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Computational finite element model updating tool for modal testing of structures

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Abstract. In this paper, the development of a new optimization software for finite element model updating of engineering structures titled as FemUP is described. The program is used for computational FEM model updating of structures depending on modal testing results. This paper deals with the FE model updating procedure carried out in FemUP. The theoretical exposition on FE model updating and optimization techniques is presented. The related issues including the objective function, constraint function, different residuals and possible parameters for FE model updating are investigated. The issues of updating process adopted in FemUP are discussed. The ideas of optimization to be used in FE model updating application are explained. The algorithm of Sequential Quadratic Programming (SQP) is explored which will be used to solve the optimization problem. The possibilities of the program are demonstrated with a three dimensional steel frame model. As a result of this study, it can be said that SQP algorithm is very effective in model updating procedure.

Keywords: finite element model updating; optimization; experimental modal analysis; operational modal analysis; SignalCAD; ModalCAD; FemUP; MATLAB; SQP algorithm

1. Introduction

Models are mathematical representations of structures. They provide a means for predicting the response characteristics of the structure without actually building it. In general, the model takes the form of a Finite Element (FE) model. In a FE model, the physical continuous domain of a complex structure is discretized into small finite elements. The FE method is extensively used in research and industrial applications. This method can produce a good representation of a true structure. However, the prediction from FE method is not always exact. Inaccuracies and errors in an FE model may arise due to some reasons. These are inaccurate estimation of the physical properties of the system, low quality mesh distribution, inadequate approximation of boundary conditions and damping and inadequate modeling of system joints.

In reality, structures always show different behavior in some way from the idealizations assumed when modeling them. The material and geometric properties may change or be uncertain

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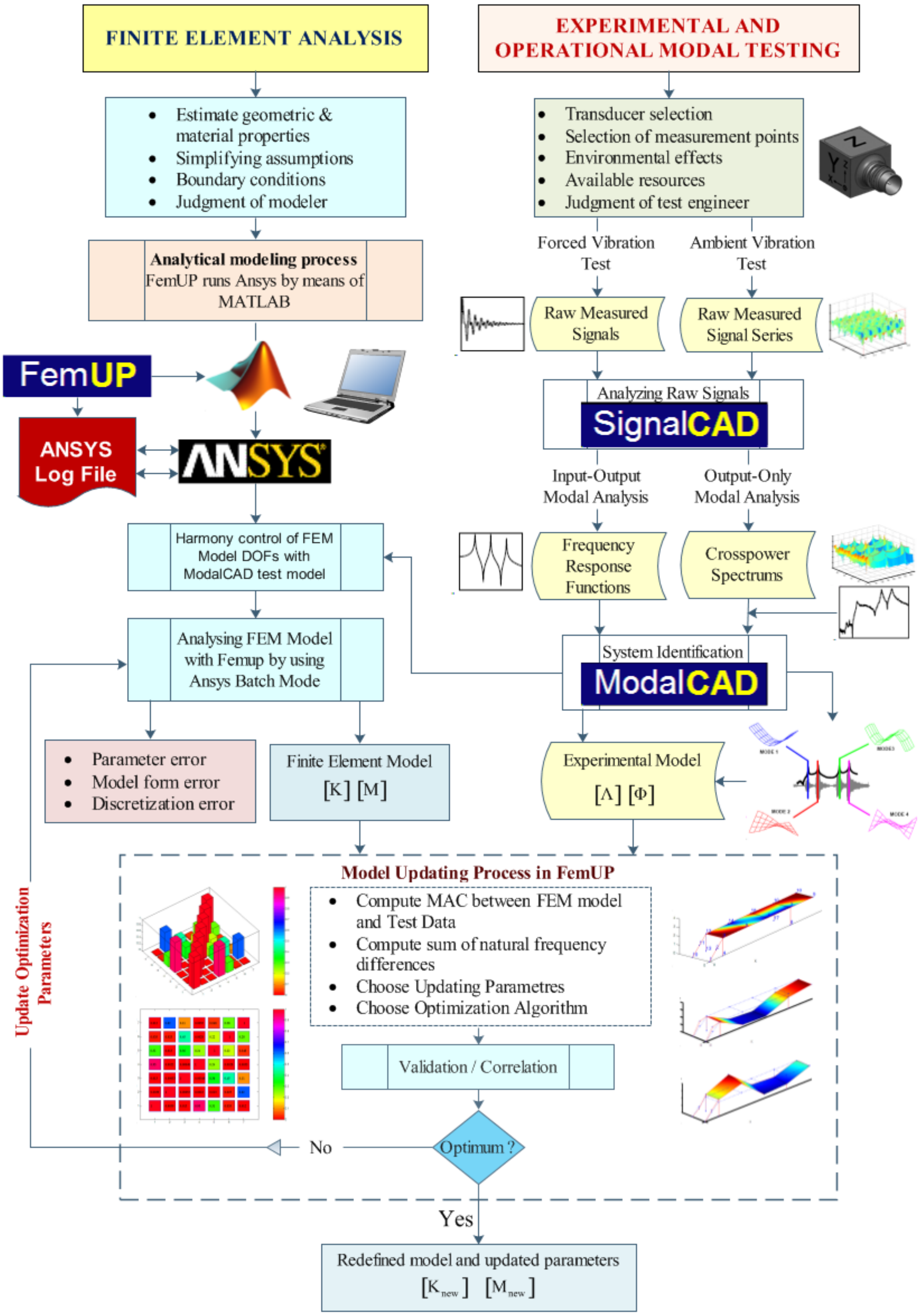


Fig. 1 The relationship between modeling, testing, system identification and model updating

and there may be nonlinearities which are not taken into account in the FE model. Therefore, FE models should be validated from some form of testing of the structure.

1.1 Modal testing and system identification

Testing is carried out to increase the knowledge and understanding of the behavior of a structure. This is achieved by observing the response of a structure to a set of known conditions. Currently, the most popular dynamic testing technique is vibration testing or experimental modal analysis (Inman 1994, Ewins 1995). Experimental and operational modal analyses are used to obtain an experimental model of a structure which describes its dynamic behavior through a set of natural frequencies, modes shapes, and damping ratios. The dynamic characteristics of the structure are obtained from a modal test of the structure during which the structure is excited and the responses of the structure are captured by a set of sensors.

Vibration measurements are taken directly from real structure, without any assumptions. Therefore, they are considered to be more reliable than their FE counterparts. Many researcher have updated or modified finite element models of the structures by using experimental results (El-Borgi *et al*. 2005, Bayraktar *et al*. 2010, Foti *et al*. 2012, Mosavi *et al*. 2012, Zhu *et al*. 2012, Ribeiro *et al*. 2012, Truong *et al*. 2012). Some computational algorithms may be used to automatically update FE models (Bakir *et al*. 2007, 2008, Okasha *et al*. 2012). Computational FE model updating is widely used for damage identification (Link *et al*. 2008, Link and Weiland, 2008, 2009, Wang and Yang 2012). Optimization techniques are used to update the analytical models by using experimental results (Jaishi and Ren 2005, 2007, Ntotsios and Papadimitriou 2008, Weng *et al*. 2010).

In this research, it is assumed that the experimental data is accurate and the FE model is modified or updated to better represent the experimental results and optimization technique is used to find optimal FE model. The relationship between modeling, testing, system identification and model updating is illustrated in Fig. 1. As shown in this figure, three new computer programs have been developed for these relationships. These programs are SignalCAD (Sahin and Bayraktar 2010), ModalCAD (Sahin and Bayraktar 2010) and FemUP. SignalCAD has been developed for digital signal processing, ModalCAD has been developed for system identification and FemUP has been developed for computational FE model updating by using MATLAB program (2009). The process of using information from an experimental model to refine an analytical model is called as the model updating. This part of the process is the subject of FemUP software.

1.2 Techniques for comparison and correlation for model updating

Correlation is initial step to assess the quality of the analytical model. Test data are assumed to be accurate and used as reference to assess the quality of the available FE model. Before updating the analytical FE model, the experimental and analytical data sets are compared to obtain some insight as to whether both sets are in reasonable agreement so that updating is at all possible. There are some techniques to compare the analytical modal data with the experimental modal data. The most often used correlation techniques are taken into account in this study. These techniques are direct natural frequency correlation, visual comparison of mode shapes and modal assurance criterion, respectively (Allemang and Brown 1982, Avitabile *et al*. 1988, Lieven and Ewins 1988, Ewins 1995, O’Callahan 1995, 1998, Maia and Silva 1997).

1.2.1 Direct natural frequency correlation

The most common approach to correlate two modal models is the direct comparison of the natural frequencies. A percentage difference can be defined as shown in Eq. (1) and an overall frequency scatter indicator may be used as presented in Eq. (2) (Jaishi 2005).

$$d_{fj} = \frac{\left|fr_{ej} - fr_{aj}\right|}{fr_{ej}} \times 100 \tag{1}$$

$$d_f = \left[\frac{\sum_{j=1}^{n_f} \left(fr_{ej} - fr_{aj} \right)}{\sum_{j=1}^{n_f} fr_{ej}^2} \right] \times 100 \tag{2}$$

Where fr_{ej} and fr_{aj} are the experimental and analytical frequencies of j-th mode respectively and n_f is the number of measured frequencies (Jaishi 2005).

1.2.2 Visual comparison of mode shapes

Visual comparison between two sets of modal data is non-quantitative graphical assessment. Simultaneous animation of one mode shape from each of the two sets is observed and direct comparison of their natural frequencies is carried out in this technique. Visual comparisons of mode shapes should be followed by numerical comparison techniques which are easy to implement in automatic correlations (Jaishi 2005).

1.2.3 Modal assurance criterion

The modal assurance criterion is generally used in automatic pairing and comparing analytical and experimental mode shapes (Allemang and Brown 1982). MAC is defined by the following equation

$$MAC_j = \frac{\left|\Phi_{aj}^T \Phi_{ej}\right|^2}{\left(\Phi_{aj}^T \Phi_{ej}\right)\left(\Phi_{ej}^T \Phi_{aj}\right)} \tag{3}$$

where Φ_{aj} is the j-th analytical modal vector that has been paired with the j-th experimental modal vector Φ_{ej}. The value of the MAC is limited between 0 and 1. A MAC value equal to 1 means a perfect correlation between analytical and experimental mode shapes. A MAC value equal to 0 indicates that the there is not any correlation between two modes. Modal assurance criterion is easy to apply and does not require mass and stiffness matrices.

The experimental and analytical modal vectors shapes must contain the same number of elements to obtain a good comparison. There must be FE nodes at the locations where the accelerometers are placed. In applications, all the analytical modes are correlated with all the experimental modes and the results are given in a matrix. The diagonal elements of the matrix should show high MAC values for a good correlation and low MAC values for uncorrelated modes.

2. Finite element model updating procedure

2.1 Initial analysis procedure

Initially, the FE model is developed using the initially estimated values for the unknown model parameters and boundary conditions. FE Modal analysis is then carried out to obtain the FE modal data. For the forced or ambient vibration testing of the structure, the optimum points for the placement of accelerometers are chosen and test data are recorded. The measured raw data is processed in SignalCAD software. In this process, frequency response functions are produced for experimental modal analysis (forced vibration testing) and crosspower spectrums are produced for operational modal analysis (ambient vibration testing). The experimental and operational modal analysis is then carried out by using Operating Vectors Method, Polyreference Time Domain Method and Complex Exponential Method to get the modal parameters by using ModalCAD software. For model updating, the experimental modal frequencies and modal vectors are exported from ModalCAD.

2.2 Optimization algorithm

Optimization is used to find a set of design parameters, $x=\{x_1, x_2, x_3 \ldots\ldots x_n\}$, that can be defined as optimal. In general, the objective function, $f(x)$, to be minimized are subjected to constraints in the form of equality constraints, $g_i(x)=0(i=1, \ldots.., m_e)$, inequality constraints, $g_i(x) \leq 0(i=m_e+1, \ldots.., m)$ and lower and upper parameter bounds $\underline{x}, \overline{x}$ respectively. The general optimization problem is stated as (Jaishi 2005)

$$\begin{aligned} &\text{minimize } f(x) \\ &x \in \Re^n \\ &\text{subject to} \quad g_i(x)=0, \quad i=1,.....,m_e \\ &\qquad\qquad g_i(x) \leq 0, \quad i=m_e+1,......,m \\ &\qquad\qquad \underline{x} \leq x \leq \overline{x} \end{aligned} \tag{4}$$

where x is the vector of design parameters, $(x \in \Re^n)$, $f(x)$ is the objective function that returns a scalar value $(f(x): \Re^n \rightarrow \Re)$, and the vector function $g(x)$ returns the values of the equality and inequality constraints evaluated at $x(g(x): \Re^n \rightarrow \Re^m)$.

2.2.1 Objective function

The least squares approach is very efficient and has become the common way to solve the updating problem (Link 1993, 1999, Link *et al*. 1996, Mottershead *et al*. 2000). An objective function f reflects the deviation between the analytical prediction and the real behavior of a structure. The FE model updating can be posed as a minimization problem to find x^* design set such that (Jaishi 2005)

$$\begin{aligned} f(x^*) &\leq f(x), && \forall x \\ \underline{x}_i \leq x &\leq \overline{x}_i, && i=1,2,3,.....n \end{aligned} \tag{5}$$

where the upper $(\bar{x}_i)$ and lower $(\underline{x}_i)$ bounds on the design variables are required. The objective function in an ordinary least squares problem is defined as a sum of squared differences (MATLAB Optimization Toolbox User's Guide 2009)

$$f(x) = \sum_{j=1}^{n_r} \left[z_j(x) - \bar{z}_j \right]^2 = \sum_{j=1}^{n_r} r_j(x)^2 \tag{6}$$

where each $z_j(x)$ represents an analytical modal quantity which is a nonlinear function of the optimization or design variables $x \in \Re^n$ and $\bar{z}$ refers to the measured modal parameters. In order to obtain a unique solution, the number of residuals n_r should be greater than the number n of unknown parameters x (Jaishi 2005).

2.2.1.1 Eigenfrequencies

The most important residual vector of the FE model updating in structural dynamics is the differences between the numerical and experimental undamped eigenfrequencies. The eigenfrequency residual r_f is formulated as (Jaishi 2005)

$$r_f(a) = \frac{\lambda_{aj} - \lambda_{ej}}{\lambda_{ej}} \quad j \in \{1, \ldots\ldots\ldots, m_f\} \tag{7}$$

with eigenvalue $\lambda_j=(2\times\pi\times f_{rj})^2$ where f_{rj} is the eigenfrequency corresponding to j-th mode. λ_{aj} and λ_{ej} are analytical and corresponding experimental eigenvalue, respectively. m_f refers to the number of identified eigenfrequencies that are used in the updating process. Relative differences are taken in r_f in order to obtain a similar weight for each eigenfrequency residual, since higher eigenfrequency gives the higher absolute difference between the analytical and experimental quantity (Jaishi 2005).

2.2.2 Constraint function

In optimization algorithm, constraint function must be produced to force the algorithm to use correlated mode pairs calculating the optimization criterion according to the objective function. The constraint function is defined as a nonlinear inequality function. The constraint function used for finite element model updating is expressed as follows

$$f(x_i) - f_{limit} \le 0 \tag{8}$$

Where; $f(x_i)$ is the dynamic parameter to be limited. This parameter may be a statement showing the relationship between experimental and analytical models. In objective function, the frequency difference is taken into account. This indicates global behavior. In constraint function, it is suitable to consider MAC matrix to hold optimization line in modal vector harmony zone.

3. Development of FemUP

3.1 Software algorithms

New computational Finite Element Model updating software which uses the MATLAB Optimization Toolbox (2009) is developed. A constrained optimization is performed using a

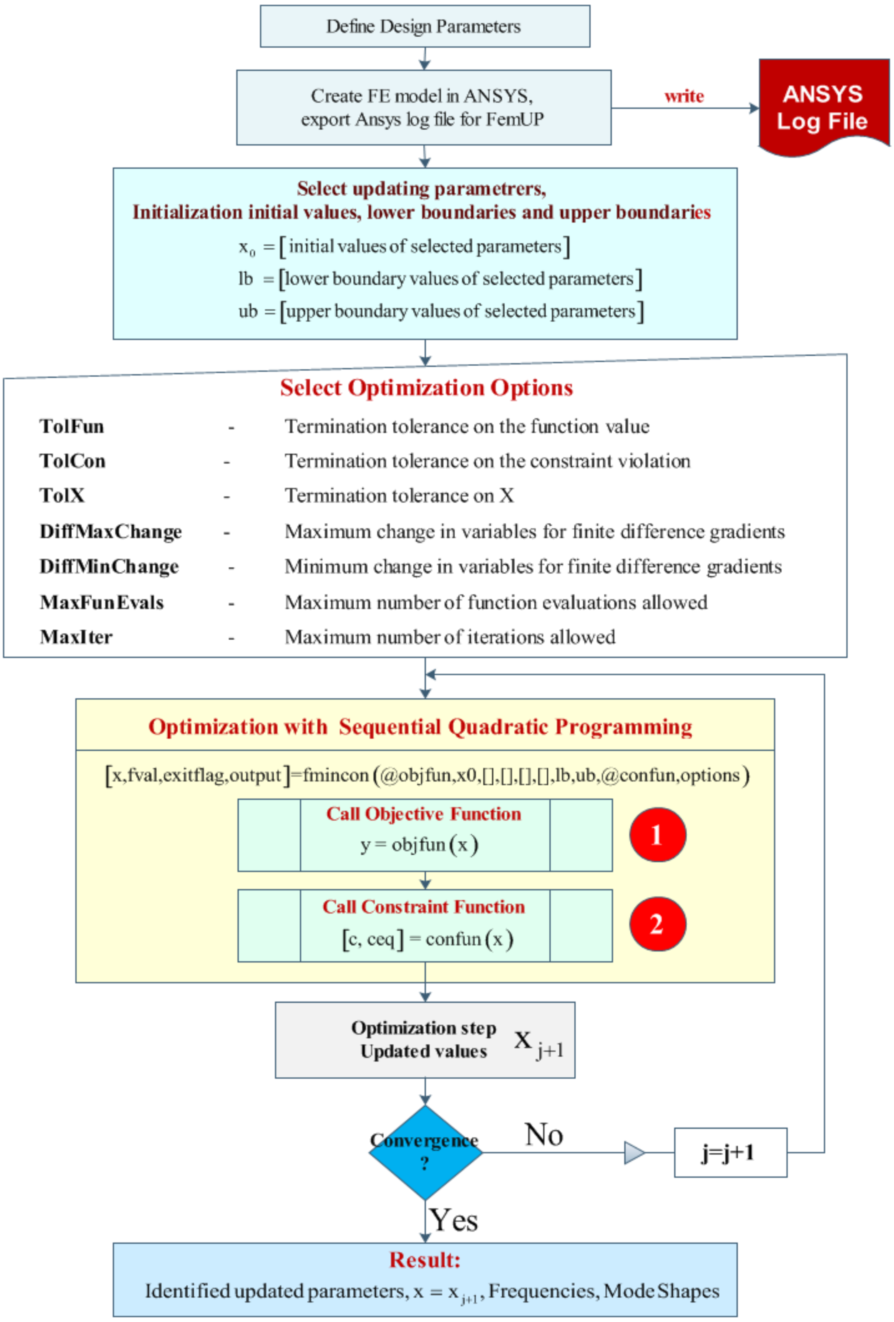


Fig. 2 The general procedure of the FE model updating method used in FemUP software

sequential quadratic programming (SQP) algorithm. The optimization algorithm is supplied with start-values, bounds, constraints and optimization criterion. The optimization criterion chosen, which is to be minimized, is the sum of the differences in natural frequency within each correlated mode pair. Constraints are used on the correlation between analytical and experimental mode shapes using the diagonal values of the MAC matrix. As indicated before, the modal assurance criterion (MAC) is a technique to detect the correlation between two sets of mode shapes. This constraint is important since it forces the algorithm to use correlated mode pairs when searching the optimization criterion. The general outline of the FE model updating procedure carried out in FemUP is shown in Fig. 2.

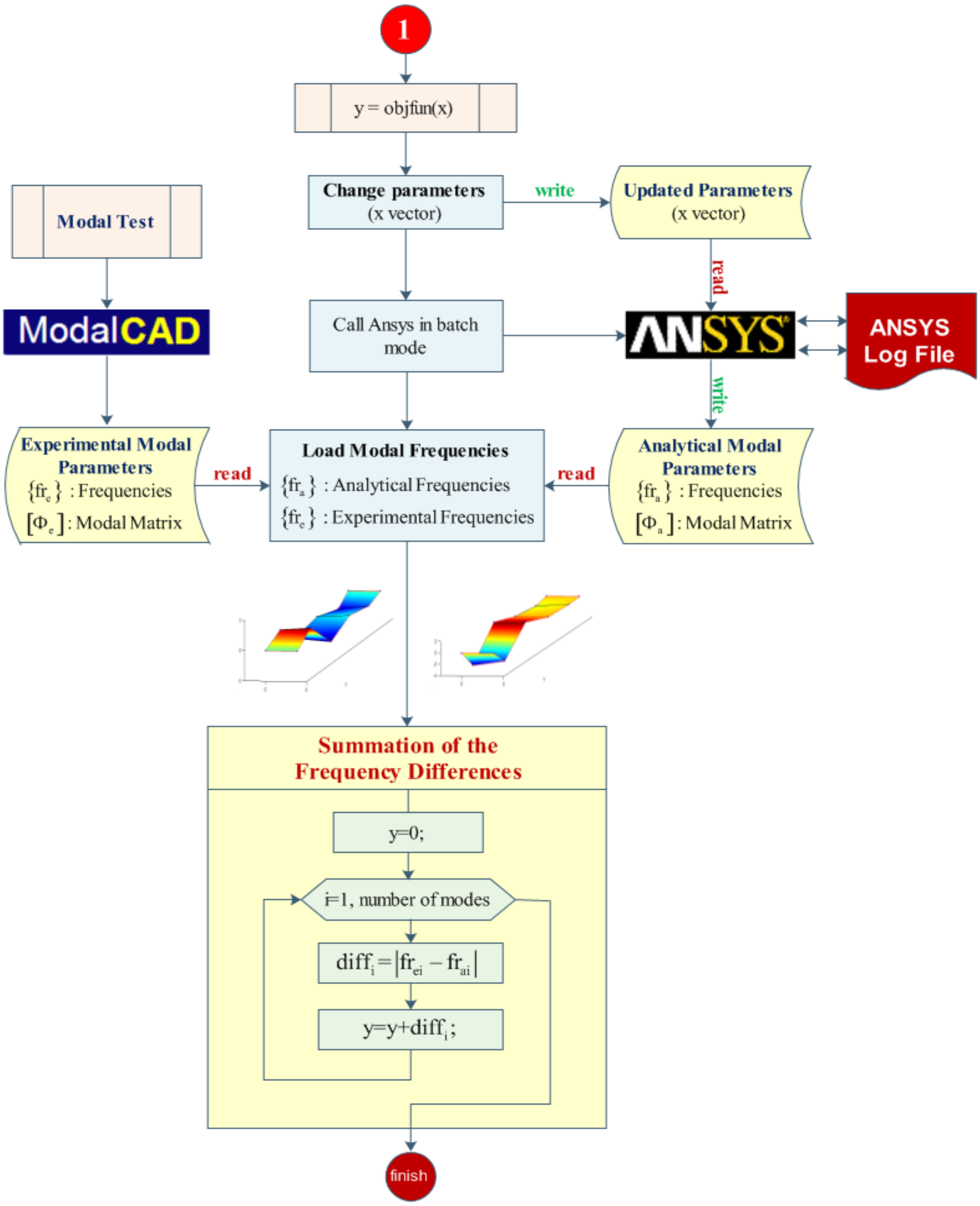


Fig. 3 The flowchart of objective function which aims to find minimum summation of frequency differences

Since natural frequencies and mode shapes must be solved for many times during the updating procedure, ANSYS (2007) and MATLAB interact with each other. The objective and constraint functions, taking advantage of MATLAB's ability of reading and writing ASCII-files, is used to transfer data between the two different software packages.

The objective function in FemUP is defined as a sum of experimental and theoretical frequency differences as shown in Fig. 3. The constraint function which includes nonlinear inequality constraints in FemUP exports a vector which consists of the differences between MAC limit selected by the user and calculated MAC values as shown in Fig. 4.

Setting appropriate tolerances for the search algorithm in the Optimization Toolbox is an important task. It usually has to be tuned for specific problems. If the tolerances are selected as tight, the algorithm is forced to make a large number of function evaluations without finding a much better solution. On the other hand, if the tolerances are selected as loosely, the search algorithm might not find the correct optimum. All parameters are scaled to be between zero and unity to set the tolerances for the optimization algorithm in a straightforward way.

SQP is a gradient-based optimization routine; therefore it only finds local optima. It can be found depending on the start-values. If the start-values are near an optimum, the search algorithm finds the optimum value faster.

3.2 Software user interface

FemUP is constructed around one main window (Fig. 5), that is divided into six main part:

1- Experimental model file loading part (compatible with ModalCAD)

2- Theoretical model file loading part (compatible with ANSYS)

3- Initial analysis and selecting update parameters part

4- Monitoring model correlation by means of correlation ratio and MAC values in a listbox part

5- Monitoring model correlation by means of MAC graphic part

6- Selecting solution algorithm, assigning minimum MAC limit for optimization and update command part.

4. Numerical application

First of all, modal testing applications of a laboratory model are provided. The example contains an experimental modal analysis of a three dimensional frame model. The raw data is processed from SignalCAD and then the spectrums are used as input data in ModalCAD. The modal parameters are calculated by means of ModalCAD program and then the analytical models are generated in ANSYS. Then FemUP software compares ModalCAD and ANSYS models and then updates the analytical model if necessary.

A three dimensional steel frame model has been produced in the laboratory and prepared for analysis outdoors. The frame model is shown in Fig. 6. It has two spans and two stories. The length of all columns and beams of the model is 0.90 m.

4.1 Experimental modal analysis of three dimensional frame structure model

The test data of the three dimensional frame model is analyzed. The data contain the measured acceleration response at each degrees of freedom of the model. The accelerometers are located on the top joints of the model as shown in Fig. 7. The channel numbers and directions are presented in Fig. 8.

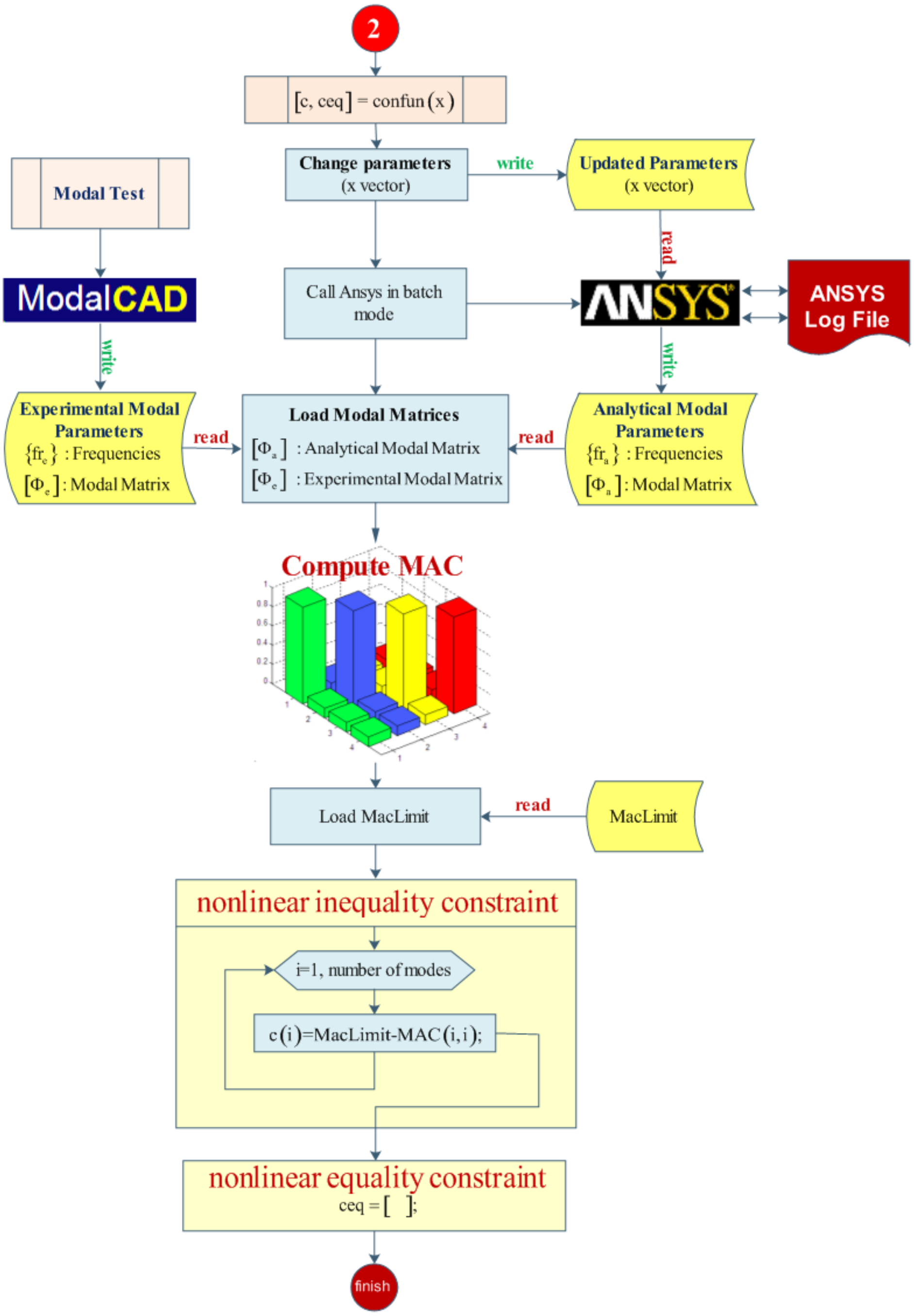


Fig. 4 The flowchart of constraint function which aims to calculate the nonlinear inequality constraint as difference between real MAC value and MAC limit chosen by user

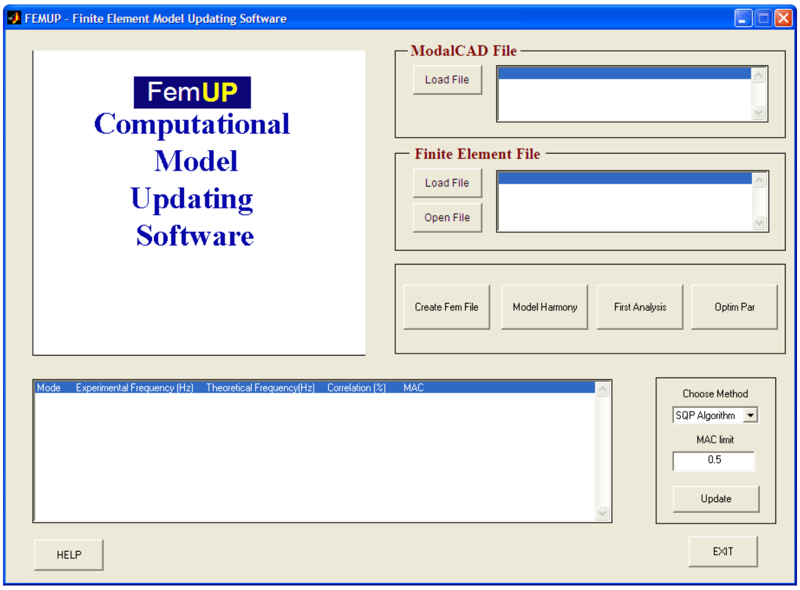


Fig. 5 FemUP main window


Fig. 6 Three dimensional frame model


Fig. 7 One of the accelerometers located on the top joints of the frame model

A single excitation point test was performed on the 3d frame model. This type of test allows for quicker data acquisition, but requires many more accelerometer channels and a longer setup time. A hammer was used to impact the model at one location and accelerometers were located at top storey of the model. Tri-axial accelerometers were used at the side corners and uni-axial

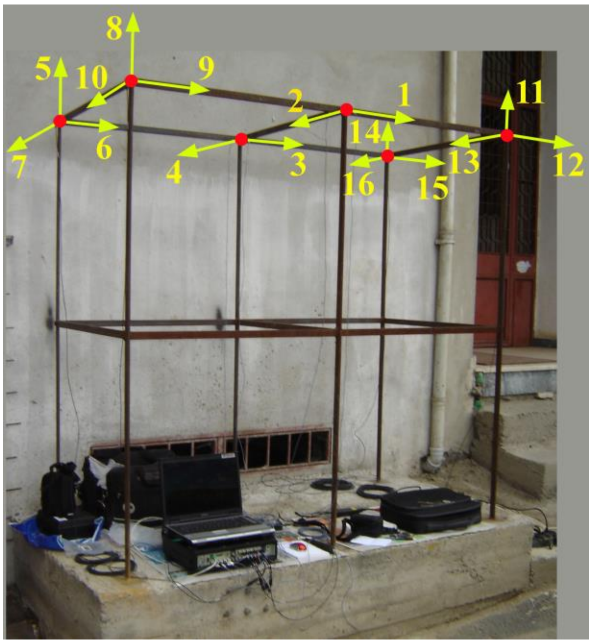


Fig. 8 The accelerometer numbers, locations and directions


Fig. 9 Data acquisition system

accelerometers were used at middle joints. Signals acquired from accelerometers are gathered in the 17 channel data acquisition system as shown in Fig. 9. During these tests the weather was cloudy sometimes a little rainy, approximately 20℃ (65-70℉), and there was negligible wind. These measurements were intended to confirm the frequencies and mode shapes determined from hammer impact test and ambient vibration test and to examine the variability introduced into estimates of dynamic properties by using various parameter identification techniques.

The impact location was on the side column of the model as shown in Fig. 10. When the beam elements were excited, it was seen that the data was poor, therefore side columns were excited for modal testing. The impact test was repeated until obtaining best results. Total five impact tests were carried out and at the end of these tests, excitation location was determined.

After the test finished, the measured raw data was processed in SignalCAD program and the frequency response functions and other spectrums were exported for ModalCAD.

Fig. 10 The 3d frame is excited with a hammer

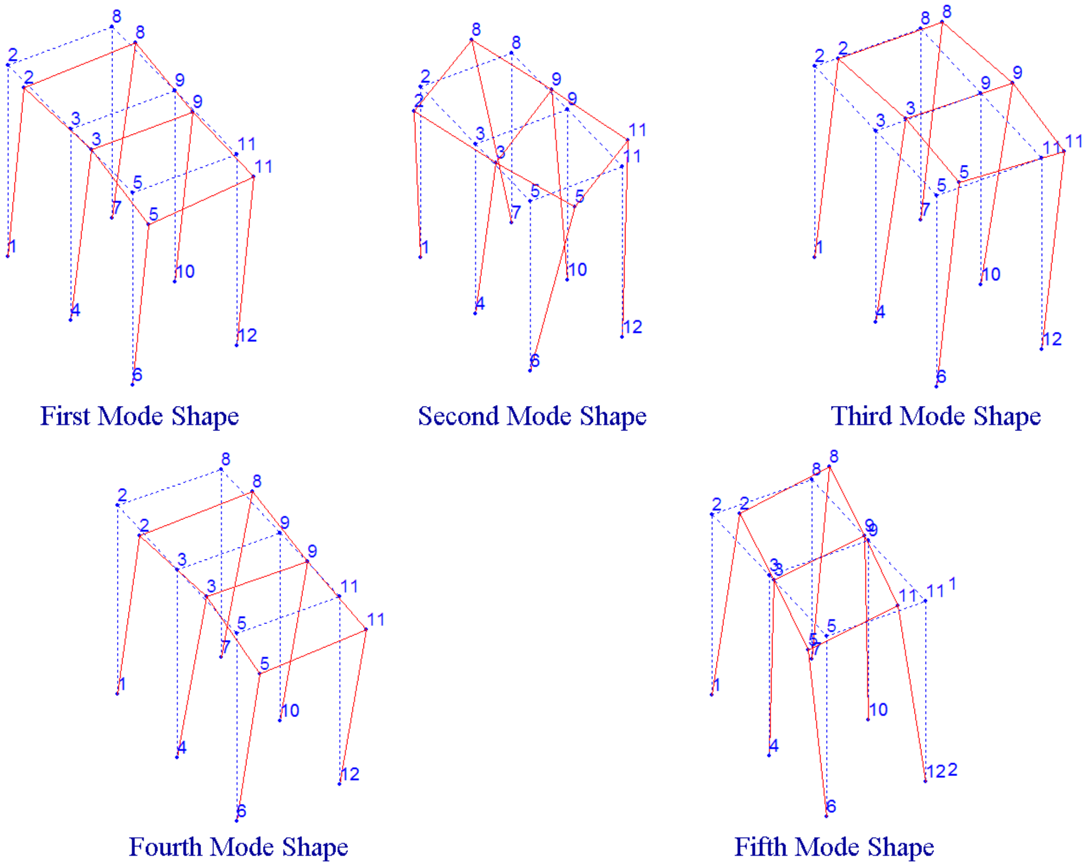
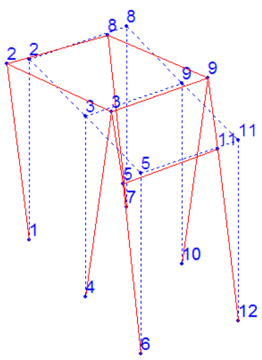
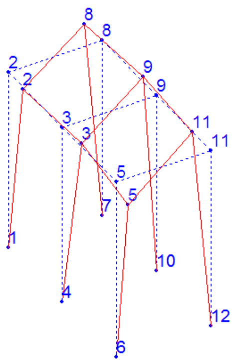


Fig. 11 Experimental mode shapes of the model obtained from ModalCAD



Sixth Mode Shape



Seventh Mode Shape

Fig. 11 Continued

Table 1 Modal frequency values obtained from all methods for experimental and operational modal analysis of 3d frame

Mode Number	Natural Frequencies (Hz) (Experimental Modal Analysis)			Natural Frequencies (Hz) (Operational Modal analysis)		
	OV Method	CE Method	PTD Method	OV Method	CE Method	PTD Method
1	4.375	4.372	4.368	4.375	4.373	4.373
2	8.875	8.845	8.845	8.875	8.861	8.872
3	10.063	10.059	10.069	10.125	10.105	10.076
4	12.500	12.487	12.487	12.500	12.493	12.499
5	13.125	13.137	13.135	13.125	13.107	13.120
6	16.563	16.539	16.539	16.500	16.481	16.511
7	17.000	16.992	16.991	17.000	17.002	16.999

The mode shapes obtained from all of the modal parameter estimation techniques used in ModalCAD are almost same. The mode shapes are presented in Fig. 11. The first, third and fourth mode shapes are translation modes, the second mode shape is torsional mode, the fifth, sixth and seventh modes are shear modes, respectively.

The modal frequency values calculated using Operating Vectors (OV), Complex Exponential (CE) and Polyreference Time Domain (PTD) methods for experimental and operational modal analyses are presented in Table 1.

When modal analysis results are investigated, it can be said that there is a good harmony in modal vectors calculated by using all different methods. Mode shapes are obtained just same in all methods. If modal frequencies are considered, it can be said that the frequencies obtained from all methods are same within trivial differences. As reference experimental model, the modal parameters calculated using Operating Vectors Method for operational modal analysis are taken into account.

4.2 Analytical modal analysis of three dimensional frame model

An analytical model of the three dimensional frame is built in ANSYS. The natural frequencies

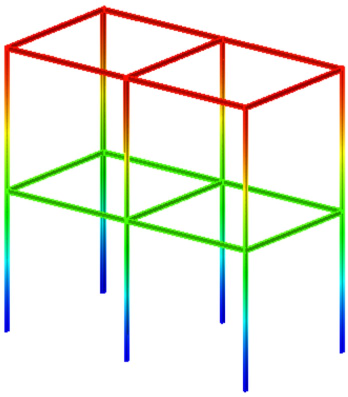


Fig. 12 Analytical frame model built in ANSYS

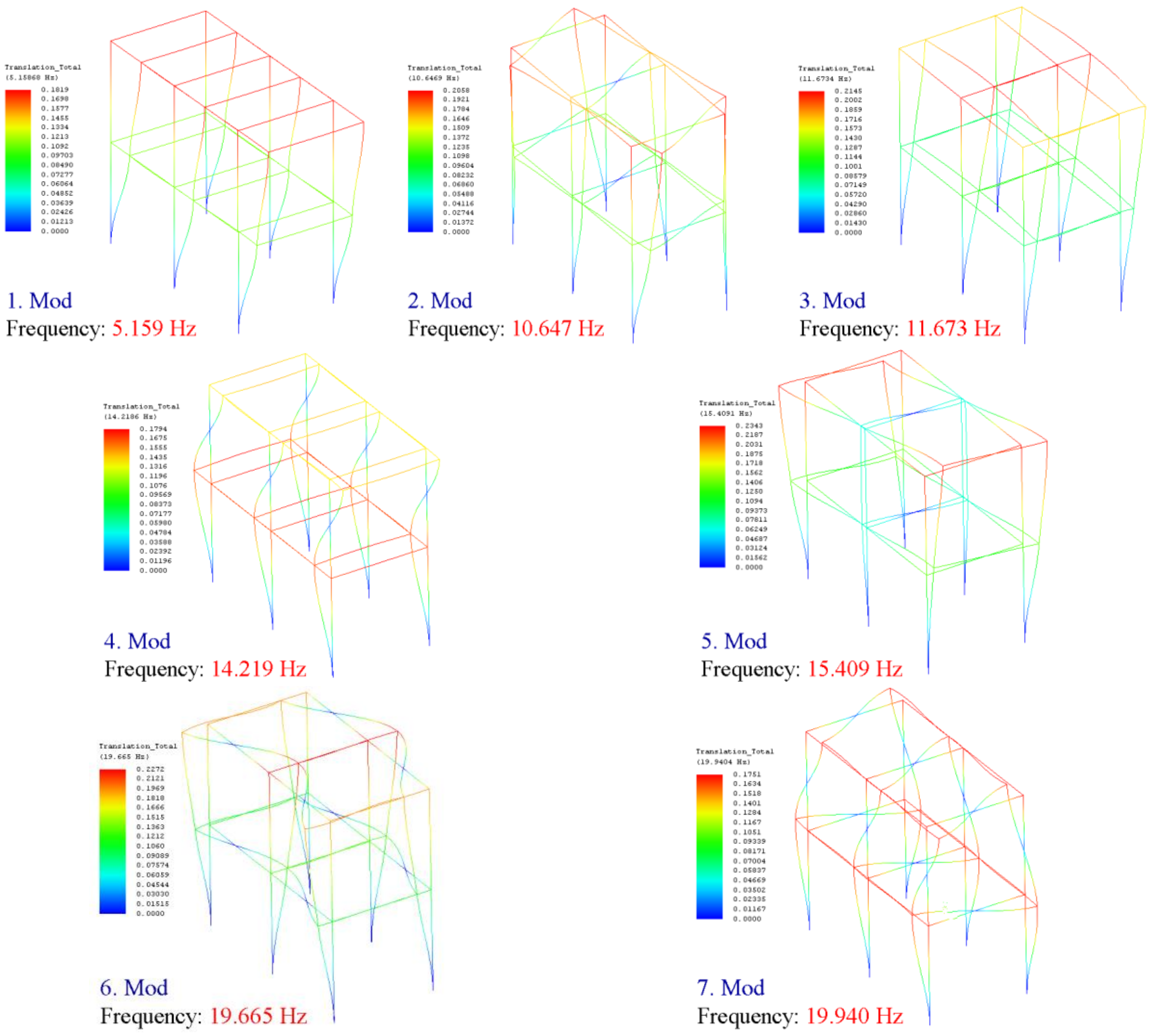


Fig. 13 Analytical mode shapes of the model obtained from ANSYS

and mode shapes are solved for by the Lanczos method. The FE model of the 3d frame is shown in Fig. 12.

The modal frequencies and modal vectors (mode shapes) are shown in Fig. 13. The first, third and fourth analytical mode shapes are translation modes, the second analytical mode shape is torsional mode, the fifth, sixth and seventh analytical modes are shear modes, respectively.

4.3 Computational FEM model updating of three dimensional frame model

A comparison between the results from an initial analytical model and the experimental results shows that there is a good harmony between modal vectors but there are important differences in modal frequencies. The analytical natural frequencies are higher than the corresponding natural frequencies obtained experimentally. These differences are based on physicals parameters. The initial comparison of theoretical model and experimental model is shown in Fig. 14.

The differences in modal frequencies between analytical and experimental model are upper than 15 per cent for all modes as shown in Table 2.

To achieve an analytical model that correlates better with the experimental results, the material properties and mesh properties are updated. There is no need to add mass and update boundary conditions in this model. In updating step, three parameters are included in the automated updating procedure. These are three material properties. Also mesh distribution has been checked for updating. The correlation between modes of this analytical model and the experimental model are calculated using the MAC matrix as shown Figs. 15-16 and Table 3.

The selected parameters for the optimization study have not been limited. The lower limit is selected as zero and the upper limit is selected as infinite. The MAC value is used as a constraint function and the MAC limit is selected as 0.5. This means that, the MAC value is not allowed to be lower than 0.5 during the optimization process. The iteration number is very low and the necessary time for completing the procedure is very short for this application. After model updating process, it can be said that the correlation is very good. All differences in natural

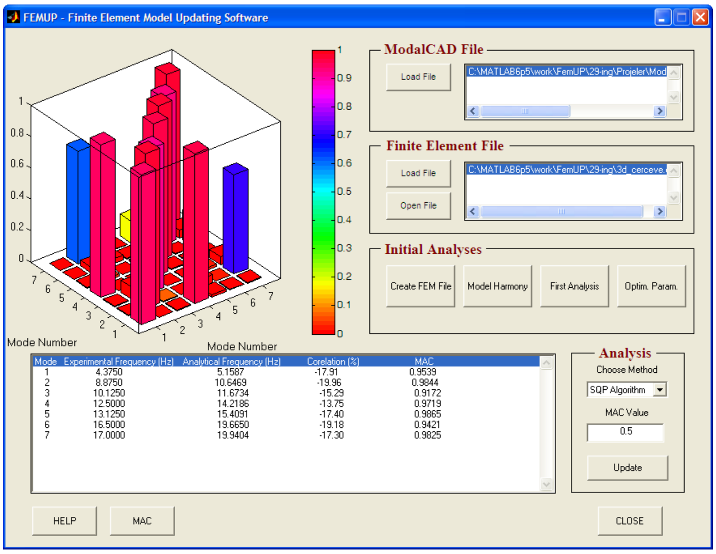


Fig. 14 FemUP is comparing the initial FEM model ad experimental model

Table 2 Initial correlation analysis results between experimental and analytical models

Mode Number	Experimental Frequencies (Hz)	Analytical Frequencies (Hz)	Error (%)	MAC
1	4.375	5.159	17.91	0.95
2	8.875	10.647	19.96	0.98
3	10.125	11.673	15.29	0.92
4	12.500	14.219	13.75	0.97
5	13.125	15.409	17.40	0.99
6	16.500	19.665	19.18	0.94
7	17.000	19.940	17.30	0.98

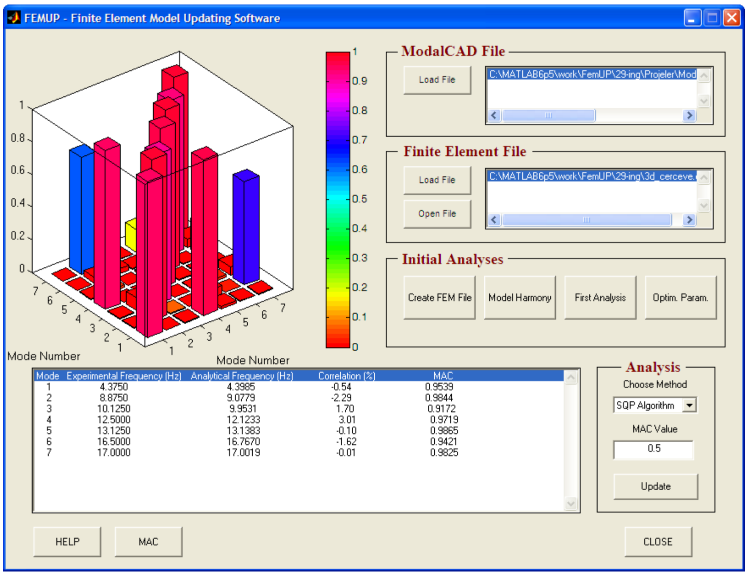


Fig. 15 FEM model have been successfully updated by FemUP

Table 3 Correlation analysis results between experimental and analytical models after model updating

Mode Number	Experimental Frequencies (Hz)	Analytical Frequencies (Hz)	Error (%)	MAC
1	4.375	4.399	0.54	0.95
2	8.875	9.078	2.29	0.98
3	10.125	9.953	1.70	0.92
4	12.500	12.123	3.01	0.97
5	13.125	13.138	0.10	0.99
6	16.500	16.767	1.62	0.94
7	17.000	17.002	0.01	0.98

frequencies are below 3 per cent. The experimental modal vectors are just same as the analytical modal vectors. This correlation may be checked from the MAC matrices as shown in Fig. 16.

The materials properties of the 3d frame model before and after the updating process are shown in Table 4. As shown in Table 4, we can see that modulus of elasticity has been changed by

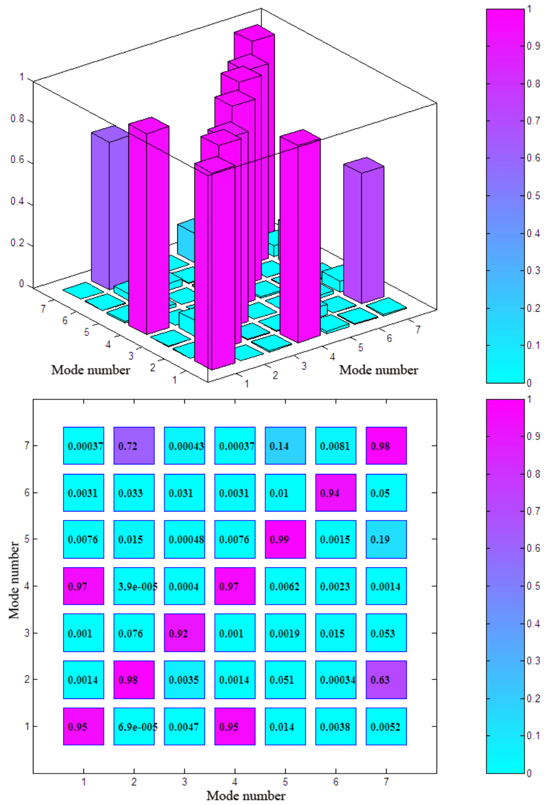


Fig. 16 Three and two dimensional MAC matrices between updated analytical and experimental model

Table 4 The model parameters before and after update

Parameter	Before Update	After Update
Modulus of Elasticity	2.75×10^{11} N/m^2	1.99921×10^{11} N/m^2
Density	7800 kg/m^3	7800 kg/m^3
Poison Raito	0.3	0.3

FemUP. This change primarily affects the frequency values. The other parameters such as density and poison ratio have not been changed. As a result of optimization study, it can be said that the most effective physical parameter of frame model for model updating is modulus of elasticity.

5. Conclusions

New optimal modal updating software, known as FemUP, has been introduced and demonstrated. The software provides the interaction of ANSYS and MATLAB with each other to perform model updating of FEM model using optimization procedures. The theoretical formulations for model updating and optimization procedures used in FemUP have been explained in detail. The structure of the software menus was described in detail in this paper, and sample applications to measured data from a three dimensional frame model were presented. Some examples of how FemUP can be used for updating a laboratory model have been shown.

In FemUP, the sequential quadratic programming algorithm in MATLAB Optimization Toolbox is used to minimize the difference between analytical and experimental natural frequencies. Constraints are used on the correlation between the analytical and experimental mode shapes using the MAC-matrix. The natural frequencies and mode shapes are solved for by ANSYS. The very good correlation between the updated analytical model and the experimental model shows that the updating procedure in FemUP works well. The main advantages with using FemUP are that:

• it forces the user to a certain level of understanding of the analysis procedure,
• it uses high level graphical user interfaces capabilities of MATLAB
• certification of routines is simplified by open functions, and
• traceability is ensured, using the same MATLAB program for each analysis.

The developed tool is very fast and efficient in comparison with the already know packages. It directly uses the ModalCAD results which is already developed in MATLAB and automatically update the FEM parameters. The FemUP uses ANSYS in batch mode and this property increases the optimization time. It also uses MATLAB optimization toolbox and it makes the solution very effective. With the presented software now available for updating FEM models related to noise and vibration analysis, also the less experienced user can take advantage of the power and flexibility of MATLAB, as a complement to an existing system, or as the only tool for analysis.

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