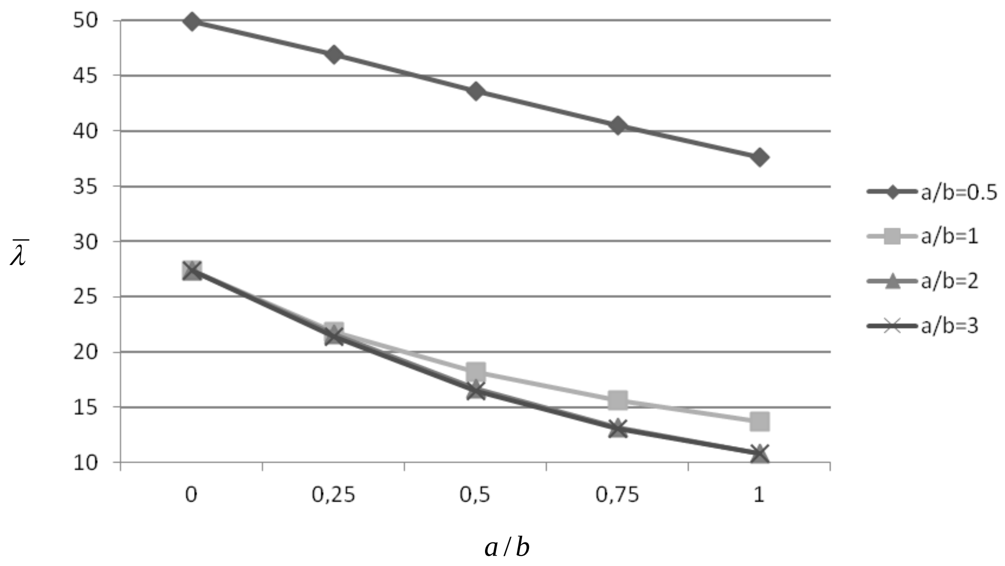

Fig. 3 Optimal critical buckling load parameters for simply supported 8-layered antisymmetric composite plates

Table 3 Optimum fibre orientations for 8-layered simply supported antisymmetric composite plates for different load ratios and plate aspect ratios ($\Delta\theta$=1°)

	a/b			
N_y/N_x	0.5	1	2	3
0	$(0/0/0/0)_{a,s}$	$(45/-45/45/-45)_{a,s}$	$(45/-45/45/-45)_{a,s}$	$(45/-45/45/-45)_{a,s}$
0.25	$(0/0/0/0)_{a,s}$	$(45/-45/45/-45)_{a,s}$	$(48/-48/90/-44)_{a,s}$	$(50/-49/90/-46)_{a,s}$
0.5	$(11/-16/3/-4)_{a,s}$	$(45/-45/45/-45)_{a,s}$	$(59/-59/90/-50)_{a,s}$	$(60/-59/90/-54)_{a,s}$
0.75	$(17/-21/1/-17)_{a,s}$	$(45/-45/45/-45)_{a,s}$	$(66/-63/90/-56)_{a,s}$	$(65/-62/90/-63)_{a,s}$
1	$(21/-26/0/-16)_{a,s}$	$(45/-45/45/-45)_{a,s}$	$(70/-67/90/-61)_{a,s}$	$(69/-66/90/-67)_{a,s}$

Fig. 3 shows the optimal critical buckling load parameters for 8-layered simply supported antisymmetric composite plates for different load ratios (N_y/N_x) and plate aspect ratios (*a/b*).

As seen from Fig. 3, as load ratio and plate aspect ratio increase, the optimal critical buckling load decreases. In Table 3, optimum fibre orientations are given for 8-layered simply supported antisymmetric composite plates for different load ratios and plate aspect ratios.

In this study, effects of number of layers the optimum results are investigated for simply supported antisymmetric composite plates subjected to biaxial compressive loads for different plate aspect ratios. As seen from Fig. 4, the optimal critical buckling load increases, as the number of layers increases. But, this effect diminishes for larger number of layers. This is due to the influence of bending-extension coupling. For multilayered plates this effect is insignificant. In Table 4, optimum fibre orientations are given for simply supported antisymmetric composite plates subjected to biaxial compressive loads for different number of layers.

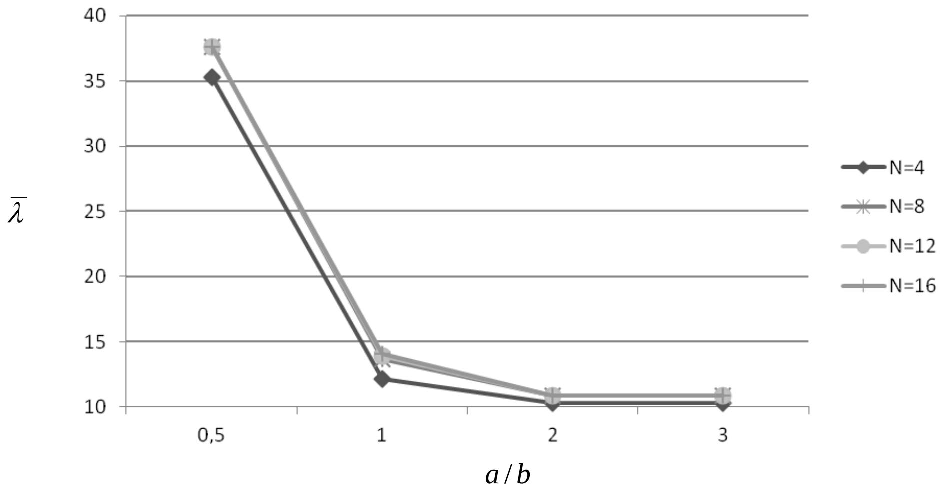

Fig. 4 Effect of number of layers on the optimal critical buckling load for simply supported antisymmetric composite plates

Table 4 Effect of number of layers on the optimum fibre orientations for simply supported antisymmetric composite plates subjected to biaxial compressive loads

	a/b			
N	0.5	1	2	3
4	$(11/-33)_{a,s}$	$(45/-45)_{a,s}$	$(72/-57)_{a,s}$	$(72/-57)_{a,s}$
8	$(21/-26/0/-16)_{a,s}$	$(45/-45/45/-45)_{a,s}$	$(70/-67/90/-61)_{a,s}$	$(69/-66/90/-67)_{a,s}$
12	$(21/-25/16/-24/0/-4)_{a,s}$	$(45/-45/45/-45/45/-45)_{a,s}$	$(70/-68/72/-66/90/-79)_{a,s}$	$(69/-67/73/-67/90/-71)_{a,s}$
16	$(21/-23/25/-23/1/-13/0/-10)_{a,s}$	$(45/-45/45/-45/45/-45/45/-45)_{a,s}$	$(69/-69/68/-71/89/-71/88/-71)_{a,s}$	$(70/-66/72/-68/72/-66/90/-82)_{a,s}$

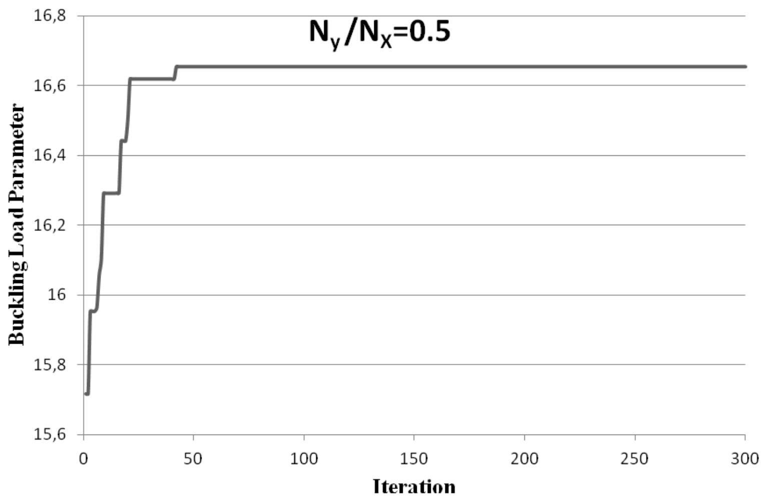

Fig. 5 Iteration stories of the optimal design for 8 layered composite plates (*a/b*=2)

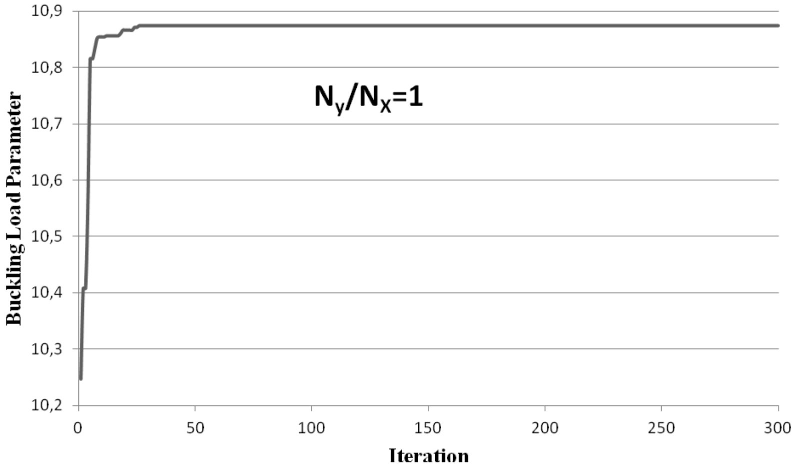

Fig. 5 Continued

Fig. 5 shows iteration histories behind the optimized designs for 8 layered composite plates subjected to different load ratios (*a/b*=2).

6. Conclusions

In this paper, applicability of the artificial bee colony algorithm (ABCA) on critical buckling load optimization of simply supported antisymetric laminated plates is investigated. The optimal stacking sequences for laminated plates is searched for by means of the ABCA. In order to perform the optimization based on this algorithm, a special code is written in MATLAB software environment. The algorithm is implemented and validated using data available in the literature. It is proved that this optimization method for buckling load calculation is a very efficient and practical design tool in the application of laminated composite plates. In addition, the critical buckling load decreases with incerease in the load ratio and plate aspect ratio. On the other hand, as the number of layer increases, the critical buckling load increases.

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