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The effect of two temperatures on a FG nanobeam induced by a sinusoidal pulse heating

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Abstract. The present investigation is concerned with the effect of two temperatures on functionally graded (FG) nanobeams subjected to sinusoidal pulse heating sources. Material properties of the nanobeam are assumed to be graded in the thickness direction according to a novel exponential distribution law in terms of the volume fractions of the metal and ceramic constituents. The upper surface of the FG nanobeam is fully ceramic whereas the lower surface is fully metal. The generalized two-temperature nonlocal theory of thermoelasticity in the context of Lord and Shulman's (LS) model is used to solve this problem. The governing equations are solved in the Laplace transformation domain. The inversion of the Laplace transformation is computed numerically using a method based on Fourier series expansion technique. Some comparisons have been shown to estimate the effects of the nonlocal parameter, the temperature discrepancy and the pulse width of the sinusoidal pulse. Additional results across the thickness of the nanobeam are presented graphically.

Keywords: thermoelasticity; two temperatures; FG nanobeam; nonlocal theory; sinusoidal pulse

1. Introduction

The classical uncoupled theory of thermo-elasticity predicts two phenomena not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms. Second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves. Biot (1956) formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory. The heat equations for both theories, however, are of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations. The generalized theory of thermoelasticity has been developed to overcome the physically unrealistic prediction of the coupled dynamical theory of thermoelasticity that thermal signals propagate with infinite speed. Lord and Shulman (1967) and Green and Lindsay (1972) established two-temperature rate dependent theories of generalized

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thermoelasticity. They introduced the thermal relaxation parameters in the basic equations of the coupled dynamical thermoelasticity theory and admit the finite value of heat propagation speed. The finiteness of the speed of the thermal signal has been found to have experiment evidence too. The generalized thermoelasticity theories are therefore more realistic, and have found much interest in recent research.

Thermoelasticity theory with two temperatures is one of the non-classical theories of thermoelastic solids. The main difference of this theory with respect to the classical one is in the thermal dependence. Chen and Gurtin (1968), Chen *et al.* (1969) formulated a theory of heat conduction in deformable bodies, which depends on two distinct temperatures: the conductive temperature φ and the thermodynamic temperature θ. For time independent situations, the difference between these two temperatures is proportional to the heat supply. The two temperatures are identical in the absence of any heat supply. However, for time-dependent problems, and for wave propagation problems in particular, the two temperatures are in general different, regardless of the presence of a heat supply. The two temperatures and the strain are found to have representation in the form of a travelling wave plus response, which occurs instantaneously throughout the body (Boley 1956). In brief, the conductive temperature and its two gradients are used to find the internal energy, entropy, stress, heat flux and thermodynamic temperature at a given material point and time. Warren and Chen (1973) investigated the wave propagation in the two-temperature theory of thermoelasticity. Quintanilla (2004 a, b) proved some theorems in thermoelasticity with two temperatures. Zenkour and Abouelregal (2014) presented the state-space approach for an infinite medium with a spherical cavity based upon two-temperature generalized thermoelasticity theory and fractional heat conduction.

Micro- and nano-mechanical resonators have attracted considerable attention recently due to their many important technological applications. Accurate analysis of various effects on the characteristics of resonators, such as resonant frequencies and quality factors, is crucial for designing high-performance components. Many authors have studied the vibration and heat transfer process of beams. Kidawa-Kukla (2003) studied the problem of transverse vibrations of a beam induced by a mobile heat source. The analytical solution to the problem is obtained using Green's functions method without considering the thermoelastic coupling effect. Boley (1972) analyzed the vibrations of a simply-supported rectangular beam subjected to a suddenly applied heat input distributed along its span. Manolis and Beskos (1980) examined the thermally induced vibration of beams exposed to rapid surface heating. They studied the effects of damping and axial loads on the structural response. Al-Huniti *et al.* (2001) investigated the thermally induced displacements and stresses of a rod using the Laplace transformation technique.

The present nanobeam is made of a ceramic-metal functionally graded material (FGM) for the purpose of thermal protection against large temperature gradients. The ceramic material provides a high temperature resistance due to its low thermal conductivity, while the ductile metal constituent prevents fracture due to its greater toughness. Gradually varying the material properties can prevent from interface cracking, delamination and residual stresses and thus maintain structural integrity to a desirable level. Ching and Yen (2006) presented numerical solutions obtained by the meshless local Petrov-Galerkin method for transient thermoelastic deformations of FG beams. Malekzadeh *et al.* (2012) presented the transient heat transfer analysis of FG hollow cylinders subjected to a distributed heat flux with a moving front boundary on its inner surface is presented. Malekzadeh and Heydarpour (2012) presented the transient thermoelastic analysis of FG cylindrical shells under moving boundary pressure and heat flux. Malekzadeh and Shojaee (2014) investigated the dynamic response of FG beams under a moving heat source. The material properties are assumed to be temperature-dependent and graded in the thickness direction. Mareishi *et al.* (2013) developed the thermo-mechanical vibrations of FG beams. Governing equations are obtained based on higher-order variation of transverse shear strain through the depth of the beam. Recently, Abbas and Zenkour (2013) presented the electro-magneto-thermoelastic analysis problem of an infinite FG hollow cylinder based upon LS theory.

This work is devoted to study the effect of the nonlocal parameter, the heat conduction and the coupling effect between temperature and strain rate in nanobeams based upon the LS model. The governing equations are written in the context of the nonlocal two-temperature generalized thermoelasticity theory. The present nanobeam is made from a ceramic-metal FGM. The solution for the generalized thermoelastic vibration of nanobeam induced by sinusoidal pulse heat with constant pulse of thermal vibration is developed. The Laplace transform method is used to determine the lateral vibration, the temperature and the displacement of the nanobeam. The effects of some parameters on thermal vibration quantities are studied and represented graphically. Some special cases are also derived. The numerical solutions of the non-dimensional governing partial differential equations of the problem have been graphically shown and some comparisons have been discussed to estimate the effect of the nonlocal parameter, the pulse parameter of heating and the parameter of two-temperature theory.

2. Basic equations of the two-temperature thermoelasticity theory

The governing equations of the linear theory of thermoelasticity with two temperatures and relaxation time are as follows:
Equations of motion

$$\sigma_{ji,j} + F_i = \rho \ddot{u}_i \tag{1}$$

Equations of entropy (Energy equations)

$$\rho \varphi_0 \dot{\eta}_i = -q_{i,i} + Q \tag{2}$$

Constitutive equations for the local theory

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} - \gamma\theta\delta_{ij} \tag{3}$$

The modified Fourier's law

$$q_i + \tau_0 \dot{q}_i = -K\varphi_{,i} \tag{4}$$

The entropy-strain-temperature relation

$$\rho\eta = \gamma\, e_{kk} + \frac{\rho C^e}{\varphi_0}\theta \tag{5}$$

The strain-displacement relations

$$2e_{ij} = u_{i,j} + u_{j,i} \tag{6}$$

In these relations, σ_{ij} are the components of the stress tensor, F_i are the components of body force vector, ρ is the material density, u_i are the components of the displacement vector, φ is the conductive temperature measured from the temperature φ_0, η is the entropy per unit volume measured from the entropy of the reference state, q_i are the components of the heat flows vector, Q is the heat supplied per unit volume from the external world, e_{ij} is the strain tensor, $\gamma=(3\lambda+2\mu)\alpha_t$ is the coupling parameters, in which, λ and μ being Lamé's coefficients and α_t being the coefficient of linear thermal expansion, $\theta=T-T_0$ denotes the thermodynamical temperature, T_0 is the reference temperature, δ_{ij} is Kronecker's delta function, K is the thermal conductivity, and τ_0 is the thermal relaxation time, which will ensure that the heat conduction equation will predict finite speeds of heat propagation. The conductive temperature φ satisfies the relation

$$\theta=\varphi-a\varphi_{,kk} \tag{7}$$

where $a>0$ is the two temperature parameter (temperature discrepancy) and $\varphi_0=T_0$, is the reference temperature.

In all of the above equations, the comma followed by a suffix denotes partial derivation with respect to the space variables and the superposed dot denotes the derivation with respect to the time t. Eqs. (1)-(7) give the basic equations of two-temperature thermoelasticity (2TT) in context of the LS theory as:

$$\begin{gathered}(\lambda+\mu)u_{i,ij}+\mu u_{i,jj}-\gamma\theta_{,i}+F_i=\rho\ddot{u}_i\\ \nabla(K\cdot\nabla\varphi)=\rho C^e\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[\frac{\partial\theta}{\partial t}+\gamma T_0\frac{\partial e_{kk}}{\partial t}-Q\right]\end{gathered} \tag{8}$$

where C^e is the specific heat at constant strain. The key element that sets the two-temperature thermoelasticity theory apart from the classical theory is the material parameter a. Specifically, in the limit as $a\rightarrow 0$ and $\varphi\rightarrow\theta$, the classical theory (one-temperature generalized thermoelasticity theory 1TT) is recovered. In what follows, Lamé's coefficients λ and μ will be given in terms of Young's modulus E and Poisson's ratio v.

3. Formulation of the problem

Let us consider a FG thermoelastic solid beam in Cartesian coordinate systems $Oxyz$. The x axis is drawn along the axial direction of the beam and the y and z axes correspond to the width and thickness, respectively (see Fig. 1). In equilibrium, the beam is unstrained, unstressed and at temperature T_0 everywhere. The small flexural deflections of the nanobeam with dimensions of length $L(0\le x\le L)$, width $b(-b/2\le y\le +b/2)$ and thickness $h(-h/2\le z\le +h/2)$ are considered. The basic governing equations of motion, balance of equilibrated force and heat conduction in the context of generalized (non-Fourier) thermoelasticity for displacement vector $\mathbf{u}(x,y,z,t)$ in the absence of body forces, external loads, extrinsic equilibrated body force and heat sources are also considered.

A new model of FGMs is presented to treat the governing equations of the thermoelastic nanobeam that subjected to a sinusoidal pulse heating. Based on this model, the effective material property $P(z)$ gradation through the thickness direction is presented by (Zenkour 2006, 2014)

$$P(z)=P_{\mathrm{m}}\,\mathrm{e}^{n_P(2z-h)/h},\quad n_P=\ln\sqrt{P_{\mathrm{m}}/P_{\mathrm{c}}} \tag{9}$$

where P_m and P_c represent the metal and ceramic properties, respectively. This study assumes that Young's modulus E, material density ρ, thermal conductivity coefficient K and the stress-

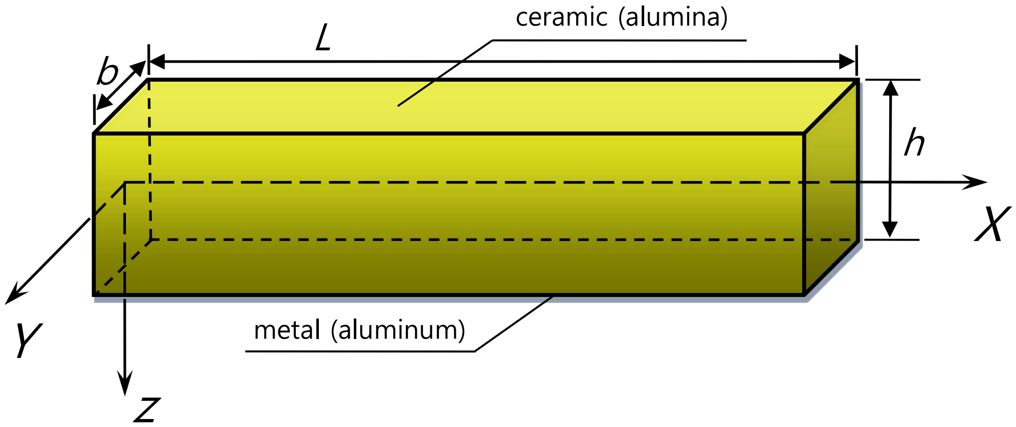


Fig. 1 Schematic diagram for the FG nanobeam

temperature modulus γ of the FGM change continuously through the thickness direction of the beam according to the gradation relation given in Eq. (9). It is to be noted that the material properties of the considered beam are metal-rich (fully metal) at the lower surface ($z=+h/2$) and ceramic-rich (fully ceramic) at the upper surface ($z=-h/2$) of the beam.

In the present study, the usual Euler–Bernoulli assumption (Mareishi *et al.* 2013) is adopted, i.e., any plane cross-section, initially perpendicular to the axis of the beam, remains plane and perpendicular to the neutral surface during bending. Thus, the displacements are given by

$$u=-z\frac{\partial w}{\partial x},\quad v=0,\quad w(x,y,z,t)=w(x,t) \tag{10}$$

where w is the lateral deflection. Substituting Eq. (10) with the aid of Eq. (9), into the heat conduction equation given in Eq. $(8)_2$ for the nanobeam without the heat source (Q=0), one gets

$$K_{\mathrm{m}}\,\mathrm{e}^{n_K(2z-h)/h}\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}+\frac{2n_K}{h}\frac{\partial\varphi}{\partial z}\right)=$$

$$=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left\{\rho_{\mathrm{m}}C^e_{\mathrm{m}}\,\mathrm{e}^{n_{\rho C^e}(2z-h)/h}\frac{\partial}{\partial t}\left[\varphi-a\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}\right)\right]-z\gamma_{\mathrm{m}}\,\mathrm{e}^{n_\gamma(2z-h)/h}T_0\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right\} \tag{11}$$

where K_m, ρ_m, γ_m and C^e_{m} are, respectively, the thermal conductivity coefficient, the material density, the thermal modulus, and the specific heat per unit mass at constant strain of the metal material. Note that the parameters n_K, n_γ and $n_{\rho C^e}$ are given according to Eq. (9) in terms the properties of ceramic and metal materials, and

$$\gamma_{\mathrm{m}}=E_{\mathrm{m}}\alpha_{\mathrm{m}}/(1-2\nu_{\mathrm{m}}),\quad \rho_{\mathrm{m}}C^e_{\mathrm{m}}=K_{\mathrm{m}}/\chi_{\mathrm{m}} \tag{12}$$

in which α_m, E_m, ν_m and χ_m are the thermal expansion coefficient, Young's modulus, Poisson's ratio and the thermal diffusivity of the metal material, respectively.

There is no heat flow across the upper and lower surfaces of the beam (thermally insulated), so that $\partial\varphi/\partial z$ should be vanish at the upper and lower surfaces of the beam z=±*h*/2. For a very thin beam (nanobeam), assuming that the conductive temperature varies sinusoidally along the thickness direction. That is

$$\varphi(x,z,t) = \varphi_1(x,t)\sin\left(\frac{\pi z}{h}\right) \tag{13}$$

Now, substituting Eq. (13) into Eq. (11) and integrating the resulting equation with respect to *z* through the beam thickness from −*h*/2 to *h*/2, yields

$$\frac{\partial^2\varphi_1}{\partial x^2} = \left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left[\bar{\mu}_{\rho C^e}\eta\left(1-a\frac{\pi^2}{h^2}-a\frac{\partial^2}{\partial x^2}\right)\varphi_1 - \frac{\bar{\mu}_\gamma T_0 h\gamma_{\mathrm{m}}}{K_{\mathrm{m}}}\frac{\partial^2 w}{\partial x^2}\right] \tag{14}$$

where $\eta = \rho_{\mathrm{m}}C^e_{\mathrm{m}}/K_{\mathrm{m}}$, $\bar{\mu}_{\rho C^e} = \mu_{\rho C^e}/\mu_K$ and $\bar{\mu}_\gamma = \mu_\gamma/\mu_K$ in which

$$\mu_K = \frac{2n_K(1+\mathrm{e}^{-2n_K})}{\pi^2+4n_K^2},\ \mu_{\rho C^e} = \frac{2n_{\rho C^e}\left(1+\mathrm{e}^{-2n_{\rho C^e}}\right)}{\pi^2+4n^2_{\rho C^e}},\ \mu_\gamma = \frac{n_\gamma(1+\mathrm{e}^{-2n_\gamma})-1+\mathrm{e}^{-2n_\gamma}}{4n_\gamma^2} \tag{15}$$

The nonlocal theory assumes that stress at a point depends not only on the strain at that point but also on strains at all other points of the body. Here, the one-dimensional constitutive equation gives the uniaxial tensile stress only, according to the differential form of the nonlocal constitutive relation proposed by Eringen (1972, 1983), Eringen and Edelen (1972), as

$$\sigma_x - \xi\frac{\partial^2\sigma_x}{\partial x^2} = -E_{\mathrm{m}}\left[\mathrm{e}^{n_E(2z-h)/h}z\frac{\partial^2 w}{\partial x^2} + \alpha_{\mathrm{m}}\,\mathrm{e}^{n_{E\alpha}(2z-h)/h}\left(1-a\frac{\pi^2}{h^2}-a\frac{\partial^2}{\partial x^2}\right)\varphi\right] \tag{16}$$

where $n_{E\alpha} = \ln\sqrt{E_m\alpha_m/E_c\alpha_c}$ in which α_c and E_c are the thermal expansion coefficient and Young's modulus of the ceramic material, respectively. The nonlocal parameter is $\xi=(e_0L)^2$ in which e_0 is a constant appropriate to each material and L is the internal characteristic length. A conservative estimate of the nonlocal parameter is generally e_0L<2.0 nm for a single wall carbon nanotube (Wang and Wang 2007).

The flexure moment of the cross-section is given, with the aid of Eqs. (13) and (16), by

$$M - \xi\frac{\partial^2 M}{\partial x^2} = -bh^2E_{\mathrm{m}}\left[h\mu_E\frac{\partial^2 w}{\partial x^2} + \alpha_{\mathrm{m}}\mu_{E\alpha}\left(1-a\frac{\pi^2}{h^2}-a\frac{\partial^2}{\partial x^2}\right)\varphi_1\right] \tag{17}$$

where

$$\begin{aligned}\mu_E &= \frac{(n_E^2+2)(1-\mathrm{e}^{-2n_E})-2n_E(1+\mathrm{e}^{-2n_E})}{8n_E^3}\\ \mu_{E\alpha} &= \frac{n_{E\alpha}(4n_{E\alpha}^2+\pi^2)(1-\mathrm{e}^{-2n_{E\alpha}})+(\pi^2-4n_{E\alpha}^2)(1+\mathrm{e}^{-2n_{E\alpha}})}{(\pi^2+4n_{E\alpha}^2)^2}\end{aligned} \tag{18}$$

The differential equation of thermally induced lateral vibration of the beam may be expressed in the form

$$\frac{\partial^2 M}{\partial x^2} = \frac{1-\mathrm{e}^{-2n_\rho}}{2n_\rho} \rho_{\mathrm{m}} A \frac{\partial^2 w}{\partial t^2} \tag{19}$$

where $A=bh$ is the cross-section area. Substituting Eq. (17) into Eq. (19), one can get the motion equation of the beam as

$$\frac{\partial^4 w}{\partial x^4} + \frac{1-\mathrm{e}^{-2n_\rho}}{2n_\rho \mu_E \varepsilon^2 h^2} \left(\frac{\partial^2 w}{\partial t^2} - \xi \frac{\partial^4 w}{\partial x^2 \partial t^2} \right) + \frac{\alpha_{\mathrm{m}} \bar{\mu}_{E\alpha}}{h} \left(1 - a \frac{\pi^2}{h^2} - a \frac{\partial^2}{\partial x^2} \right) \frac{\partial^2 \varphi_1}{\partial x^2} = 0 \tag{20}$$

where $\bar{\mu}_{E\alpha} = \mu_{E\alpha} / \mu_E$ and $\varepsilon = \sqrt{E_{\mathrm{m}} / \rho_{\mathrm{m}}}$.

The initial and boundary conditions should be considered to solve the present problem. The initial conditions of the problem are taken as

$$w(x,t)\big|_{t=0} = \left.\frac{\partial w(x,t)}{\partial t}\right|_{t=0} = 0, \quad \varphi_1(x,t)\big|_{t=0} = \left.\frac{\partial \varphi_1(x,t)}{\partial t}\right|_{t=0} = 0 \tag{21}$$

These conditions are supplemented by considering the two ends of the nanobeam are simply-supported

$$w(x,t)\big|_{x=0,L} = 0, \qquad \left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=0,L} = 0 \tag{22}$$

Let us also consider the nanobeam is loaded thermally by sinusoidal pulse heating incidents into the surface of the nanobeam x=0 with pulse width t_0. In this case, the conductive temperature is considered as

$$\varphi_1(x,t)\big|_{x=0} = f(t) = \begin{cases} \sin\left(\dfrac{\pi}{t_0} t \right), & 0 \le t \le t_0 \\ 0, & t > t_0,\ t < 0 \end{cases} \tag{23}$$

In addition, the conductive temperature at the end boundary should satisfy the following relation

$$\frac{\partial \varphi_1}{\partial x} = 0 \quad \text{on} \quad x = L \tag{24}$$

The preceding governing equations can be put in nondimensional forms using the following dimensionless parameters

$$\begin{gathered} (x', L', u', w', z', h') = \eta\varepsilon(x, L, u, w, z, h), \quad (t', \tau_0', t_0') = \eta\varepsilon^2 (t, \tau_0, t_0), \\ (\xi', a') = \eta^2 \varepsilon^2 (\xi, a), \quad \theta_1' = \frac{\theta_1}{T_0} \end{gathered} \tag{25}$$

So, the governing equations and the thermodynamical temperature θ in the dimensionless forms are simplified as (dropping the primes for convenience)

$$\frac{\partial^4 w}{\partial x^4} + A_1\left(\frac{\partial^2 w}{\partial t^2} - \xi\frac{\partial^4 w}{\partial x^2 \partial t^2}\right) + A_2\left(c - a\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 \varphi_1}{\partial x^2} = 0 \tag{26}$$

$$\frac{\partial^2 \varphi_1}{\partial x^2} = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left[A_3\left(c - a\frac{\partial^2}{\partial x^2}\right)\frac{\partial \varphi_1}{\partial t} - A_4\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right] \tag{27}$$

$$\theta = \sin\left(\frac{\pi z}{h}\right)\left(c - a\frac{\partial^2}{\partial x^2}\right)\varphi_1 \tag{28}$$

where

$$c = 1 - a\frac{\pi^2}{h^2},\quad A_1 = \frac{1 - e^{-2n_\rho}}{2n_\rho \mu_E h^2},\quad A_2 = \frac{\alpha_m \bar{\mu}_{E\alpha} T_0}{h},\quad A_3 = \bar{\mu}_{\rho C^e},\quad A_4 = \frac{\bar{\mu}_\gamma h \gamma_m}{\eta K_m} \tag{30}$$

4. Solution procedure

The closed-form solution of the governing and constitutive equations may be possible by adapting the Laplace transformation method. Applying the Laplace transform to Eqs. (26) and (27), one gets the field equations as

$$\left(\frac{d^4}{dx^4} - A_1 s^2 \xi\frac{d^2}{dx^2} + A_1 s^2\right)\bar{w} = A_2\frac{d^2}{dx^2}\left(a\frac{d^2}{dx^2} - c\right)\bar{\varphi}_1 \tag{31}$$

$$qA_4\frac{d^2\bar{w}}{dx^2} = \left(qcA_3 - (1 + qA_3 a)\frac{d^2}{dx^2}\right)\bar{\varphi}_1 \tag{32}$$

where an over bar symbol denotes its Laplace transform, s denotes the Laplace transform parameter and

$$q = s(1 + \tau_0 s) \tag{33}$$

Eliminating $\bar{\varphi}_1$ or $\bar{w}$ from Eqs. (31) and (32) gives the following differential equation for either $\bar{w}$ or $\bar{\varphi}_1$

$$\left[\frac{d^6}{dx^6} - A\frac{d^4}{dx^4} + B\frac{d^2}{dx^2} - C\right]\{\bar{w}, \bar{\varphi}_1\} = 0 \tag{34}$$

where the coefficients A, B and C are given by

$$\begin{gathered} A = \frac{cq(A_3 + A_2 A_4) + A_1 s^2 \xi(1 + aA_3 q)}{1 + aq(A_3 + A_2 A_4)} \\ B = \frac{A_1 s^2[1 + A_3 q(a + c\xi)]}{1 + aq(A_3 + A_2 A_4)},\quad C = \frac{A_1 A_3 c q s^2}{1 + aq(A_3 + A_2 A_4)} \end{gathered} \tag{35}$$

Introducing m_i (i=1,2,3) into Eq. (34), one gets

$$[(D^2 - m_1^2)(D^2 - m_2^2)(D^2 - m_3^2)]\{\bar{w}, \bar{\varphi}_1\} = 0 \tag{36}$$

where $D = \mathrm{d}/\mathrm{d}x$ and m_1^2, m_2^2 and m_3^2 are the roots of the characteristic equation

$$m^6 - Am^4 + Bm^2 - C = 0 \tag{37}$$

These roots are given by

$$\begin{gathered} m_1^2 = \tfrac{1}{3}[2p_0 \sin(q_0) + A], \quad m_2^2 = -\tfrac{1}{3} p_0[\sqrt{3}\cos(q_0) + \sin(q_0)] + \tfrac{1}{3} A \\ m_3^2 = \tfrac{1}{3} p_0[\sqrt{3}\cos(q_0) - \sin(q_0)] + \tfrac{1}{3} A \end{gathered} \tag{38}$$

where

$$p_0 = \sqrt{A^2 - 3B}, \quad q_0 = \tfrac{1}{3}\sin^{-1}\left(-\frac{2A^3 - 9AB + 27C}{2p_0^3}\right) \tag{39}$$

The solution of the governing Eq. (36) in the Laplace transformation domain can be represented as

$$\{\bar{w}, \bar{\varphi}_1\} = \sum_{i=1}^{3} \left(\{C_i, F_i\} \mathrm{e}^{-m_i x} + \{C_{i+3}, F_{i+3}\} \mathrm{e}^{m_i x} \right) \tag{40}$$

where C_i and F_i are parameters depending on s. The compatibility between these two equations and Eqs. (31) and (32), gives

$$F_i = \beta_i C_i, \quad F_{i+3} = \beta_i C_{i+3}, \quad \beta_i = \frac{m_i^4 + A_1 s^2}{A_2(a m_i^4 - c m_i^2)} \tag{41}$$

So

$$\{\bar{\varphi}_1, \bar{w}\} = \sum_{i=1}^{3} \{\beta_i, 1\} \left(C_i\, \mathrm{e}^{-m_i x} + C_{i+3}\, \mathrm{e}^{m_i x} \right) \tag{42}$$

Then, the thermodynamical temperature θ in the Laplace domain with the aid of the above equations, becomes

$$\bar{\theta} = \sin\left(\frac{\pi z}{h}\right) \sum_{i=1}^{3} \beta_i (c - a m_i^2) \left(C_i\, \mathrm{e}^{-m_i x} + C_{i+3}\, \mathrm{e}^{m_i x} \right) \tag{43}$$

Also, the axial displacement after using Eq. (42) takes the form

$$\bar{u} = -z \frac{\mathrm{d}\bar{w}}{\mathrm{d}x} = z \sum_{i=1}^{3} m_i \left(C_i\, \mathrm{e}^{-m_i x} - C_{i+3}\, \mathrm{e}^{m_i x} \right) \tag{44}$$

In addition, the strain will be

$$\bar{e} = \frac{\mathrm{d}\bar{u}}{\mathrm{d}x} = -z \sum_{i=1}^{3} m_i^2 \left(C_i\, \mathrm{e}^{-m_i x} + C_{i+3}\, \mathrm{e}^{m_i x} \right) \tag{45}$$

After using Laplace transform, the boundary conditions, in their dimensionless forms, are given by

$$\left.\bar{w}(x,s)\right|_{x=0,L}=0, \quad \left.\frac{\mathrm{d}^2\bar{w}(x,s)}{\mathrm{d}x^2}\right|_{x=0,L}=0$$
$$\left.\bar{\varphi}_1(x,s)\right|_{x=0}=\frac{\pi t_0}{\pi^2+t_0^2s^2}=\bar{G}(s), \quad \left.\frac{\mathrm{d}\bar{\varphi}_1(x,s)}{\mathrm{d}x}\right|_{x=L}=0 \tag{46}$$

Substituting Eq. (42) into the above boundary conditions, one obtains six linear equations in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \mathrm{e}^{-m_1L} & \mathrm{e}^{-m_2L} & \mathrm{e}^{-m_3L} & \mathrm{e}^{m_1L} & \mathrm{e}^{m_2L} & \mathrm{e}^{m_3L} \\ m_1^2 & m_2^2 & m_3^2 & m_1^2 & m_2^2 & m_3^2 \\ m_1^2\,\mathrm{e}^{-m_1L} & m_2^2\,\mathrm{e}^{-m_2L} & m_3^2\,\mathrm{e}^{-m_3L} & m_1^2 e^{m_1L} & m_2^2\,\mathrm{e}^{m_2L} & m_3^2\,\mathrm{e}^{m_3L} \\ \beta_1 & \beta_2 & \beta_3 & \beta_1 & \beta_2 & \beta_3 \\ -m_1\beta_1\,\mathrm{e}^{-m_1L} & -m_2\beta_2\,\mathrm{e}^{-m_2L} & -m_3\beta_3\,\mathrm{e}^{-m_3L} & m_1\beta_1\,\mathrm{e}^{m_1L} & m_2\beta_2\,\mathrm{e}^{m_2L} & m_3\beta_3\,\mathrm{e}^{m_3L} \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \bar{G}(s) \\ 0 \end{Bmatrix} \tag{47}$$

The solution of the above system of linear equations gives the unknown parameters C_i and C_{i+3}. This completes the solution of the problem in the Laplace transform domain.

5. Inversion of the Laplace transforms

To get the solutions for stress, strain, temperature, induced magnetic field and induced electric field, we have applied the Laplace inversion formula to Eqs. (42)-(45). These have been done numerically using a method based on the Fourier series expansion technique. In this method, a function in the Laplace domain is inverted to the time domain through the sum

$$f(t)=\frac{\mathrm{e}^{\zeta t}}{t}\left[\tfrac{1}{2}\mathrm{Re}[\bar{F}(\zeta)]+\mathrm{Re}\sum_{n=0}^{N}\left(\bar{F}\left(\zeta+\frac{in\pi}{t}\right)(-1)^n\right)\right] \tag{48}$$

where *Re* is the real part and *i* is the imaginary number unit. For faster convergence, numerical experiments have shown that the value that satisfies the above relation is $\zeta \approx 4.7/t$ (Tzou 1996).

6. Numerical results

In terms of the Riemann-sum approximation defined in Eq. (48), numerical Laplace inversion is performed to obtain the non-dimensional lateral vibration, temperature and displacement in the nanobeam. In the present work, the thermoelastic coupling effect is analyzed by considering a beam made of an FGM. The *aluminum* as lower metal surface and *alumina* as upper ceramic surface are used for the present nanobeam. The material properties are assumed to be:
Metal (*aluminum*)

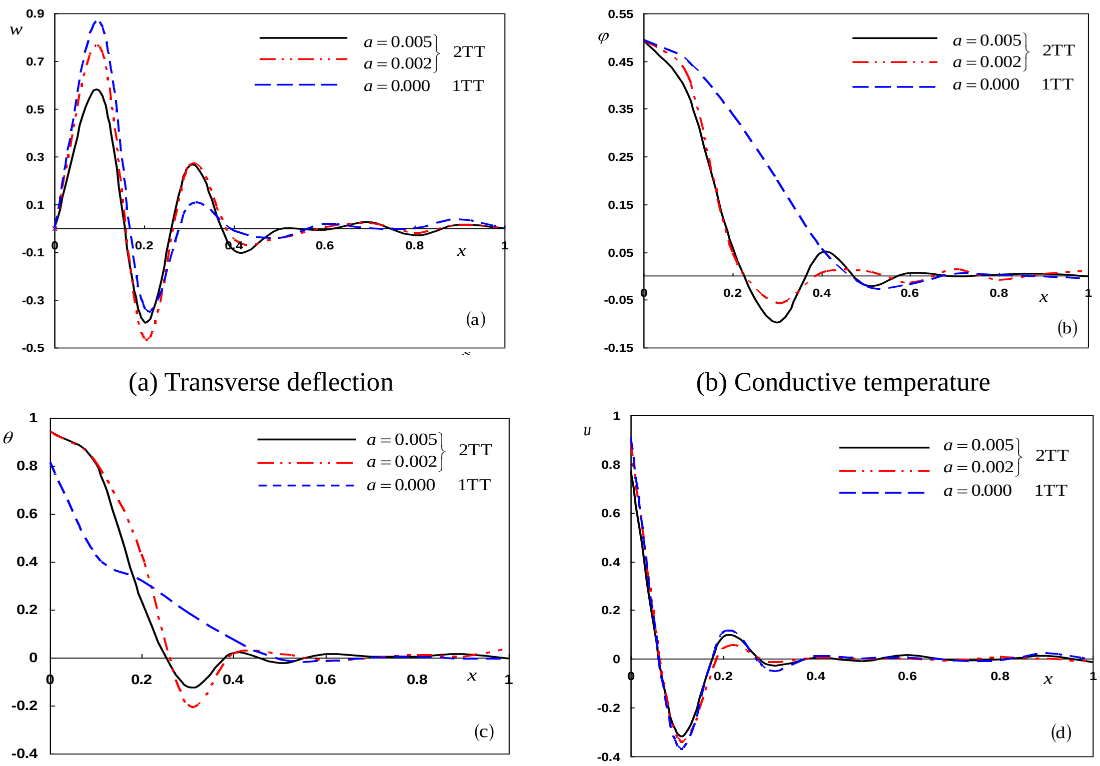
$$\begin{aligned} &E_{\mathrm{m}} = 70\,\mathrm{GPa}, \quad \nu_{\mathrm{m}} = 0.35, \quad \rho_{\mathrm{m}} = 2700\,\mathrm{Kg/m^3} \\ &\alpha_{\mathrm{m}} = 23.1\times10^{-6}\,\mathrm{K^{-1}}, \;\; \chi_{\mathrm{m}} = 84.18\times10^{-6}\,\mathrm{m^2/s}, \;\; K_{\mathrm{m}} = 237\,\mathrm{W/(m\,K)} \end{aligned} \tag{49}$$

Ceramic (*alumina*)

$$\begin{aligned} &E_{\mathrm{c}} = 116\,\mathrm{GPa}, \quad \nu_{\mathrm{c}} = 0.33, \quad \rho_{\mathrm{c}} = 3000\,\mathrm{Kg/m^3} \\ &\alpha_{\mathrm{c}} = 8.7\times10^{-6}\,\mathrm{K^{-1}}, \;\; \chi_{\mathrm{c}} = 1.06\times10^{-6}\,\mathrm{m^2/s}, \;\; K_{\mathrm{c}} = 1.78\,\mathrm{W/(m\,K)} \end{aligned} \tag{50}$$

The reference temperature of the nanobeam is T_0=293 K. The dimensionless variables defined in Eq. (25) are plotted for a wide range of nanobeam length when L=1, h=0.1 and t=0.12 sec. The thickness position is assumed, except otherwise stated, to be $z=h/6$. In what follows, the nonlocal parameter $\bar{\xi}$ ($\bar{\xi}=10^6\xi$) is used. It should be less than 4($\mu\mu$m^2). The local theory of the beam is given when $\bar{\xi}=0$.

Some plots consider the present quantities through the length of the nanobeam and others take into account both the length and thickness directions. Eq. (48) is used to invert the Laplace transforms in Eqs. (42)-(45) and the conductive temperature, the dynamical temperature and the displacement distributions are represented graphically with respect to x($0\le x\le1$). Figs. 2-5 represent the curves predicted for these quantities.



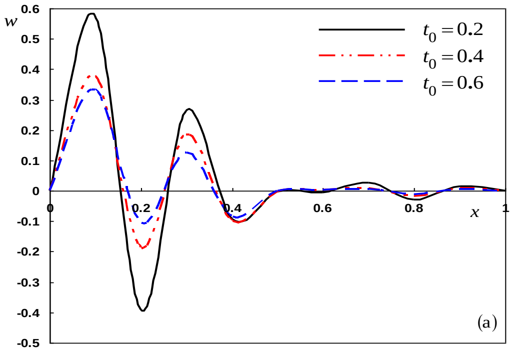
(a) Transverse deflection

(b) Conductive temperature

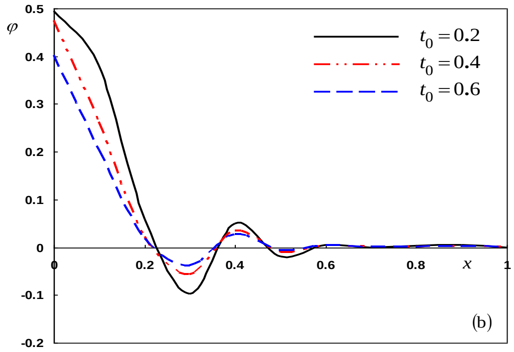
(c) Dynamical temperature

(d) Axial displacement

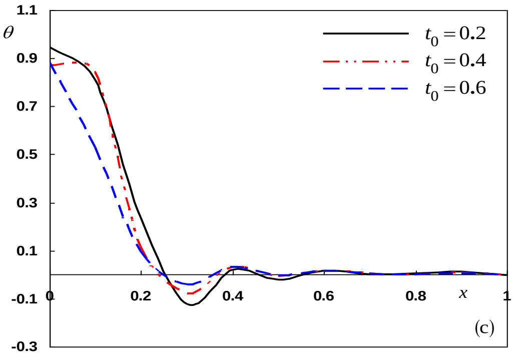
Fig. 2 The field quantities of the FG nanobeam for different values of temperature discrepancy a



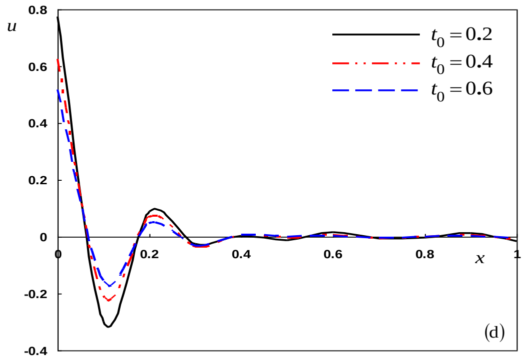
(a) Transverse deflection



(b) Conductive temperature



(c) Dynamical temperature



(d) Axial displacement

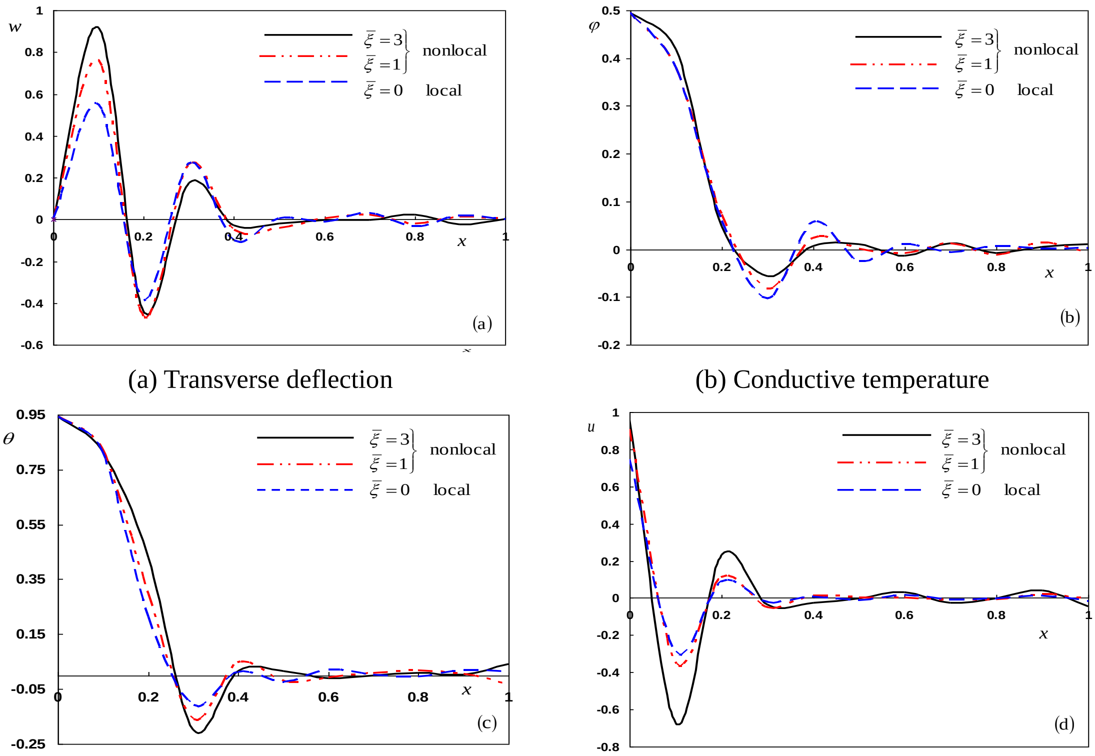
Fig. 3 The field quantities of the FG nanobeam for different values of pulse width parameter t_0

In fact, numerical calculations are carried out for three cases. The first one is to investigate how the non-dimensional conductive temperature, thermodynamic temperature and displacement vary with the different values of the non-dimensional temperature discrepancy *a*. Note that the case of *a*=0 indicates the old situation (one temperature 1TT theory) and the cases of *a*=0.005 and 0.002 indicate the two-temperature theory (2TT). In this case, one assumed that the characteristic time of the pulse width t_0=0.2 and the non-local parameter $\bar{\xi}$ =1.

The second case is to investigate how the non-dimensional conductive temperature, thermodynamic temperature and displacement vary with different values of the characteristic time of the pulse width t_0 when the temperature discrepancy parameter remains constant (*a*=0.005). In this case, one assumed that the characteristic non-local parameter $\bar{\xi}$ =1.

The third case is to investigate the non-dimensional lateral vibration, temperature and displacement for various values of the nonlocal parameter $\bar{\xi}$ when the pulse width remains constant (t_0=0.2) and the temperature discrepancy parameter *a*=0.005.

In the first case, we consider three different values of the temperature discrepancy parameter *a* as shown on Fig. 2. The variation of *a* is very sensitive to the response of all field quantities. When *a*=0, all quantities get different behaviors. This shows the difference between the one temperature generalized thermoelasticity of LS and the two-temperature generalized thermoelasticity. Also, Fig. 2 shows that this parameter has significant effect on all the field quantities. The waves reach



(a) Transverse deflection (b) Conductive temperature

(c) Dynamical temperature (d) Axial displacement

Fig. 4 The field quantities of the FG nanobeam for different values of nonlocal parameter $\bar{\xi}$

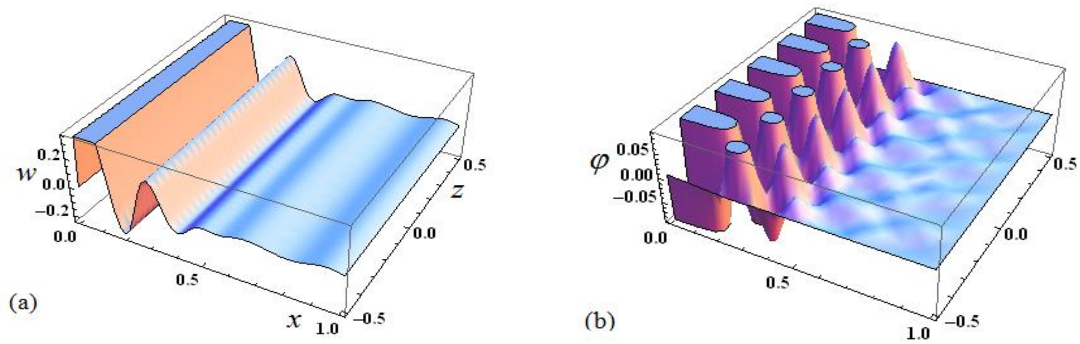
the steady state depending on the value of the temperature discrepancy *a*. Fig. 2(a) depicts the distribution of the lateral vibration *w* through the length of the beam. It always begins at the zero value and non-uniformly vibrates through the beam length to vanish once again at the end of the beam. This satisfies the boundary condition at beam boundaries.

Fig. 2(b) shows the variation of the conductive temperature φ versus distance *x*. It is observed that the conductive temperature φ decreases as the axial distance *x* increases to move in the direction of wave propagation. The conductive temperature of 2TT model may be differing than those of 1TT theory.

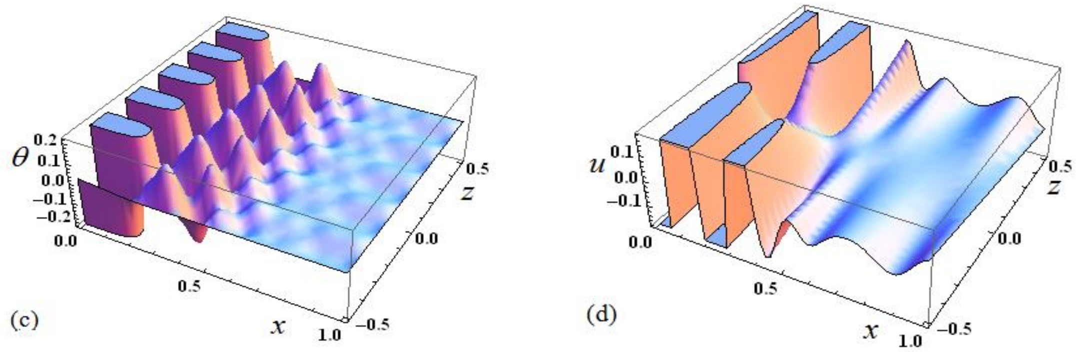
Fig. 2(c) exhibits the space variation of thermodynamic temperature θ in which we observe that a significant difference in the thermodynamic temperature for the value of the non-dimensional two-temperature parameter *a* where the case of a=0 indicates one-type temperature and the case of a>0 indicates two-type temperature. From this figure it can be observed that a>0 corresponds a slower rate of decay than the case when a=0.

Fig. 2(d) shows that the axial displacement *u* moves directly in the direction of wave propagation. A significant difference in the displacement is noticed for different values of the non-dimensional two-temperature parameter *a*. Once again, the behavior of 2TT model may be differing than that of 1TT model near the boundary plane x=0.

In the second case, one considered different values of the time of the pulse width, i.e., t_0=0.2, 0.4 and 0.6 with the constant parameter a=0.005. From Fig. 3, it is found that, the pulse width



(a) Transverse deflection (b) Conductive temperature



(c) Dynamical temperature (d) Axial displacement

Fig. 5 Distributions of the field quantities through the axial and thickness directions of the FG nanobeam

parameter has significant effects on all the field quantities. The pulse width makes the difference between the results in the context of the two-temperature generalized thermoelasticity theory. The case of t_0=0.2 gives absolutely different behaviour comparing with the other cases t_0=0.4 and t_0=0.6.

In the third case we restrict our attention to the behaviour of the nonlocal parameter $\bar{\xi}$ when the pulse width remains constant (t_0=0.2). The variations of the field quantities with various values of the nonlocal parameter are depicted in Fig. 4. It can be seen that the effect of nonlocal parameter on the deflection, temperatures and displacement is highly significant. In general, the field variables are increase with the increasing of the nonlocal parameter.

Additional and interesting case is added in Fig. 5. The three dimensions of all field quantities of the nanobeam are presented at constant value of the pulse width parameter t_0=0.2, the non-local parameter $\bar{\xi}=1$, the temperature discrepancy parameter a=0.005 and wide range of thickness $-0.5 \le z/h \le 0.5$. When the thickness increases all the studied field quantities increase and it is very obvious at the peak points of the curves.

7. Conclusions

The vibration characteristics of the deflection, conductive temperature, the thermodynamics temperature and the displacement of an Euler-Bernoulli nanobeam induced by a sinusoidal pulse heating in the context of nonlocal two-temperature theory of thermoelasticity have been investigated. The effects of the nonlocal parameter, the pulse width parameter and time parameter t_0 as well as the two-temperature parameter a of thermal vibration on the field quantities are investigated. It is found that all of theses parameters have significant effects on the behaviors of the studied field quantities. Numerical technique based on the Laplace transformation has been used. Some numerical examples of a FGM nanobeam are presented to show the differences due to the presence of a nonlocal nanoscale.

In engineering application, one can choose appropriate gradients to make the FG nanobeams be safe in structural integrity when subjected to high-temperature change of the inner or outer environment. The significant differences in the physical quantities are observed for all the one-temperature models and two-temperature models. Two-temperature theory is more realistic than the one-temperature theory in the case of generalized thermoelasticity.

The paper also concludes the governing equation of motion for a nonlocal nanobeam can be formed by replacing the bending moment term in the classical equation of motion with an effective nonlocal bending moment as presented herewith. This work is expected to be useful to design and analyze the wave propagation properties of nanoscale devices with terahertz frequency range.

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