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A multi-crack effects analysis and crack identification in functionally graded beams using particle swarm optimization algorithm and artificial neural network

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Abstract. In the first part of this paper, the influences of some of crack parameters on natural frequencies of a cracked cantilever Functionally Graded Beam (FGB) are studied. A cantilever beam is modeled using Finite Element Method (FEM) and its natural frequencies are obtained for different conditions of cracks. Then effect of variation of depth and location of cracks on natural frequencies of FGB with single and multiple cracks are investigated. In the second part, two Multi-Layer Feed Forward (MLFF) Artificial Neural Networks (ANNs) are designed for prediction of FGB's Cracks' location and depth. Particle Swarm Optimization (PSO) and Back-Error Propagation (BEP) algorithms are applied for training ANNs. The accuracy of two training methods' results are investigated.

Keywords: multiple cracks; functionally graded beam; artificial neural network; particle swarm optimization; identification

1. Introduction

In mechanical systems, crack is a damage that if develops, may cause catastrophic failure. Therefore identification of cracks is a very important issue. In recent years, from non-destructive methods of study cracked structures, researchers have paid great attention to vibration analysis methods. Many numerical, analytical, and experimental studies have been done in this field. A Crack causes a local flexibility in structure which has some influences on dynamic behavior. For example, it changes the mode shapes and reduces natural frequencies. Analysis of these effects can was used for crack detection by Douka *et al.* (2003). Dimarogonas (1996) studied methods of investigation of cracked structures. Dimarogonas (1976), Paipetis and Dimarogonas (1986) modeled a crack using local flexibility and evaluated the equivalent stiffness utilizing fracture mechanics. Adams and Cawley (1979) developed an experimental technique to estimate crack depth and location using natural frequencies. In another investigation Chan and Dimarogonas

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(1980) presented methods which relate crack depth to natural frequencies when the crack location is known. These methods can be used for detection of cracks in different structures. Goudmunson (1982) presented a method for prediction of changes of natural frequencies caused by faults such as cracks, notches, etc. Shen and Taylor (1991) presented a method based on minimizing difference between measured data and data obtained from analytical study for identification of cracks in an Euler-Bernoulli beam. Masoud *et al.* (1998) studied vibrational characteristics of a fixed-fixed beam which contains a symmetric crack considering coupling effect of crack depth and axial load.

In recent decade some investigations have been done on vibrational behavior of multi-cracked structures. Sekhar (2008) summarized different papers on multiple cracks, the respective influences, and identification methods in some structures such as beams, pipes, rotors etc. Lee (2009) used FEM to solve forward problem in a multi-cracked beam. In this paper an inverse problem is solved iteratively for locations and depths of the cracks using the Newton-Raphson method. Patil and Maiti (2003) identified multiple cracks using frequency measurements. Their procedure presented a linear relationship explicitly between changes in natural frequencies and damage parameters. Mazanoglu and Yesilyurt (2009) performed a vibration analysis of multi-cracked variable cross section beams using the Rayleigh–Ritz approximation method. Binici (2005) presented a parametric study on the effect of cracks and axial force levels on eigenfrequencies. A new method for natural frequency analysis of beams with an arbitrary number of cracks has been developed by Khiem and Lien (2001). Cam *et al.* (2008) studied vibrations of cracked beam as a result of impact shocks to obtain information about crack location and depth of beams. Also Lu and Liu (2012) proposed a response sensitivity-based model updating method to identify both a single and multiple cracks of a beam from dynamic responses.

A new technique often used for identification of damage in the recent two decades is artificial neural network. Wu *et al.* (1992) used multi-layer feed forward neural network to identify the location of fault in a simple frame. Wang and He (2007) developed a numerical simulation and model experiment upon a hypothetical concrete arch dam for the crack identification based on reduction of natural frequencies using a statistical neural network. In another study, Kao and Hung (2003) presented a two-step method for detection of cracks using ANN. The first step is identification of damaged and undamaged system situations and the second step is damage detection in structures. In the second step a trained ANN is applied to produce free vibration response of system. After that changes of amplitude and periods between results are compared. Also Chen *et al.* (2003) applied the ANN for damage detection in structures when excitation signal is not available.

Optimization algorithms have been frequently applied for identification of damages in structures in recent years. Xiang *et al.* (2008) presented a method for detection of crack location and depth in shafts. They construct Rotating Rayleigh-Timoshenko and Rayleigh-Euler beam elements of B-Spline Wavelet on the interval to discretize slender shaft and stiff disc, respectively. They modeled a cracked shaft using wavelet-based elements to obtain precise frequencies. Then they used the first three natural frequencies of the structures in crack detection process and identified the normalized crack location and depth by means of GA. In another work, He *et al.* (2001) formulated crack identification of a shaft as an optimization problem and using GA to search for a solution.

A new type of materials that are generally made of a mixture of ceramic and a metal to satisfy demand of a ultra-high-temperature environment and to eliminate interface problems are known as Functionally Graded Materials (FGMs). Since the concept of FGMs has been introduced in 1980s, these new kinds of materials have been employed in many engineering application fields, such as space vehicles, aircrafts, defense industries, electronics and biomedical sectors, to eliminate stress concentrations, enhance bonding strength and relax residual stresses. Because of the high cost of productions and importance of applications of FGMs, study behavior of the cracked Functionally Graded (FG) structures is very important. Yu and Chu (2009) employed a *ρ*-version of FEM to evaluate transverse vibration characteristics of a cracked FGB. Kitipornchai *et al.* (2009) investigated nonlinear vibration of cracked FGBs using Timoshenko beam theory and von-Karman geometric nonlinearity. They assumed that material properties vary exponentially through beam thickness. Yang and Chen (2008) presented a theoretical investigation in free vibration and elastic buckling of cracked FGBs containing open edge cracks by using a rotational spring model and the Bernoulli-Euler beam theory. They obtained analytical solutions of natural frequencies, critical buckling load, and corresponding mode shapes for FGBs containing open edge cracks with different end supports. In another study a non-linear dynamic analysis of FGB with pinned-pinned supports due to a moving harmonic load using Timoshenko beam theory with the von--Karman's non-linear strain-displacement relationships was presented by Simsek (2010). He assumed that material properties of a beam vary along thickness according to a power-law form.

In this paper, the effects of some crack factors on natural frequencies of a cantilever FGB with single and multiple cracks are investigated using FEM. The results of the FGB are then compared with a homogeneous beam. After that a procedure is presented for the crack identification in FGBs. In this procedure BEP and PSO algorithms are applied separately for ANN training for crack detection in the FGB. Finally the results of these training methods are analyzed.

2. Mathematical background

In this paper, a crack is assumed to be perpendicular to beam surface and always remains open. The cracked beam can be treated as two sub-beams which are connected by an elastic rotational spring with no mass and no length at the crack section according to the rotational spring model. The following formulation relates the cracked section bending stiffness K_t to the flexibility G.

$$K_t = \frac{1}{G} \tag{1}$$

According to Broek (1986) the flexibility of the beam due to the presence of edge crack can be calculated from

$$\frac{1-\mu^2}{E(z)} K_1^2 = \frac{M_1^2}{2} \frac{dG}{da}, \tag{2}$$

Where M_1 is the cracked section bending moment. Stress Intensity Factor (SIF) under mode *I* is a function of crack depth, beam geometry, external loading and material properties which it was derived by Erdogan and Wu (1997) for an FGM stripe with an open edge crack under bending as

$$K_1 = -\frac{4\sqrt{a}v(z)}{1+\bar{\mu}} \sum_{i=0}^{\infty} \alpha_i T_i\left(\frac{2z-a}{a}\right), \tag{3}$$

Where T_i is the first kind Chebyshev polynomial and $\bar{\mu} = (3-4\mu)$ By evaluating boundary

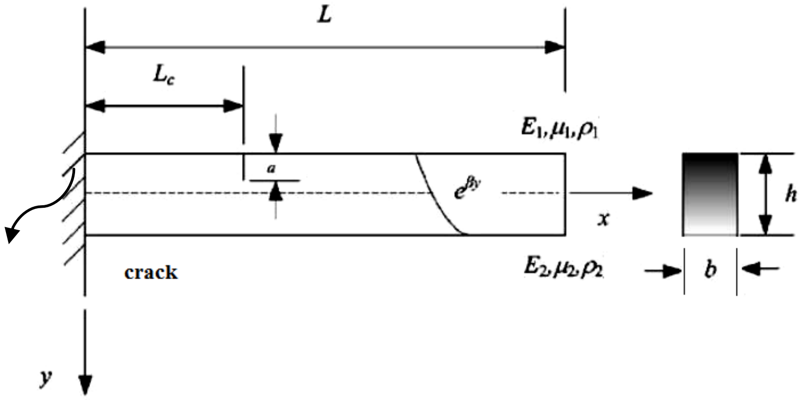


Fig. 1 Geometry of the Cracked FGB

Table 1 Characteristics of the FGB

Length-Height ratio (*L/h*)	20
(E_2/E_1) ratio	0.01
(E_1) (GPa)	70
Density (ρ_1) (kg/m3)	2780
Poisson's ratio	0.33

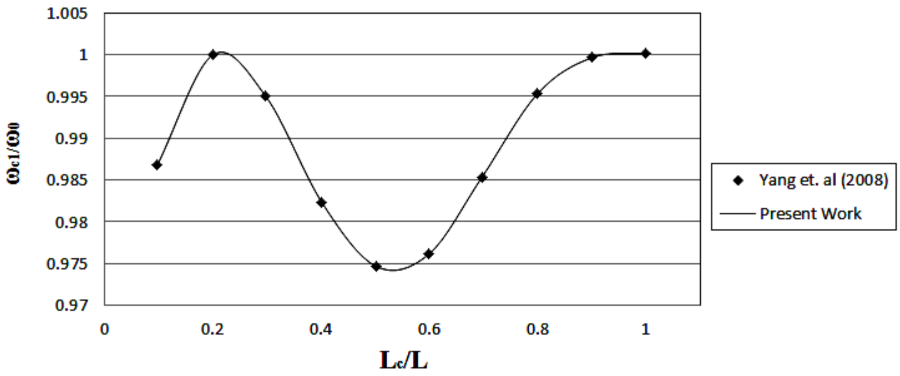
integrals using the Gaussian quadrature and solving the resulting functional equation by a collocation method, the constants α_i can be determined.

3. Modal analysis using finite element method

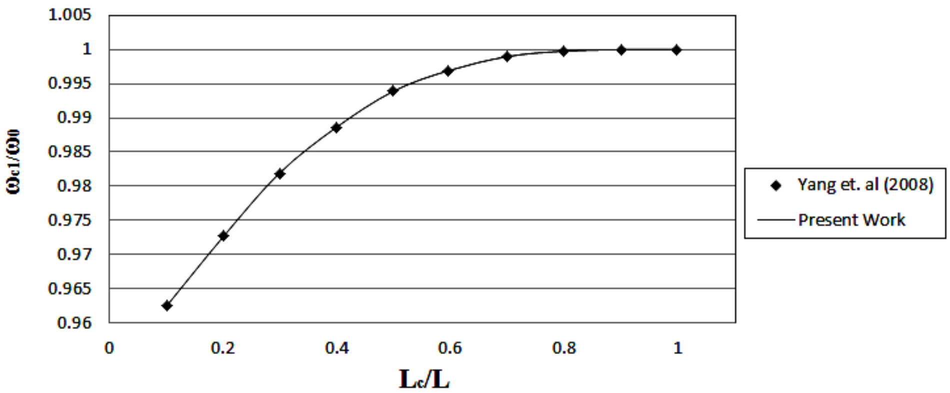
Considered structure in this paper is a cantilever FGB. The elastic modulus *E*, shear modulus μ and mass density ρ are taken to be in the form of Eq. (4). In these equations, E_0, μ_0 and ρ_0 are values of these parameters at *y*=0, and $\beta=ln(E_2/E_1)/h$ is a constant representing material gradient, where *h* is height of the beam and E_1 and E_2 are elastic modulus in top and bottom of the FGB. A schematic view of the FGB has been shown in Fig. 1. Table 1 shows material and geometrical characteristic of the FGB.

$$E(y) = E_0 e^{\beta y},\ \mu(y) = \mu_0 e^{\beta y},\ \rho(y) = \rho_0 e^{\beta y} \tag{4}$$

In following examples, 2D FEM of the FGB with and without crack is established using a commercial package ANSYS. The FGB is assumed to consist of 30 layers with same heights containing isotropic homogeneous materials. The material properties are constant in each layer and are equal to the value of middle point of the corresponding layer. The multi-layer plate structure is discretized using a 8-node quadrilateral plane stress element (PLANE 183), which has two degrees of freedom at each node. Three first natural frequencies of structure in different conditions of the crack in this study were obtained using a modal analysis of ANSYS. Then the calculated data is validated in comparison with the result of paper of Yang *et al*. (2008) for cracked and un-cracked



(a)



(b)

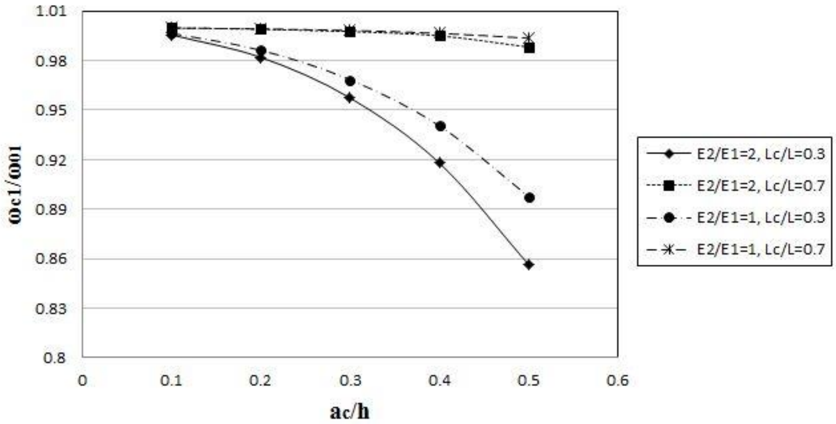
Fig. 2 Data validation in comparison with Yang *et al*. for cracked FGM beam: non-dimensional natural frequency of cracked FGB versus non-dimensional crack location (a) First natural frequency (b) Second natural frequency

FGB which is shown in Fig. 2. The properties of structure used in paper of Yang *et al*. have been tabulated in Table 1.

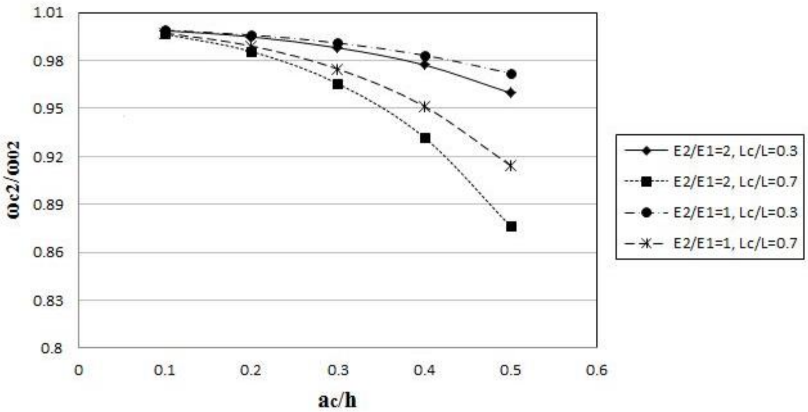
4. Effects of cracks' parameters on natural frequencies of FGB

Variations of different cracks parameters such as depth, location, and number of cracks are considered to assess their effects on natural frequencies of structure.

Figs. 3(a)-(c) indicate effect of variation of location of the crack on first three natural frequencies of beam for two different constant depths of cracks. In this figure both FG and homogeneous beams have been investigated. It should be mentioned that the natural frequencies have been considered based on the ratio of data of beam in cracked form to un-cracked form. As can be seen from Fig. 3(a), the non-dimensional natural frequencies for both two depths of cracks have a monotonic growth with the increase of crack location for both FG and homogeneous beams.



(a)



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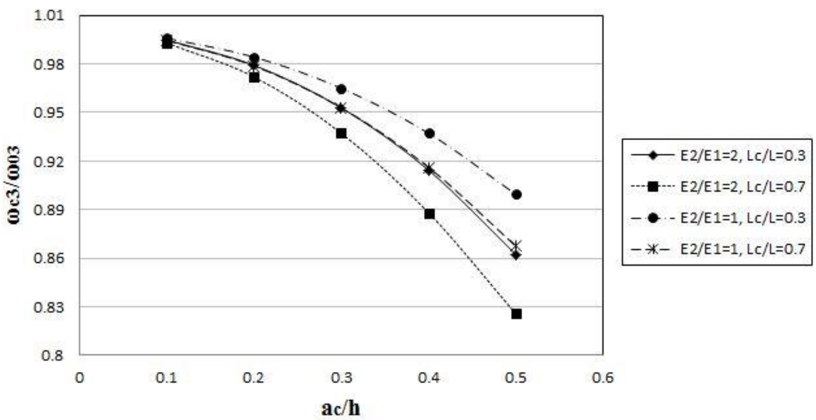
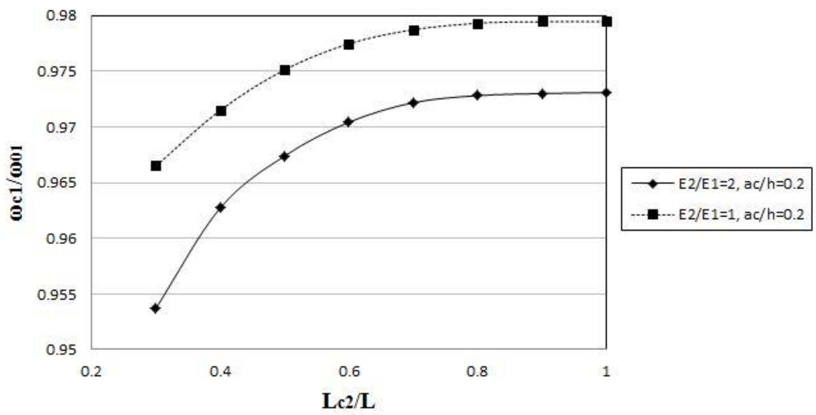
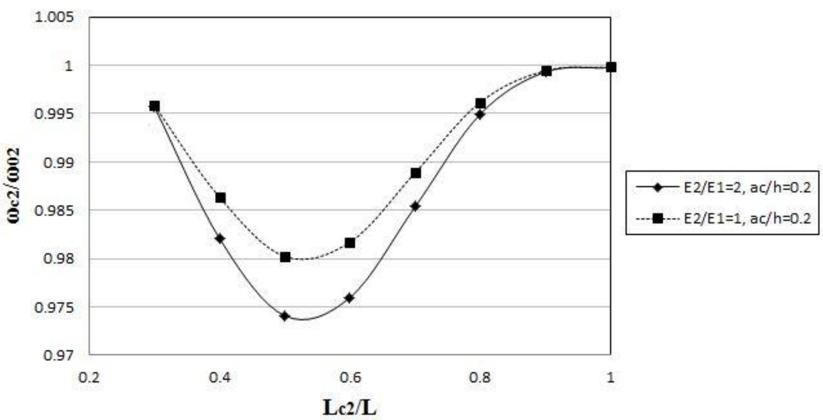


(c)

Fig. 4 The first three non-dimensional natural frequencies of the cantilever FGB versus non-dimensional crack depth for different crack locations and material properties



(a)



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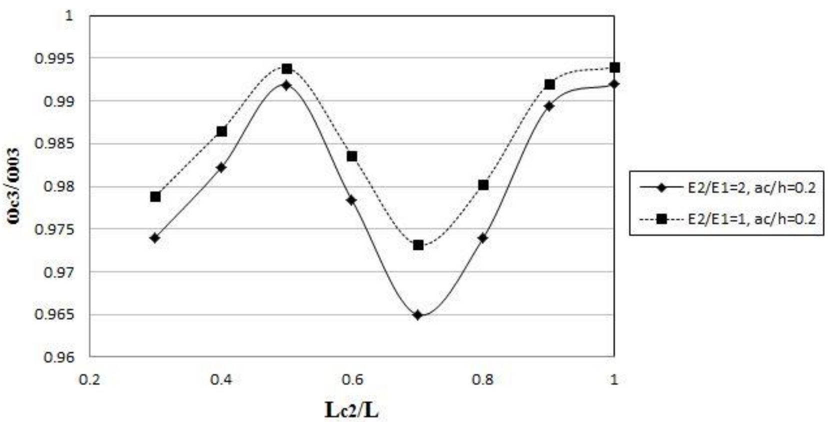
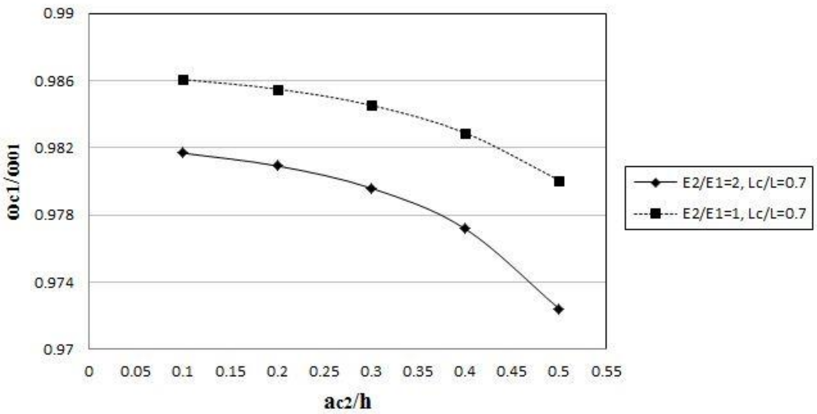
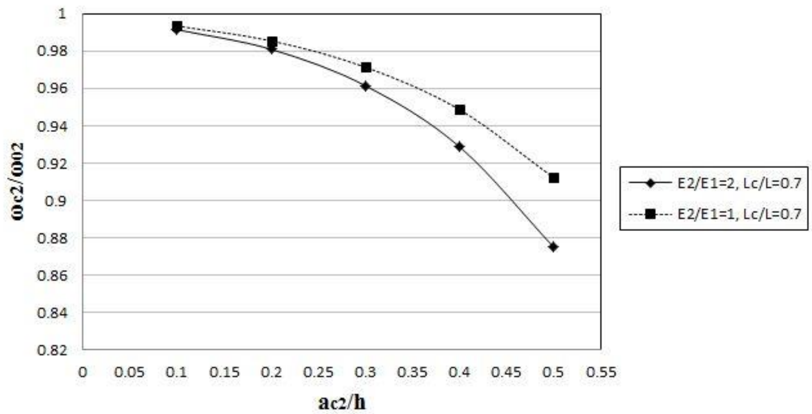


(c)

Fig. 5 The first three non-dimensional natural frequencies of the cantilever FGB with two cracks versus the second non-dimensional crack location for different material properties



(a)



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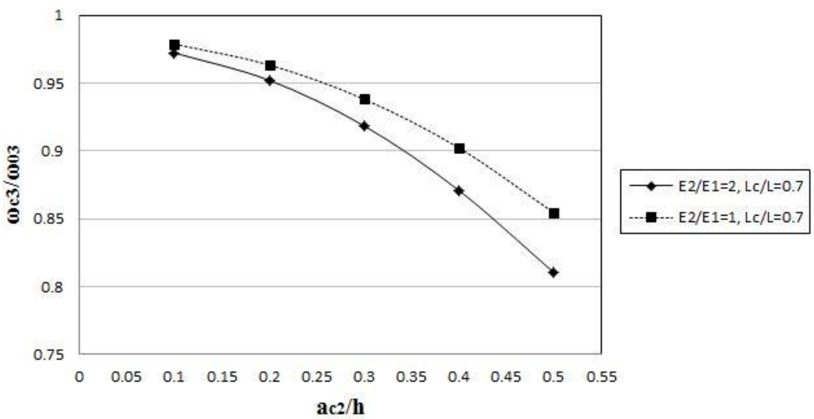


(c)

Fig. 6 The first three non-dimensional natural frequencies of the cantilever FGB with two cracks versus the second non-dimensional crack depth for different material properties

Figs. 4(a)-(c) illustrate effects of variation of the crack depth on first three natural frequencies of beam for two different constant crack locations. As can be seen from Fig. 4, the non-dimensional natural frequencies for both locations of cracks have a monotonic decline with the increase of non-dimensional depth of crack for both FG and homogeneous beams.

Figs. 5 (a)-(c) show the influences of second crack location on first three natural frequencies of beam with two cracks for two different constant depths of cracks. In this part, depth and location of one of the cracks has been considered to be constant, and the location of the second crack has been changed along the length of beam. As can be seen from Figs. 5(a)-(c), the non-dimensional natural frequencies for both depths of cracks have a monotonic rise with the increase of non-dimensional location of cracked FGB. Also as shown in Fig. 5(b) and 5(c), with increase of location of the crack, the second and third non-dimensional natural frequencies of beam for both cracks depths and for both material types of beam (FG and homogeneous) does not have a monotonous trend.

Figs. 6 (a)-(c) indicate effect of variation of the second crack depth on first three natural frequencies of the beam with two cracks. This time depth and location of one crack is constant, and the depth of the second crack is changed. As can be seen from Fig. 6, the non-dimensional natural frequencies have a monotonic decrease with the increase of non-dimensional depth of the crack for both FG and homogeneous beams.

From Figs. 3-6 it can be concluded that, curves of homogeneous beam are always higher than FGB.

5. Crack identification using soft computing methods

5.1 Artificial Neural Network

The ANN used in this study is a MLFF neural network consists of an input layer, some hidden layers and an output layer. A schematic of the MLFF neural network is shown in Fig. 7. Usually, knowledge in ANNs is stored as a set of connection weights. Process of modification of connection weights, using a suitable learning method is called training. In this study two distinct

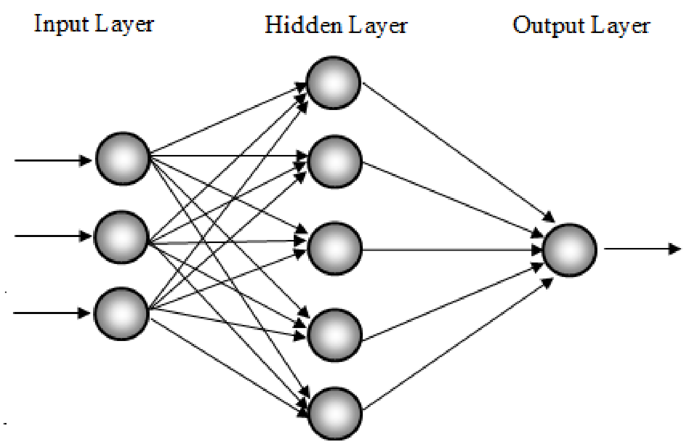


Fig. 7 Schematic diagram of a typical MLFF neural-network architecture

ANNs are employed for prediction of the crack locations and depths. The ANNs consist of one input layer with 3 neurons, one hidden layer with 6 neurons and one output layer with one neuron. Transfer functions for neurons of hidden and output layers are defined as Eq. (5), called Tansig function.

$$f(n)=\frac{2}{[1+\exp(-2n)]}-1; \tag{5}$$

5.2 Back-Error propagation algorithm

The BEP is the most widely used learning algorithm of MLFF neural networks. This learning method was proposed by McClelland and Rumelhart (1986) in a ground-breaking study originally focused on cognitive computer science.

In this work the structure of the ANN includes three layers: input, hidden, and output layers.

Values of w_{ab} represent weights between the input and the hidden layer. Values of w_{bc} represent weights between the hidden and the output layer. The operation of BEP consists of three stages:

1- Feed-forward stage

$$v = w_{bc}(n).y(n); \tag{6}$$

$$o(n)=\varphi(v(n)))=\frac{2}{1+\exp[-v(2n)]}; \tag{7}$$

Where o is the output, u is the input, y is the output of hidden layer and φ is the activation function.

2- Back-propagation stage

$$\delta(n)=e(n).\varphi[v(n)]=[d(n)-o(n)].[o(n)].[1-o(n)]; \tag{8}$$

Where δ represents local gradient function, e shows error function, o and d means actual and desired outputs, respectively.

1- Adjust weighted value

$$w_{ab}(n+1)=w_{ab}(n)+\Delta w_{ab}(n)=w_{ab}(n)+\eta\delta(n).o(n); \tag{9}$$

Where η is the learning rate. Repeating these three stages led to a value of the error function that will be zero or a constant value.

5.3 Particle swarm optimization algorithm

PSO is a population based stochastic optimization technique developed by Eberhart and Kennedy (1995), inspired by social behavior of bird flocking or fish schooling. This algorithm shares many similarities with evolutionary computation techniques such as GA. The optimization procedure is started with a population of random solutions and searches for optima by updating generations. Unlike GA, PSO has no evolution operators such as mutation and crossover. In PSO, potential solutions that are named “particles” fly through the space of problem by following current optimum particles. Each particle keeps track of its coordinates in the space of problem which are associated with the best solution (best fitness) it has achieved up to now, called “*pbest*”.

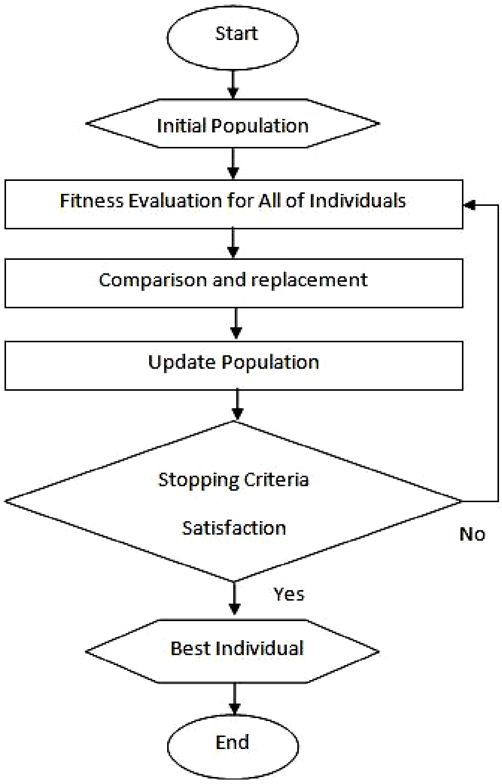


Fig. 8 Flowchart of the PSO

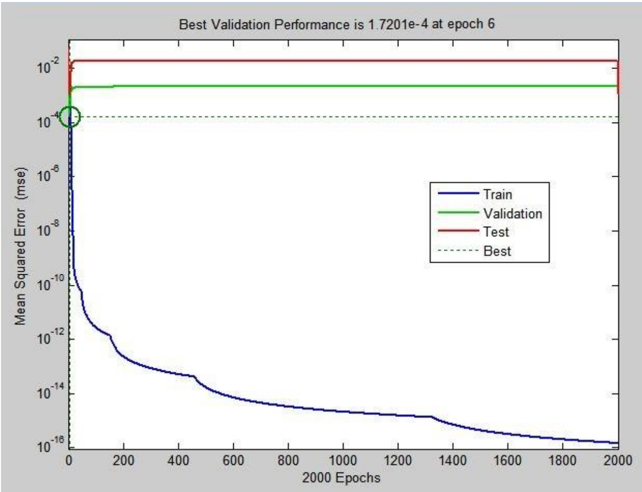
Another "*best*" value that is tracked using PSO is the best value, achieved so far by any particle in the neighbors of the particle. This parameter is called "*lbest*". When a particle takes all the population as its topological neighbors, a global best, named "*gbest*", is the best value. The PSO concept at each time step consists of changing the velocity of each particle toward its "*pbest*" and "*lbest*" locations. Acceleration is weighted using a random term, by separate random numbers being generated for acceleration toward "*lbest*" and "*pbest*" locations. A schematic of optimization procedure of PSO is illustrated in Fig. 8.

5.4 Crack detection procedure

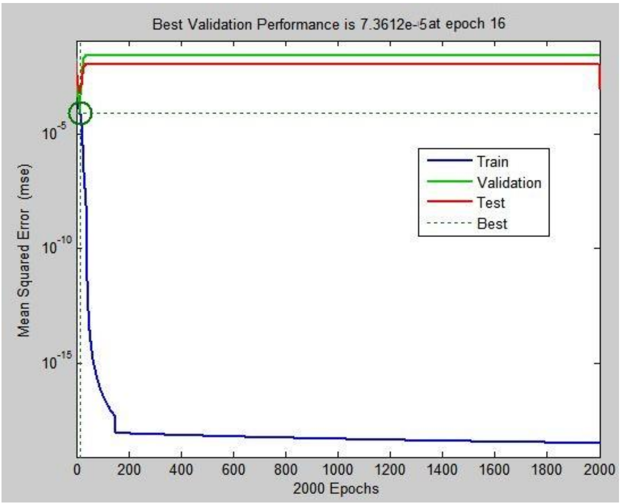
For detection of crack, first of all, MLFF ANNs with 3, 6 and 1 neuron in input, hidden and output layers are created. First three natural frequencies of the FGB in 22 different crack conditions are applied to ANNs as the input and the corresponding locations and depths are applied as targets to first and second networks, respectively. The data applied to ANNs are obtained using FEM on the FGB that are modelled by 20 different layers. Training the ANNs is done using two different algorithms of BEP and PSO using MATLAB. The BEP algorithm is used with 2000 iterations. In ANN training using the BEP method, 70%, 20% and 10% of data were used for train, verification and test of network. BEP training procedure for prediction of location and depth of cracks using MATLAB, are plotted in Figs. 9(a) and 9(b), respectively. In training

Table 2 The predicted crack location and depth using ANN and different training methods

Crack Number		1st	2nd	3rd	4th	Average
Actual	*Depth*	0.3	0.4	0.25	0.35	
	Location	0.5	0.3	0.4	0.4	
BEP	*Depth*	0.287	0.396	0.248	0.368	
	Location	0.536	0.303	0.375	0.394	
	Error of Depth (%)	1.33	0.357	0.235	1.793	0.93
	Error of Location (%)	3.59	0.31	2.51	0.61	1.75
PSO	*Depth*	0.279	0.391	0.245	0.344	
	Location	0.562	0.299	0.394	0.415	
	Error of Depth (%)	2.09	0.87	0.42	0.54	0.98
	Error of Location (%)	6.18	0.01	0.54	1.57	2.10

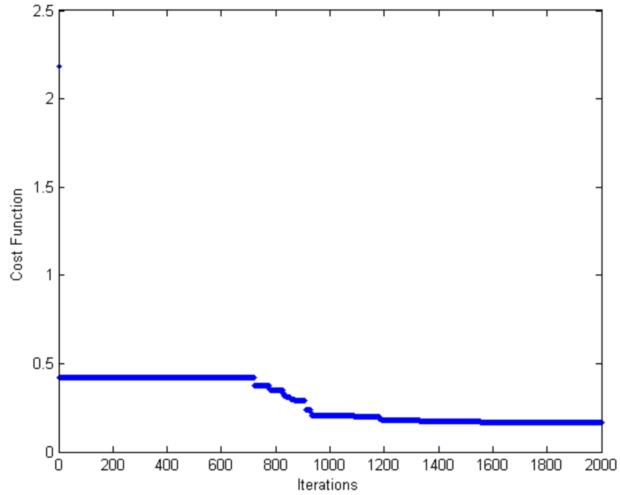


(a) location of cracks

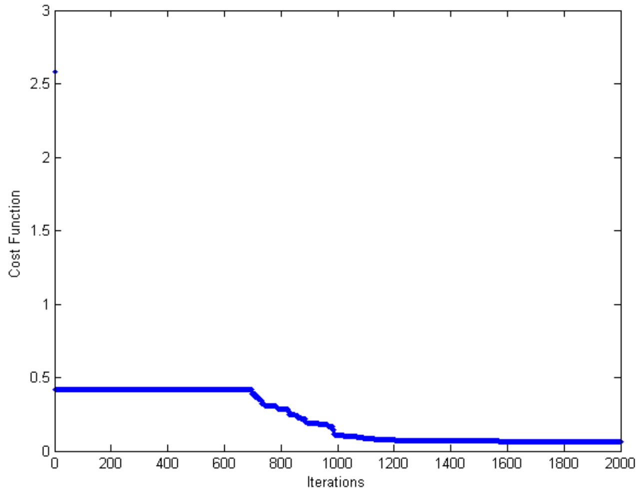


(b) depth of cracks

Fig. 9 BEP Training procedures of ANNs for identification of cracks



(a) location of cracks



(b) depth of cracks

Fig. 10 PSO based ANNs training for identification of cracks

process using PSO algorithm, weights of ANNs were considered as optimization variables and sum of square of differences between the output and the target values of ANNs were assumed as cost functions. Considered PSO algorithm has a population size of 100, 2000 iterations and 31 optimization variables. The trained ANNs are tested for 4 different conditions of cracks as tabulated in Table 2. Training procedure of ANNs using PSO algorithm for prediction of location and depth of cracks are illustrated in Figs. 10(a) and 10(b), respectively.

As can be found from this table, average errors in prediction of crack locations for the BEP and PSO algorithms are 1.75% and 2.10%, respectively. Location Errors were computed as differences between actual and predicted location of crack on length of beam in percent. Also the average errors in prediction of cracks depths for BEP and PSO algorithms are 0.93% and 0.98%, respectively. Depth Errors were computed as differences between actual and predicted depth of cracks on depth of beam in percent. Therefore, it can be concluded that for the considered ANNs and assumed condition, the ANNs are trained by these training methods predict the cracks' depths more accurate than cracks’ locations. Also as can be seen in Table 2, there is good agreements between actual and predicted results.

6. Conclusions

In the first part of this study, the effects of some crack factors on natural frequencies of a cracked cantilever FG, and homogeneous beams is studied. A cantilever beam is modeled using the FEM and the results are validated using an available reference. Then its natural frequencies are calculated for different crack conditions. In the case of FGB with a single crack, the influences of crack location and depth on natural frequencies of FG and homogeneous beam are investigated. The same procedure is done for variation of crack locations and depths for multi-cracked FGB. In the second part, a procedure consisting of three steps based on ANN for detection of crack location and depth in FGB is presented. In the first step, three natural frequencies of a FGB for different locations and depths of cracks are calculated using the FEM that was obtained in the first part of this study. In the second step, two MLFF neural networks are created and the BEP and PSO algorithms are used to train them based on the FEM data. In the third step, trained ANNs are used to predict the characteristics of some cracks in the FGB. The first ANN is used to predict the crack locations and the second ANN for the crack depths. Finally it is concluded that for the assumed condition of training algorithm and considered ANNs, both algorithms can predict the cracks' locations and depth accurately. Furthermore, the crack depth can be predicted more accurately than the crack locations using both training methods.

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