Transverse and longitudinal partial interaction in composite bolted side-plated reinforced-concrete beams

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Abstract. A procedure is being developed for bolting plates to the sides of existing reinforced concrete beams to strengthen and stiffen them. Unlike standard composite steel and concrete beams in which there is longitudinal-partial-interaction at the steel/concrete interface (that is slip along the length of the beam), composite bolted side-plated reinforced-concrete beams are unique in that they also exhibit transverse-partial-interaction, that is slip transverse to the length of the beam. In this work, the fundamental mathematical models for transverse-partial-interaction and its interaction with longitudinal-partial-interaction are developed. The fundamental models are then further developed to determine the number of connectors required to resist the transverse forces and to limit the degree of transverse-partial-interaction in bolted side-plated reinforced concrete beams.

Key words: plated beam; partial-interaction; reinforced-concrete beam.

1. Introduction

For the last two decades, bolting and gluing steel plates to the tension face of reinforced concrete (RC) beams and slabs to strengthen and stiffen them has become quite common practice in the process of repair and maintenance in the construction industry (Oehlers and Moran 1990, Oehlers 1992, Jones, Swamy & Charif 1988). A procedure for bolting steel plates to the sides of existing reinforced concrete beams is being developed by the authors, as this technique can be used to not only increase the flexural rigidity but also the flexural and shear strengths without loss of ductility.

There is much research on the effect of longitudinal-partial-interaction in standard composite beams (Newmark, Siess and Viest 1951, Oehlers and Sved 1995, Oehlers and Bradford 1995), that is the slip between the concrete slab and steel beam, that occurs in a longitudinal plane

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and in the longitudinal direction. However, bolted side-plated RC beams, such as that shown in Fig. 1, are unique in that they not only exhibit longitudinal-partial-interaction but are subject to vertical slip between the plates and the beam and, therefore, also exhibit transverse-partial-interaction. The aim of this paper is first to develop the fundamental equations for this new concept of transverse-partial-interaction, which can be used as the corner stone for future non-linear research on this subject, and then show how the fundamental equations can be used to develop design rules to cope with transverse-partial-interaction. Methods for determining the transverse forces in composite bolted side-plated beams are first developed. These are then used to quantify the transverse slip, from which a transverse-partial-interaction design procedure is developed for determining the number of connectors required.

2. Transverse-partial-interaction

Transverse-partial-interaction affects both the strength and stiffness of a composite structure. Consider the composite plated beam in Fig. 1(a) which is constructed with the soffits of the beam and plate in line as shown in Fig. 1(b), where both the reinforced-concrete beam and the plate are simply supported at the ends, and where the shear stiffness of the bolt shear connection is K_{si}.

As the applied load $2F$ in Fig. 1(a) is gradually increased, the relative vertical deformation of the plate to the reinforced-concrete beam, that is the vertical slip s_v, depends on the shear stiffness of the shear connection K_{si}. For example when $K_{si} = 0$, that is when there are no bolt shear connectors, then the applied load $2F$ only deflects the reinforced-concrete beam as shown in Fig. 1(c). This is referred to as no-transverse-interaction and will be denoted by the degree of transverse-partial-interaction $\varphi_v = 0$. In contrast when $K_{si} \rightarrow \infty$, which occurs when the plate is glued to the reinforced-concrete beam, then there is no relative vertical movement between the beam and the plate as shown in Fig. 1(d), and this will be referred to as full-transverse-interaction and is denoted by a degree of transverse-partial-interaction of $\varphi_v = 1$. Bolt shear connec-
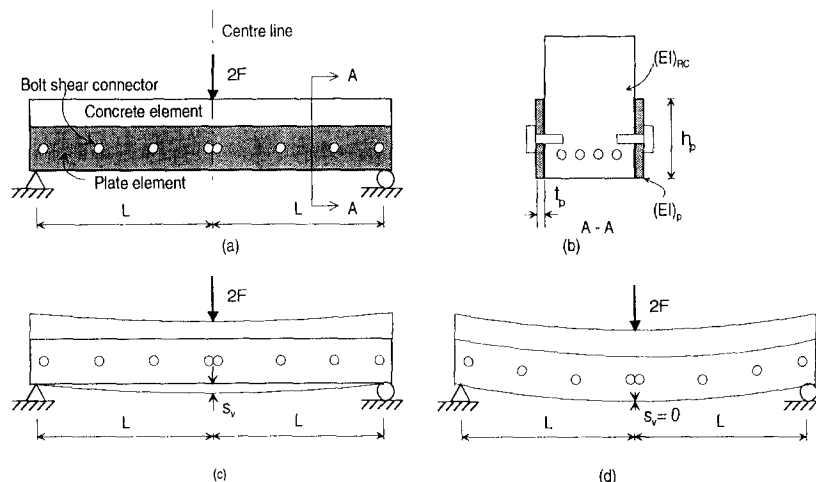

Fig. 1 Degree of transverse-interaction: (a) Simply supported bolted side-plated beam; (b) Cross-section A-A; (c) No-transverse-interaction: $K_{si} = 0$, $\varphi_v = 0$; (d) Full-transverse-interaction: $K_{si} \rightarrow \infty$, $\varphi_v = 1$.
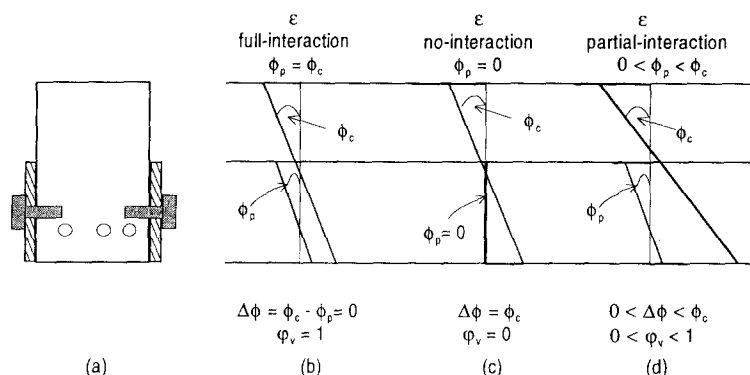
Fig. 2 Variation in strength and stiffness due to transverse-partial-interaction.

tors are mechanical shear connections and, hence, in reality $0<K_{si}<\infty$ so that $0<\varphi_v<1$.

From a comparison of Figs. 1(c) and (d), it can be seen that $\varphi_v=f(s_v)$. As the vertical slip s_v can be derived by integrating the difference between the curvatures in the plate and reinforced-concrete beam $\Delta\phi$ twice, it can be seen that $\varphi_v=f(\Delta\phi)$. This is illustrated in Fig. 2 which shows the strain profiles ε in a beam that exhibits longitudinal-partial-interaction (that is longitudinal slip) as well as transverse-partial-interaction. When there is full-interaction, that is $K_{si}\to\infty$, it can be seen in Fig. 1(d) that $s_v=0$, so that the distributions of curvatures in the plate ϕ_p and in the beam ϕ_c are the same as shown in Fig. 2(b); hence, $\varphi_v=1$ when $\Delta\phi=0$. In contrast when $K_{si}=0$ in Fig. 1(c), then the plate is not deformed so that $\phi_p=0$ as in Fig. 2(c), therefore, $\varphi_v=0$ when $\Delta\phi=\phi_c$. Partial interaction occurs between these two extremes as shown in Fig. 2(d).

It can be seen in Fig. 2(c) that when $\varphi_v=0$ then the plate is unstrained so that the composite plated beam has its weakest strength and stiffness. However when $\varphi_v=1$ in Fig. 2(b), then the plate has its maximum strains in comparison to those in the concrete and so the plated beam is at its strongest and stiffest. The partial-interaction strength and stiffness will lie between these two extremes and, hence, depend on the degree of vertical interaction φ_v as illustrated in Fig. 2(d). The aim of this paper is to develop the fundamental equations for transverse-partial-interaction in terms of $\Delta\phi$ and hence φ_v, so that the degree of transverse-partial-interaction can eventually be used to quantify the flexural strength and stiffness of composite bolted side-plated reinforced-concrete beams.

3. Transverse force in composite bolted side-plated beams

The vertical forces in side-plated beams are determined in two stages. In the first stage, the beam is assumed to have longitudinal-partial-interaction but transverse-full-interaction, that is longitudinal slip can occur in the beam in Fig. 1(d) but vertical slip cannot occur. The effect of vertical slip will then be introduced in the second stage.

3.1. Longitudinal-partial-interaction and transverse-full-interaction

Consider the simply supported composite plated beam in Fig. 1(d) in which a point load
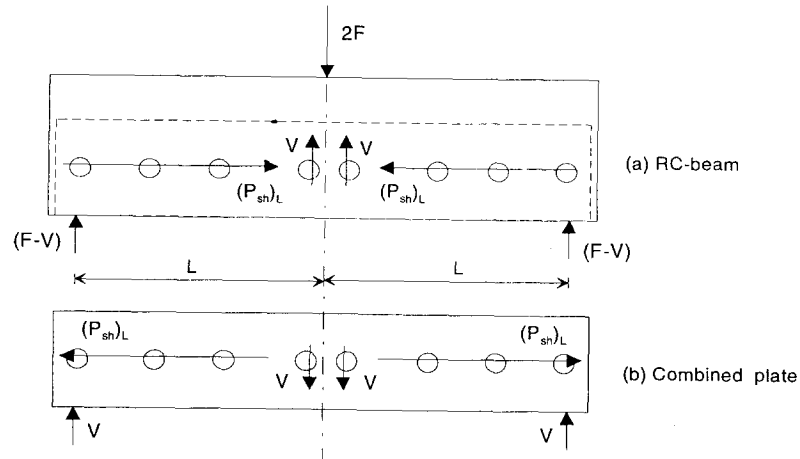
Fig. 3 Idealised forces in the plate and in the RC-beam.

2F is applied to the concrete element at the mid-span and where both the RC beam and the side plates are simply supported at their ends. Furthermore, let us assume that there is a single line of shear connectors that is in line with the centroid of the plate. Research on the fracture of shear connectors in composite beams (Oehlers and Sved 1995) has shown that a good method for modelling the composite beam is to assume that both the concrete and steel elements remain linear elastic, whereas, all the shear connectors are plastic, that is fully loaded. In this paper, we will use this model to develop the fundamental equations for side plated beams.

The idealised forces in the plate and concrete elements are shown separately in Fig. 3. The force $(P_{sh})_L$ is the strength of the shear connection in the shear span of length L as it has been assumed that the connectors are fully loaded longitudinally. It needs to be remembered that this is the strength of the shear connection in both plates in the shear span of length L. Let us define the part of the applied load $2F$ that passes through the plate element as $2V$. As the longitudinal slip of the bolts at mid-span must be zero, it is assumed that the connectors at mid-span resist all of the vertical load $2V$ as shown in Fig. 3(b). In the following analysis, we will determine the vertical force for the simplest case in which there is longitudinal-partial-interaction (i.e., longitudinal slip) and transverse-full-interaction (i.e., no vertical slip) as shown in Fig. 1(d).

A shear span of length x of the plated beam is shown in Fig. 4(a). Both plates in Fig. 1(b) are combined as an equivalent single side plate of thickness $2t_p$ in Fig. 4(b) and the strength of the shear connectors in this shear span of length x is $(P_{sh})_x$. Hence on this shear span, there is an axial tensile force of $(P_{sh})_x$ in the plate element and an axial compressive force in the concrete element of the same magnitude. These forces are shown in Fig. 4(a) to act at the neutral axes of the elements which are at a distance h_e apart.

Let us define the moment acting at the neutral axis of the equivalent plate element of thickness $2t_p$ in Fig. 4(b) as $(M_p)_x$ as shown in Fig. 4(a), the moment acting at the neutral axis of the concrete element as $(M_{RC})_x$, and the applied moment acting at the end of the shear span of length x as $(M_{app})_x$. Due to longitudinal-partial-interaction between the plate and the concrete elements, their neutral axes do not coincide and, therefore, there is a slip-strain between the strain profiles of the elements ε_c and ε_p as shown in Fig. 4(c). As the case of transverse-full-interaction between the elements is being considered, the curvatures of the plate element ϕ_p
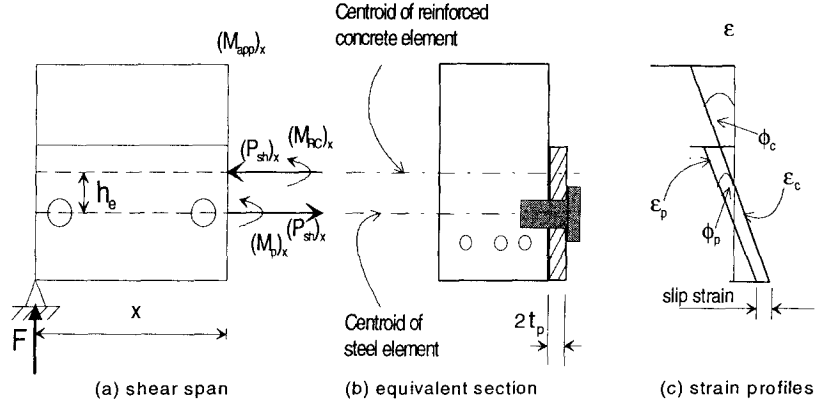
Fig. 4 Elastic-plastic analysis for simply supported side-plated bolted RC beam.
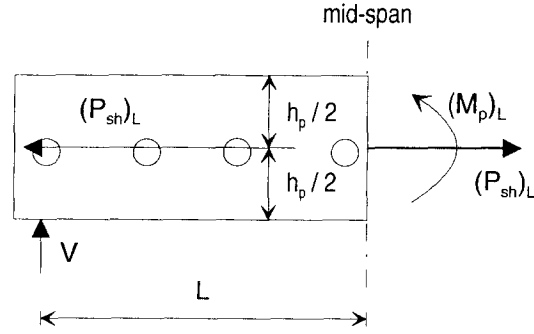

Fig. 5 Stress resultants on combined plate element.

and concrete element ϕ_c in Fig. 4(c) are the same. Therefore, from the compatibility of the curvatures

$$\frac{(M_{RC})_x}{(EI)_{RC}} = \frac{(M_p)_x}{(EI)_p} = \frac{(M_{RC})_x + (M_p)_x}{(EI)_{RC} + (EI)_p} \tag{1}$$

where $(EI)_{RC}$ and $(EI)_p$ are the flexural rigidities of the concrete and plate element respectively. From equilibrium of the moments shown in Fig. 4(a), we get.

$$(M_{app})_x = (M_{RC})_x + (M_p)_x + (P_{sh})_x h_e \tag{2}$$

From Eqs. (1) and (2) and substituting m for $(EI)_{RC}/(EI)_p$, gives

$$(M_{p,vfi})_x = \frac{(M_{app})_x - (P_{sh})_x h_e}{m + 1} \tag{3}$$

$$(M_{RC,vfi})_x = \frac{m[(M_{app})_x - (P_{sh})_x h_e]}{m + 1} \tag{4}$$

where the subscript *vfi* in $M_{p,vfi}$ and in $M_{RC,vfi}$ is to remind the reader that we are dealing with vertical or transverse-full-interaction in the analysis.

The combined stress resultants acting on the plate element in Fig. 3(b) and 4(a) are shown in Fig. 5. As the connectors are acting through the centroid of the plate element of depth h_p,

they do not contribute to the moment in the plate and hence

$$(M_{p,vfi})_L = V_{fi}L \tag{5}$$

where V_{fi} is the vertical shear force in the shear connection when there is full-transverse-interaction.

Substituting Eq. (5) into Eq. (3) and noting from Fig. 3(a) that for a mid-span cross-section when $x = L$ then $(M_{app})_L = FL$ and $(P_{sh})_L = q_{sh}L$, where q_{sh} is the shear flow strength of the connectors for the combined plate, gives

$$V_{fi} = \frac{F - q_{sh}h_e}{m+1} \tag{6}$$

Eq. (6) gives an upper bound to the vertical force in the shear connectors which will reduce to zero as the degree of transverse-partial-interaction reduces to zero. In the following analysis, the effect of the degree of transverse-partial-interaction on the vertical force is determined.

3.2. Longitudinal and transverse partial-interaction

Consider the beam in Fig. 1(c) when there is now both longitudinal and transverse-partial-interaction between the plate and the concrete elements as shown in Fig. 2(d). In this case, the element curvatures, ϕ_c and ϕ_p in Fig. 4(c) are not the same. If the difference between the element curvatures is $\Delta\phi$, then

$$\Delta\phi = \phi_c - \phi_p = \frac{M_{RC,vpi}}{(EI)_{RC}} - \frac{M_{p,vpi}}{(EI)_p} \tag{7}$$

where subscript *vpi* refers to vertical-partial-interaction. From Eqs. (2) and (7), we can derive in terms of the $\Delta\phi$ the following moments acting at the centroids of the concrete and steel elements when there is vertical-partial-interaction.

$$M_{RC,vpi} = \frac{m(M_{app} - P_{sh}h_e) + \Delta\phi \ (EI)_{RC}}{m+1} \tag{8}$$

$$M_{p,vpi} = \frac{M_{app} - P_{sh}h_e - \Delta\phi \ (EI)_{RC}}{m+1} \tag{9}$$

At a cross-section of the length x from the beam support as shown in Fig. 4(a)

$$(M_{app})_x = Fx \tag{10}$$

$$(P_{sh})_x = q_{sh}x \tag{11}$$

where $(M_{app})_x$ is the applied moment at the cross-section x, and $(P_{sh})_x$ is the force in shear connectors along the shear span x.

In this model, it is assumed that the steel and concrete elements are linear elastic and in this case of a single point load at the mid-span of the simply supported beam in Fig. 1(b), there is a linear variation in $(M_{RC})_x$ and $(M_p)_x$ as can be seen in Fig. 3. Hence, the difference between the element curvatures $(\Delta\phi)$ is a linear function of the shear span. If $\Delta\phi_{max}$ is the maximum of the differences between element curvatures which occurs at $x = L$ and as $\Delta\phi = 0$ at $x = 0$, then

$$\Delta\phi = \frac{\Delta\phi_{max}x}{L} \tag{12}$$

Substituting Eqs. (10) to (12) into Eqs. (8) and (9) and simplifying, gives

$$(M_{RC,vpi})_x = \frac{[m(F - q_{sh}h_e) + \Delta\phi_{max}L^{-1}(EI)_{RC}]x}{m+1} = k_c\,x \tag{13}$$

$$(M_{p,vpi})_x = \frac{[F - q_{sh}h_e - \Delta\phi_{max}L^{-1}(EI)_{RC}]x}{m+1} = k_p\,x \tag{14}$$

Let V_{pi} be the transverse shear force acting on the connectors at mid-span and at the supports when there is vertical-partial-interaction. Hence from Fig. 5

$$(M_{p,vpi})_L = V_{pi}L \tag{15}$$

Applying Eq. (14) at $x=L$, substituting into Eq. (15) and rearranging gives

$$V_{pi} = k_p = \frac{F - q_{sh}h_e - \Delta\phi_{max}L^{-1}(EI)_{RC}}{m+1} \tag{16}$$

Further substituting Eq. (6) into Eq. (16) gives

$$V_{pi} = V_{fi} - \frac{\Delta\phi_{max}}{L\sum\overline{EI}} \tag{17}$$

in which

$$\sum\overline{EI} = \frac{1}{(EI)_{RC}} + \frac{1}{(EI)_p}$$

Let the degree of the transverse-partial-interaction φ_v be defined as the ratio of the vertical shear force acting on the connectors due to partial-transverse-interaction to that due full-transverse-interaction, that is $\varphi_v = V_{pi}/V_{fi}$. Hence, from Eq. (17) the degree of transverse-partial-interaction is given by

$$\varphi_v = 1 - \frac{\Delta\phi_{max}}{LV_{fi}\sum\overline{EI}} = 1 - \frac{(m+1)\Delta\phi_{max}}{L(F - q_{sh}h_e)\sum\overline{EI}} \tag{18}$$

which can be rearranged to give the difference in curvature as a function of the degree of transverse interaction as

$$\Delta\phi_{max} = \frac{L(1 - \varphi_v)(F - q_{sh}h_e)\sum\overline{EI}}{m+1} \tag{19}$$

It can be seen in Eq. (18) that transverse-full-interaction is reached (i.e., $\varphi_v = 1$) when the element curvatures ϕ_c and ϕ_p in Fig. 4(c) are the same (i.e., $\Delta\phi_{max}=0$). Furthermore, it can be seen from Eq. (19) that the maximum difference in the element curvatures is linearly dependent of the degree of transverse interaction.

In the next section, the relationship between the transverse displacement distribution of the elements along the shear span and the degree of transverse-partial-interaction is established.

4. Transverse slip in composite bolted side-plated beams

Dividing Eqs. (13) and (14) by the flexural rigidity of their elements gives the following variations in curvatures of each elements

$$\frac{d^2y_c}{dx^2} = \frac{k_c x}{(EI)_{RC}}$$ (20)

$$\frac{d^2y_p}{dx^2} = \frac{k_p x}{(EI)_p}$$ (21)

where y_c and y_p are the vertical displacements of the concrete and plate elements respectively and the constants k_c and k_p are defined in Eqs (13) and (14). Bearing in mind that in both of the elements shown in Fig. 3, the vertical displacements at the supports and the slopes at mid-span is zero, then integrating Eqs. (20) and (21) twice gives the following variation in vertical displacements

$$y_c(x) = \left[\frac{x^3}{6} + \frac{L^2 x}{2} \right] \frac{k_c}{(EI)_{RC}}$$ (22)

$$y_p(x) = \left[\frac{x^3}{6} + \frac{L^2 x}{2} \right] \frac{k_p}{(EI)_p}$$ (23)

The difference between Eqs. (22) and (23) is the following vertical slip s_v between the elements.

$$s_v(x) = \left[\frac{x^3}{6} + \frac{L^2 x}{2} \right] \left[\frac{k_p}{(EI)_p} - \frac{k_c}{(EI)_{RC}} \right]$$ (24)

which has the maximum value at mid-span of

$$s_{v,max} = \frac{2L^3}{3} \left[\frac{k_p}{(EI)_p} - \frac{k_c}{(EI)_{RC}} \right]$$ (25)

Furthermore, substituting the values of k_p and k_c from Eqs. (13) and (14) into Eq. (25) gives

$$s_{v,max} = \frac{\Delta\phi_{max}L^2}{3}$$ (26)

which can also be determined directly by integrating Eq. (12) twice.

Substituting Eq. (19) into Eq. (26) gives the following maximum transverse slip in terms of the degree of transverse-interaction

$$s_{v,max} = \frac{1}{3(m+1)} L^3 (1 - \varphi_v)(F - q_{sh}h_c)\sum \overline{EI}$$ (27)

It is worth noting that if the maximum slip capacity of a bolt shear connector is $s_{sh,f}$, then substituting this value for $s_{v,max}$ in Eq. (27) gives the minimum permissible degree of transverse interaction $(\phi_{v,min})_f$ as

$$(\phi_{v,min})_f = 1 - \frac{3(m+1)\, s_{sh,f}}{L^3(F - q_{sh}h_c)\sum \overline{EI}}$$ (28)

Hence, it is necessary to have a degree of transverse-interaction greater than $(\phi_{v,min})_f$, otherwise, premature failure due to fracture of the bolt shear connector will occur.

5. Number of shear connectors required to resist transverse forces

The load/slip behaviour of a bolt shear connector (Ahmed 1996) has the same characteristics as stud shear connectors in composite steel and concrete beams (Oehlers & Coughlan 1986). Each type of connector has a linear response then a plastic plateau. If the initial linear stiffness of an individual connector is K_{si} and there are $N_{v,pi}$ connectors in a group resisting the shear force V_{pi} then $V_{pi} = (N_{v,pi} K_{si})$. s_v, so that the number of connectors required is

$$N_{v,pi} = \frac{V_{pi}}{s_{v,max} K_{si}} \qquad (29)$$

Substituting Eqs. (17) and (27) into Eq. (29) gives

$$N_{v,pi} = \frac{3}{K_{si} L^2} \left[\frac{V_{fi}}{\Delta \phi_{max}} \frac{1}{L \sum \overline{EI}} \right] \qquad (30)$$

Hence, the number of connectors required depends on how much the designer wishes to restrict the difference in the curvatures. Alternatively, substituting $V_{fi}/\Delta \phi_{max}$ from Eq. (18) into Eq. (30) gives the number of connectors required in terms of the degree of transverse-partial-interaction

$$N_{v,pi} = \frac{3 \phi_v}{K_{si} L^3 (1 - \phi_v) \sum \overline{EI}} \qquad (31)$$

It can be seen in Eq. (31) that the number of shear connectors required when there is transverse-partial-interaction depends upon the stiffness of the individual connector, the degree of transverse partial interaction, the beam length, and the material and geometric properties of the composite beam.

We are dealing with a linear elastic analysis procedure as the number of shear connectors depends on the elastic stiffness of the shear connector K_{si}. It can, therefore, be seen that the design procedure that is being proposed requires that the longitudinal shear force $(P_{sh})_L$ in Fig. 3 is resisted by the plastic shear strength of the connection. Whereas and in contrast, the vertical shear force V in Fig. 3 is resisted by the elastic stiffness of the shear connection. The design procedure, therefore, uses a combination of elastic theory and plastic theory. In order to ensure that the vertical shear force V is resisted by the elastic stiffness, it is necessary that the minimum degree of transverse partial interaction is

$$(\phi_{v,min})_e = 1 - \frac{3(m+1) s_{sh,e}}{L^3 (F - q_{sh} h_e) \sum \overline{EI}} \qquad (32)$$

where $s_{sh,e}$ is the slip at which the connector behaviour starts to become plastic, that is the maximum slip below which the stiffness is K_{si}.

It can, therefore, be seen that in Eq. (31) the range of the degree of transverse-interaction is valid for $(\phi_{v,min})_e < \phi_v < 1$. It is also worth noting that the minimum degree of interaction in Eq. (32) which is based on linear elastic stiffness is greater than that required in Eq. (28) which

ensures that fracture does not occur.

6. Conclusions

A technique is being developed for bolting plates to the sides of existing reinforced concrete beams to both strengthen and stiffen them. This paper develops the fundamental principles that are essential to the understanding of the behaviour of this new form of construction, and which are now being used in the formation of design rules. The following conclusions were drawn from this research.

1) In this form of composite construction, the shear connection is subjected to not only longitudinal-partial-interaction but also to the unique concept of transverse-partial-interaction. It is necessary that this new phenomenon should be taken into account in designing this type of structure as the transverse-partial-interaction affects both the strength and stiffness of the composite structure.

2) The longitudinal shear forces can be resisted by the plastic capacity of the shear connectors. However, the elastic stiffness of the shear connectors has to be used to resist the transverse-partial-interaction, in order to restrict the difference in curvatures between the steel and concrete components.

3) The concept of the degree of transverse-partial-interaction has been introduced, that can be used to restrict the difference in curvature between the steel and concrete elements and, hence, can be used to prevent plasticity or fracture of the shear connectors that resist the vertical or transverse forces.

4) Fundamental simple mathematical models for transverse-partial-interaction and its interaction with longitudinal-partial-interaction have been developed in a form that will eventually be used to develop design rules for this relatively new form of construction.

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Notations

E	= Young modulus
EI	= Flexural rigidity
F	= half magnitude of the applied point load at the mid-span
h_e	= distance between neutral axes of the plate and concrete element
h_p	= depth of plate
I	= second moment of area
K_{si}	= shear stiffness of bolt shear connection
k_c	= concrete element parameter given by Eq. (13)
k_p	= plate element parameter given by Eq. (14)
L	= half span of beam
M_{app}	= applied moment
M_{RC}	= moment at the neutral axis of the concrete element
M_p	= moment at the neutral axis of the combined plate element
m	= ratio of element rigidities: $m = (EI)_{RC}/(EI)_p$
$N_{v,pi}$	= number of shear connectors to resist V_{pi} when there is transverse-partial-interaction
$(P_{sh})_L$	= strength of shear connection in a half span
$(P_{sh})_x$	= strength of shear connection in a shear span of length x
q_{sh}	= shear flow strength of shear connection: connector strength per unit length of beam
RC	= reinforced concrete
$s_{sh,e}$	= slip below which the connectors are elastic
$s_{sh,f}$	= slip above which the connectors fracture
s_v	= vertical or transverse slip
$s_{v,max}$	= maximum vertical or transverse slip
t_p	= plate thickness
V	= vertical shear force acting on the shear connectors
V_{fi}	= V with full-transverse-interaction
V_{pi}	= V with partial-transverse-interaction
x	= distance from the beam support to the considered cross-section
y_c, y_p	= vertical or transverse displacements of the concrete and plate element respectively
ε	= strain profile
$\varepsilon_c, \varepsilon_p$	= strain in the concrete and plate element respectively
ϕ_c, ϕ_p	= the concrete and plate element curvatures respectively
$\Delta\phi$	= difference in the element curvatures ($\Delta\phi = \phi_c - \phi_p$)
$\Delta\phi_{max}$	= maximum difference in the element curvatures
φ_v	= degree of vertical-interaction or transverse-interaction
$(\varphi_{v,min})_e$	= φ_v above which the connectors remain elastic
$(\varphi_{v,min})_f$	= φ_v below which the connectors fracture

Suffices:

L	= at mid-span
p	= of combined plate element
RC	= of reinforced-concrete element
v_{fi}	= in a beam with transverse-full-interaction
v_{pi}	= in a beam with transverse-partial-interaction
x	= at a distance x from a support