Structural Engineering and Mechanics, *Vol. 50, No. 3 (2014) 323-347*
DOI: http://dx.doi.org/10.12989/sem.2014.50.3.323

Harmony search based, improved Particle Swarm Optimizer for minimum cost design of semi-rigid steel frames

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(Received July 20, 2013, Revised January 15, 2014, Accepted March 5, 2014)

Abstract. This paper proposes a Particle Swarm Optimization (PSO) algorithm, which is improved by making use of the Harmony Search (HS) approach and called HS-PSO algorithm. A computer code is developed for optimal sizing design of non-linear steel frames with various semi-rigid and rigid beam-to-column connections based on the HS-PSO algorithm. The developed code selects suitable sections for beams and columns, from a standard set of steel sections such as American Institute of Steel Construction (AISC) wide-flange W-shapes, such that the minimum total cost, which comprises total member plus connection costs, is obtained. Stress and displacement constraints of AISC-LRFD code together with the size constraints are imposed on the frame in the optimal design procedure. The nonlinear moment-rotation behavior of connections is modeled using the Frye-Morris polynomial model. Moreover, the P-Δ effects of beam-column members are taken into account in the non-linear structural analysis. Three benchmark design examples with several types of connections are presented and the results are compared with those of standard PSO and of other researches as well. The comparison shows that the proposed HS-PSO algorithm performs better both than the PSO and the Big Bang-Big Crunch (BB-BC) methods.

Keywords: harmony search; Particle Swarm Optimization; semi-rigid connections; steel frames; optimal sizing design

1. Introduction

In the analysis and design of steel frames, in order to model the actual behavior of beam-to-column connections, it is convenient to use one of the two simplified extremes of fully rigid and perfectly pinned behavior. In spite of its simplicity, however, such a modeling cannot lead to a realistic prediction of response of a structure. This is due to the fact that, these connections posses some flexural stiffness between two extremes, i.e., are semi-rigid connections. This semi-rigid behavior is nonlinear in nature, as well. Consequently, to take into account the effects of the actual behavior of the beam to column connections on the response of a frame, in its analysis and design the moment-rotation behavior of connections have to be modeled by using suitable relationships.

Various models including linear, polynomial, cubic B spline, power and exponential models are proposed based on $M-\theta$ relations for several connections (Frye and Morris 1975, Abdalla and Chen 1995). Moreover, analysis and design of steel frames with semi-rigid connections have been

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

extensively investigated (Kaveh and Moez 2008, Ihaddoudène *et al*. 2009, Gorgun 2013, Valipour and Bradford 2013). As a more convenient activity of research in the field of structural optimization, optimal design of steel frames with semi-rigid connections has also been investigated by means of mathematical programming techniques (Alsalloum and Almusallam 1995, Simoes 1996) and of meta-heuristics (Kameshki and Saka 2003, Hayalioglu and Degertekin 2005, 2010, Rafiee *et al*. 2013).

Steel specifications such as British Standard, BS5950 (1990), Eurocode3 (1992) and American Institute of steel construction (AISC) have investigated the semi-rigid behavior of beam to column connections. AISC-ASD specification (2010) describes three types of steel constructions: rigid, simple (unrestrained) and semi-rigid (partially restrained) framing, whereas, in AISC-LRFD (2010) two types of steel construction namely FR (fully restrained) and PR (partially restrained) types are described. The behavior of the construction type PR, which is considered to be semi-rigid, is described on the basis of experimental and numerical studies.

In structural optimization, a number of efficient meta-heuristic optimization algorithms mimicking natural phenomena and physical processes, have been applied. One of the well-known optimization algorithms, which has already received a lot of attention, is the particle swarm optimization (PSO) algorithm. The PSO was proposed by Kennedy and Eberhart in 1995 and is based on the simulation of a simplified social model. PSO algorithm was proved to be of high computation efficiency, easy implementation and stable convergence. Recently, Groenwold and coworkers (2002, 2003) used PSO for optimal size and shape design of truss structures. Later, Perez and Behdinan (2007) applied improved PSO for optimal design of truss size. Then, Li *et al*. (2007, 2009) applied a heuristic particle swarm optimization for optimum design of pin-connected structures and truss structures with discrete variables. Lately, a two-stage particle swarm optimization was utilized by Luh and Lin (2011) to solve truss-structure optimization problem achieving minimum weight objective under stress, deflection, and kinematic stability constraints. Furthermore, PSO was used for optimum design of unbraced steel frames by Doğan and Saka (2012). As a combined use of meta-heuristics, in 2012, Kaveh and Talatahari developed a hybrid CSS and PSO algorithm for optimal design of structures.

Another famous optimization algorithm, namely, harmony search (HS) was proposed by Geem *et al*. in 2001, inspiring the performance process of natural music. The use of HS in searching for solutions to various optimization problems has been resulted in effective results (Lee and Geem 2005, Geem 2007, Cheng *et al*. 2008, Mun and Geem 2009). Together with these studies, HS has also been utilized to optimize the design of structures in a number of researches and the results demonstrated its robustness. Among these work, those in which the main purpose is to minimize the weight of the structure can be summarized as follows: Degertekin (2008), Saka (2009), Saka and Erdal (2009). In addition, Degertekin and Hayalioglu (2010) studied the minimum cost design of steel frames by developing an algorithm on the basis of harmony search.

In this study a Particle Swarm Optimization (PSO) algorithm is proposed, which is improved by making use of the Harmony Search (HS) approach and called HS-PSO algorithm. In the HS-PSO algorithm, the harmony memory (HM) is created and improved using particle swarm optimization. On the other hand, the new off-springs generated by PSO, which can be considered as new improvised harmonies, are improved through the concepts used in harmony search.

A computer code is developed based on HS-PSO, for optimal sizing design of steel frames with various semi-rigid and rigid connections. The developed code selects suitable sections for beams and columns such that the total member plus connection cost of the frame, is minimized, while the stress and displacement constraints of AISC-LRFD code together with the size constraints are imposed on the frame in the optimal design procedure. The *P*-Δ effects of beam-column members are taken into account in the non-linear structural analysis. The behavior of semi-rigid beam to column connections are assumed to be defined by the Frye-Morris polynomial model (Frye and Morris 1975), whereas, the column bases are supposed to be rigid. Three benchmark design examples with several types of connections are presented and the results are compared with those of standard PSO. The results show the efficiency of the HS-PSO algorithm proposed herein in comparison with the standard PSO. In addition, the comparison of results of this work with those obtained by Rafiee *et al*. (2013) shows that the HS-PSO performs better than Big Bang-Big Crunch (BB-BC) method in all cases.

2. Optimization algorithms

In order to yield to an illuminated representation of the proposed algorithm, namely, the HS-PSO algorithm, in the first two subsections of this part of the paper, we have a brief review on particle swarm optimization and harmony search algorithms, respectively. Then in last subsection the HS-PSO algorithm is represented in detail.

2.1 Particle Swarm Optimization (PSO)

In 1995, Kennedy and Eberhart first introduced the particle swarm optimization (PSO) method, which is derived from the social-psychological theory, and has been found to be robust in structural optimization problems. PSO is a kind of population-based optimization algorithm. The population of PSO is called a swarm (or flock) while each individual in the population of PSO is called a particle (or bird). In PSO each particle is treated as a valueless particle in *n*-dimensional search space, and keeps track of its coordinates in the problem space associated with the best solution (Kennedy and Eberhart, 1995). This evaluating value is called *pbest*. Another best value that is tracked by the global version of the particle swarm optimizer is the overall best value, and its location, obtained so far by any particle in the group, this is called *gbest*. The PSO concept consists of changing the velocity of each particle toward its *pbest* and *gbest* locations.

In a flock of *m* particles or birds, the *i*th particle is represented as $\mathbf{x}^i = \left(x_1^i, x_2^i, ..., x_n^i\right)$ in the *n*-dimensional search space. The best previous position of this particle is recorded and represented as $\mathbf{pbest}^i = \left(pbest_1^i, pbest_2^i, ..., pbest_n^i\right)$. The index of best particle among all the particles in the flock is represented by **gbest**. Furthermore, the rate of the change in position i.e. the velocity for particle *i* is represented as $\mathbf{v}^i = \left(v_1^i, v_2^i, ..., v_n^i\right)$. Kennedy and Eberhart (1995) originally proposed that the position $\mathbf{x}^i$ of the *i*th particle be updated as

$$\mathbf{x}_{t+1}^i = \mathbf{x}_t^i + \mathbf{v}_{t+1}^i \tag{1}$$

whereas, the term $\mathbf{v}^i$ is updated as

$$\mathbf{v}_{t+1}^i = \omega \mathbf{v}_t^i + c_1 r_1 \left(\mathbf{pbest}_t^i - \mathbf{x}_t^i\right) + c_2 r_2 \left(\mathbf{gbest}_t - \mathbf{x}_t^i\right) \tag{2}$$

where, the subscript *t* denotes a unit integer for pseudo-time increment, or in other words, is the pointer of iterations (generations). $\mathbf{pbest}_t^i$ and $\mathbf{gbest}_t$ are, respectively, the best ever position

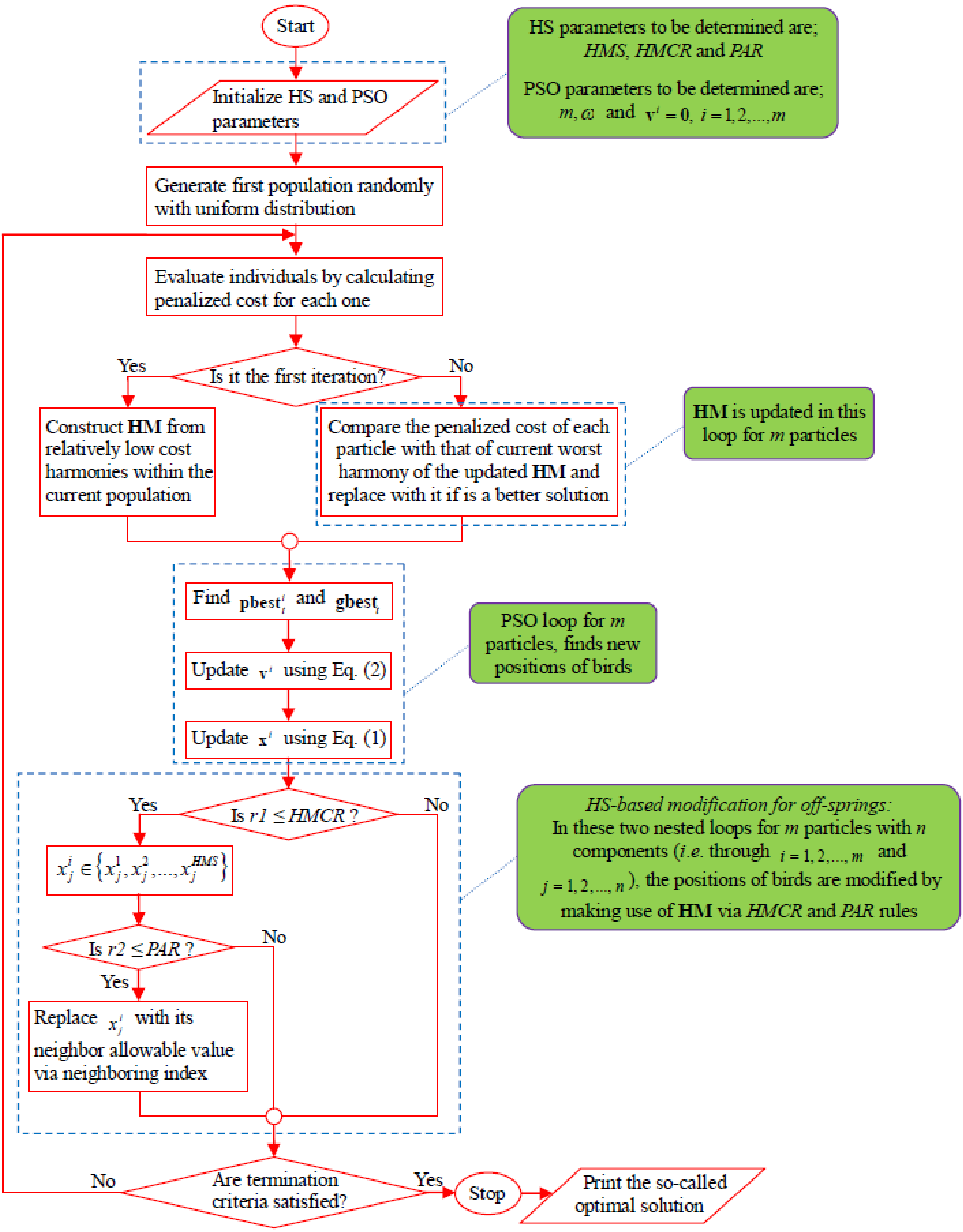


Fig. 1 Flow chart of the HS-PSO algorithm

of particle i and the global best position in the swarm associated with time t. According to the common characteristic of meta-heuristic methods i.e., the randomness, the r_1 and r_2 correspond to uniform random numbers in the interval 0 to1. It should be noted that the initial velocity of each particle equals zero.

Moreover, as it is initially proposed by Kennedy and Eberhart, the multipliers c_1 and c_2 should be selected to be equal to 2. This is because it allows a mean of unity when multiplied by r_1 or r_2. As a consequence of use of such cognitive and social scaling factors, birds overfly the target half the time.

In Eq. (2), the previous velocity of the particle $\mathbf{v}_t^i$, is multiplied by ω, which is introduced by Shi and Eberhart (1998) and called inertia term. They suggested that ω be selected from range $0.8< \omega<1.4$. In its original form, however, we have $\omega=1$.

In an optimal sizing design problem, the position of each bird is represented by the design variables $\mathbf{x}$, while the velocity of each bird $\mathbf{v}$ influences the incremental change in the position of each bird, and hence the design variables.

2.2 Harmony Search (HS)

Harmony search is developed based on the analogy between the performance process of natural music and seeking for solutions of optimum design problems. The method consists of following steps:

Step 1. *Initialize the HS parameters* by determining harmony memory size (*HMS*), harmony memory consideration rate (*HMCR*), pitch adjusting rate (*PAR*).

Step 2. *Initialize harmony memory (HM)* by filling **HM** matrix with randomly generated designs as the size of the harmony memory (*HMS*). *HMS* is similar to the total number of individuals in the population matrix of the genetic algorithm.

$$\mathbf{HM} = \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^{HMS} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_n^1 \\ x_1^2 & x_2^2 & \cdots & x_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_n^{HMS} \end{bmatrix} \tag{3}$$

As it is evident from Eq. (3), each row in the HM matrix represents a steel design (in our problem) denoted by the vector $\mathbf{x}^i = \left(x_1^i, x_2^i, \ldots, x_n^i\right)$, where $i=1,2,\ldots,HMS$ and the subscript n corresponds to the n-dimensional search space.

Step 3. *Improvise a new harmony* by making use of three rules, HM consideration, pitch adjustment and random generation. The HM consideration can be stated by Eq. (4), as follows

$$\begin{cases} x_j^{new\ harmony} \in \left\{x_j^1, x_j^2, \ldots, x_j^{HMS}\right\} & r1 \le HMCR \\ x_j^{new\ harmony} \in \mathbf{X}_j^{allowable} & r1 > HMCR \end{cases} \tag{4}$$

where $r1$ is a random number uniformly distributed over the interval [0,1] and is generated for $x_j^{new\ harmony}$. This equation implies that, with the probability of *HMCR* the j-th component of the $\mathbf{x}^{new\ harmony}$ vector is selected randomly from the j-th column of **HM** matrix, while, it is selected randomly from the set of allowable values for the *j*-th design variable, with the probability of 1-*HMCR*.

Once the *j*-th component of the new harmony is obtained using the above-mentioned procedure, the pitch adjusting rule is applied for the case where the component is chosen from HM, i.e., when $r1 \leq HMCR$ holds, which can be formulated as

$$\text{pitch adjusting decision for } x_j^{new\ harmony} = \begin{cases} Yes & r2 \leq PAR \\ No & r2 > PAR \end{cases} \tag{5}$$

If the pitch adjustment decision for the variable under consideration is *Yes*, then $x_j^{new\ harmony}$ is replaced with its neighbor allowable value. Otherwise, the current value is not changed. A random number $r2$ uniformly distributed over the interval [0, 1] is generated for $x_j^{new\ harmony}$. The selection of neighbor value is determined by neighboring index, which is usually selected to be equal to ±1 with equal probability.

Step 4. *Update the harmony memory* by comparing the new harmony with the worst design in the **HM**. If the new harmony is better, the new design is included in the **HM** and the existing worst harmony is excluded from the **HM**.

Step 5. Repeat Steps 3 and 4 until the termination criteria are satisfied.

Supplementary details about harmony search algorithm can be found in Lee and Geem (2005).

2.3 Proposed algorithm: Harmony Search based Particle Swarm Optimizer (HS-PSO)

In the HS-PSO algorithm, presented herein, the harmony memory (HM) is created and improved using particle swarm optimization. On the other hand, the new off-springs generated by PSO, which can be considered as new improvised harmonies, are improved through the concepts used in harmony search. In other words, in HS-PSO the random generation rule, which is used in HS, is removed and instead the PSO is applied, and at the same time, the new position of each bird in the flock is changed by making use of HM consideration rate and pitch adjustment rules.

In a standard PSO algorithm, when a particle sees a location which is better than its current *pbest*, interchanges them. This change means that thereafter that *pbest* plays no role in optimization process, whereas that position may be better than the *pbest* values of other particles up to that iteration. To overcome this drawback, in HS-PSO algorithm, a memory of best positions (harmonies) is considered. On the other hand, in a standard HS algorithm, harmony memory is constructed in a random manner with no proper strategy to move the individuals toward the feasible domain. It seems that, the particle swarm concept may be a proper strategy, which can result in a better memory of harmonies within a relatively small number of iterations.

That is to say, we have a swarm of harmonies which flies toward better solutions by adopting the strategy of moving toward the best ever seen position of each bird and that of all birds from PSO, together with the scheme of considering a harmony memory and pitch adjustment from HS.

The HS-PSO algorithm can be explained by the flowchart of Fig. 1. This figure is self-explanatory; however, some points should be noted, as follows

- In hybrid HS-PSO algorithm $HMS \leq m$ must be satisfied.
- The random numbers $r1$ and $r2$ used in Eq. (2) is generated independent from those needed for HS-based modification for off-springs.
- The random numbers $r1$ and $r2$ used for HS-based modification for off-springs are generated

for each component of each particle independently.

3. Problem formulation of minimum cost design of semi-rigid steel frames

In this study, the goal of the optimization problem is to minimize the cost of steel frame design. The total cost of a steel frame with semi-rigid beam to column connections, considering member and connection costs, is defined by Xu and Grierson (1993) as follows

$$Z(\mathbf{x}) = \sum_{i=1}^{n} W_i \, A_i + \sum_{i=1}^{NB} \sum_{j=1}^{2} \left(\beta_{ij} R_{ij} + \beta_{ij}^{0} \right) \tag{6}$$

where A_i and W_i are the ith member cross-section area and weight coefficient, respectively (W_i= material density × member length), R_{ij} and β_{ij} are the connection rotational stiffness and cost coefficient, and β_{ij}^{0} is the cost of a pinned connection having zero rotational stiffness. The j-subscripts in Eq. (6) correspond to two ends of the semi-rigid beam member and n and NB denote the total number of members (n-dimensional search space) and beams in a frame, respectively.

The values of β_{ij} for two ends of a semi-rigid beam member are assumed to be equal and calculated as

$$\beta_i = \frac{0.225 \, W_i \, A_i}{S_i} \tag{7}$$

where S_i is rotational stiffness of a connection which is a estimated value depending on the stiffness of the connection, equal for the both ends of a beam and lies in the range 2.26×10^{5} kN.mm/rad to 5.65×10^{8} kN.mm/rad as it is suggested by Xu and Grierson (1993) and the equal value for β_{i1}^{0} and β_{i2}^{0} are accepted to be equal to

$$\beta_i^{0} = 0.125 \, W_i \, A_i \tag{8}$$

As it is usually involved in an optimization problem, some constraints should be imposed on the problem during the optimization procedure, which divide the search space into feasible and infeasible domains. The optimum design problem of a steel frame with semi-rigid connections has the following constraints:

a) The strength constraints of AISC-LRFD (2001) considering the interaction of bending moment and axial force can be formulated in the normalized form, for ith member of the frame, as follows

$$V_i^{IER} = \begin{cases} \left(\dfrac{P_u}{\phi P_n} \right)_i + \dfrac{8}{9} \left(\dfrac{M_{ux}}{\phi_b M_{nx}} \right)_i - 1.0 & \dfrac{P_u}{\phi P_n} \geq 0.2 \\ \dfrac{1}{2} \left(\dfrac{P_u}{\phi P_n} \right)_i + \left(\dfrac{M_{ux}}{\phi_b M_{nx}} \right)_i - 1.0 & \dfrac{P_u}{\phi P_n} < 0.2 \end{cases} \tag{9}$$

where P_u and P_n are required and nominal strength of a member (tensile or compressive), respectively and ϕ is resistance reduction factor, which is equal to 0.9 for the member in tension and 0.85 for compressive ones. Moreover, M_{ux} and M_{nx} are notations for required and nominal flexural strength of the member about its major axis, respectively and reduction factor that corresponds to bending is denoted by ϕ_b (equal to 0.9). The nominal strength of a compressive member is calculated based on AISC-LRFD (2001) as follows

$$P_n = AF_{cr} \tag{10}$$

$$F_{cr} = \begin{cases} 0.658^{\lambda_c^2} F_y & \lambda_c \le 1.5 \\ \dfrac{0.877}{\lambda_c^2} F_y & \lambda_c > 1.5 \end{cases} \tag{11}$$

$$\lambda_c = \frac{K\,L}{r\,\pi}\sqrt{\frac{F_y}{E}} \tag{12}$$

where A is cross-sectional area; F_y is yield stress; and E is modulus of elasticity of steel member. L and r are the member length and radius of gyration, respectively. The effective length factor, which is denoted by K in Eq. (12), is needed in stability evaluation of the columns in the frame. K-factor of columns in an unbraced semi-rigid frame is calculated following the relations proposed by Kishi *et al*. (1997).

b) The displacement normalized constraints including the constraints of inter-storey drift and top storey sway can be formulated in general form of

$$V_j^d = \frac{\left|\delta_j\right|}{\delta_j^u} - 1.0, \quad j = 1, 2, \ldots g \tag{13}$$

where δ_j is the displacement of the jth restricted displacements among the total number of g and δ_j^u is its allowable upper bound limit determined by the code of practice.

c) The other group of constraints imposed on the optimization problem in this study arises from the size adaptations of beams and columns relative to each other. This group consists of two constructional considerations: one consideration implies that flange width of a beam must be smaller than the same value for column in all joints, whereas, the other one considers the fact that the column of each storey cannot be smaller in depth compared to its above storey column. These two constraints can be formulated, respectively, as

$$V_p = \frac{b_f^{bp}}{b_f^{cp}} - 1.0, \quad p = 1, 2, \ldots nj \tag{14}$$

$$V_q = \frac{d_c^{uq}}{d_c^{lq}} - 1.0, \quad q = 1, 2, \ldots nc \tag{15}$$

where b_f^{bp} and b_f^{cp} are the value of flange width for beam and column in node number p among the total number of nj nodes, respectively (nj is the total number of nodes of frame except the supports). The d_c^{uq} and d_c^{lq} are notations for depths of column sections of upper and lower floor in a node, respectively. *nc* is the total number of columns in the frame excluding ones for first storey.

The optimum design problem, considered in the present work, is a constrained problem; we can transform it into an unconstrained one using a penalty function. Here we use the penalty function suggested by Rajeev and Krishnamoorthy (1992), so the objective function of the problem can be computed as

$$\varphi(\mathbf{x}) = Z(\mathbf{x})\left[1 + C\left(\sum_{i=1}^{n} v_i^{IER} + \sum_{j=1}^{g} v_j^{d} + \sum_{p=1}^{nj} v_p + \sum_{q=1}^{nc} v_q\right)\right] \tag{16}$$

where $\boldsymbol{Z}(\mathbf{x})$ is calculated by Eq. (6); C is a penalty constant, which is equal to 10 in this work; v_i^{IER}, v_j^d, v_p and v_q are the violations of normalized interaction equation ratio, displacement, and size considerations for beams and columns, respectively and are computed using Eqs. (17)-(20).

$$v_i^{IER} = \max\left(0, V_i^{IER}\right) \tag{17}$$

$$v_j^{d} = \max\left(0, V_j^{d}\right) \tag{18}$$

$$v_p = \max\left(0, V_p\right) \tag{19}$$

$$v_q = \max\left(0, V_q\right) \tag{20}$$

In this work, two termination criteria are used to stop the optimal design process. The first criterion stops the algorithm when a predetermined number of iterations (generations) are performed, whereas, the second one terminates the process before reaching the maximum iteration number, if lighter frame is not found during a specified number of successive generations. If one of these criteria is satisfied, the algorithm is terminated and the so-called optimal solution is printed.

4. Nonlinear analysis of steel frames with semi-rigid beam to column connections

In a structural optimization problem, each structural design (individual) is evaluated through its analysis, which leads to structural response and makes it possible to evaluate the penalty function. On the other hand, it is obvious that the actual complex behavior of a structure must be simplified for analysis by feasible modeling of it. Among the numerous experimental and numerical studies on the modeling of semi-rigid beam-to-column connections, the model proposed by Frye and Morris (1975) is adopted for use in this work, due to its easy-to-implement characteristic. This odd-power polynomial model is reasonably good for simulation of the nonlinear $M-\theta$ behavior of connections and has been presented as

$$\theta = c_1(\kappa M) + c_2(\kappa M)^3 + c_3(\kappa M)^5 \tag{21}$$

where θ is the connection rotation and M denotes the moment acting on the connection. The

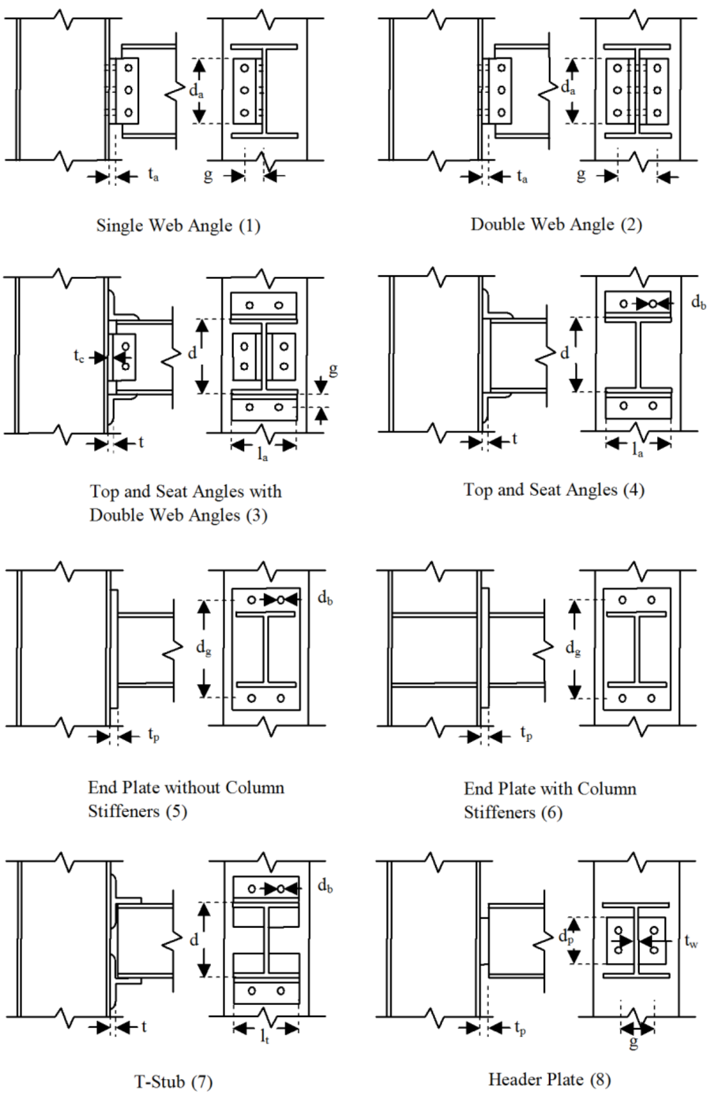


Fig. 2 Semi-rigid beam-to-column connection types

parameter κ is the standardization factor determined by the connection type and geometry, and c_1, c_2 and c_3 are curve-fitting constants obtained by using the method of least squares.

For several types of beam to column connections, which are shown in Fig. 2, the values of the constants c_1, c_2 and c_3 and the parameter κ for each type, are illustrated in Table 1 (Faella *et al*. 2000). The schematic $M-\theta$ curves for these eight types of connections are drawn in Fig. 3 according to Chen *et al*. (1996).

In the analysis procedure of the steel frames with semi-rigid beam to column connections, we consider the nonlinear $M-\theta$ behavior of semi-rigid connections, and the geometrical nonlinearity of beam-column members.

Table 1 The Curve fitting constants and standardization parameters for Frye-Morris polynomial model

Connection type	Curve fitting constants			Standardization parameter, (κ)
	c_1	c_2	c_3	
1	4.28×10^{-3}	1.45×10^{-9}	1.51×10^{-16}	$\kappa = d_a^{-2.4}\, t_a^{-1.81}\, g^{0.15}$
2	3.66×10^{-4}	1.15×10^{-6}	4.57×10^{-8}	$\kappa = d_a^{-2.4}\, t_a^{-1.81}\, g^{0.15}$
3	2.23×10^{-5}	1.85×10^{-9}	3.19×10^{-12}	$\kappa = d^{-1.287}\, t^{-1.128}\, t_c^{-0.415}\, l_a^{-0.694}\, g^{1.35}$
4	8.46×10^{-4}	1.01×10^{-4}	1.24×10^{-8}	$\kappa = d^{-1.5}\, t^{-0.5}\, l_a^{-0.7}\, d_b^{-1.5}$
5	1.83×10^{-3}	1.04×10^{-4}	6.38×10^{-6}	$\kappa = d_g^{-2.4}\, t_p^{-0.4}\, d_b^{-1.5}$
6	1.79×10^{-3}	1.76×10^{-4}	2.04×10^{-4}	$\kappa = d_g^{-2.4}\, t_p^{-0.6}$
7	2.10×10^{-4}	6.20×10^{-6}	-7.60×10^{-9}	$\kappa = d^{-1.5}\, t^{-0.5}\, l_t^{-0.7}\, d_b^{-1.1}$
8	5.10×10^{-5}	6.20×10^{-10}	2.40×10^{-13}	$\kappa = d_p^{-2.3}\, t_p^{-1.6}\, t_w^{-0.5}\, g^{1.6}$

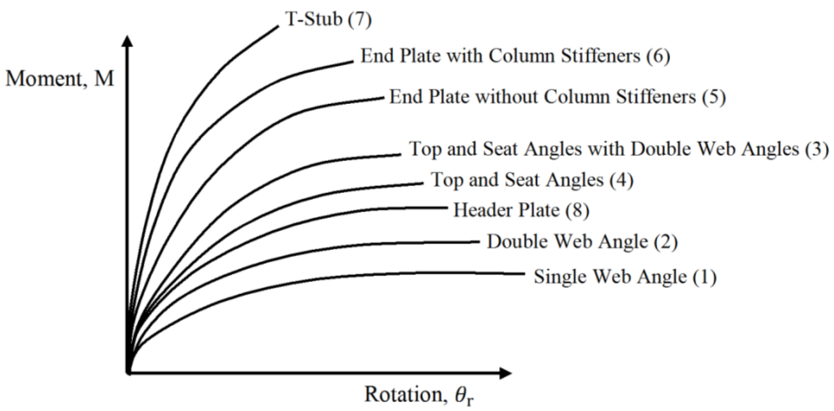


Fig. 3 Moment-Rotation curves of semi-rigid connection types

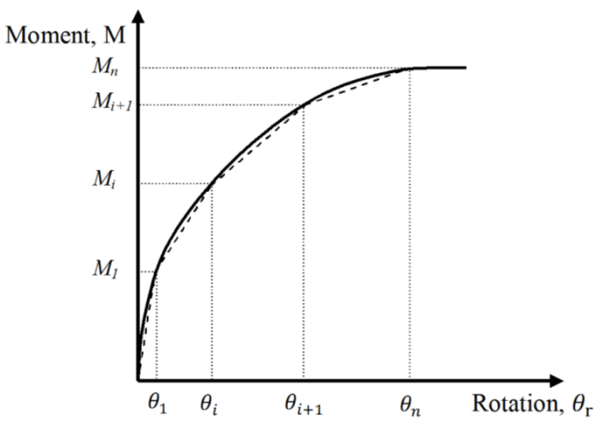


Fig. 4 Secant stiffness values of load increments

Table 2 The fixed connection size parameters and adopted rotational stiffness values

Connection type	Fixed connection size parameters (cm)	S_i values in Eq. (7) (kN.mm/rad)
1	t_a=2.54, g=11.43	85×10^6
2	t_a=2.858, g=25.4	113×10^6
3	t=2.54, t_c=2.54, g=11.43	282×10^6
4	t=2.54, d_b=2.858	226×10^6
5	t_p=2.54, d_b=2.858	339×10^6
6	t_p=2.54	395×10^6
7	t=3.81, d_b=2.858	452×10^6
8	t_p=2.54, g=25.4	141×10^6

In this study, the displacements method is used to analyze the structure, wherein, the stiffness matrix of the structure is constructed through assembling of the stiffness matrices of members in the global coordinates. In order to consider the P-Δ effects into account in the analyses of frames, an incremental approach is applied, such that, in each increment the stiffness matrices are updated using most recently computed axial force values for beam-column elements, in an iterative procedure until the convergence is achieved. Moreover, the secant stiffness approach is applied to consider the semi-rigid connection stiffness nonlinearity of beam members. The connection secant stiffness values corresponding to all load increments are shown in Fig. 4. In each set of iterations convergence criterion is controlled by comparing of the difference between end forces of members with applied incremental loads so that to be smaller than a determined tolerance. A convergent solution of a load increment forms an initial estimate for the next iteration, and the iterative process continues until all load increments are considered. The solutions for all load increments are accumulated to obtain a total nonlinear response.

5. Numerical examples

In this study three steel frames with semi-rigid beam-to-column connections are solved. These examples have been solved by Rafiee *et al*. (2013) using Big Bang-Big Crunch algorithm. Following this reference, in these examples the A36 steel grade is used for all of the members and the sections for these members are selected among a total number of 273 standard sections of American Institute of Steel Construction wide flange W shapes. The first example is a nine-storey single-bay frame (as a small size frame); the second frame consists of ten stories with four bays (as a median frame) and the third example has 24 stories and three bays (as a large scale frame).

In each example the eight types of connections as shown in Fig. 2 are used as semi-rigid beam-to-column connections. In the present work, due to simplification of the problem, some of the connection size parameter values required in Frye-Morris polynomial model of $M-\theta$ curve is considered to be fixed during the optimum design procedure. These fixed values are selected according to Table 2, whereas, the values of angle length, beam height, the vertical distance between bolt groups, web thickness of beam are calculated based on dimensions of W-shape section assigned to the beam member throughout the optimal design procedure. The last column of Table 2, gives the estimated rotational stiffness values, S_i for each type of semi-rigid connections. These are the case for all of the design examples considered herein.

On the other hand, as it is evident from Fig. 1, in the HS-PSO algorithm for a *HMCR* value of zero the algorithm is simplified to the PSO and therefore one can see the *HMCR* as a *HSCR* parameter, which determines the contribution of harmony search (HS) scheme in the HS-PSO. In this study for the minimum cost design of steel frames, the *PAR* value is chosen to be 0.4 for all the examples according to (Degertekin and Hayalioglu, 2010), whereas, to determine the optimum value of *HMCR*, the first example is solved for a range of values of *HMCR* from 0.1 to 0.8 with a step of 0.1, and the optimal *HMCR* is found to be equal to 0.7. This value is used for the rest of the examples.

5.1 Nine-storey, single-bay frame

The geometry, member grouping and the service loading conditions for the nine-storey, one-bay frame are illustrated in Fig. 5. The applied loads W, W_1 and W_2 are equal to 17.8 kN, 27.14 kN/m and 24.51 kN/m, respectively. In order to impose the fabrication conditions on the construction of the frame, the 27 members of this frame are separated to seven groups of members. Table 3 presents the optimal designs developed by the HS-PSO algorithm for the 9-storey frame. In order to investigate the effect of column sizes on optimal designs, this example is solved again with the column sections limited to W14 sections. The optimal designs developed by the HS-PSO algorithm

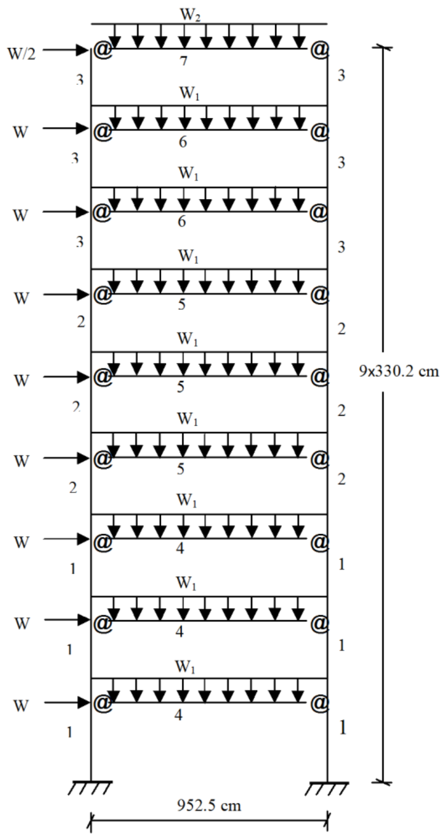


Fig. 5 Nine-storey, single-bay frame

Table 3 The optimal *W*-shape sections of nine-storey, single-bay frame via HS-PSO

Group no.	Semi-rigid connection types								Rigid connection
	1	2	3	4	5	6	7	8	
1	40×215	40×149	40×149	40×149	40×149	33×118	30×90	44×230	24×62
2	30×90	24×62	24×68	24×76	24×55	24×55	24×55	40×149	24×55
3	24×68	12×40	14×34	16×36	14×34	14×30	14×30	24×62	14×30
4	21×44	21×44	21×44	21×44	21×44	21×48	21×55	21×44	24×55
5	30×90	21×50	24×55	24×68	21×50	24×55	24×55	24×62	21×50
6	21×44	16×45	21×44	21×44	18×46	21×44	21×44	24×55	21×44
7	21×50	18×35	18×40	18×40	18×40	18×35	18×35	21×44	18×35

Table 4 The optimal *W*-shape sections of nine-storey, single-bay frame via HS-PSO when columns are *W*14

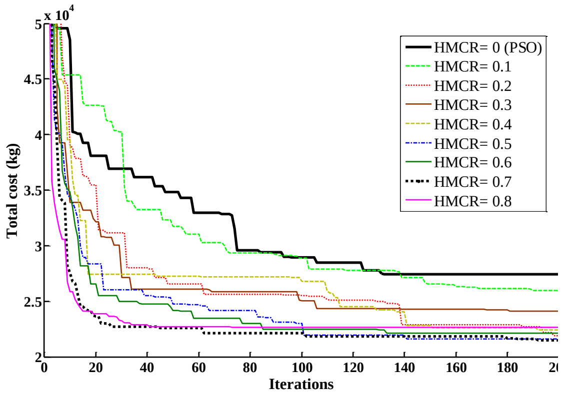
Group no.	Semi-rigid connection types								Rigid connection
	1	2	3	4	5	6	7	8	
1	14×426	14×109	14×90	14×90	14×99	14×99	14×90	14×176	14×68
2	14×145	14×74	14×61	14×90	14×68	14×74	14×61	14×90	14×48
3	14×48	14×30	14×30	14×34	14×30	14×30	14×43	14×30	14×30
4	30×90	24×68	30×90	30×90	24×68	24×68	24×68	33×118	24×68
5	36×135	24×55	24×68	24×68	24×55	24×55	24×55	36×135	24×55
6	21×44	18×46	21×44	21×44	18×46	18×46	16×45	21×44	21×44
7	18×40	18×35	18×35	18×40	18×35	18×35	16×40	21×44	18×35

in this case are listed in Table 4. The global sway corresponding to the roof level is limited to a maximum value of 154 mm.

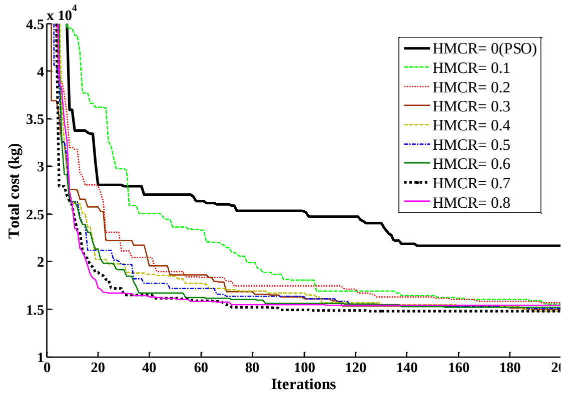
According to Xu and Grierson (1993), the cost of a steel member with *W*-section is increased by approximately 70% if its end connections are rigid jointed, so the total cost of the rigidly-connected members of the frame is obtained multiplying the weight values of those members by 1.70. Fig. 6 shows the convergence history for the optimum design of this frame with connection types 1 and 7. As it is clear from this figure, an *HMCR* value of 0.7 results in better results, i.e., in lighter frames with good convergence.

The minimum cost values presented in Table 5 shows that in the most cases PSO leads to low cost frames compared to BB-BC, however, in the cases of rigid connection and semi-rigid connection type 3 the BB-BC gives frames with lower costs. In addition, HS-PSO algorithm designs the frames so that their costs are lower both than those of BB-BC and of PSO algorithms for all connections. This reduction in cost varies in a range of 8% (for connection type 3) through 72% (for type 7) with an average of 44%, if one compares HS-PSO with BB-BC. The comparison of HS-PSO with PSO, however, gives the reduction percents of 12% (for connection types 3 and rigid connection) and 32% (for type 7) with an average value of 18%.

Moreover, it is clear from the results that, limiting the columns to *W*14 sections will increase the frame total cost, weight and top storey sway values; but the percentage of increase in total cost is bigger than that of weight, this is due to the increase in connection cost to overcome the lateral loads. This fact is evident from Fig. 7 as well, in which the mean used / capacity ratio is drawn versus the connection types. This figure depicts that limiting the column sizes to *W*14 sections changes the optimal solution such that in which the storey drift values are dominant with compared to member stress values.



(a) Connection type 1



(b) Connection type 7

Fig. 6 The convergence history of nine-storey, single-bay frame

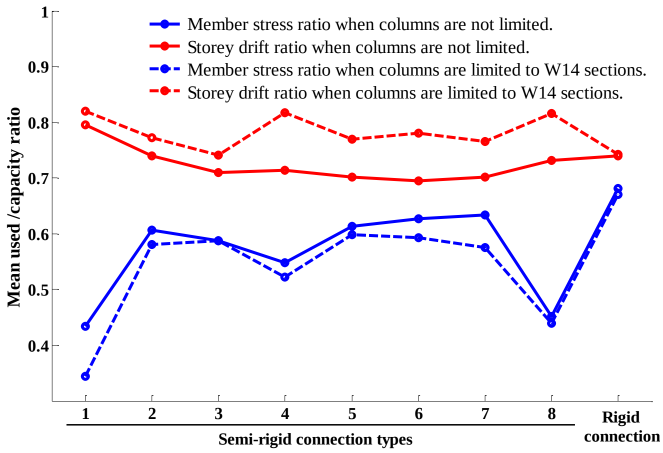


Fig. 7 The used / capacity ratio diagram for nine-storey, single-bay frame

5.2 Ten-storey, four-bay frame

The second design example is a 10-storey, 4-bay frame with 90 members. Fig. 8 shows the twelve groups of members, acting loads and dimensions for this frame. The values of loads are: W=44.49 kN, W_1=47.46 kN/m, W_2=42.91 kN/m. The values of top storey sway for this frame is restricted to 158 mm based on AISC-LRFD specifications.

In this example, the investigation on the effect of column sizes on optimal designs is conducted by solving it again with the columns restricted to W14 sections as well. The optimum design procedure for the 10-storey frame results in the W-sections, which are listed in Table 6. The convergence history for the optimum design of ten-storey frame with rigid and third semi-rigid connections are shown in Fig. 9. The minimum cost results obtained for this frame are shown in Table 7. These results demonstrates that PSO leads to low cost frames compared to BB-BC except for the semi-rigid connection type 3, in which BB-BC gives frames with lower costs. Furthermore, HS-PSO algorithm designs the frames so that their costs are lower both than those of BB-BC and of PSO algorithms for all connections, as it is the case for the first example.

Table 5 The optimal results for nine-storey, single-bay frame

		Semi-rigid connection types								Rigid type
		1	2	3	4	5	6	7	8	
BB-BC (Rafiee *et al.* 2013)	Total cost (*kg*)	40,520	36,235	16,881	25,786	33,488	35,799	53,601	46,146	19,861
	Weight (*kg*)	38,718	32,617	14,809	23,956	30,804	33,481	43,450	44,527	11,683
	Top storey sway (*mm*)	56	55	66	76	54	65	44	71	73
PSO (present work)	Total cost (*kg*)	27,417	21,455	17,570	19,173	18,373	18,410	21,662	25,932	16,100
	Weight (*kg*)	24,331	16,867	15,051	16,905	15,630	16,045	18,988	23,338	11,926
	Top storey sway (*mm*)	67	66	68	71	70	71	68	73	72
HS-PSO (present work)*	Total cost (*kg*)	21,486	17,886	15,464	16,499	15,773	14,970	14,787	21,757	14,877
	Weight (*kg*)	18,693	13,182	13,468	14,288	12,901	12,136	11,590	19,722	10,529
	Top storey sway (*mm*)	79	73	70	71	70	69	69	73	73
HS-PSO (present work)**	Total cost (*kg*)	34,681	20,456	16,848	17,972	16,774	16,323	16,392	26,523	16,168
	Weight (*kg*)	29,668	13,281	13,797	14,864	12,805	12,983	12,803	21,376	11,281
	Top storey sway (*mm*)	81	76	73	81	76	77	76	81	74

* For the case where columns are not limited.

** For the case where columns are limited to W14 sections.

Table 6 The optimal W-shape sections of ten-storey, four-bay frame via HS-PSO

Group no.	Semi-rigid connection types								Rigid connection
	1	2	3	4	5	6	7	8	
1	14×342	14×132	14×82	14×109	14×82	14×132	14×109	14×132	14×74
2	14×233	14×120	14×132	14×145	14×132	14×120	14×132	14×159	14×132
3	14×120	14×61	14×61	14×74	14×74	14×99	14×74	14×90	14×61
4	14×193	14×109	14×90	14×90	14×82	14×82	14×90	14×90	14×82
5	14×53	14×48	14×53	14×53	14×68	14×68	14×38	14×53	14×43
6	14×68	14×61	14×53	14×53	14×48	14×48	14×48	14×53	14×48
7	14×48	14×43	14×48	14×61	14×61	14×68	14×38	14×43	14×43
8	14×61	14×48	14×61	14×48	14×43	14×30	14×48	14×53	14×43
9	30×90	21×48	24×55	24×62	21×44	21×44	21×44	30×90	21×44
10	30×90	16×50	21×44	24×55	18×46	21×44	18×46	30×90	21×44
11	21×44	16×45	21×44	21×44	16×45	16×45	21×44	21×44	21×44
12	18×40	18×40	21×44	18×40	16×45	18×46	18×40	18×46	18×40

The comparison of HS-PSO with BB-BC shows that the reduction in cost varies in a range of 49% (for connection type 4) to 77% (for types 2 and 7) with an average of 63%. Moreover, this reduction varies in a range of 41% (for type 4) through 60% (for types 2 and 3) with an average

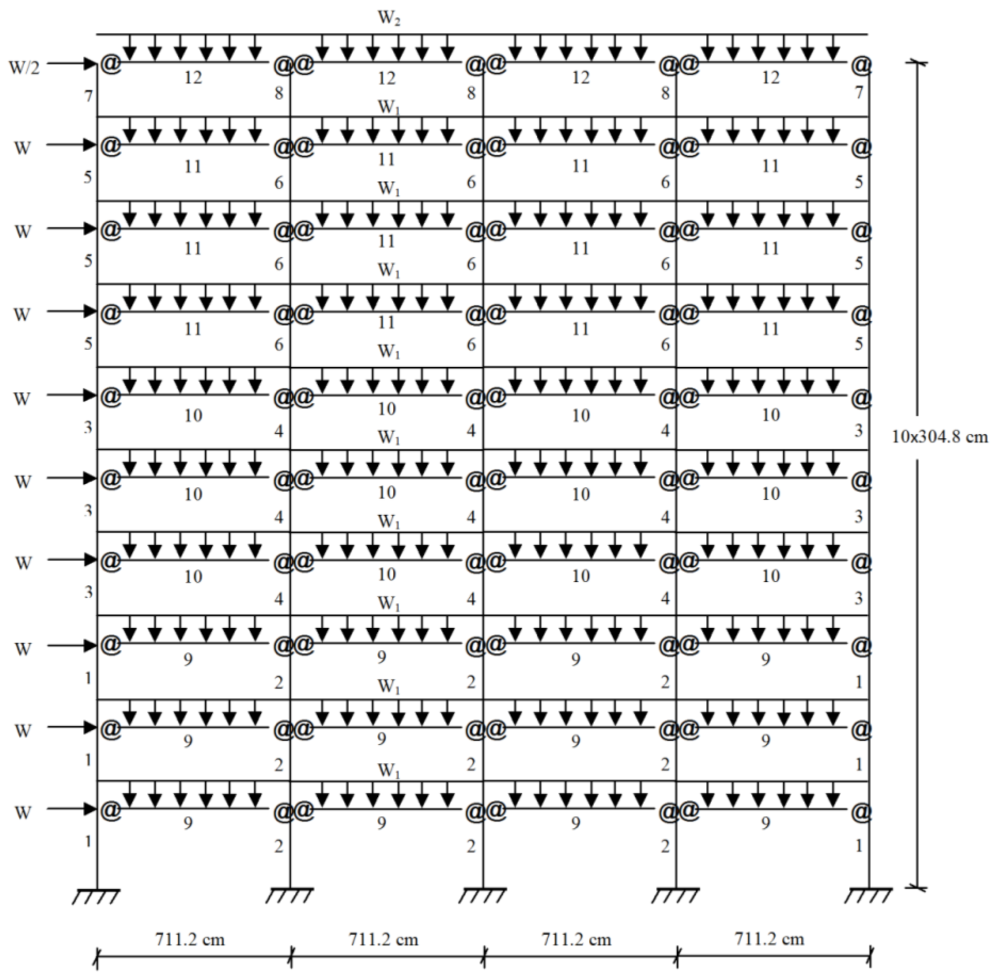
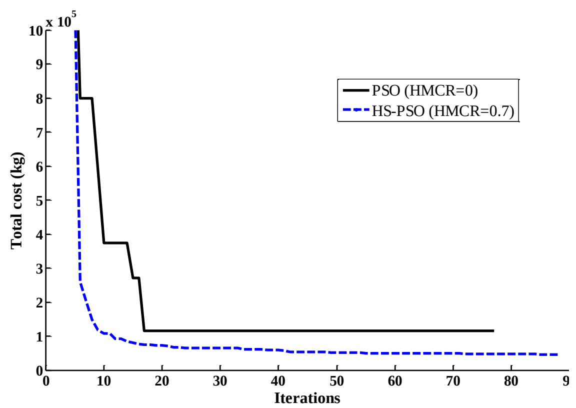
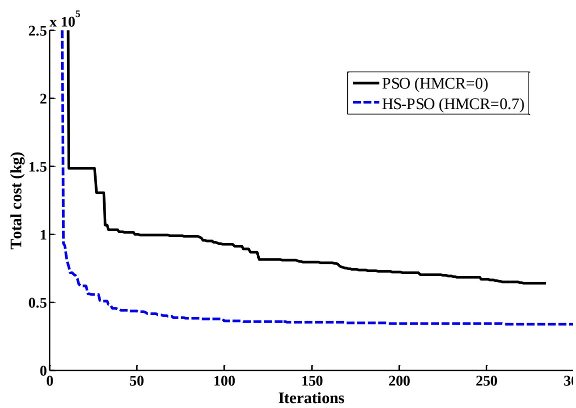


Fig. 8 Ten-storey, four-bay frame



(a) Connection type 3



(b) Rigid connection type

Fig. 9 The convergence history of ten-storey, four-bay frame

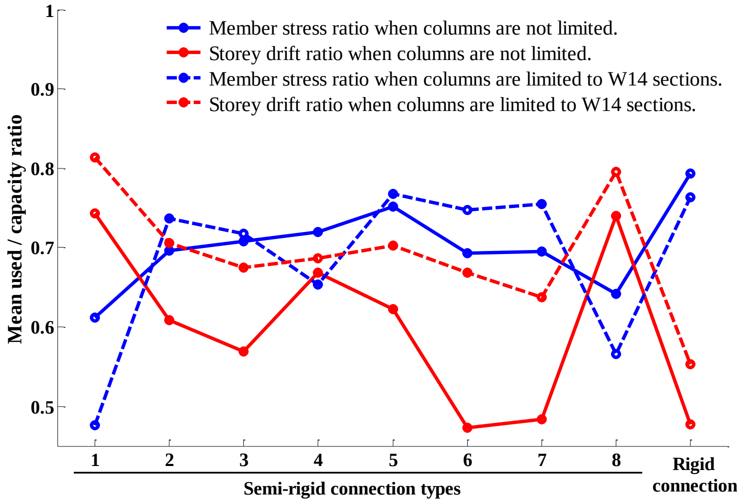


Fig. 10 The used / capacity ratio diagram for ten-storey, four-bay frame

value of 51%, if HS-PSO and PSO are compared. Furthermore, the results show that, limiting the columns to *W*14 sections will increase the importance of lateral displacements in finding optimal solutions. The used / capacity ratio diagram for this frame is shown in Fig. 10, in which the dashed red line is completely on top of solid one (similar to previous example) but the solid red line has intersections with blue one (in contrary to nine storey frame). This implies that in this frame member stresses are dominant in comparison with storey drifts. This may be due to the number of bays in this frame which provides enough lateral stiffness.

5.3 Twenty four-storey, three-bay frame

The topology, service loading conditions, four beam groups and sixteen column groups of 24-storey, 3-bay frame consisting of a total number of 168 members are shown in Fig. 11. Applied loads including point (W) and uniformly distributed (W_1 through W_4) loads have the values of W=25.628 kN, W_1=4.378 kN/m, W_2=6.362 kN/m, W_3=6.917 kN/m and W_4=5.954 kN/m.

In this frame, each of the four beam element groups may choose from all 273 W-shapes, while the 16 column element groups are limited to *W*14 sections. AISC-LRFD limits the top storey sway of this frame to a maximum value of 456 mm. Tables 8-9 show the optimal designs (*W*-shape sections) obtained using HS-PSO algorithm and minimum cost values for this frame, respectively.

The comparison of results listed in Table 9 shows that the HS-PSO reduces the costs in ranges of 5% (for type 5) to 59% (for type 7) and 8% (for types 2 and 8) to 23% (for type 7) relative to BB-BC and PSO algorithms, respectively. Also the corresponding average reduction ratios are 18% and 14%, respectively. In addition, the comparison demonstrates that the BB-BC performs better than PSO if the connection types 2, 4, 5 and 6 are used; otherwise the PSO gives better results. However, in all cases the HS-PSO algorithm posses the best performance. It should be noted that in the 24-storey frame with connection type 1, both the BB-BC and PSO algorithms give an infeasible design, whereas, the design obtained using HS-PSO is a feasible one. The convergence history of optimal design procedure of this frame with connection types 2 and 6 are also shown in Fig. 12.

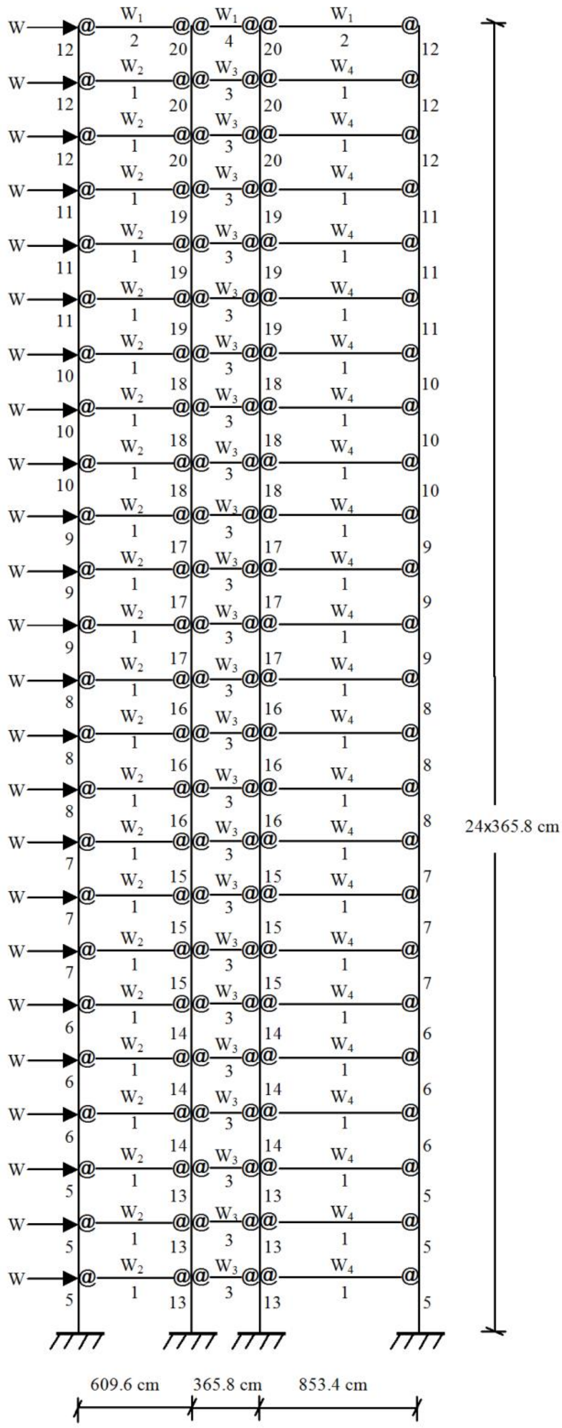
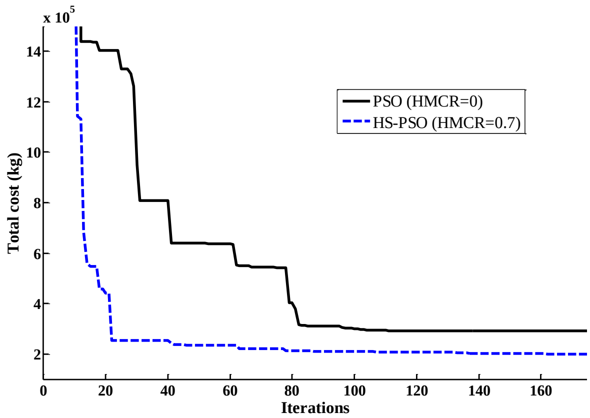


Fig. 11 Twenty four-storey, three-bay frame

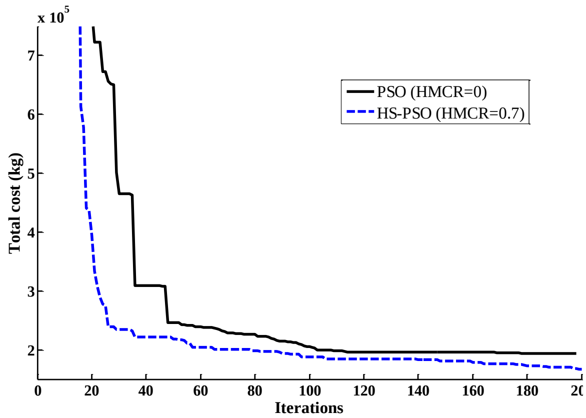
Table 7 The optimal results for ten-storey, four-bay frame

		Semi-rigid connection types								Rigid type
		1	2	3	4	5	6	7	8	
BB-BC (Rafiee *et al*. 2013)	Total cost (kg)	140,744	237,050	106,868	93,255	123,743	113,055	204,773	136,881	173,655
	Weight (kg)	128,418	195,578	100,254	87,432	111,865	103,357	150,274	126,120	102,150
	Top storey sway (mm)	67	25	35	58	37	40	26	56	25
PSO (present work)	Total cost (kg)	112,077	137,027	115,401	81,765	93,044	88,375	97,494	112,199	79,335
	Weight (kg)	99,261	103,364	97,242	73,160	82,861	76,577	83,181	100,949	63,862
	Top storey sway (mm)	82	29	34	45	47	36	41	62	36
HS-PSO (present work)*	Total cost (kg)	58,939	55,118	46,328	47,788	46,407	46,469	47,328	53,489	46,771
	Weight (kg)	52,196	43,746	40,040	41,853	38,532	37,950	38,737	47,018	33,628
	Top storey sway (mm)	76	62	58	68	63	48	49	75	48
HS-PSO (present work)**	Total cost (kg)	77,340	53,332	44,786	48,962	45,415	46,005	45,288	61,771	48,149
	Weight (kg)	65,672	39,435	37,972	41,590	36,988	38,288	36,845	51,376	35,125
	Top storey sway (mm)	83	72	69	70	71	68	65	81	56

* For the case where columns are not limited.
** For the case where columns are limited to W14 sections.



(a) Connection type 2



(b) Connection type 6

Fig. 12 The convergence history of twenty four-storey, three-bay frame

Table 8 The optimal W-shape sections of twenty four-storey, three-bay frame via HS-PSO

Group no.	Semi-rigid connection types								Rigid connection
	1	2	3	4	5	6	7	8	
1	44×290	24×62	33×118	33×118	24×62	27×84	27×84	40×235	24×55
2	10×33	10×17	10×12	10×17	10×15	10×12	6×20	10×15	21×57
3	44×290	30×90	30×108	33×118	30×90	24×62	21×55	40×211	27×94
4	21×50	10×39	12×79	18×106	6×8.5	12×14	12×14	24×207	14×34
5	14×730	14×193	14×283	14×342	14×233	14×257	14×257	14×398	14×193
6	14×665	14×193	14×211	14×283	14×193	14×211	14×211	14×342	14×176
7	14×370	14×176	14×193	14×193	14×193	14×193	14×211	14×283	14×132
8	14×257	14×159	14×193	14×159	14×176	14×145	14×145	14×233	14×132
9	14×257	14×99	14×145	14×132	14×132	14×132	14×120	14×233	14×109
10	14×211	14×74	14×120	14×132	14×109	14×132	14×109	14×176	14×82
11	14×211	14×68	14×109	14×109	14×74	14×99	14×68	14×159	14×61
12	14×211	14×61	14×99	14×90	14×53	14×68	14×68	14×99	14×43
13	14×730	14×426	14×426	14×398	14×370	14×370	14×233	14×370	14×257
14	14×730	14×398	14×370	14×370	14×311	14×257	14×233	14×342	14×233
15	14×605	14×342	14×211	14×211	14×311	14×193	14×159	14×211	14×193
16	14×283	14×211	14×159	14×176	14×193	14×159	14×132	14×211	14×176
17	14×233	14×145	14×145	14×176	14×120	14×159	14×109	14×176	14×120
18	14×233	14×109	14×145	14×109	14×120	14×120	14×109	14×132	14×90
19	14×233	14×90	14×109	14×109	14×109	14×99	14×99	14×99	14×68
20	14×211	14×90	14×99	14×90	14×99	14×68	14×68	14×90	14×90

Table 9 The optimal results for twenty four-storey, three-bay frame

		Semi-rigid connection types								Rigid type
		1	2	3	4	5	6	7	8	
BB-BC (Rafiee *et al.* 2013)	Total cost (kg)	502,197	202,737	267,414	249,806	171,868	176,864	385,074	383,738	233,430
	Weight (kg)	381,754	139,161	236,249	211,149	140,536	150,362	359,372	297,834	137,312
	Top storey sway (mm)	204	245	170	184	237	231	240	190	238
PSO (present work)	Total cost (kg)	479,401	206,100	233,390	250,771	183,522	194,011	202,175	369,633	168,151
	Weight (kg)	364,271	127,581	194,883	210,154	148,650	150,122	172,881	284,020	136,680
	Top storey sway (mm)	207	237	182	189	229	225	226	189	224
HS-PSO (present work)	Total cost (kg)	505,366	189,791	205,473	210,296	162,582	165,828	156,161	341,798	139,727
	Weight (kg)	384,890	135,368	172,004	175,521	133,930	137,054	125,589	261,722	111,170
	Top storey sway (mm)	200	245	194	208	238	217	221	203	242

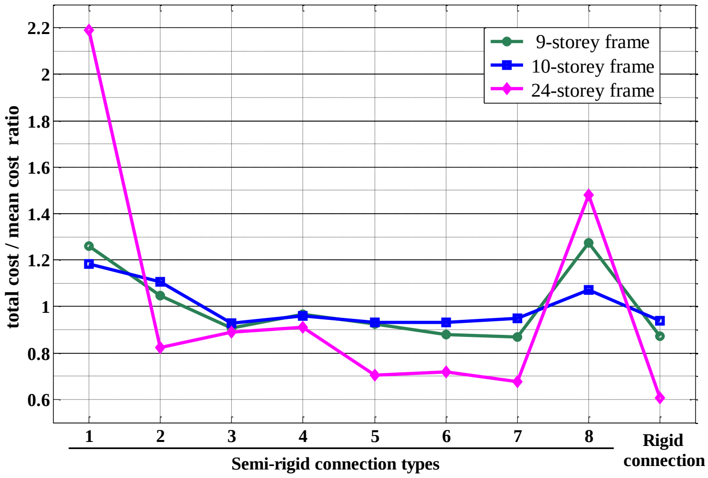


Fig. 13 Minimum total cost ratios of frames

To provide an illuminated comparison of frames with different connection, in Fig. 13 the results of examples are shown. In this figure the total cost values of frames are divided by the average cost of that example obtained through HS-PSO. The cost of third example, which is a large scale frame, is very sensitive to the connection type. The figure depicts that the frames with rigid connection and with semi-rigid connection types 7, 6 and 5 (less flexible types) lead to relatively low cost frames, whereas, the connection types 1, 2 and 8 (more flexible types) result generally in high costs. There are, however, cases where a more flexible connection gives a low cost frame with compared to less flexible types, such as the third example with second connection type.

6. Conclusions

Harmony search (HS) and particle swarm optimization (PSO) are the well-known meta-heuristic optimization algorithms and are proposed inspiring the performance process of natural music and the social behavior of birds respectively. In this study a hybrid HS and PSO algorithm, called HS-PSO is developed and a discrete algorithm based on HS-PSO presents for optimal design of steel frames with rigid and semi-rigid beam-to-column connections. The algorithm finds the member cross-sections so that the minimum total cost comprising utilized section costs as well as connection construction costs. American Institute of Steel Construction (AISC) wide-flange (W) shape standard steel sections are used. Stress and displacement constraints of AISC-Load and Resistance Factor Design (LRFD) specification are considered as the design constraints. Also, in order to find more practical design, size constraints for beams and columns adaptation are imposed on the frame in the optimal design procedure. The P-Δ effects of beam-column members are considered in the non-linear frame analyses. The nonlinear moment-rotation behavior of semi-rigid connections is modeled using the Frye-Morris polynomial model.

Three benchmark design examples with several types of connections are investigated and the results of PSO and HS-PSO are compared. The results are compared with those reported in literature using Big Bang-Big Crunch (BB-BC) algorithm, as well. The comparisons lead to the following concluding remarks

• In some cases BB-BC algorithm leads to more economic frames than those obtained through PSO, whereas there are cases where the PSO gives frames with lower costs.

• In all the examples with all of the rigid and semi-rigid connection types the HS-PSO gives the frames with lower cost in comparison with both the BB-BC and PSO algorithms.

• The convergence of the HS-PSO is better than that of PSO, such that in the same number of iterations the HS-PSO reaches better solutions than the PSO, while the premature convergence is prevented.

• A *HMCR* value of 0.7 is found to contribute properly the harmony memory to the HS-PSO.

• The results show that, the particle swarm concept can be a proper strategy, which leads to a better memory of harmonies within a relatively small number of iterations.

Furthermore, the comparison of total cost values of frames with different beam-to-connections depicts that

• Among the various types of connections utilized in the design examples, the rigid connection and semi-rigid connection types 7, 6 and 5 (less flexible types) lead to more economic frames compared to other types.

• The connection types 1, 2 and 8 (more flexible types) result generally in high cost frames.

• There are, however, cases where a more flexible connection provides better interaction between the constraints imposed on the response of frame and eventually gives a low cost frame with compared to less flexible types.

• The significant variations in the optimal designs and minimum costs of frames with different types of connections (especially for large scale frames), shows the key role of connection modeling in the real behavior and response of the frame and in the corresponding minimum costs.

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