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The use of topology optimization in the design of truss and frame bridge girders

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Abstract. It is shown that topology optimization is a valuable tool for the design of bridge girders. This paper is a follow-up to (Kutyłowski and Rasiak 2014) and it includes an analysis of truss members' outer dimensions dictated by the standards. Moreover, a frame bridge girder mapped from a selected topology is compared with a typical frame girder on the basis of (Kutyłowski and Rasiak 2014). The analysis shows that topology optimization by means of the proposed algorithm yields a topology from which one can map a frame bridge girder requiring less material for its construction than the typical frame girder currently used in bridge construction.

Keywords: topology optimization; design of frame bridge girders; topology and design parameters analysis

1. Introduction

This paper deals with the application of topology optimization in design. It is generally based on (Kutyłowski and Rasiak 2014), represents the minimum compliance approach (Bendsøe 1989, Ramm *et al*. 1994, Bendsøe and Sigmund 1999, Bendsøe and Sigmund 2003,) and is a follow-up to (Kutyłowski and Rasiak 2014) where an original way of determining an optimum topology for a truss construction, based on body compliance minimization, and a way of mapping the obtained topology into a truss bridge girder construction were presented. This paper is additionally based on (Kutyłowski 2002, Kutyłowski and Rasiak 2008). As regards mapping, two principal tasks: the mapping of a topology into a system of members and the development of a way of mapping the "thickness" of the individual topology elements into the size of a given truss member's cross section are demonstrated. Moreover, the amounts of material used for the girder in standard design conditions are compared taking in consideration the relevant codes (Code 1, Code 2, Code 3) and paper (Kmita *et al*. 1989). Thanks to the assumptions concerning the mapping of a topology into a truss system the amount of material used for the girder obtained through topology optimization was found to be comparable with that consumed for a typical girder. The aim was to demonstrate that even for the most conservative way of modelling, the material consumption is at least

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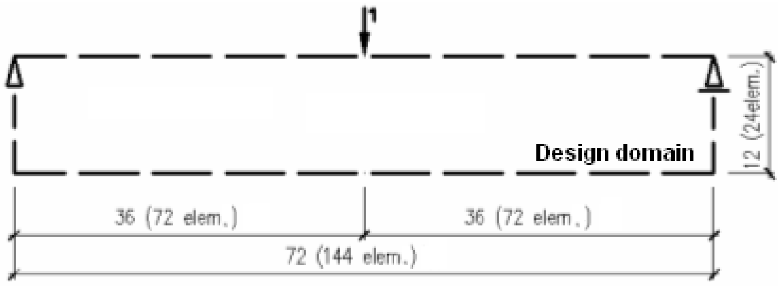


Fig. 1 Analyzed design domain

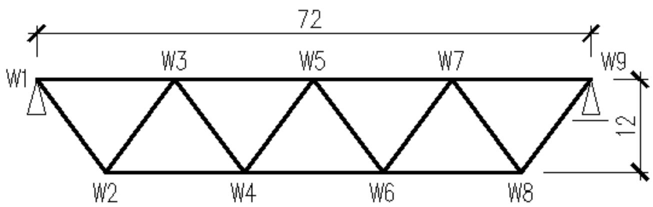


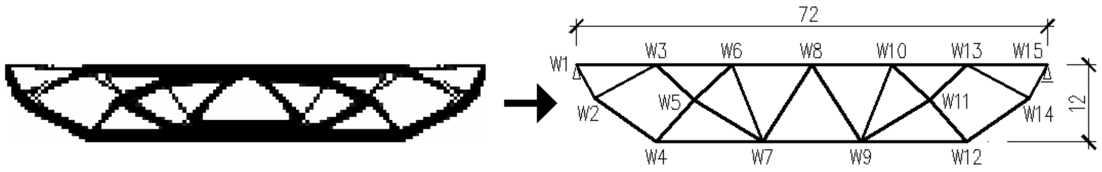
Fig. 2 Reference truss girder (A)

comparable. If modelling taking into account the nodes created in the course of optimization were used, the theoretical length would be shorter whereby the material consumption for a truss obtained from a topology would have to be lower than for a typical truss girder.

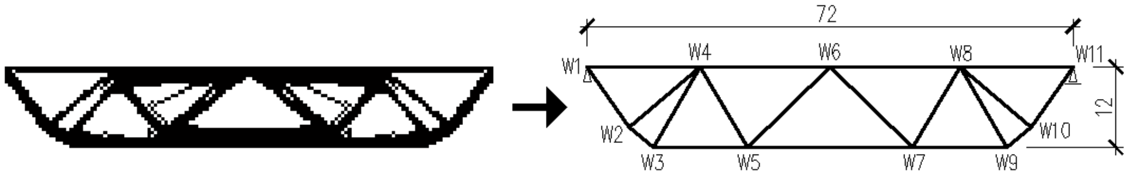
It is worth mentioning that the field of topology optimization (in the general sense and as regards the considered problem) has been developing rapidly in recent years. This applies not only to the theory (e.g., Lee *et al*. 2010), where dynamic problems are considered in the context of the uncertainty of material properties, and (Huang and Xie 2010) dealing with the problems with regard to material and geometrical nonlinearity), but also to the practice (e.g., Lee and Park 2011, Yang *et al*. 2011), Kutyłowski and Rasiak (2014) or the present contribution).

In the present paper the truss girder is subjected to further analysis. First an optimum size is selected for each member and then material consumption for the selected sizes is analyzed. The results are compared with those for typical truss girders used in practice. Then calculations are done for the frame girder construction for one selected topology. The characteristic feature of the truss girder model considered so far was that the load from the bridge deck was transferred through the nodes. In the frame construction model the load from the deck of the deck bridge is transferred continuously to the top flange. The bridge girders designed today are of the frame type, which means that the results of this research can be directly applied in practice. It is shown that the material consumption for a frame girder obtained through topology optimization is lower than that for a typical frame girder.

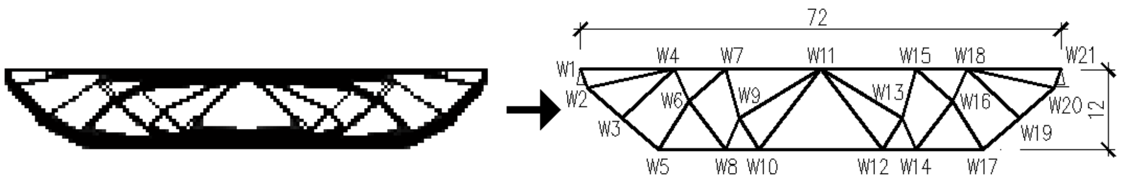
The design area considered in this paper and in (Kutyłowski and Rasiak 2014) is shown in Fig. 1. A typical truss girder is shown in Fig. 1 and girders B1, B2 and B3 (also analyzed in (Kutyłowski and Rasiak 2014)) obtained from topology transformation are shown in Fig. 3. The same load, adopted in accordance with the relevant standards, and the same box and tubular cross sections are used in both papers. are used. As regards the design principles, the present research is based on design standards (Code 1, Code 2, Code 3).



(B1) Girder obtained by approximating topology b1



(B2) Girder obtained by approximating topology b2



(B3) Girder obtained by approximating topology b3

Fig. 3 Topologies and corresponding deck trusses

2. Analysis of truss member's outer dimensions

All the analyzed truss members had the same outer dimension (Kutyłowski and Rasiak 2014). It seemed natural to do calculations for at least two outer dimensions. An outer dimension of 0.5 m and 0.8 was adopted for respectively the cross braces (except for the support ones) and the other members. This did not have any noticeable effect on material consumption. In addition, an analysis was carried out to determine the optimal outer dimension for each of the members. The optimal outer dimensions were then used in calculations. Cross-sectional areas were determined through statical strength calculations (in which the effect of local stability was taken into account). The selected outer dimensions of the cross sections were close to the ones actually used in design. The values of the adopted outer dimensions were rounded off to 10 cm, amounting to 0.8, 0.7, 0.6, 0.5, 0.4, 0.3 and 0.2 [m]. Larger or smaller dimensions were not used since they are not typical in bridge construction practice. The results of calculations for both box cross sections (KS") and tubular cross sections (OR") are compiled in the tables below.

It should be noted that the optimal (i.e., minimal) cross-sectional areas for the particular bars were obtained for different outer dimensions. Sometimes for relatively small outer dimensions (0.20 and 0.30) the bearing capacity conditions were not satisfied with regard to buckling stress or because slenderness was not equal to 150.

The table 5 shows steel volumes for the selected optimal cross sectional areas for typical truss A and trusses B1, B2 and B3. In the case of the members with box cross sections (KS"), a slight increase (by 0.86% relative to truss A) was noted for truss B1 while small losses (amounting to respectively 1.61% and 0.47%) were noted for trusses B2 and B3.

Table 1 Cross-sectional (KS”) areas [cm^2] of truss A bars for different outer dimensions

Member	Compressed bars (for given outer dimension)						Tensioned bars
	0.30 m	0.40 m	0.50 m	0.60 m	0.70 m	0.80 m	
W1-W2							217.70
W2-W4							261.23
W4-W6							359.38
W1-W3	-	246.87	**185.48**	209.25	252.39*	298.92*	
W3-W5	-	698.20	449.96	397.21	**372.37**	413.53*	
W2-W3	389.69	280.83	256.89	**246.00***	292.25*	344.81*	
W3-W4							81.79
W4-W5	130.87	**104.69**	136.87*	172.66*	210.78*	250.74*	

104.69 – optimal cross-sectional areas for the particular bars, shown in table 5.
* – results in which the effect of local buckling had to be taken into account.

Table 2 Cross-sectional (KS”) areas [cm^2] of truss B1 bars for different outer dimensions

Member	Compressed bars (for given outer dimension)							Tensioned bars
	0.20 m	0.30 m	0.40 m	0.50 m	0.60 m	0.70 m	0.80 m	
W1-W2								216.21
W2-W4								239.84
W4-W7								318.40
W7-W9								359.40
W1-W3	-	166.54	**129.42**	147.35	185.01*	225.86*	268.83*	
W3-W6	-	284.35	221.16	**205.36**	228.49*	270.02*	321.53*	
W6-W8	-	-	434.73	395.23	375.46	**365.58**	412.81*	
W2-W3	263.20	139.22	**125.08**	148.19*	186.64*	229.09*	273.57*	
W3-W5	**36.52**	47.88*	69.42*	93.17*	118.32*	145.11*	173.70*	
W4-W5	337.56	218.86	205.87	**200.31**	228.50*	270.76*	323.33*	
W5-W6	278.01	212.17	201.19	**195.70**	225.33*	268.73*	321.89*	
W5-W7	**27.20**	38.98*	55.22*	73.31*	92.84*	113.47*	135.29*	
W6-W7								58.83
W7-W8	-	**76.49**	88.86*	117.25*	148.29*	181.38*	215.81*	

125.08 – optimal cross-sectional areas for the particular bars, shown in table 5.
* – results in which the effect of local buckling had to be taken into account.

The results show that the steel volumes for the approximated trusses (B1, B2, B3) are at a level characteristic of a typical construction. Sometimes a slight gain is achieved while in some cases there is a small loss. Thus the topology optimization results confirm the optimum material consumption for the currently used typical trusses.

The tables below show normalized the cross sectional areas of the elements of topologies b1, b2 and b3 and bar structures B1, B2 and B3 while individual values are shown in the figures.

Good agreement between the results and the topologies was obtained for most top and bottom flange bars of the approximated trusses. For example, for member W4-W7 in structure B3 the ratio was respectively 0.51 to 0.41 and for member W1-W2 in truss B2 it was 0.60 to 0.57. Good agreement was also found for the cross braces, e.g. for member W1-W2 in truss B3 and member W5-W7 in truss B1.

Table 3 Cross-sectional (KS”) areas [cm^2] of truss B2 bars for different outer dimensions

Member	Compressed bars (for given outer dimension)						Tensioned bars
	0.30 m	0.40 m	0.50 m	0.60 m	0.70 m	0.80 m	
W1-W2							216.63
W2-W3							218.08
W3-W5							248.36
W5-W7							359.40
W1-W4	-	207.59	**168.05**	203.43*	246.54*	291.94*	
W4-W6	-	750.72	450.43	381.14	**358.04**	400.09*	
W2-W4	**80.23**	90.40*	119.57*	151.58*	184.96*	220.67*	
W3-W4	253.03	205.38	**190.59**	219.71*	263.56*	314.73*	
W4-W5							80.13
W5-W6	201.76	**136.49**	148.77*	185.20*	226.12*	268.39*	

190.59 – optimal cross-sectional areas for the particular bars, shown in table 5.
* – results in which the effect of local buckling had to be taken into account.

Table 4 Cross-sectional (KS”) areas [cm^2] of truss B3 bars for different outer dimensions

Member	Compressed bars (for given outer dimension)							Tensioned bars
	0.20 m	0.30 m	0.40 m	0.50 m	0.60 m	0.70 m	0.80 m	
W1-W2								198.17
W2-W3								247.19
W3-W5								247.08
W5-W8								287.25
W8-W10								299.52
W10-W12								359.40
W1-W4	-	138.16	**106.84***	130.84*	164.22*	199.94*	237.15*	
W4-W7	-	204.47	189.40	**182.69**	216.82*	262.87*	313.88*	
W7-W11	-	-	454.65	387.05	362.48	**350.20***	400.27*	
W2-W4	-	176.55	**149.58**	160.92*	195.76*	239.67*	285.32*	
W3-W4								0.93
W4-W6	**52.10**	56.97*	80.48*	108.47*	138.21*	170.03*	202.99*	
W5-W6	334.54	224.29	209.08	**205.28**	231.88*	272.92*	325.88*	
W6-W7	230.86	192.10	181.99	**180.31**	215.33*	261.70*	312.46*	
W6-W8								13.07
W7-W9								45.33
W8-W9	**20.65***	35.04*	51.18*	68.86*	87.63*	107.73*	128.42*	
W9-W10								50.49
W9-W11	68.82	**48.54***	67.76*	89.31*	112.89*	138.07*	164.13*	
W10-W11	249.03	**86.24**	90.80*	119.55*	150.74*	183.93*	219.46*	

106.84 – optimal cross-sectional areas for the particular bars, shown in table 5.
* – results in which the effect of local buckling had to be taken into account.

The above results show that quite good agreement with the topology was obtained for the cross braces and the top and bottom flange bars, particularly when the results are compared with the ones reported in (Kutyłowski and Rasiak 2014). There is neither marked improvement nor deterioration in agreement with the topology, nevertheless the proper matching of outer

Table 5 Volumes of steel in structures A, B1, B2 and B3 for optimal cross sectional areas of members

	Steel volume [m^3]
	KS"
A	5.55
B1	5.50
Gain/loss [%]	0.86
B2	5.64
Gain/loss [%]	-1.61
B3	5.57
Gain/loss [%]	-0.47

"+" and"-" represent respectively gain and loss

Table 6 Normalized cross sectional areas of members of topology b1 and bar structure B1 (cross sections KS")

Member	Topology b1	Truss B1 (KS")
W1-W2	**0.31**	0.59
W2-W4	**0.51**	0.66
W4-W7	**0.47**	0.87
W7-W9	**1.00**	0.98
W1-W3	**0.19**	0.35
W3-W6	**0.48**	0.56
W6-W8	**0.71**	1.00
W2-W3	**0.29**	0.34
W3-W5	**0.16**	0.10
W4-W5	**0.20**	0.55
W5-W6	**0.38**	0.54
W5-W7	**0.33**	0.07
W6-W7	**0.09**	0.16
W7-W8	**0.21**	0.21

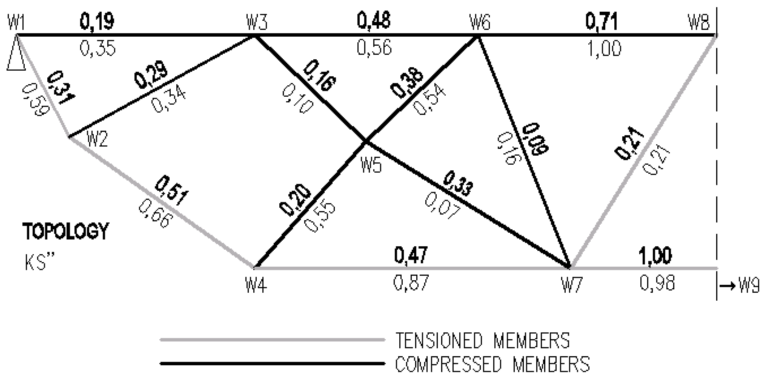


Fig. 4 Normalized cross sectional areas of members of topology b1 and of corresponding bars of structure B1 for cross sections KS"

Table 7 Normalized cross sectional areas of members of topology b2 and bar structure B2 (cross sections KS”)

Member	Topology b2	Truss B2 (KS”)
W1-W2	**0.57**	0.60
W2-W3	**0.35**	0.61
W3-W5	**0.81**	0.69
W5-W7	**1.00**	1.00
W1-W4	**0.29**	0.47
W4-W6	**0.95**	1.00
W2-W4	**0.22**	0.22
W3-W4	**0.21**	0.53
W4-W5	**0.32**	0.22
W5-W6	**0.43**	0.38

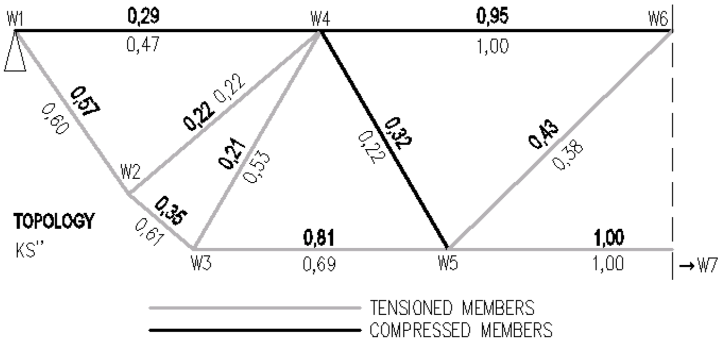


Fig. 5 Normalized cross sectional areas of members of topology b1 and of corresponding bars of structure B2 for cross sections KS”

dimensions and cross-sectional areas, based on the proportions between topology members, may significantly speed up the design process. Referring to the difficulties encountered in topology approximation (Kutyłowski and Rasiak 2014), due to problem complexity, it should be noted that the latter may be the cause of poorer agreement between the results.

The above agreement between the normalized truss cross-sectional areas (KS”) and the topologies is comparable with that for similar bars and box cross sections (KS1’, KS2’), reported in (Kutyłowski and Rasiak 2014). A similar level of agreement was achieved for all the trusses. The outer dimensions of cross sections KS1’ and KS2’ were respectively 0.50 m and 0.80 m and those of cross sections KS” (considered here) were in a range of 0.20-0.70 [m]. For example, for truss B bottom flange, some members (e.g. W7-W9) with cross section KS” (Fig. 4) show better agreement than in the case of KS1’/KS2’ while other members (W1-W2, W2-W4, W4-W7) show slightly worse agreement. In the case of the top flange members, better results were obtained for W1-W3 and W3-W6 while for members W6-W8 the value of 1.0 was obtained in both cases. The cross braces with cross section KS1’ or KS2’ and KS” show better agreement with the topology for KS” in the case of members W2-W3, W3-W5 and W7-W8. For the other cross braces the agreement is better for KS1’ or KS2’. Similarly in the case of trusses B2 and B3, some bars show better agreement for cross sections KS” while other for KS1’ or KS2’, but the

Table 8 Normalized cross sectional areas of members of topology b3 and bar structure B3 (cross sections KS”)

Member	Topology b3	Truss B3 (KS”)
W1-W2	**0.29**	0.55
W2-W3	**0.56**	0.69
W3-W5	**0.54**	0.69
W5-W8	**0.88**	0.80
W8-W10	**0.74**	0.83
W10-W12	**1.00**	1.00
W1-W4	**0.19**	0.30
W4-W7	**0.48**	0.51
W7-W11	**0.87**	0.97
W2-W4	**0.16**	0.42
W3-W4	**0.16**	0.01
W4-W6	**0.16**	0.14
W5-W6	**0.26**	0.57
W6-W7	**0.22**	0.50
W6-W8	**0.22**	0.04
W7-W9	**0.08**	0.13
W8-W9	**0.16**	0.06
W9-W10	**0.16**	0.14
W9-W11	**0.17**	0.14
W10-W11	**0.18**	0.24

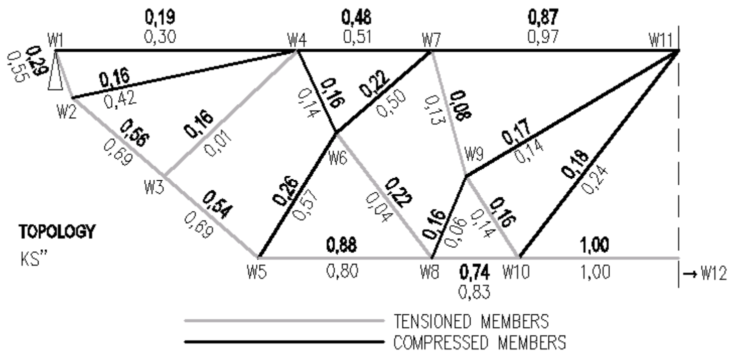


Fig. 6 Normalized cross sectional areas of members of topology b3 and of corresponding bars of structure B3 for cross sections KS”

differences are generally small. It should be noted that cross sections KS1’/KS2’ are described by two outer dimensions: 0.50/0.80 [m] and so the one closer to the value read off the topology was selected for comparisons. Sometimes the outer dimensions of cross sections KS1’/KS2’ and KS”, and consequently their normalized area values, were identical.

The case described in (Kutyłowski and Rasiak 2014) and above is the least favourable since then in the model of load transfer through nodes (at the most) the same level of steel consumption as for truss A was achieved for the B trusses. Also a certain degree of agreement between the cross sections obtained from the topology and the ones determined for trusses A and B on the basis of the standard requirements was noted. However, if one assumes the alternative (closer to reality) way of determining the theoretical length of the particular bars, then certainly material gains for trusses B (in comparison with truss A) and better agreement between the cross sections will be obtained.

Until now structures in which the load from the deck is transferred solely through nodes have been used. Today structures with the same deck as described here but with a different way of transferring the load from the deck are designed. The deck is in a continuous manner fixed to the top flange of the girder. Thus besides axial forces also bending moments occur in the structure. Such bar systems will be referred to as frame structures.

3. Calculations and analysis of frame strutures

3.1 Asumptions

Structure B1 was chosen for further analysis. It will be compared with a typical frame girder A. Both girders constitute a deck bridge with moving loads. Similar to (Lee and Park 2011) the frame structure problem is solved below.

The same types of standard load as previously (Kutyłowski and Rasiak 2014), i.e. loading with rolling stock, the deck weight, the track superstructure weight and the insulation weight ($4*P$, p, g_{max}/g_{min}), were used. The dimensions of typical railway bridge deck components were adopted in order to come as close to reality as possible. The cross section is shown in the figure below.

The structure is continuous: there are no hinges in the joints between the members. The joints are rigid and the deck is continuously joined to the girder. This means that besides (compressive or tensile) forces also bending moments and shearing forces occur in the girders.

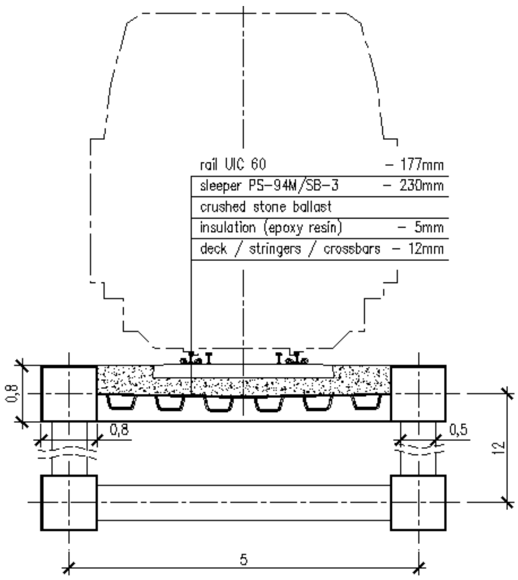


Fig. 7 Analyzed closed-grid deck

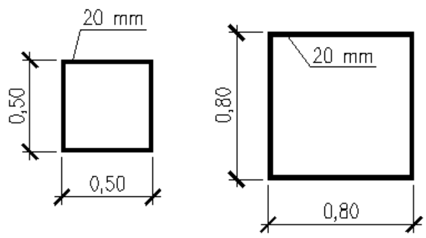


Fig. 8 Analyzed cross sections of bars in: inner cross braces (a) and top and bottom flange support cross braces (b)

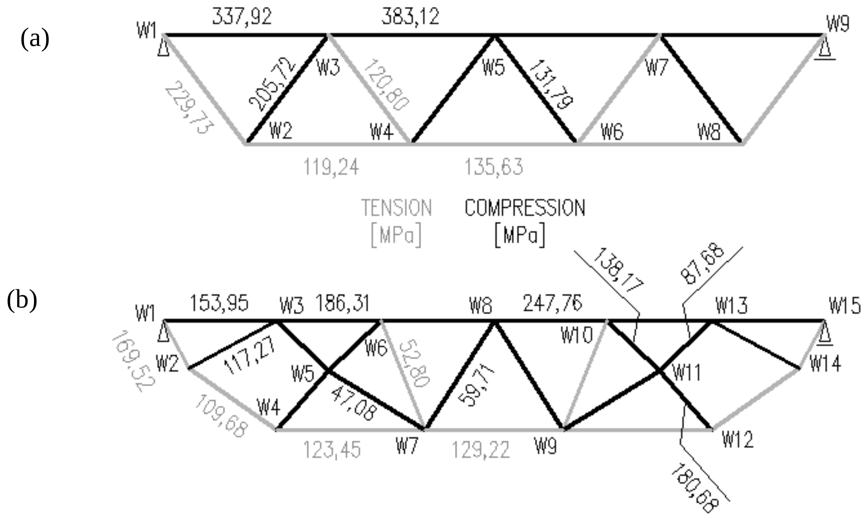


Fig. 9 Stresses [MPa] in frame structure A (a) and B1 (b) for cross section wall sheet gauge of 20 mm

The same assumptions concerning the cross sections of the frame structure's particular bars as previously (Kutyłowski and Rasiak 2014) were adopted, i.e., R=200 MPa and allowable slenderness for compressed elements λ_{max}=150 and for tensioned elements λ_{max}=200. The bars of the top and bottom flanges and the support cross braces were assumed to have square box cross sections with an outer dimension of 0.80 m. In the case of the inner cross braces, the dimension was 0.50 m. The same wall thickness of 20 mm (Fig. 8) was used in the two types of cross sections.

Under the above assumptions calculations were done for a typical frame A and frame B1 based on optimization results. The calculations yielded the values of (compressive and tensile) axial forces and moments. Then the maximum stresses in the particular members of the structure were determined.

3.2 Results of preliminary static strength calculations for girders (first step)

The maximum stresses in the initial cross sections of the two frame structures are compiled in the tables below. They were determined for a dominant axial force and the associated moment and for a dominant moment and the associated axial force as well as for intermediate values. The maximum stresses in each of the bars are shown in Fig. 9 and in the figures below.

Table 9 Stresses in members of frame structure A

Member	Stress [MPa]
W1-W2	**229.73**
W2-W4	119.24
W4-W6	135.63
W1-W3	**-337.92**
W3-W5	**-383.12**
W2-W3	**-205.72**
W3-W4	120.80
W4-W5	-131.79

"+" and "-" represent respectively tension and compression
229.73 – stress exceeding the assumed strength of the steel

Table 10 Stresses in members of frame structure B1

Member	Stress [MPa]
W1-W2	169.52
W2-W4	109.68
W4-W7	123.45
W7-W9	129.22
W1-W3	-153.95
W3-W6	-186.31
W6-W8	**-247.76**
W2-W3	-117.27
W3-W5	-87.68
W4-W5	-180.68
W5-W6	-138.17
W5-W7	-47.08
W6-W7	52.80
W7-W8	-59.71

"+" and "-" represent respectively tension and compression
247.76 – stress exceeding the assumed strength of the steel

The volumes of steel used to construct the frame structures amounted to respectively: 13.19 m^3 for structure A and 15.40 m^3 for B1. The difference in steel volume is due to the large number of inner cross braces in the frame obtained from optimization. It should be noted that in the first step the cross sections of the particular members were assumed as shown in Fig. 8. Taking into account the number of cross braces, this gives a much larger volume for frame structure B1. Nevertheless, the allowable stress was not exceeded in the bars of frame B1, except for bars W6-W7 and W8-W10. In some cases (W5-W7, W6-W7, W7-W8), substantial bearing capacity margins were found.

It should be noted that the maximum stress in many bars of frame A exceeds the allowable value in a larger number of members (as previously, the strength of the steel is 200 MPa) while in structure B1 the stress values are much lower.

3.3 Analysis of static strength calculation results for girders after optimization of cross sections (step two)

After the preliminary selection of cross sections, the thickness of the walls was changed (while preserving the outer dimensions of the particular types of bars) to obtain stress values closest to the steel strength (but not exceeding it). In the members in which the stress was exceeded the thickness was increased while in the other bars it was reduced. The wall thicknesses of the particular cross sections are compiled in the table below. Initially, for both outer cross-sectional dimensions they amounted to 20 mm in all the members. The reduction or increase in the thickness of the cross sections consisted in thinning or thickening the wall thickness by 1 mm. This assumption agrees with reality, which means that the assumed sheet gauges are realistic.

Stress values a little closer to the design strength of the steel were obtained for structure B1. Since the differences are not large, one can assume that the general level of stress in all the bars in the two frames is similar. It should be noted that for most bars of frame A it is possible to further

Table 11 Outer dimensions and thicknesses of cross sections of bars in frame structure A

Member	Dimensions	
	Outer dimension [m]	Wall thickness [mm]
W1-W2	0.80	24
W2-W4	0.80	12
W4-W6	0.80	14
W1-W3	0.80	36
W3-W5	0.80	42
W2-W3	0.50	21
W3-W4	0.50	12
W4-W5	0.50	13

Table 12 Outer dimensions and thicknesses of cross sections of bars in frame structure B1

Member	Dimensions	
	Outer dimension [m]	Wall thickness [mm]
W1-W2	0.80	18
W2-W4	0.80	12
W4-W7	0.80	13
W7-W9	0.80	13
W1-W3	0.80	16
W3-W6	0.80	21
W6-W8	0.80	27
W2-W3	0.50	12
W3-W5	0.50	9
W4-W5	0.50	19
W5-W6	0.50	15
W5-W7	0.50	7
W6-W7	0.50	5
W7-W8	0.50	7

Table 13 Changes in stress in comparison with changes in cross-sectional areas of bars in frame structure A

Member	Stress [MPa]	Change in stress [%]	Change in cross-sectional area [%]
W1-W2	168.24	-24.77	19.38
W2-W4	178.06	49.32	-39.38
W4-W6	183.38	35.21	-29.46
W1-W3	-184.65	-45.36	76.31
W3-W5	-194.17	-49.32	104.08
W2-W3	-165.74	-19.43	4.78
W3-W4	108.25	-10.39	-39.00
W4-W5	-111.81	-15.17	-34.05

In the case of stress, “+” and “-” stand for respectively tension and compression

Table 14 Changes in stress in comparison with changes in cross-sectional areas of bars in frame structure B1

Member	Stress [MPa]	Change in stress [%]	Change in cross-sectional area [%]
W1-W2	197.90	16.75	-9.77
W2-W4	184.41	68.14	-39.38
W4-W7	181.10	46.70	-34.42
W7-W9	194.00	50.14	-34.42
W1-W3	-195.47	26.97	-19.59
W3-W6	-193.23	3.71	4.87
W6-W8	-198.97	-19.69	33.79
W2-W3	-178.70	52.38	-39.00
W3-W5	-182.95	108.67	-53.97
W4-W5	-173.40	-4.03	-4.80
W5-W6	-189.87	37.41	-24.22
W5-W7	-108.26	129.96	-64.05
W6-W7	129.90	146.01	-74.22
W7-W8	-135.16	126.37	-64.05

In the case of stress, “+” and “-” stand for respectively tension and compression

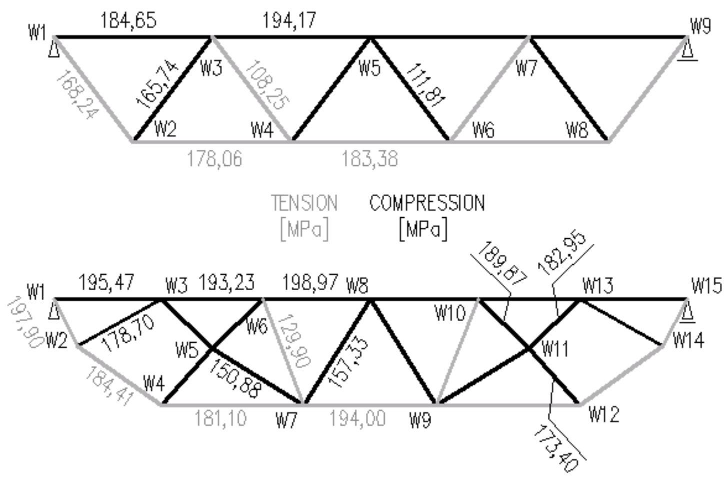


Fig. 10 Stresses [MPa] in frame structure A (a) and B1 (b) for changed cross section wall sheet gauges

optimize their cross sections by removing material. In the case of structure B1, the number of bars whose cross-sectional areas can be further reduced is similar. It should be noted that the reduction in the thickness of the walls of the boxes, aimed at getting closer to the practical strengths, amounts to 1-2 mm. This shows that the difference in the volume of steel between structure A and structure B1 in the next attempt to get closer to limit R=20 MPa would not change significantly, which means that the change would be insignificant in comparison with the global material consumption for both frames and so the above results for the two structures can be reliably compared.

After the adjustment of the cross-sectional areas for frame A the maximum stresses in the particular bars decreased in all the members except for bars W2-W4, W4-W6 and the corresponding symmetric bars. The decrease amounted to 10-49 [%] while the increase ranged from 35 to 49 [%] relative to the original values. It should also be noted how the stress changed depending on the cross-sectional area of the particular structural members. In the bottom flange members, a change in the cross-sectional area resulted in a relatively larger correction in the stress level (e.g., in member W1-W2 the cross-sectional area increased by 19.38% while the stress decreased by 24.77%). In the top flange the change in cross-sectional area by a certain percentage value was caused by a much smaller change in stress than in the case of the bottom flange (as regards stress, the percentages are also relatively lower than those for the cross sectional area). In some inner cross braces (members W3-W4, W4-W5) a reduction in cross sectional area resulted in a decrease in stress. This is due to the specific character of such frame structures and the role the cross brace bars play in them. An analysis of the calculation results for a frame with the continuous transfer of load from the deck is quite complicated. A change in the cross section of even one of the members causes a change in the magnitude of the internal forces (axial forces and bending moments). The cross braces ensure the proper distribution of the forces between the top flange and the bottom flange. As the cross sectional areas of members W3-W4 and W4-W5 were reduced by 35-40 [%], the stresses in them decreased by 10-15 [%] (Table 14). This shows that the design of such structures is quite complicated.

One can say that bridge structures should be optimally designed already at the initial stage of the design process. On the basis of his/her experience the designer selects the sizes of the cross sectional areas. Then they are verified and (after possible corrections) approved. All detailed calculations are done in the further stages of the design process. A similar approach was adopted in order to obtain the results presented in this section.

The change in the size of the cross-sectional areas in structure B1 resulted in an increase in the maximum stress in all its members, except for bars W6-W8, W4-W5 and the corresponding symmetric bars. The increase and the decrease amounted to respectively 3-220% and 4-19% relative to the initial values. Such large increases (not exceeding the design strength) indicate that the structure has been optimally designed. Similarly as for typical frame A, a stress correction relatively larger than the one in the cross-sectional area occurred in all the bottom flange bars (e.g., in bar W2-W4 the cross-sectional area was reduced by 9.77% and as a result the stress increased by 68.14%). Similarly as for typical frame A, in most of the top flange bars the obtained stress percentages are lower than the percentage change in the cross-sectional area, except for bars W1-W3 and W13-W15. When comparing percentage changes for the inner cross braces, the same regularity as in the case of the bottom flange becomes apparent, except for bars W4-W5 and W11-W12. In bars W4-W5 and W11-W12, whose cross-sectional areas were reduced, the maximum stresses decreased.

Table 15 Steel volumes in structures A and B1 for frame models with continuous load transfer from deck

	Steel volume [m^3]
	KS_{des}
A	15.60
B1	11.15
Gain [%]	27.61

A comparison of the percentage changes in maximum stress with the changes in cross-sectional area shows that increasing or reducing the cross-sectional areas produces better results in frame structure B1. The stress values change faster than in the case of typical frame A.

After the cross-sectional areas had been changed to “more optimum” (and so satisfying the strength conditions) the volumes of steel needed to construct the particular frames amounted to: 15.60 m^3 for structure A and 11.15 m^3 for B1. A considerable increase (by as much a 18.29%) in the amount of needed material is observed for the typical structure (A). In B1 the decrease amounts to 28.60%.

By comparing the adjusted steel volumes for the typical frame (A – 15.60 m^3) and the one obtained through the approximation of the optimization result one can find the optimal solution. The gain in consumed material for structure B1 in comparison with typical structure A amounted here to as much as 27.61% (Table 15). It is apparent that greater material benefits were obtained for the continuous frame with bending moments, axial forces and external moving loads taken into account. Of course, the steel volumes needed to construct trusses with cross sections KS1’ and KS2’ are markedly smaller (amounting to respectively 5.44 m^3 and 5.25 m^3) than in the case of the frame structure (11.15 m^3), but this cannot be the subject of comparison since different loading schemes (at midspan and a moving load on the span, respectively) were used in the two cases and in addition, the truss and the frame are two different structures.

Subsequently, the thicknesses of the normalized cross-sectional areas of the topology b1 elements (Fig. 3) were compared with those (denoted by KS_{des}) of the frame (in which moments, axial forces and moving loads were taken into account) and referred to cross sections KS1’ and KS2’ determined for the previously analyzed structure (Kutyłowski and Rasiak 2014) in which only tensile or compressive forces had been taken into account. A direct comparison of the normalized cross sectional areas KS1’ and KS2’ with KS_{des} is not recommended, even though the results for the reduced cross sections (Kutyłowski and Rasiak 2014) were obtained for the designed structure through normalization for the two types of outer dimension. The results are included in Ttable 16 and in Fig. 11 to provide a general reference.

In most cases, the normalized values of cross sections KS_{des} are close to the proportions obtained from topology b1, but in some bars (e.g., W1-W2, W7-W9, W1-W3) the differences are considerable. This may be due to mainly the different loading schemes (a concentrated force and a bridge load on the structure, respectively) adopted in the two versions of calculations and to problems mentioned earlier (Kutyłowski and Rasiak 2014), such as the loss of material in member W1-W3 and the way of reading off the length of the particular bars, dictated by, among other things, the size of the nodes.

It appears from the general comparison that as regards the inner cross braces, the proportions of cross-sectional areas KS_{des} are more similar than in the case of the truss with cross sections KS1’, KS2’with bending moments not taken into account. If the top flange and the bottom flange are taken into account, the sometimes considerable differences are due to the specificity of the loading schemes used in the two cases. However, one should treat such comparisons with reserve since they are based on structures obtained under too many different initial assumptions.

It follows from the above analysis that the outer dimensions of the cross sections should be properly matched: larger for the bottom flange and the support cross braces and smaller for the internal cross braces.

Table 16 Normalized cross-sectional areas of topology B1 members and corresponding bars of structure B1 for cross sections KS_{des}, KS1' and KS2'

ember	Topology b1	Cross sections with designed material amount (two outer dimensions of square cross sections) KS_{des}	KS1' (0,50 m)	KS2' (0,80 m)
W1-W2	**0.31**	0.67		**<u>0.52</u>**
W2-W4	**0.51**	0.45		**<u>0.58</u>**
W4-W7	**0.47**	0.49		**<u>0.77</u>**
W7-W9	**1.00**	0.49		**<u>0.87</u>**
W1-W3	**0.19**	0.60		**<u>0.65</u>**
W3-W6	**0.48**	0.78		**<u>0.78</u>**
W6-W8	**0.71**	1.00		**<u>1.00</u>**
W2-W3	**0.29**	0.28	**<u>0.36</u>**	
W3-W5	**0.16**	0.21	**<u>0.23</u>**	
W4-W5	**0.20**	0.44	**<u>0.49</u>**	
W5-W6	**0.38**	0.35	**<u>0.47</u>**	
W5-W7	**0.33**	0.17	**<u>0.18</u>**	
W6-W7	**0.09**	0.12	**<u>0.14</u>**	
W7-W8	**0.21**	0.17	**<u>0.28</u>**	
Steel vol. [m^3]	-	11.15	6.59	

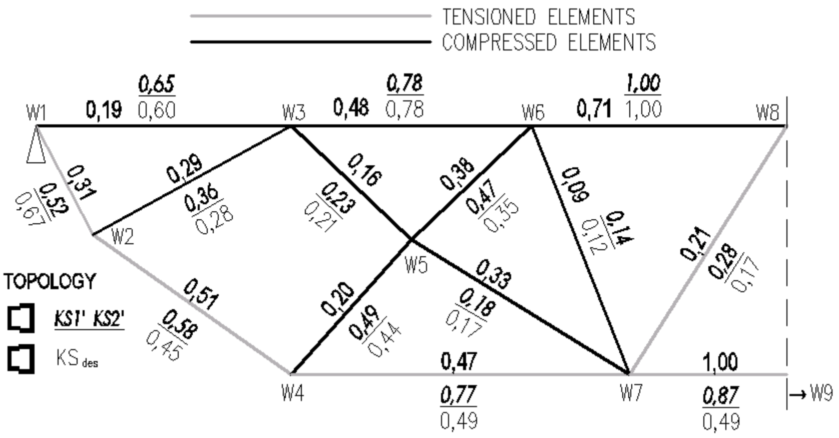


Fig. 11 Normalized cross-sectional areas of topology b1 members and corresponding bars of structure B1 for cross sections KS_{des} (moments, axial forces) and reduced cross sections KS1' and KS2' (axial forces)

From Table 16 and Fig. 11 one can conclude that cross sections KS_{des} are generally closer to the topology than cross sections KS1' and KS2'.

In the course of the design process, the cross sections of the girders of structure A and B1 were verified in accordance with the standard guidelines. From the designer's point of view it is essential to check the local stability of the cross sections of the frame's particular bars. In solid-wall cross sections (which the closed box sections used in the calculations are) the general condition of stability under bending and compression is checked as follows

$$\sigma \leq \frac{R}{m_{sn}} \tag{1}$$

It should be noted that the stresses in compressed bars W5-W7/W9-W11 and W7-W8/W8-W9 in structure B1 (Table 14) must be compared with the steel's design strength divided by coefficient m_{sn}=1.29 (200/1.29=155.04 MPa). The limit value is not exceeded and it can be said that the design strength of the steel in the above bars of the frame has been optimally used.

To sum up, the assumption of the continuous transfer of the standard bridge load from the deck to the girder brought considerable gains (in comparison with the typical structure) in the volume of steel for the frame approximated on the basis of the optimization result. This is of major consequence for the application of topology optimization in the design process. The possibilities here as regards the approximation of optimization results are quite extensive. Also the choice of cross-sectional areas and outer dimensions for the bars on the basis of the thickness of the topology bars is possible already at the conceptual stage whereby considerable design time savings can be made thanks to the quicker selection of the cross sections.

4. Checking proportions between cross sections of approximated truss bars and topology bars under identical loading

In order to check the relationship between the thicknesses of the particular topology elements and the cross-sectional areas (selected as a result of the design process) of the truss bars, calculations were performed for truss structures with only axial (tensile and compressive) forces taken into account, loaded as shown in Fig. 12. As regards its application point, direction and sense, the load is identical as the one used in the design area in the process of topology optimization. Concentrated force P amounts to 10000 kN (Fig. 12), which corresponds to a unit value of the force applied to the design area (Fig. 1). Since normalized values of the cross-sectional areas are analyzed, the differences in the nominal load values are of no importance.

The calculation results are compiled in the three tables (17, 18 and 19) below. The results are for square box sections with an outer dimension of 0.50 m. Column 2 shows the normalized cross-sectional areas for the topology (in the successive tables for respectively topology b1, b2 and b3). Column 3 shows the normalized cross-sectional areas for girders B1, B2 and B3 loaded as shown in Fig. 12. The values in brackets in this column are also for girders B1, B2 and B3, but under the standard load as shown in Fig. 13.

It is apparent that in the case of trusses B1 and B3, the highest cross-sectional area values in comparison with the previous calculations (with the standard load on the girder) are located in the same members of the topologies and the approximated structures. In the case of truss B 2, the cross-sectional area in bar W5-W7 changed from 0.80 to 0.94.

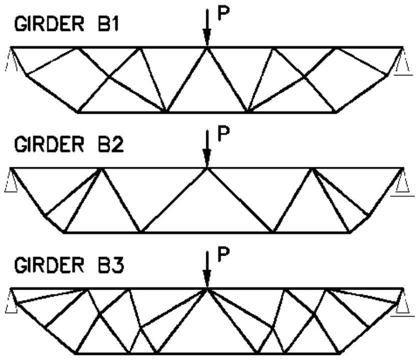


Fig. 12 Method of loading girders B1, B2 and B3 with unit force

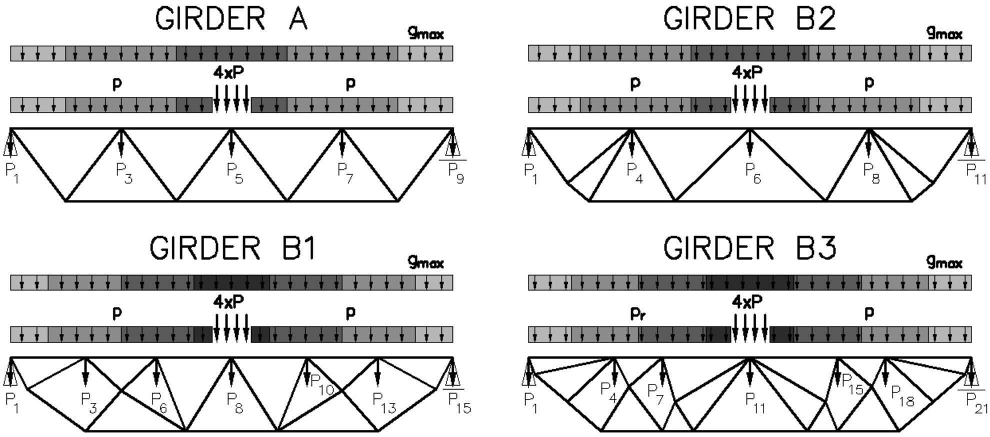


Fig. 13 Standard way of applying load to truss girders A, B1, B2 and B3

In Tables 17, 18 and 19 the positive and negative changes in the normalized box cross-sectional areas of the members under the unit force load (as shown in Fig. 12) relative to similar structures loaded in accordance with the standard and the scheme used in Kutyłowski and Rasiak (2014) (Fig. 13) are marked with respectively the plus sign and the minus sign.

The positive changes mean here an improvement in the agreement between the normalized cross-sectional areas of the members of the approximated trusses and the ones read off the topologies for the loading scheme shown in Fig. 12 and the scheme shown in Fig. 13, whereas the negative changes mean a deterioration in this agreement. For example, in table 17 for bar W1-W2 the relative cross section was determined here to be 0.38, which is closer to the figure obtained for the topology (0.31) than to the one (0.55) for dimensionally similar cross sections KS1' of truss B1 to which the bridge standard load was applied. Since 0.38 is closer to 0.31 than 0.55, the trend is positive.

Generally, in the case of truss B1 one can observe a tendency towards positive changes in most of the members, except for three cross braces. In one case, the proportions did not change. Also in the other analyzed structures (B2, B3) an improvement in most of the bars in comparison with the corresponding topologies was noted. However, in some members the proportions changed for the worse or there were no changes at all.

Table 17 Normalized cross-sectional areas of members of topology b1 and bar structure B1

Member	Topology b1	KS1' – square box section with outer dimension of 0.50 m
1	2	3
W1-W2	**0.31**	0.38 (0.55) +
W2-W4	**0.51**	0.42 (0.61) +
W4-W7	**0.47**	0.55 (0.81) +
W7-W9	**1.00**	1.00 (0.91) +
W1-W3	**0.19**	0.23 (0.37) +
W3-W6	**0.48**	0.52 (0.52)
W6-W8	**0.71**	0.96 (1.00) +
W2-W3	**0.29**	0.23 (0.37) +
W3-W5	**0.16**	0.14 (0.24) +
W4-W5	**0.20**	0.34 (0.51) +
W5-W6	**0.38**	0.37 (0.50) +
W5-W7	**0.33**	0.18 (0.19) –
W6-W7	**0.09**	0.25 (0.15) –
W7-W8	**0.21**	0.46 (0.30) –
Steel vol. [m^3]	-	**9.81**

( ) – values for dimensionally similar cross sections KS1' of truss B1 under the bridge standard load are given in brackets;
+ / – stand for respectively a positive/negative change relative to the topology values.

Table 18 Normalized cross-sectional areas of members of topology b2 and bar structure B2

Member	Topology b2	KS1' – square box section with outer dimension of 0.50 m
1	2	3
W1-W2	**0.57**	0.38 (0.48) –
W2-W3	**0.35**	0.38 (0.48) +
W3-W5	**0.81**	0.44 (0.55) –
W5-W7	**1.00**	0.94 (0.80) +
W1-W4	**0.29**	0.29 (0.37) +
W4-W6	**0.95**	1.00 (1.00)
W2-W4	**0.22**	0.17 (0.27)
W3-W4	**0.21**	0.33 (0.42) +
W4-W5	**0.32**	0.36 (0.18) +
W5-W6	**0.43**	0.56 (0.33) –
Steel vol. [m^3]	-	**11.09**

( ) – values for dimensionally similar cross sections KS1' of truss B2 under the bridge standard load are given in brackets;
+ / – stand for respectively a positive/negative change relative to the topology values.

By comparing the results for the standard load and the load exactly corresponding to the one used in the design area during optimization one can evaluate the effect of loading method simplification. In the case of such simplification, this effect is sometimes significant and the use of a better equivalent of the given loading scheme may lead to better results. On the other hand, by simplifying the loading scheme one can simplify the process of optimization and still obtain

Table 19 Normalized cross-sectional areas of members of topology b3 and bar structure B3

Member	Topology b3	KS1' – square box section with outer dimension of 0.50 m
1	2	3
W1-W2	**0.29**	0.35 (0.51) +
W2-W3	**0.56**	0.44 (0.64) –
W3-W5	**0.54**	0.44 (0.64)
W5-W8	**0.88**	0.51 (0.74) –
W8-W10	**0.74**	0.65 (0.77) –
W10-W12	**1.00**	1.00 (0.93) +
W1-W4	**0.19**	0.19 (0.34) +
W4-W7	**0.48**	0.39 (0.47) –
W7-W11	**0.87**	0.85 (1.00) +
W2-W4	**0.16**	0.25 (0.42) +
W3-W4	**0.16**	0.01 (0.01)
W4-W6	**0.16**	0.05 (0.28) +
W5-W6	**0.26**	0.37 (0.53) +
W6-W7	**0.22**	0.35 (0.47) +
W6-W8	**0.22**	0.15 (0.03) +
W7-W9	**0.08**	0.22 (0.12) –
W8-W9	**0.16**	0.18 (0.18)
W9-W10	**0.16**	0.29 (0.14) –
W9-W11	**0.17**	0.22 (0.23) +
W10-W11	**0.18**	0.37 (0.31) –
Steel vol. [m^3]	-	**9.97**

( ) – values for dimensionally similar cross sections KS1' of truss B3 under the bridge standard load are given in brackets;
+ / – stand for respectively a positive/negative change relative to the topology values.

relatively good results. This is of particular importance for bridge structures whose load capacity to a considerable degree depends on the weight of the moving vehicles. From among the factors having a bearing on the calculation results one should also mention the way in which the particular members of the topology are approximated. It should be noted that the use of identical loading schemes somewhat improved the agreement between the results, but the sensitivity to a change in loading was rather low in most of the bars, which means that the unit load is a good approximation of the real load. The differences between the normalized cross-sectional area values are acceptable. Thus it is possible to load the topology with a unit force to obtain a distribution of the thickness of the particular bars (cross-sectional areas), which will be similar to the one produced by the real load.

5. Conclusions

The possibilities of the practical application of the optimization algorithm in conceptual design have been demonstrated. Conceptual design is often the most important stage in the design process, during which the basic overall dimensions of the structure and the needed amounts of material are determined and the economics of individual solutions are assessed.

The analysis was carried out for steel bridge trusses under the standard load. The effect of buckling was taken into account and thoroughly examined. Comparisons were made for three different approximated trusses and a typical girder used in design practice. Also different types of cross sections were considered.

Structures more optimal than the typical ones can be obtained through optimization. When buckling is taken into account, the stress level in the individual elements may be lower in the approximated girders than in the typical girder designs.

Topology optimization can also be helpful in assessing the amount of material needed for the particular elements of the truss girder, whereby the design process can be significantly shortened.

Also buckling may have a smaller effect (than the one determined in this work) in trusses obtained by means of the optimization algorithm, due to the fact that larger nodes, and consequently shorter reduced buckling lengths of the bars, occur in the obtained topologies.

The above calculations were made for a linear case. In the case of geometrical and/or material nonlinearity , the optimal topology will be slightly different, since in such cases the structures are stiffer (Huang and Xie 2010), and the total amount of material can be slightly larger. In the linear case, the amount of material needed may be the same or even smaller when stronger inclusions are added (Kutyłowski 2009).

Trusses with a deck of the same type as the one considered in this work are designed today, but the way in which the load from the deck is applied to the truss is different: the deck is fixed to the truss's top flange in a continuous way whereby besides axial forces also bending moments and shearing forces occur in the truss. Appropriate calculations have been just completed and the obtained results are promising.

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Code 2 PN-82/S-10052, *Obiekty mostowe. Konstrukcje stalowe, Projektowanie*, (Bridges. Steel structures. Designing).

Code 3 PN-90/B-03200, *Konstrukcje stalowe, Obliczenia statyczne i projektowanie*, (Steel structures. Static calculations and designing).