Structural Engineering and Mechanics, *Vol. 52, No. 1 (2014) 149-172*
DOI: http://dx.doi.org/10.12989/sem.2014.52.1.149

Damage identification of 2D and 3D trusses by using complete and incomplete noisy measurements

M. Rezaiee-Pajand* and M.S. Kazemiyan[a]

Department of Civil Engineering, Ferdowsi University of Mashhad, Azadi Sq., Mashhad, Khorasan Razavi, Iran

(Received November 30, 2013, Revised April 17, 2014, Accepted July 7, 2014)

Abstract. Four algorithms for damage detection of trusses are presented in this paper. These approaches can detect damage by using both complete and incomplete measurements. The suggested methods are based on the minimization of the difference between the measured and analytical static responses of structures. A non-linear constrained optimization problem is established to estimate the severity and location of damage. To reach the responses, the successive quadratic method is used. Based on the objective function, the stiffness matrix of the truss should be estimated and inverted in the optimization procedure. The differences of the proposed techniques are rooted in the strategy utilized for inverting the stiffness matrix of the damaged structure. Additionally, for separating the probable damaged members, a new formulation is proposed. This scheme is employed prior to the outset of the optimization process. Furthermore, a new tactic is presented to select the appropriate load pattern. To investigate the robustness and efficiency of the authors' method, several numerical tests are performed. Moreover, Monte Carlo simulation is carried out to assess the effect of noisy measurements on the estimated parameters.

Keywords: damage assessment; updated matrix; Sherman-Morrison-Woodbury formula; truss structure; optimization; Monte Carlo simulation; incomplete measurement

1. Introduction

Recently, damage diagnosis of structures has emerged as an active field in civil, mechanical and aerospace engineering, enabling the continuous inspection of the safety and integrity of structures. Damage identification includes three main levels: detection, localization and quantification. During the past few years, a large amount of damage diagnosis approaches has been presented. Some of these techniques are based on finite element method. The basic idea of them is that structural responses and modal parameters, notably stress, strain, displacement, natural frequencies and mode shapes are the functions of physical properties of the structure (stiffness, mass and damping). As a consequence, changes in physical characteristics lead to alteration in the aforementioned responses and parameters. These damage detection algorithms aim to update the FEM model to match the analytical and the measured data. In this way, physical parameters of the

∗Corresponding author, Professor, E-mail: mrpajand@yahoo.com
[a]Graduate Student, E-mail: Sadegh_kazemiyan@yahoo.com

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

damaged structure can be identified. Based on the type of the measured data, these strategies are divided into two categories, namely the static and dynamic techniques. In the latter group, modal parameters such as frequencies and mode shapes are applied, while responses such as displacements and strains are measured in static methods. It is worth mentioning; some of the damage identification schemes use both static and dynamic data to monitor the structures.

Some researchers provided a summary of the literature pertaining to the dynamic methods (Mottershead and Friswell 1993, Doebling *et al.* 1996, Salawu 1997, Roeck 2003, Carden and Fanning 2004, Yan *et al.* 2007, Friswell 2007, Li and Chen 2013). In these surveys, the characteristics and the formulations of various dynamic approaches were described. It should be reminded that the changes in modal properties are dependent on changes in the mass, stiffness and damping properties. It is worth emphasizing; these techniques are more complicated than static ones since static responses are only dependent on the stiffness characteristics. Furthermore, Shih *et al.* (2011) proposed a dynamic tactic for damage identification of truss bridges. Modal flexibility and modal strain energy changes were deployed in a companion with the natural frequencies in this scheme. Yun (2012) proposed a strategy for damage detection of structures under the ambient vibration loading. Moreover, the genetic algorithm was deployed in this procedure. Nobahari and Seyedpoor (2013) utilized the concept of flexibility matrix and strain energy to detect damage in structures. Majumdar *et al.* (2013) deployed ant colony optimization approach for damage assessment of beams. In this method, the natural frequencies were employed.

Dynamic strategies have some drawbacks. In general, the frequencies of the structures will show low sensitivity to damage, unless the large levels of damage occur or very precise devices are utilized for measurements. Moreover, it is very difficult to measure the mode shapes of the structures. To obtain precise mode shapes, sufficient control of excitation is required, which is hard to achieve in the field. In addition, the effect of boundary conditions also has a crucial impact on measured vibration frequencies and mode shapes.

In the following paragraph, the state- of- the- art of the static damage detection techniques will be summarized. Sheena *et al.* (1982) proposed a method in which the elements of the stiffness matrix were updated to minimize the difference between the actual stiffness matrix, and the analytical one subjected to the measured displacement constraints. Note that the displacements were measured in specific locations in this approach. In addition, it was assumed that the measurements were noise-free. Sanayei and Nelson (1986) were suggested a scheme to estimate the stiffness properties of structural elements. In this strategy, displacements were essential to be measured at the same location as the applied external loads. Consequently, this tactic was impractical. Hence, Sanayei and Onipede (1991) removed this drawback. In this technique, the applied force DOFs and measured displacement DOFs were not required to be totally overlapped. Sanayei and Saletnik (1996a, part I) identified damage in structures by measuring the strains. It should be added that there was no need to measure the strains of all the elements. Additionally, they evaluated the effect of noisy data on their methods (Sanayei and Saletnik 1996b, part II). In addition, Sanayei *et al.* (1997) constructed a small-scale frame model to prove the ability of their proposed approaches. Hajela and Soeiro (1990a, b) classified the existing parameter identification tactics into three categories, namely equation error, output error and minimum deviation strategies. Moreover, they utilized both dynamic and static responses to identify damage in structures. Banan *et al.* (1994a, Part I, 1994b, Part II) presented two algorithms for damage detection of structures, and they used recursive quadratic programming method for solving the damage identification problem. Hjelmstad and Shin (1997) proposed a scheme to detect damage by measuring the static responses. Note that this algorithm is based on the research carried out by Banan *et al.* (1994a, Part

I, 1994b, Part II). Liu and Chian (1997) suggested a damage detection technique for trusses by deploying a new formulation which separated the stiffness parameters of the stiffness matrix from other elements of this matrix. Chou and Ghaboussi (2001) assessed damage in structures by employing GA. Rezaiee-Pajand and Saliani (2004) developed the proposed method by Liu and Chian (1997). Their technique could identify damage in trusses by measuring displacements. In addition, they compared the capability of several existing damage detection tactics. Bakhtiari-Nejad *et al.* (2005) used sequential quadratic programming (SQP) to estimate the location and severity of damage in structures. They applied an approximate formula based on the second-order approximation of Taylor series in their formulation. Shenton lll and Hu (2006) proposed a method employing dead load redistribution to detect damage in the structures. Kouchmeshky *et al*. (2007) presented a co-evolutionary algorithm for damage identification of structures. The problem of damage detection employing minimum test data was studied in their work. Rezaiee-Pajand and Aftabi Sani (2008) suggested a tactic to locate damage and estimate its severity. The related non-linear programming was solved by applying a pseudo-dual simplex method. Due to the fact that the mentioned scheme worked with binary variable, the design variables were converted to the binary ones. Yang and Sun (2010) identified the location and severity of damage in structures by deploying the flexibility disassembly technique. This approach identified damage by applying a simple algorithm in which optimization methods were not required. Besides, Abdo (2012) analytically studied the relationship between damage properties (location and severity) and changes in static displacement curvature. Moreover, Seyedpoor and Yazdanpanah (2013) suggested a new damage indicator based on static strain energy. Recently, Rezaiee-Pajand *et al.* (2014) proposed a new algorithm for damage identification of 2D and 3D frames.

In this paper, four algorithms for damage detection of trusses are presented. The suggested techniques identify the location and magnitude of damage in structures by minimizing the differences between the analytical displacements and the measured ones. In a common way, the damage is modeled as a reduction in the elements' cross-sectional areas. Hence, these schemes are not able to localize concentrated damage. The formulation of the problem leads to a non-linear optimization problem which is then solved by successive quadratic programming. In each iteration of the proposed techniques, the stiffness matrix of the damaged structure is estimated and inverted. Recall that the harmed structure's stiffness matrix is assumed to be the sum of the stiffness matrix of the undamaged structure and a perturbation matrix. The differences between the suggested strategies result from the algorithms deployed for inverting the estimated matrix.

The first algorithm uses Cholesky decomposition to invert the estimated stiffness matrix. The second and third ways employ the formulas based on the first-order and the second-order approximation of the Taylor series to calculate the inversion of the estimated matrix, respectively. The last approach deploys Sherman- Morrison- Woodbury formula. The suggested methods have two stages. At the first one, the probable damaged members will be identified by utilizing a new procedure. Then, the cross-sectional areas of these members will be calculated. Both complete and incomplete measurements can be applied in these algorithms. Keeping in mind, it is not always possible to measure displacements of all the DOFs. In fact, measuring devices may have errors. In other words, errors may exist in measurements. In this case, the proposed algorithms can never converge to the exact solutions. To investigate the effect of the measurement errors on the estimated parameter, a Monte Carlo experiment is carried out in this work. It is important to note that the speed and the accuracy of the static damage detection methods rely on the applied load cases (Hajela and Soeiro 1990b, Liu and Chian 1997, Bakhtiari-Nejad *et al*. 2005, Kouchmeshky *et al*. 2007). Therefore, a new load calculation which produces equally stressed members in trusses is proposed.

This paper is organized as follows. Section 2 is devoted to introduce a new load pattern for static damage diagnosis. In Section 3, a method for identifying the probable damaged members is presented. Section 4 deals with the non-nonlinear constrained optimization problem required to be solved. In Section 5, four formulations for damage identification are proposed. The incomplete and noisy measurements are discussed in Section 6. Then, the capability and the efficiency of the suggested algorithms are evaluated by performing several numerical samples in Section 7. Finally, Section 8 summarizes the conclusions.

2. Load case determination

Characteristic properties of structural members such as Young's modulus and cross-sectional areas are unknown in damage identification problems, and the static responses such as strains and displacements are known. The selection of the load cases which approximately have similar effects on all elements is an important problem in static damage identification techniques. Under a specific load case, limitation of the load path and slight contribution of some elements to the static response may lead to concealment of some harmed elements in the damage detection process. Usually, a load case which produces equally stressed members in structure is appropriate for damage detection. Utilizing this ideal load case increases the prospect of reaching to the exact solution (Hajela and Soeiro 1990b, Bakhtiari-Nejad *et al*. 2005). It should be reminded that the damage in members, which participate a little in the load-bearing process of the structure, may be concealed.

In this section, a new technique for obtaining a load case which induces an equal stress distribution in trusses is proposed. The suggested approach uses the finite element method. Accordingly, the stress vector can be achieved as follows

$$\sigma = \boldsymbol{B}\,\boldsymbol{D} \tag{1}$$

in which stress matrix, stress and displacement vector are denoted by $\boldsymbol{B}$, $\boldsymbol{D}$ and $\boldsymbol{\sigma}$, respectively. Note that the displacement vector has *nd* entries, and the number of the entries of the stress vector is *nel*. Additionally, the stress matrix is a $nel \times nd$ matrix. *nd* and *nel* are the number of degrees of freedom and number of members, respectively. The following relation can be written with the help of Eq. (1) and assuming that $\boldsymbol{B}^T\boldsymbol{B}$ is invertible

$$\boldsymbol{B}^T \sigma = \boldsymbol{B}^T \boldsymbol{B}\,\boldsymbol{D} \tag{2}$$

Presuming an equal distribution of stress in the structure leads to the coming result

$$\boldsymbol{D} = (\boldsymbol{B}^T \boldsymbol{B})^{-1} \boldsymbol{B}^T \sigma \tag{3}$$

The corresponding nodal load vector is obtained as below

$$\boldsymbol{F} = \boldsymbol{K}\,\boldsymbol{D} \tag{4}$$

where $\boldsymbol{F}$ and $\boldsymbol{K}$ are the load and displacement vector, respectively. It should be mentioned that the proposed load case in this section is an ideal type of loading, and it is not always possible to utilize this load condition in a realistic structure (Hajela and Soeiro 1990b).

3. Identification of the probable damaged members

In the damage detection problem of trusses, the unknowns are the cross-sectional areas of the structure. As a result, the number of unknowns is equal to the number of members. On the other hand, the responses of the damaged structure, such as displacements and strains, are known. In general, the number of unknowns is more than equations. Thus, the problem is under-determinate, and it has no unique solution. To solve this problem, the optimization techniques are utilized (Chou and Ghaboussi 2001). To reduce the effect of solution's non-uniqueness, two tactics are employed by researchers: 1- reducing the number of unknowns 2-increasing the related equations. By utilizing several load cases the number of equations will be increased, and the non-unique solution problem may be remedied (Kouchmeshky *et al*. 2007). However, using several load cases is not practical.

In this section, a new technique to detect the undamaged members is presented. This method is based on the static nodal equilibrium of the structures. By deploying this approach, the number of the unknown parameters will be decreased. The equilibrium equation of the trusses has the coming appearance

$$\boldsymbol{KD} = \boldsymbol{F} \tag{5}$$

where $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{D}$ is the displacement vector, and $\boldsymbol{F}$ is the load vector. The number of DOFs is nd. It is clear that $\boldsymbol{K}$ is $nd\times nd$ matrix. $\boldsymbol{D}$ and $\boldsymbol{F}$ are $nd\times 1$ vector. It should be added that Eq. (5) includes the nodal equilibrium equations of truss. The stiffness matrix of the structure can be rewritten as the subsequent form (Doebling *et al*. 1998)

$$\boldsymbol{K} = \boldsymbol{APA}^T \tag{6}$$

In the last relationship, $\boldsymbol{A}$ is a $nd\times nel$ matrix, which is named the stiffness connectivity matrix. By utilizing connectivity and geometrical data of the structure, $\boldsymbol{A}$ is established. The number of truss members is nel. $\boldsymbol{P}$ is a $nel\times nel$ diagonal matrix. The diagonal entries of this matrix are the stiffness parameters of the truss' members. Utilizing Eq. (6) leads to the succeeding result

$$\boldsymbol{A}\,\boldsymbol{PA}^T\boldsymbol{D} = \boldsymbol{F} \tag{7}$$

Due to damage, the member's stiffness will be reduced. Hence, the static responses under the specific load case will be altered. Therefore, the equilibrium equation of the damaged structure will be changed to the below form

$$\boldsymbol{K}_d\boldsymbol{D}_d = \boldsymbol{F} \tag{8}$$

In this equality, $\boldsymbol{D}_d$ and $\boldsymbol{K}_d$ are the stiffness matrix and the displacement vector of the damaged structure, respectively. $\boldsymbol{F}$ is the load vector. $\boldsymbol{K}_d$ is a $nd\times nd$ matrix. $\boldsymbol{D}_d$ and $\boldsymbol{F}$ are the $nd\times 1$ vector. By utilizing Eq. (6), the next results will be found

$$\boldsymbol{K}_d = \boldsymbol{A}(\boldsymbol{P}+\delta\boldsymbol{P})\boldsymbol{A}^{\mathrm{T}} \tag{9}$$

$$\boldsymbol{A}(\boldsymbol{P}+\delta\boldsymbol{P})\boldsymbol{A}^{\mathrm{T}}\boldsymbol{D}_d = \boldsymbol{F} \tag{10}$$

where $\delta\boldsymbol{P}$ is a $nel\times nel$ diagonal matrix, and its main diagonal entries are the amounts of reduction in the stiffness parameters of the members. $\boldsymbol{D}_d$ is the displacement vector of the damaged structure.

It should be reminded that the amounts of reduction in the structural stiffness parameters are unknown, and the static responses are known. As it was mentioned so far, in this paper, damage is modeled as a reduction in cross-sectional areas of the members. This common way of modeling damage is not capable of detecting concentrated damage. Nevertheless, successively subdividing the members can remedy this limitation. Note that $\boldsymbol{A}$ doesn't change due to damage.

Obviously, Eqs. (5) and (8) are systems of equations. Each of them is a nodal force equilibrium equation in translational direction. Based on relations (7) and (10), equations associated with a specific node are dependent on the cross-sectional areas of the elements connected to the mentioned node. The cross-sectional areas are unknown in unhealthy structure. If the aforesaid parameters are estimated correctly, the system of nodal equilibrium equations of the damaged structure will be satisfied. In practice, damage leads to deterioration in only few members within the structure, and others remain unharmed. To identify the healthy ones, it is first presumed that all the members are undamaged. In other words, the cross-sectional areas of the unharmed structures are employed in Eq. (8). If all the members connected to the specific node are healthy, the nodal equilibrium equations of the mentioned node will be satisfied by inserting the unharmed cross-sectional areas into Eq. (8). In this way, the nodes whose elements are healthy can be identified. Then, the undamaged members can be easily detected by finding the members related to these nodes. Obviously, the unharmed elements connected to two damaged nodes cannot be detected by using the presented tactic. Accordingly, employing this scheme leads to the division of the structural members into two groups: 1-probable damaged members 2- unharmed members. The cross-sectional areas of the members which belong to the first group are unknowns, and their cross-sectional areas are estimated through the optimization process. On the other hand, the cross-sectional areas of the members included in the second category are known prior to the outset of the optimization procedure. This is because the category contains healthy members. Cross-sectional areas of these members do not change after the occurrence of damage. By utilizing this strategy, the number of unknown parameters will be reduced. Consequently, the effect of solution non-uniqueness decreases.

4. Optimization based damage detection

Some of the damage identification methods calculate the cross-sectional areas of the damaged members by minimizing the differences between the analytical responses and the measured responses. These techniques are named output error approaches (Hajela and Soeiro 1990b). As it was mentioned, the displacements of the structure are known, and the amounts of reduction in the cross-sectional areas are unknown in damage identification problem. As a result, the stiffness matrix of the damaged structure is unknown. To obtain the unknown parameters, some damage detection techniques minimize the difference between the measured displacements and analytical displacements. It is obvious that the strains and displacements of the structure are related. Therefore, the strains can be measured instead of displacements. To detect the severity and the location of damage in structures, the following error function is employed

$$\boldsymbol{R}_{\mathrm{i}} = \boldsymbol{D}_{\mathrm{d}}{}^{\mathrm{a}} - \boldsymbol{D}_{\mathrm{d}}{}^{\mathrm{m}} \tag{11}$$

In this relation, the subscript i denotes the number of load cases, the measured displacement vector and the analytical displacement vector are shown by $\boldsymbol{D}_d{}^m$ and $\boldsymbol{D}_d{}^a$, respectively. These vectors have *nd* entries. The objective function has the coming form

$$g = \sum_{i=1}^{nlc} \boldsymbol{R}_i^{\mathrm{T}} \boldsymbol{R}_i \tag{12}$$

In this formula, g denotes the goal function. Based on Eq. (9), the stiffness matrix of the damaged structure is a function of the unknown parameters. Consequently, the analytical displacements are a function of the amounts of reduction in the cross-sectional areas. It should be added; the number of load cases is *nlc*. By deploying the above-cited cost function, the non-linear optimization problem can be gained as below

$$\begin{cases} Min\ g\,(\,diag(\,\boldsymbol{\delta P}\,)) \\ diag\,(\,\boldsymbol{\delta P}\,) \le 0 \end{cases} \tag{13}$$

where $diag(\boldsymbol{\delta P})$ is a $nel\times 1$ vector, which contains the diagonal entries of $\boldsymbol{\delta P}$. It should be reminded that the diagonal entries of $\boldsymbol{\delta P}$ are the amounts of reduction in the cross-sectional areas, which are the unknown parameters of the problem. It is important to note that the inequality constraints are applied to guarantee that the change in the stiffness matrix is always negative, because a positive change can never be induced by damage.

In the following section, four algorithms for damage identification of trusses are presented. These approaches establish the discussed non-linear optimization problem to estimate the severity and location of damage. However, these tactics utilize four various formulations to construct the goal function in the optimization process. It is worth emphasizing; if the cross-sectional areas of the damaged members are calculated correctly, then $\boldsymbol{D}_d^{a}$ will be equal to $\boldsymbol{D}_d^{m}$. Consequently, the objective function will be zero. In this study, the successive quadratic method is applied to solve the optimization problem. Prior to deploying these techniques, the probable damaged members are identified by applying the proposed scheme in the third section.

5. Damage identification methods

Due to damage, the cross-sectional areas of the damaged members degrade. It is assumed that the stiffness matrix changes by the amount of $\Delta\boldsymbol{K}$. Consequently, the damaged stiffness matrix can be written as below

$$\boldsymbol{K_d} = \boldsymbol{K} + \boldsymbol{\Delta K} \tag{14}$$

In this equality, $\Delta\boldsymbol{K}$ is the perturbation matrix and has $nd\times nd$ entries. This matrix demonstrates the changes in the stiffness matrix of the undamaged structure. Utilizing Eqs. (6) and (9) results in the subsequent perturbation matrix

$$\boldsymbol{\Delta K} = \boldsymbol{A}\boldsymbol{\delta P}\,\boldsymbol{A}^{\mathrm{T}} \tag{15}$$

It should be added that the probable damaged members are identified by employing the suggested method in the third section before the outset of the optimization procedure. Note that the severity of damage is estimated in the minimization process. After detecting the members, which belongs to the group of the undamaged members, the columns of $\boldsymbol{A}$ related to these members will be eliminated. Consequently, the number of columns of this matrix will be reduced. $\boldsymbol{A}_m$ denotes the new obtained matrix which has $nd \times nm$ entries. The number of members which pertains to the group of the probable damaged members is equal to nm. Moreover, the rows and columns of $\boldsymbol{\delta P}$ which are related to the category of the undamaged members will be omitted. As a result, a new matrix, which is named $\boldsymbol{\delta P}_m$, and has $nm \times nm$ entries, will be obtained. Then, $\boldsymbol{A}$ and $\boldsymbol{\delta P}$ are replaced with $\boldsymbol{A}_m$ and $\boldsymbol{\delta P}_m$ in the aforementioned formulas, respectively. Hence, Eq. (15) can be rewritten as the coming shape

$$\boldsymbol{\Delta K} = \boldsymbol{A}_m \boldsymbol{\delta P}_m \boldsymbol{A}_m^{\mathrm{T}} \tag{16}$$

According to the aforesaid equations, the analytical displacement vector of the damaged structure can be expressed in the succeeding shape

$$\boldsymbol{D}_{\mathrm{d}}^{a} = (\boldsymbol{K} + \boldsymbol{\Delta K})^{-1} \boldsymbol{F} \tag{17}$$

By deploying this formula, the next error vector is found

$$\boldsymbol{R}_{\mathrm{i}} = (\boldsymbol{K} + \boldsymbol{\Delta K})^{-1} \boldsymbol{F} - \boldsymbol{D}_{\mathrm{d}}^{\mathrm{m}} \tag{18}$$

The objective function can be established by substituting Eq. (18) into Eq. (12). In minimization process, initial values are assigned to the design variables in the first iteration. Recall that design variables are the diagonal entries of $\boldsymbol{\delta P}_m$. Based on the optimization method, initial

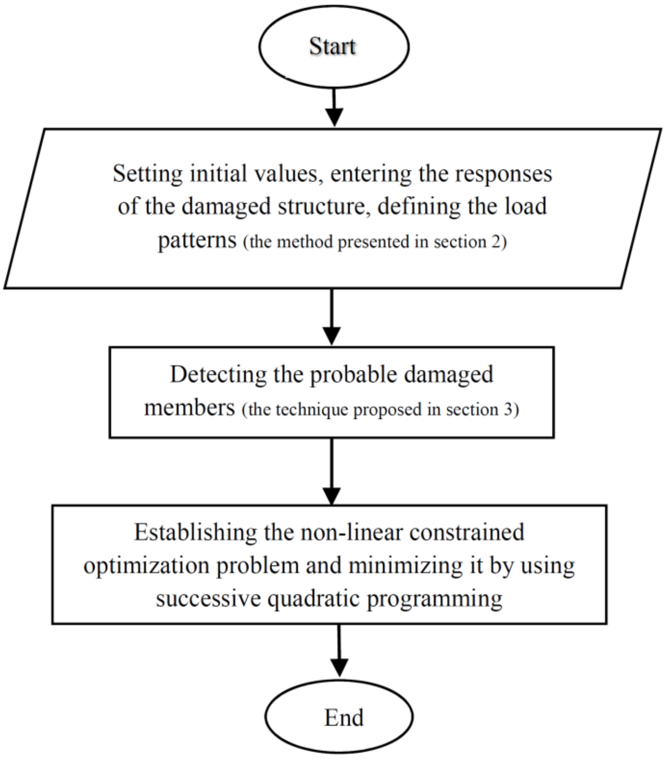

Fig. 1 The flowchart of the damage identification process

values are modified and assigned to the design variables in the subsequent iterations. By deploying Eq. (9), the stiffness matrix of the damaged structure is estimated in each iteration. It is worth emphasizing that the sum of the perturbation matrix and the stiffness matrix of the undamaged structure are inverted. After inverting this matrix, the analytical displacements and the value of the objective function are obtained in each cycle. This process is repeated until the objective function is minimized. The identification procedure is summarized in Fig. 1.

It should be reminded; the successive quadratic algorithm is deployed in this work (Rezaiee-Pajand and Sarafrazi 2005). Based on Eq. (18), it is obvious that the sum of the stiffness matrix and the perturbation matrix should be inverted to calculate the value of the goal function in each iteration. Various techniques can be used to invert the sum of two matrices. Accordingly, four strategies for constructing the objective function are presented in this section. The first scheme takes advantage of Cholesky decomposition for inverting the matrices. This strategy is named direct method. Briefly, it is denoted by DM. Recall that the inversion of the sum of two matrices can be exactly achieved with the help of Cholesky decomposition.

Additionally, Taylor series can be employed for calculating the approximate inversion of the sum of two matrices. If the first two sentences of the Taylor series are deployed, the first-order approximation will be obtained. This approximate formula has the coming form

$$(\boldsymbol{K}+\boldsymbol{\Delta K})^{-1} \cong \boldsymbol{K}^{-1}-\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1} \tag{19}$$

By substituting this relationship into Eq. (17), the analytical displacement vector of the damaged structure will be computed approximately as follows

$$\boldsymbol{D_d}^{a} \cong (\boldsymbol{K}^{-1}-\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1})\boldsymbol{F} \tag{20}$$

The above relation can be rewritten in the subsequent shape

$$\boldsymbol{D_d}^{a} \cong \boldsymbol{D}-\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{D} \tag{21}$$

By applying this equation, the error vector is obtained as below

$$\boldsymbol{R}_{\mathrm{i}} = (\boldsymbol{D}-\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{D})-\boldsymbol{D}_{\mathrm{d}}^{\mathrm{m}} \tag{22}$$

Substituting Eq. (16) into this equality will yield the next result

$$\boldsymbol{R}_{\mathrm{i}} = (\boldsymbol{D}-\boldsymbol{K}^{-1}\boldsymbol{A}_m\,\boldsymbol{\delta P}_m\,\boldsymbol{A}_m^{\mathrm{T}}\boldsymbol{D})-\boldsymbol{D}_d^{\,m} \tag{23}$$

By employing Eq. (12), the objective function is established. Then, the cross-sectional areas of the damaged members will be calculated by minimizing the goal function.

In another way, by utilizing the first three sentences of the Taylor series, an approximate formula will be obtained to invert the sum of two matrices. This second-order approximation is given below

$$(\boldsymbol{K}+\boldsymbol{\Delta K})^{-1} \cong \boldsymbol{K}^{-1}-\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1}+\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1} \tag{24}$$

By substituting the last equality into Eq. (17), the succeeding analytical displacement vector will be derived

$$\boldsymbol{D}_d^{\,a} \cong (\boldsymbol{K}^{-1}-\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1}+\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1})\boldsymbol{F} \tag{25}$$

This relationship can be rewritten as the subsequent form

$$\boldsymbol{D}_d^{\ a} \cong \boldsymbol{D} - \boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{D} + \boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{D} \tag{26}$$

Introducing this equation into Eq. (11) results in

$$\boldsymbol{R}_{\mathrm{i}} = (\boldsymbol{D} - \boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{D} + \boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{K}^{-1}\boldsymbol{\Delta K}\,\boldsymbol{D}) - \boldsymbol{D}_{\mathrm{d}}^{\ \mathrm{m}} \tag{27}$$

Inserting Eq. (16) into the last relation gives the succeeding formula

$$\boldsymbol{R}_{\mathrm{i}} = (\boldsymbol{D} - \boldsymbol{K}^{-1}\boldsymbol{A}_m\,\boldsymbol{\delta P}\,\boldsymbol{A}_m^{\ \mathrm{T}}\,\boldsymbol{D} + \boldsymbol{K}^{-1}\boldsymbol{A}_m\,\boldsymbol{\delta P}\,\boldsymbol{A}_m^{\ \mathrm{T}}\,\boldsymbol{K}^{-1}\boldsymbol{A}_m\,\boldsymbol{\delta P}\,\boldsymbol{A}_m^{\ \mathrm{T}}\,\boldsymbol{D}) - \boldsymbol{D}_{\mathrm{d}}^{\ \mathrm{m}} \tag{28}$$

By substituting Eq. (28) into Eq. (12), the objective function will be established. To find the cross-sectional areas of the damaged structure, the goal function should be minimized. It should be reminded that the probable harmed members are detected prior to the outset of the optimization process.

In the presented approximation approaches, the Eqs. (19) and (24) are applied to invert the sum of two matrices. These formulas calculated the inversion of the sum of two matrices by matrix multiplication. In this work, the first-order and second-order approximation of Taylor series, are denoted by FTM and STM, respectively.

Also, Sherman- Morrison- Woodbury formula can be used to accurately invert the sum of two matrices; as a result, the sum of the undamaged structure's stiffness matrix and the perturbation matrix can be inverted by utilizing this formula (Henderson and Searle 1981, Hager 1989). Therefore, the usage of this identity is helpful to set up the above-cited objective function. By minimizing the goal function, the cross-sectional areas of the damaged members can be calculated.

As mentioned previously, the stiffness matrix changes due to damage. By applying the Sherman- Morrison- Woodbury formula, the inversion of the damaged matrix can be written as follows

$$(\boldsymbol{K} + \boldsymbol{\Delta K})^{-1} = (\boldsymbol{K} + \boldsymbol{A}_m\,\boldsymbol{\delta P}_m\,\boldsymbol{A}_{\mathrm{m}}^{\mathrm{T}})^{-1} = \boldsymbol{K}^{-1} - \boldsymbol{K}^{-1}\boldsymbol{A}_{\mathrm{m}}\,(\boldsymbol{\delta P}_m^{-1} + \boldsymbol{A}_{\mathrm{m}}^{\mathrm{T}}\,\boldsymbol{K}^{-1}\,\boldsymbol{A}_m)^{-1}\boldsymbol{A}_{\mathrm{m}}^{\mathrm{T}}\,\boldsymbol{K}^{-1} \tag{29}$$

It should be added that $(\boldsymbol{\delta P}_m^{-1} + \boldsymbol{A}_{\mathrm{m}}^{\mathrm{T}}\,\boldsymbol{K}^{-1}\,\boldsymbol{A}_m)$ is called the capacitance matrix. The capacitance matrix must be invertible. Note that the number of rows and columns of this matrix is dependent on the number of rows and columns of $\boldsymbol{A}_m$, $\boldsymbol{A}_m^{\ T}$ and $\boldsymbol{\delta P}_m^{-1}$. It is obvious that $\boldsymbol{\delta P}_m$ and $\boldsymbol{K}$ should be invertible. The stiffness matrix of the stable structure is always invertible. $\boldsymbol{\delta P}_m$ is a diagonal matrix. Therefore, by inverting its diagonal entries, the inversion of this matrix will be obtained. It is worth mentioning that the proposed method in the third section sometimes identifies undamaged members as the probable damaged ones. Consequently, $\boldsymbol{\delta P}_m$ will not be invertible in this case. To solve this problem, a lower bound, which is small relative to the undamaged cross-sectional areas, is always presumed for the unknown parameters. The lower bounds show a very small reduction in cross-sectional areas of the damaged members. By employing this technique, $\boldsymbol{\delta P}_m$ is always invertible.

On the other hand, being invertible is not essential for $\Delta\boldsymbol{K}$. In each iteration, the inversion of $(\boldsymbol{K}+\Delta\boldsymbol{K})$ can be calculated by using Eq. (29). Obviously, the only matrix which is inverted is the capacitance matrix. If the rank of the perturbation matrix is less than the stiffness matrix, the number of columns and rows of the capacitance matrix will be less than the stiffness matrix. As a result, the sum of two matrices can be inverted faster. Note that Eq. (29) is an accurate formula for calculating the inversion of sum of two matrices.

In practice, only a few members will degrade due to damage. Thus, only the rows and columns of the perturbation matrix which are dependent on these members are not zero. Therefore, the rank of $\Delta\boldsymbol{K}$ is less than the stiffness matrix; as a result, it is efficient to use Eq. (29) to invert the sum of the perturbation matrix and the undamaged structure's stiffness matrix.

Substituting Eq. (29) into Eq. (17) results in

$$\boldsymbol{D}_d{}^a = (\boldsymbol{K}^{-1} - \boldsymbol{K}^{-1}\boldsymbol{A}_m(\delta\boldsymbol{P}_m^{-1} + \boldsymbol{A}_m^T\boldsymbol{K}^{-1}\boldsymbol{A}_m)^{-1}\boldsymbol{A}_m^T\boldsymbol{K}^{-1})\boldsymbol{F} \tag{30}$$

This equation can be rewritten in the subsequent form

$$\boldsymbol{D}_d{}^a = \boldsymbol{D} - \boldsymbol{K}^{-1}\boldsymbol{A}_m(\delta\boldsymbol{P}_m^{-1} + \boldsymbol{A}_m^T\boldsymbol{K}^{-1}\boldsymbol{A}_m)^{-1}\boldsymbol{A}_m{}^T\boldsymbol{D} \tag{31}$$

By employing the above-cited relations, the error vector is obtained as follows

$$\boldsymbol{R}_i = (\boldsymbol{D} - \boldsymbol{K}^{-1}\boldsymbol{A}_m(\delta\boldsymbol{P}_m^{-1} + \boldsymbol{A}_m^T\boldsymbol{K}^{-1}\boldsymbol{A}_m)^{-1}\boldsymbol{A}_m^T\boldsymbol{D}) - \boldsymbol{D}_d{}^m \tag{32}$$

By utilizing this equation, the objective function is established. To calculate the cross- sectional areas of the damaged members, the objective function should be minimized. To be brief, the proposed method in this section will be shown by SMWM. Note that Sherman- Morrison-Woodbury formula has other forms, which are mentioned in Appendix A (Henderson and Searle 1981, Hager 1989).

6. Incomplete and noisy measurements

As previously remarked, nodal displacements of trusses should be measured to detect damage by using the aforesaid tactics. In most cases, it is not always possible to measure all the nodal displacements due to the economical and practical limitations. Hence, an appropriate formulation for damage assessment should be able to employ both complete and incomplete measurements. To achieve this purpose, Boolean matrix is deployed in this study (Banan *et al*. 1994a, Bakhtiari-Nejad *et al*. 2005). By means of this matrix, the measured displacement can be extracted from the complete displacement vector. To this end, the analytical displacement vector is pre-multiplied by Boolean matrix. Afterwards, the error vector can be written in the following shape

$$\boldsymbol{R}_i = \boldsymbol{Q}\boldsymbol{D}_d{}^a - \boldsymbol{D}_d{}^m \tag{33}$$

in which $\boldsymbol{Q}$ denotes Boolean matrix. Recall that $\boldsymbol{D}_d^m$demonstrates the measured displacements vector. When incomplete measurements are applied, this vector has *nmd* entries. The number of measured DOFS equals *nmd*. In cases that all the nodal displacements are measured *nmd* is equal to *nd*. It should be noted that the analytical displacement vector can be calculated by utilizing one of the proposed techniques. By deploying the aforementioned equation and the presented methods, the error vector and the objective function can be established. In other words, the discussed non-linear optimization problem is set up and solved by measuring some of the nodal displacements.

Selection of measurement locations plays a key role in damage detection with incomplete measurements. The selected locations should be sensitive to any change in the structural parameters. In addition, they should be insensitive to noise in measurements. Extensive researches have been conducted to find the appropriate measurement locations (Sanayei *et al*. 1992, Sanayei and Saletnik 1996b, Banan *et.al* 1994b, Bakhtiari-Nejad *et al*. 2005). This issue is not investigated in this study.

In addition to incompleteness of measurements, the existence noise affects the accuracy of the obtained results. In an actual test or field experiment, no matter how accurate the measurements are, it is expected that there would be some errors due to the existence of noise in measurements. These errors influence the value of the estimated cross-sectional areas. Also, they may cause damage identification algorithms to diverge. To simulate the aforesaid errors in numerical samples, noise is simulated by adding a series of pseudo-random numbers on the theoretically computed displacements. In this work, it is presumed that the errors are proportional and distributed uniformly (Sanayei and Saletnik 1996b). Usually, Monte Carlo analysis is carried out to investigate the effect of input error on the estimated parameters (Sanayei *et al*. 1992, Sanayei and Saletnik 1996b, Banan *et.al* 1994b, Liu and Chian 1997, Bakhtiari-Nejad *et al*. 2005).

7. Numerical examples

In this section, several numerical tests are performed to assess the accuracy and efficiency of the proposed algorithms. First, a damage scenario is presumed. Then, by using the structural analysis, the responses of the damaged structure are calculated. In other words, the structural analysis is used instead of static tests. By obtaining the displacement vector of the damaged structure, the damage identification procedure begins. To detect the magnitude and the location of damage in trusses, the four proposed techniques are employed. It is worth emphasizing; the conditions on the optimization problem are similar in all the methods. Both complete and incomplete measurements are used. Note that the selected measurement locations are the same for all the suggested algorithms.

For comparing the ability of the presented schemes in damage identification with complete and incomplete measurements, the accuracy and the efficiency of the presented approaches are compared. As it was mentioned, the differences between the proposed methods result from the tactics used by each scheme to invert the stiffness matrix in each iteration. To compare the accuracy of the above-cited strategies an error index is employed. For the *i*-th scheme, this index is defined as below

$$EI_i = 100 \times \frac{1}{nel} \times \sum_{j=1}^{nel} \left| \left(\frac{P_{Ex}(j) - P_{Es}(j)}{P_{Ex}(j)} \right) \right| \tag{34}$$

where $\boldsymbol{P}_{Ex}$ and $\boldsymbol{P}_{Es}$ denote $nel \times 1$ vectors, which include exact and estimated cross-sectional area of the elements, respectively. In addition, the maximum and minimum percentage errors produced in estimating the cross-sectional areas are calculated. To compare the speed of the above-cited methods, a speed index is defined as follows

$$FI_i = \frac{T_{\min}}{T_i} \times 100 \tag{35}$$

in which $T_{\min}$ denotes the consumed time of the fastest method, and the consumed time of the *i*-th technique is shown by T_i.

Note that only the time consumed during the optimization process is measured. It should be added that the estimated parameters are shown in tables. Also, the number of iterations, the values of the objective function, error index and speed index are tabulated. Furthermore, the maximum and minimum percentage errors produced in estimating the cross-sectional areas are illustrated.

Finally, to investigate the capability of the proposed strategies in detecting damage with the presence of noise, 1%, 2% and 3% proportional error distributed uniformly are applied to the measured displacements. Then, 100 Monte Carlo observations are performed for each example (Sanayei *et al*. 1992, Bakhtiari-Nejad *et al*. 2005). In other words, the number of observations, which is denoted by *NOBS*, is equal to 100. For each unknown parameter, *NOBS* values exist. Hence, $nm \times NOBS$ values are produced. For ease of comparison, it is essential to decrease this large number of values to single parameters. For this purpose, the grand mean percentage error (*GM*), and a grand standard deviation percentage error (*GSD*) are used (Sanayei and Saletnik 1996b). These parameters are computed as below

$$GM = \frac{1}{NOBS \times nm} \sum_{j=1}^{nm} \sum_{k=1}^{NOBS} E_{j,k} \tag{36}$$

$$GSD = \sqrt{\left[\frac{1}{NOBS \times nm - 1}\right] \sum_{j=1}^{nm} \sum_{k=1}^{NOBS} (E_{j,k} - GM)^2} \tag{37}$$

in which

$$E_{j,k} = 100 \frac{P_k(j) - P_{Ex}(j)}{P_{Ex}(j)} \tag{38}$$

In this equality, the *k*-th observation of the parameter *j* and the true values of this parameter are shown by $P_k(j)$ and $P_{Ex}(j)$, respectively. In fact, the above-mentioned parameters are system mean and standard deviation, respectively

Furthermore, the mean of the estimated parameters are calculated by employing the coming equation

$$\boldsymbol{P}_M = \frac{1}{NOBS} \sum_{k=1}^{NOBS} \boldsymbol{P}_k \tag{39}$$

In this relation, $\boldsymbol{P}_M$ and $\boldsymbol{P}_k$ are $nel \times 1$ vectors. $\boldsymbol{P}_M$ includes the mean of the estimated parameters. In addition, the entries of $\boldsymbol{P}_k$ are the estimated parameters in the *k*-th Monte Carlo test. To assess the ability of the proposed algorithms in the presence of the simulated measurement errors, the mean values compared with the obtained results from the noise-free measurements. Besides, the coefficients of variation of the Mont Carlo simulation are computed (Billinton and Li 1994). For each estimated parameter, this coefficient equals the standard deviation of the observed values to the mean of them. Finally, the error index is deployed to assess the ability of the authors' techniques in presence of noise.

It is worth emphasizing; the method proposed in the third section for identification of the probable damaged members is utilized prior to the outset of the optimization process. In this way, the number of unknowns may decrease. Note that any other strategies can be deployed for finding the probable unhealthy members such as the tactic presented by Seyedpoor and Yazdanpanah (2013).

Keeping in mind, the suggested load case in the second section is employed during the damage

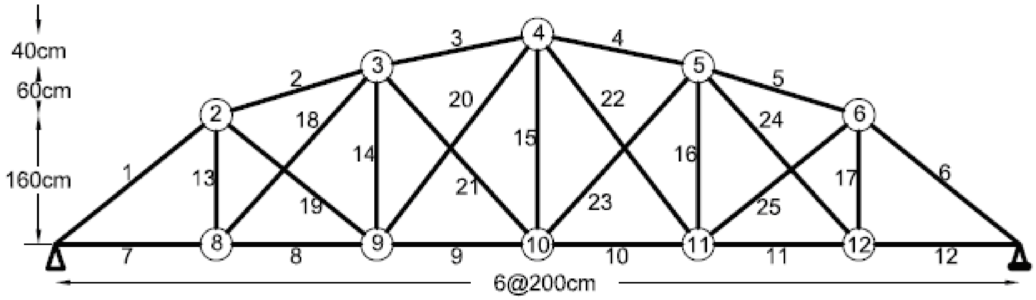

Fig. 2 Planar 25- members truss

Table 1 The cross-sectional areas of the planar 25-members truss

Element	Cross- sectional area (cm^2)
1-6	18
7-12	15
13-17	10
25-18	12

Table 2 The applied loads of the planar 25- members truss

Node	Applied loads in the horizontal direction	Applied loads in the vertical direction
2	-10.05	18.86
3	-0.33	23.52
4	0.00	28.87
5	0.33	23.52
6	10.05	18.86
7	23.24	0.00
8	-6.46	-15.10
9	1.64	-21.61
10	0.00	-22.21
11	-1.64	-21.61
12	6.46	-15.10

identification process. It should be reminded that the accuracy of the static damage identification tactics depends on the applied load pattern. Any other load cases can be applied in numerical samples. However, some damaged members may be concealed due to their slight contributions in structural responses, and the accuracy of the approaches may degrade. In other words, every load pattern which induces equal stress in all members can lead to appropriate results.

7.1 Planar 25- members truss

One of the trusses which is commonly used in bridges and roofs is shown in Fig. 2 (Rezaiee-Pajand and Saliani 2004). This structure consists of twenty five elements, and its number of degrees of freedom is twenty one. Thus, this is an indeterminate truss.

The cross-sectional areas of the above-mentioned truss are listed in Table 1. It should be added that Young's modulus is equal to 2×10^5 kg/cm^2. After the occurrence of damage, the cross-sectional areas of elements 1-6 degrade.

By using the proposed technique, the applied load vector is obtained. This load case produces the constant stress of value 800 kg/cm^2 in all members. The applied loads are tabulated in Table 2.

After determining the load case, the probable damaged members are specified by utilizing the suggested approach in the third section. The cross-sectional areas of these members are unknown, and they are estimated through optimization procedure. Applying this technique leads to the placement of members 1-6 in the group of the probable harmed members. The cross-sectional areas of these members are calculated by employing the proposed methods. At this stage, all nodal displacements are used to detect damage in this structure. The damaged members' cross-sectional areas are demonstrated in Table 3. Note that other members are undamaged and their cross-sectional areas haven't deteriorated.

It is obvious that damaged members are identified correctly by the usage of the algorithm presented in the third section. The direct method accurately calculates the cross–sectional areas of these members. On the other hand, FTM and STM schemes are not able to obtain the correct cross-sectional areas since they employ Taylor series to invert the sum of the perturbation matrix and the stiffness matrix. As it was expected, findings demonstrate that the second-order approximation of Taylor series is more accurate than the first-order approximation. As it was mentioned so far, SMWM technique uses an exact formula to invert the sum of two matrices called Sherman-Morrison- Woodbury formula. Therefore, this algorithm can converge to the correct solutions.

Moreover, the efficiency and accuracy of the above-mentioned strategies are compared. The number of iterations, the value of objective function, the speed index and the error index of each method are presented in Table 4. Besides, the maximum and minimum percentage errors produced in estimating the cross-sectional areas are illustrated.

Obviously, all the proposed approaches converge. It is worth emphasizing; SMWM and DM method calculates the cross-sectional areas correctly. This is because they use accurate tactics to

Table 3 The results of damage identification in planar 25-members truss

Element	Cross - sectional area before damage	Correct cross-sectional area after damage	Estimated Cross -sectional area by DM	Estimated Cross -sectional area by FTM	Estimated Cross -sectional area STM by	Estimated Cross - sectional area by SMWM
1	18	14	14	12.86	13.83	14
2	18	14	14	12.97	13.85	14
3	18	13	13	11.19	12.69	13
4	18	13	13	11.17	12.69	13
5	18	12	12	9.35	11.05	12
6	18	9.5	9.5	1.90	7.74	9.5

Table 4 The comparison of efficiency and accuracy of the proposed methods

Method	Number of iterations	Speed index	Error index	Minimum percentage error	Maximum percentage error	The value of objective function
DM	40	30.55	0.00	0.00	0.00	1.22×10^{-10}
FTM	26	100	5.83	0.00	80.05	1.22×10^{-13}
STM	39	61.31	1.19	0.00	18.50	5.68×10^{-10}
SMWM	40	66.45	0.00	0.00	0.00	1.22×10^{-10}

Table 5 The results of damage identification in planar 25-members truss by using noisy measurements

Element	Cross - sectional area before damage	Correct cross-sectional area after damage	Estimated Cross -sectional area by DM	Estimated Cross -sectional area by FTM	Estimated Cross -sectional area STM by	Estimated Cross -sectional area by SMWM
1	18	14	14.04	12.92	13.86	14.04
2	18	14	13.53	12.19	13.32	13.52
3	18	13	12.99	11.23	12.68	13.01
4	18	13	12.94	11.10	12.62	12.92
5	18	12	11.91	9.18	11.39	11.94
6	18	9.5	9.41	1.54	7.59	9.40
7	15	15	14.85	14.85	14.85	14.85
8	15	15	14.85	14.85	14.85	14.85
9	15	15	14.85	14.85	14.85	14.85
10	15	15	14.85	14.85	14.85	14.85
11	15	15	14.85	14.85	14.85	14.85
12	15	15	14.84	14.83	14.84	14.82
13	10	10	9.42	9.40	9.42	9.43
14	10	10	9.20	9.17	9.21	9.21
15	10	10	9.41	9.40	9.42	9.41
16	10	10	9.44	9.48	9.45	9.47
17	10	10	9.67	9.66	9.67	9.70
18	12	12	11.88	11.88	11.88	11.88
19	12	12	11.87	11.87	11.87	11.87
20	12	12	11.88	11.88	11.88	11.88
21	12	12	11.85	11.87	11.85	11.86
22	12	12	11.88	11.88	11.88	11.88
23	12	12	11.85	11.86	11.85	11.85
24	12	12	11.87	11.87	11.87	11.87
25	12	12	11.80	11.80	11.80	11.80

invert the sum of two matrices. It should be added that SMWM algorithm needs less computational efforts than DM approach. Furthermore, the algorithms utilized Taylor series are approximate. In other words, they are unable to reach to the exact responses. It should be noted that STM method is more accurate than FTM technique. However, it is slower, in comparison to FTM scheme. Clearly, FTM tactic is the least accurate method. It should be added that the consuming times of the four aforesaid schemes are infinitesimal in this test. Nevertheless, the differences between the consuming times of the proposed strategies are tangible.

Up to now, the noise-free measurements are utilized to identify damage in this truss. All the four presented methods converge. The obtained results are compared to investigate the capability of the suggested techniques. At this stage, it is presumed that the measurements are noisy. 1%, 2% and 3% proportional errors distributed uniformly are added to all the measurements, respectively. Firstly, the presented scheme in section three is applied to identify the probable harmed members. Then, the four proposed strategies are deployed, and 100 Monte Carlo observations are performed for each algorithm. Afterwards, the mean of the estimated parameters is calculated. For brevity, only the results of the case, in which 1% percent error is applied, are demonstrated in the following table. The results corresponding to cases with 2% and 3% errors are shown in Appendix B. Also, the estimated parameters' coefficients of variation of the Mont Carlo simulation are presented in the aforesaid appendix.

It is obvious that all the members are identified as the probable damaged members. In other words, healthy members are mistakenly placed in the group of the probable harmed members. This is rooted in the fact that the existence of noise in measurements perturbs all the nodal equilibrium equations. Based on Table 5, it is obvious that the proportional errors produced in estimation of the cross-sectional areas of undamaged members are small.

As it was expected, the suggested techniques cannot converge to the exact solution, when measurements are noisy. To be able to compare the accuracy of the suggested techniques in damage detection by utilizing the noisy measurements, error indices are calculated. Error index of SMWM and DM techniques equal 1.96 and 1.99, respectively. Error index of the FTM and STM tactic are 8.02 and 3.21, respectively. It is obvious that SMWM and DM schemes can estimate cross-sectional areas more accurately than FTM and STM strategies in the presence of noise on the measurements.

To investigate the effect of noise percentage on the performance of the authors' methods, the grand mean percentage error (GSDs) of the proposed techniques is used. GSDs are compared in the Fig. 3.

Among the authors' approaches, FTM method has the poorest performance in presence of noise. On the other hand, DM and SMWM tactics lead to more accurate results, in comparison to other strategies. Additionally, it is obvious that GSDs of DM approach is very close to SMWM algorithm. As it was expected, increasing the noise percentage results in more inaccurate results.

7.2 Three-dimensional 26-members truss

A three-dimensional truss which has twenty six members and twenty four degrees of freedom is shown in Fig. 4. Other researchers assessed damage in this structure (Yang and Sun 2010). The

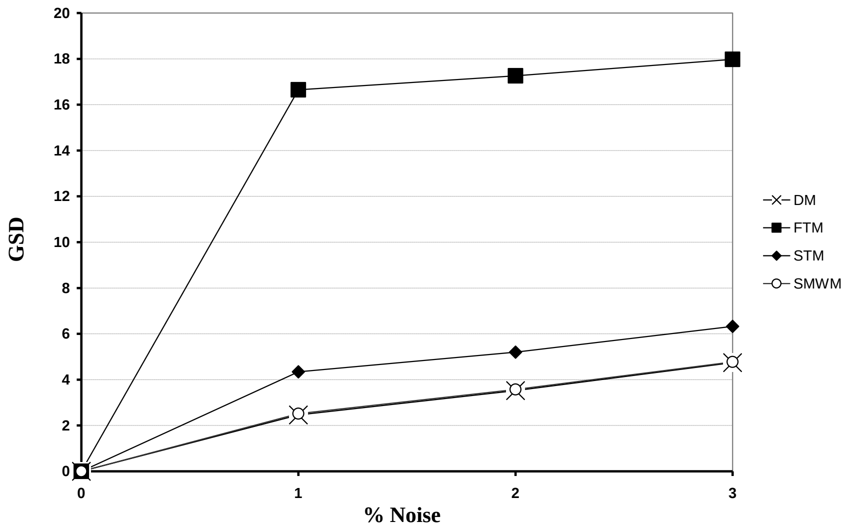

Fig. 3 Comparison of GSds

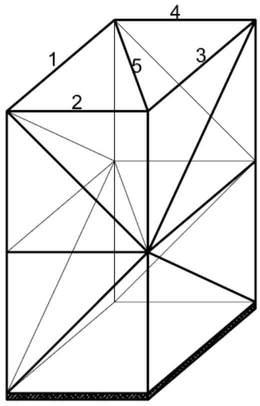

Fig. 4 Three-dimensional twenty six members truss

Table 6 The results of damage identification in 3D 26-members truss

Element	Cross - sectional area before damage	Correct cross-sectional area after damage	Estimated Cross -sectional area by DM	Estimated Cross -sectional area by FTM	Estimated Cross -sectional area STM by	Estimated Cross -sectional area by SMWM
1	40	20	20	0.25	15.34	20
2	40	30	30	26.75	29.47	30
3	40	35	35	34.32	34.94	35
4	40	38	38	37.90	38	38
5	40	25	25	15.79	23.06	25
14	40	40	40	40	40	40
16	40	40	40	40	40	40

cross-sectional areas of its members are 40 cm^2, and Young' modulus equals 2×10^5 kg/cm^2. The cross-sectional areas of members 1-5 are reduced because of being damaged. It should be noted that the damaged members are specified.

To detect damage in this structure, a load case, which generates the equal stress with the value of 800 kg/cm^2 in all members, is employed. Then, the horizontal displacements of the first story's nodes in x and y directions are measured (*xy* plane is the horizontal plane, and the *yz* plane is the vertical one). In addition, the displacements of the second story's nodes in *x* and *z* direction are measured. In fact, all the nodal displacements are not measured.

By applying the proposed method in the third section, the members 1-5, 14 and 16 are placed in the group of the probable harmed members. As a result, their cross-sectional areas should be estimated by using the suggested approaches. The results of analysis are listed in Table 6. It should be added that the cross-sectional areas of other members are known prior to using the presented algorithms. Their cross-sectional areas do not alter due to damage.

Clearly, FTM and STM algorithm are not accurate. In each iteration, these approaches calculate the inversion of the estimated matrix by using Taylor series. Therefore, they are unable to accurately estimate the cross-sectional areas of the members which belong to the category of the probable harmed members. When FTM method is used, the estimated cross-sectional area of member 1 is obviously far smaller than the correct value. It is important to note that this technique

Table 7 The comparison of efficiency and accuracy of the proposed methods

Method	Number of iterations	Speed index	Error index	Minimum percentage error	Maximum percentage error	The value of objective function
DM	46	35.83	0.00	0.00	0.00	2.64×10^{-10}
FTM	27	100	5.72	0.00	98.75	5.60×10^{-16}
STM	41	63.55	1.27	0.00	23.33	2.64×10^{-10}
SMWM	46	74.56	0.00	0.00	0.00	2.64×10^{-10}

produces more errors in comparison to others. Since STM tactic utilizes the second-order approximation of Taylor series, it is more accurate than the FTM method. SMWM and DM approach are precise; consequently, they are able to converge to the exact responses. It is obvious that these strategies calculate the unknown parameters correctly. To compare the accuracy and speed of the proposed methods, the error and speed indices are utilized. The findings are shown in Table 7.

Note that DM and SMWM approach have no errors, while the others have. Although FTM algorithm is computationally less expensive than other methods, it has more errors than them. It is obvious that STM technique is more accurate than FTM tactic. Moreover, SMWM strategy needs fewer arithmetic operations than the DM scheme and converges faster. It should be added that the less the rank of the perturbation matrix is, the less arithmetic operations the SMWM method requires. It should be reminded; the consuming times of the presented techniques are infinitesimal in this numerical example. However, the differences between the consuming times of these tactics are tangible.

At this stage, the capability of the suggested techniques in presence of noise is investigated. For this purpose, it is assumed that 1% proportional error distributed uniformly is added to all the measurements. Then, 100 Mont Carlo tests are performed, and the mean values of the parameters are computed. The error index of the SMWM, DM, STM and FTM techniques are 2.14 and 2.13, 3.88 and 10.19, respectively. Obviously, FTM and STM method performed more weakly than the other two tactics in presence of noise. It should be mentioned that the effects of 2% and 3% error on the obtained results were studied. In addition, GSDs of the proposed tactics were investigated. The findings were compatible with the results achieved in the previous sample. Hence, the results are not illustrated for brevity.

8. Conclusions

In this paper, four algorithms were presented to detect the magnitude and location of damage in trusses. These tactics are capable of damage identification by deploying both complete and incomplete measurements. The aforesaid approaches have two stages. At the first phase, the probable harmed members are separated from other members. The cross-sectional areas of these members are unknown. The author's techniques were applied to estimate them. Note that the remaining members are healthy and their cross-sectional areas are known. Moreover, a new formulation for calculating a proper load case was suggested. This load case is an ideal one which produces equally stressed members.

By investigating the results of numerical samples, the accuracy and efficiency of the proposed techniques were compared. Among the presented procedures, the direct tactic and the SMWM approach utilize accurate inversion of matrices. As a result, they are able to converge to the exact solutions. The findings highlight that the damage detection based on Sherman- Morrison-Woodbury method is more efficient than the direct inversion. In other words, the SMWM technique is less computationally expensive than the direct one. It is worth emphasizing; the less the rank of the perturbation matrix is, the fewer arithmetic operations required by SMWM method. On the other hand, the other two ways are approximate approaches. In other words, they are not able to converge to the exact solutions. This is because the first and second-order approximations of Taylor series are applied in these schemes to invert a sum of two matrices. As any analyzer can expect, the strategy utilized the first-order approximation of Taylor series has more errors in comparison to the other approaches, but it needs less computational efforts than the other suggested formulations, and it is the fastest tactic.

To investigate the capability of the suggested approaches in presence of noise in measurements, Mont Carlo test was carried out. Based on the findings, it is proven that SMWM and DM method perform more accurately, in comparison with the other two schemes, when noisy measurements are used.

References

Abdo M.A.B. (2012), “Parametric study of using only static response in structural damage detection”, *Eng. Struct.*, **34**, 124-131.

Bakhtiari-Nejad, F., Rahai, A. and Esfandiari, A. (2005), “A structural damage detection method using static *noisy data*”, *Eng. Struct.*, **27**, 1784-1793.

Banan, M.R., Banan, M.R. and Hjelmstad, K.D. (1994a), “Parameter estimation of structures from static response, I: Computational Aspects”, *J. Struct. Eng.*, ASCE, **120**(11), 3243-3258.

Banan, M.R., Banan, M.R. and Hjelmstad, K.D. (1994b), “Parameter estimation of structures from static response, II: Numerical Simulation Studies”, *J. Struct. Eng.*, ASCE, **120**(11), 3259-3283.

Billinton, R. and Li, W. (1994), *Reliability Assessment of Electric Power Systems Using Mont Carlo Methods*, Plenum Press, New York, NY, USA.

Carden, E.P. and Fanning, P. (2004), “Vibration based condition monitoring: a review”, *Struct. Hlth. Monit.*, **3**(4), 355-377.

Chou, J.H. and Ghaboussi, J. (2001), “Genetic algorithm in structural damage detection”, *Comput. Struct.*, **79**, 1335-1353.

Doebling, S.W., Peterson, L.D. and Alvin, K.F. (1998), “Experimental determination of local structural stiffness by disassembly of measured flexibility matrices”, *J. Vib. Acoust.*, **120**, 949-957.

Dobeling, S.W., Peterson, L.D. and Alvin, K.F. (1998), “Experimental determination of local structural stiffness by disassembly of measured flexibility matrices”, *J. Vib. Acoust.*, **120**, 949-957.

Friswell, M.I. (2007), “Damage identification using inverse methods”, *Philos. T. Roy. Soc. A.*, **365**, 393-410.

Hager, W.W. (1989), “Updating the inverse of a matrix”, *SIAM Rev.*, **31**(2), 221-239.

Hajela, P. and Soeiro, F.J. (1990a), “Structural damage detection based on static and modal analysis”, *AIAA J.*, **28**(6), 1110-1115.

Hajela, P. and Soeiro, F.J. (1990b), “Recent developments in damage detection based on system identification methods”, *Struct. Optim.*, **2**, 1-10.

Henderson, H.V. and Searle, S.R. (1981), “On deriving the inverse of a sum of matrices”, *SIAM Rev.*, **23**(1), 53-60.

Hjelmstad, K.D. and Shin, S. (1997), “Damage detection and assessment of structures from static response”, *J. Eng. Mech.*, ASCE, **123**(6), 568-576.

Kouchmeshky, B., Aquino, W., Bongard, J.C. and Lipson, H. (2007), “Co-evolutionary algorithm for structural damage identification using minimal physical testing”, *Int. J. Numer. Meth. Eng.*, **69**, 1085-

1107.
Li, Y.Y. and Chen, Y. (2013), "A review on recent development of vibration-based structural robust damage detection", *Struct. Eng. Mech.*, **45**(2), 159-168.
Liu, P.L. and Chian, C.C. (1997), "Parameter identification of truss structures using static strains", *J. Struct. Eng*., ASCE, **123**(7), 927-933.
Majumdar, A., De, A., Maity, D. and Maiti, D.K. (2013), "Damage assessment of beams from changes in natural frequencies using ant colony optimization", *Struct. Eng. Mech*., **45**(3), 391-410.
Mottershead, J.E. and Friswell, M.I. (1993), "Model updating in structural dynamics: a survey", *J. Sound. Vib*., **167**(2), 347-375.
Nobahari, M. and Seyedpoor, S.M. (2013), "An efficient method for structural damage localization based on the concepts of flexibility matrix and strain energy of a structure", *Struct. Eng. Mech*., **46**(2), 231-244.
Rezaiee-Pajand, M. and Aftabi Sani, A. (2008), "Truss damage detection utilizing Pseudo-Dual simplex method", *Int. J. Eng. Sci*., **19**(8), 77-87. (in Persian)
Rezaiee-Pajand, M., Kazemiyan, M.S. and Aftabi, S.A. (2014), "Static damage identification of 3D and 2D frames", *Mech. Base. Des. Struct.*, **42**(1), 70-96.
Rezaiee-Pajand, M. and Saliani, M. (2004), "Damage assessment of plane trusses by static finite element method", Journal of Tehran University College of Engineering (In Persian), **38**(6), 759-775.
Rezaiee-Pajand, M. and Sarafrazi, S.R. (2005), *System Engineering and Optimization*, Ferdowsi University of Mashhad Press, Mashhad, Iran.
Roeck, G.D. (2003), "The state-of-the-art of damage detection by vibration monitoring: the SIMCES Experience", *J. Struct. Control*, **10**, 127-134.
Salawu, O.S. (1997), "Detection of structural damage through changes in frequency: a review", *Eng. Struct.*, **19**(9), 718-723.
Sanayei, M., Imbaro, G.R., McClain, A.S. and Brown, L.C. (1997), "Structural model updating using experimental static measurements", *J. Struct. Eng*., ASCE, **123**(6), 792-798.
Sanayei, M. and Nelson, R.B. (1986), "Identification of structural element stiffness from incomplete static test data", Society of Automotive Engineering, Technical Paper Ser, SAE-861793.
Sanayei, M. and Onipede, O. (1991), "Damage assessment of structures using static test data", *AIAA J.*, **29**, 1174-1179.
Sanayei, M., Onipede, O. and Babu, S.R. (1992), "Selection of noisy measurement locations for error reduction in static parameter identification", *AIAA J.*, **30**(9), 2299-2308.
Sanayei, M. and Saletnik, M.J. (1996a), "Parameter estimation of structures from static strain measurements I: Formulation", *J. Struct. Eng.*, ASCE, **122** (5), 555-562.
Sanayei, M. and Saletnik, M.J. (1996b), "Parameter estimation of structures from static strain measurements II: Error sensitivity analysis", *J. Struct. Eng.*, ASCE, **122**(5), 563-572.
Seyedpoor, S.M. and Yazdanpanah, O. (2013), "An efficient indicator for structural damage localization using the change of strain energy based on static noisy data", *Appl. Math. Model.*, doi: http://dx.doi.org/10.1016/j.apm.2013.10.072.
Sheena, Z., Unger, A. and Zalmanovich A. (1982), "Theoretical stiffness matrix correction by static test results", *Israel J. Tech*., **20**, 245-253.
Shenton, lll, H.W. and Hu, X. (2006), "Damage identification based on dead load redistribution: methodology", *J. Struct. Eng*., ASCE, **132**(8), 1254-1263.
Shih, H.W., Thambiratnam, D.P. and Chan, T.H.T. (2011), "Damage detection in truss bridges using vibration multi-criteria approach", *Struct. Eng. Mech.*, **39** (2), 187-206.
Yan, Y.J., Cheng, L., Wu, Z.Y. and Yam L.H. (2007), "Development in vibration-based structural damage detection technique", *Mech. Syst. Sig. Pr.*, **21**, 2198-2211.
Yang, Q.W. and Sun, B.X. (2010), "Structural damage localization and quantification using static test data", *Struct. Hlth. Monit.*, **10**(4), 381-389.
Yun, G.J. (2012), "Detection and quantification of structural damage under ambient vibration environment", *Struct. Eng. Mech.*, **42**(3), 425-448.

Appendix A: Sherman-Morrison-Woodbury formulas

The presented formulas in this section are applied to invert a sum of two matrices. It is important to note that these equations calculate the inversion of a sum of two matrices accurately. They are named Sherman- Morrison- Woodbury formulas. In this article, the third one is utilized.

$$[\boldsymbol{A}-\boldsymbol{U}\boldsymbol{V}]^{-1}=\boldsymbol{A}^{-1}+\boldsymbol{A}^{-1}\boldsymbol{U}(\boldsymbol{I}-\boldsymbol{V}\boldsymbol{A}^{-1}\boldsymbol{U})^{-1}\boldsymbol{V}\boldsymbol{A}^{-1} \tag{A.1}$$

In this relation, $\boldsymbol{A}$ is $n\times n$ matrix, and matrix $\boldsymbol{U}$ has $n\times m$ entries. Furthermore, $\boldsymbol{V}$ is a $m\times n$ matrix. $\boldsymbol{I}$ is a $m\times m$ identity matrix. If matrix $\boldsymbol{U}$ is a column vector, and matrix $\boldsymbol{V}$ is a row vector, the last equation will have the below shape

$$[\boldsymbol{A}-\boldsymbol{U}\boldsymbol{V}]^{-1}=\boldsymbol{A}^{-1}+\alpha\boldsymbol{A}^{-1}\boldsymbol{U}\boldsymbol{V}\boldsymbol{A}^{-1},\ \alpha=\frac{1}{(1-\boldsymbol{V}\boldsymbol{A}^{-1}\boldsymbol{U})} \tag{A.2}$$

By generalizing Eq. (A.1), it can lead to the coming form

$$[\boldsymbol{A}-\boldsymbol{U}\boldsymbol{D}^{-1}\boldsymbol{V}]^{-1}=\boldsymbol{A}^{-1}+\boldsymbol{A}^{-1}\boldsymbol{U}(\boldsymbol{D}-\boldsymbol{V}\boldsymbol{A}^{-1}\boldsymbol{U})^{-1}\boldsymbol{V}\boldsymbol{A}^{-1} \tag{A.3}$$

In this formula, matrix $\boldsymbol{U}$ has $n\times m$ entries, and matrix $\boldsymbol{V}$ has m rows and n columns. As it was mentioned so far, Eq. (A.3) is utilized in this article.

Appendix B: The results obtained from Mont Carlo analysis

For the first numerical sample, the mean of the estimated parameters is calculated for 2% and 3% proportional error. Also, the corresponding coefficients of variation of the Mont Carlo simulation are presented.

Table B.1 The results of damage identification in planar 25-members truss, 2% proportional error

Element	Cross - sectional area before damage	Correct cross-sectional area after damage	Estimated Cross -sectional area by DM	Estimated Cross -sectional area by FTM	Estimated Cross -sectional area STM by	Estimated Cross -sectional area by SMWM
1	18	14	14.06	12.95	13.88	14.06
2	18	14	13.50	12.12	13.30	13.49
3	18	13	13.02	11.22	12.67	13.03
4	18	13	12.93	11.10	12.62	12.90
5	18	12	11.96	9.10	11.34	11.95
6	18	9.5	9.35	1.40	7.55	9.37
7	15	15	14.85	14.85	14.85	14.85
8	15	15	14.85	14.85	14.85	14.85
9	15	15	14.85	14.85	14.85	14.85
10	15	15	14.85	14.85	14.85	14.85
11	15	15	14.83	14.82	14.81	14.80
12	15	15	14.68	14.68	14.72	14.73
13	10	10	9.41	9.34	9.36	9.39

Table B.1 Continued

14	10	10	9.19	9.07	9.17	9.17
15	10	10	9.36	9.27	9.31	9.32
16	10	10	9.39	9.31	9.28	9.35
17	10	10	9.53	9.49	9.53	9.59
18	12	12	11.88	11.88	11.87	11.88
19	12	12	11.84	11.83	11.84	11.84
20	12	12	11.86	11.86	11.86	11.86
21	12	12	11.80	11.82	11.80	11.81
22	12	12	11.83	11.82	11.83	11.84
23	12	12	11.79	11.83	11.80	11.80
24	12	12	11.79	11.80	11.80	11.80
25	12	12	11.69	11.73	11.72	11.71

Table B.2 The results of damage identification in planar 25-members truss, 3% proportional error

Element	Cross - sectional area before damage	Correct cross- sectional area after damage	Estimated Cross -sectional area by DM	Estimated Cross -sectional area by FTM	Estimated Cross -sectional area STM by	Estimated Cross -sectional area by SMWM
1	18	14	14.07	12.97	13.89	14.08
2	18	14	13.46	12.02	13.25	13.44
3	18	13	13.02	11.20	12.65	13.03
4	18	13	12.93	11.09	12.61	12.88
5	18	12	11.98	8.93	11.31	11.96
6	18	9.5	9.30	1.33	7.49	9.34
7	15	15	14.85	14.85	14.85	14.85
8	15	15	14.85	14.85	14.85	14.85
9	15	15	14.85	14.84	14.85	14.84
10	15	15	14.83	14.85	14.83	14.83
11	15	15	14.79	14.75	14.77	14.72
12	15	15	14.51	14.54	14.56	14.63
13	10	10	9.35	9.23	9.25	9.31
14	10	10	9.10	8.92	9.05	9.06
15	10	10	9.24	9.09	9.17	9.18
16	10	10	9.26	9.12	9.10	9.20
17	10	10	9.38	9.33	9.39	9.48
18	12	12	11.86	11.86	11.86	11.87
19	12	12	11.79	11.78	11.80	11.80
20	12	12	11.82	11.82	11.81	11.83
21	12	12	11.75	11.78	11.74	11.76
22	12	12	11.76	11.75	11.76	11.79
23	12	12	11.74	11.78	11.75	11.76
24	12	12	11.70	11.70	11.70	11.71
25	12	12	11.59	11.66	11.65	11.63

Table B.3 The coefficients of variation of the Mont Carlo simulation for 25 planar truss

	1% proportional error				2% proportional error				3% proportional error			
Element	DM	FTM	STM	SMWM	DM	FTM	STM	SMWM	DM	FTM	STM	SMWM
1	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.02	0.03	0.02	0.02
2	0.01	0.02	0.01	0.01	0.02	0.04	0.02	0.01	0.02	0.05	0.03	0.03
3	0.01	0.03	0.02	0.01	0.04	0.06	0.03	0.01	0.04	0.08	0.05	0.04
4	0.02	0.04	0.02	0.02	0.06	0.09	0.05	0.02	0.06	0.13	0.07	0.06
5	0.03	0.09	0.04	0.03	0.08	0.17	0.08	0.03	0.08	0.25	0.11	0.08
6	0.01	0.26	0.03	0.01	0.04	0.57	0.05	0.01	0.04	0.84	0.08	0.04
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.01	0.01
11	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.02	0.02	0.02
12	0.00	0.00	0.00	0.01	0.04	0.02	0.01	0.01	0.04	0.04	0.03	0.04
13	0.02	0.03	0.02	0.03	0.06	0.05	0.04	0.03	0.06	0.06	0.05	0.06
14	0.02	0.03	0.03	0.03	0.07	0.06	0.05	0.03	0.07	0.08	0.07	0.07
15	0.03	0.03	0.03	0.03	0.06	0.06	0.05	0.03	0.06	0.08	0.07	0.07
16	0.03	0.04	0.04	0.04	0.08	0.07	0.07	0.04	0.08	0.10	0.09	0.08
17	0.03	0.03	0.03	0.02	0.07	0.05	0.05	0.02	0.07	0.08	0.07	0.06
18	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.01
19	0.00	0.00	0.00	0.00	0.02	0.01	0.01	0.00	0.02	0.02	0.01	0.02
20	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.01	0.02	0.01
21	0.00	0.00	0.01	0.01	0.02	0.01	0.01	0.01	0.02	0.02	0.02	0.02
22	0.00	0.00	0.00	0.00	0.02	0.01	0.01	0.00	0.02	0.02	0.02	0.02
23	0.01	0.00	0.01	0.01	0.03	0.01	0.01	0.01	0.03	0.02	0.02	0.02
24	0.00	0.00	0.00	0.00	0.03	0.02	0.02	0.00	0.03	0.03	0.03	0.03
25	0.01	0.01	0.01	0.01	0.04	0.02	0.03	0.01	0.04	0.03	0.04	0.04