Structural Engineering and Mechanics, Vol. 5, No. 3 (1997) 329-338
DOI: http://dx.doi.org/10.12989/sem.1997.5.3.329

An analytical approach for nonlinear response of elastic cable under complex loads

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Abstract. In this paper a general analytical approach is proposed to analyse the nonlinear response of elastic cable under complex loads. The effect of temperature change on the cable is also considered. From the vertical equilibrium equations of cable, the general analytical formula of vertical displacement is derived. Based on the vertical displacement formula and on the compatibility condition of the cable, the dimensionless equation with respect to cable tension is established. By means of such analytical procedures, the exact solutions of various cable problems can be obtained quickly. The example given in this paper shows that the new procedure is efficient for practical analysis and can be easily implemented by a general computer program without the superposition problem which there has always been in traditional analytical methods.

Key words: cable (ropes); tension.

1. Introduction

Cable structures provide economical solutions to large span structural problems, because of the high efficiency of steel in simple tension. They are used extensively as roofs of gymnasium, communication towers, suspension bridges, offshore structures, etc. For this reason, the analysis of cables has attracted considerable interest in the past and continues to do so. In contrast with other civil engineering structures, cable structures respond in a nonlinear fashion to both prestressing and in-service forces, due to their flexibility in bending. Single cable analysis is the base of cable structures design. Such analysis must take into consideration geometric nonlinearity.

The first half of this century has seen the development of analytical solutions for the flexible extensible cable under distributed and concentrated loads. Included in this range are the solutions by Pippard and Chitty (1942), Markland (1951), Pugsley (1957), Francis (1965), Otto (1967), Morales (1968), O'Brien (1968), Buchholdt (1970), Wilson and Wheen (1977), Krishna (1978) and others. The most general and practical form has been proposed by Irvine (1975, 1981). Although numerous efforts in analytical treatment of cable problems have been made, there has always been the

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problem of superposition of solutions because of the nonlinear cable behaviour. In fact, analytical methods can only be exployed for quite simple cable problems.

The advent of computer made the iterative methods of solution for the nonlinear problem of the cable feasible. Most numerical schemes Leonard (1972, 1973, 1988), Argyris & Scharpf (1972), *et al.* are based on the discretization of the equilibrium expressions followed by an iterative solution of the resulting nonlinear algebraic equations. The discretization procedure has been largely based upon the versatile finite element methods. They can be used to get the solution of the cable under any load pattern, however, we are always harassed by the problems of convergence and divergence.

In this paper, a new general analytical procedure is presented for the static nonlinear analysis of cable under complex loads. The deficiency of traditional analytical procedures in superposition and the embarrassment of current numerical methods in convergence and divergence problems will be overcome here. The reliability and efficiency of the new procedure will be demonstrated by the example given in this paper.

2. Problem and cable configurations

Consider a elastic cable suspended between two rigid supports at the samelevel (see Fig.1). The cable is assumed to be of uniform cross section and is made from a material of uniform density. The flexible rigidity of the cable is ignored. Let E, A, m and α be the elastic modulus, the cross-sectional area, the mass per unit length and the coefficient of expansion of the cable respectively. Concentrated load system P_i ($i=1, 2, \cdots, n-1$) and stair distributed load system q_i ($i=1, 2, \cdots, n$) are applied on the cable (see Fig. 1). In addition, the temperature change of the environment is Δt. In the following context, the analytical formulations which can be used to analyse such cable problem will be proposed.

First, the cable is divided into n sections so that the load applied on each section is constant. Let Ox_i be an orthogonal coordinate system which is referred to in the description of cable configurations. Two of them are distinguished. (a) The natural configuration C^o is static equilibrium under its own weight. The material point M has the location ${}^oM({}^os)$ in C^o, ${}^os$ being a curvilinear abscissa. The cable is assumed inextensible C^o. In addition, the ratio of sag ${}^od$ to span l is assumed to be about 1:8 or less. Let ${}^oH$ be the horizontal component of cable tension in C^o,

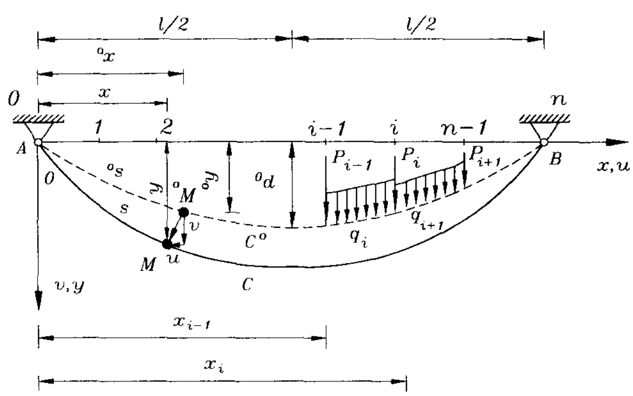

Fig. 1 Cable configurations.

then we have

$$^{o}y=\frac{1}{2}\frac{mgl^{2}}{^{o}H}\left[\frac{x}{l}-\left(\frac{x}{l}\right)^{2}\right], \tag{1}$$

where

$$^{o}H=\frac{1}{8}\frac{mgl^{2}}{^{o}d} \tag{2}$$

Because $x\approx{}^{o}x$, the superscript 0 is omitted for ^{o}x. (b) The deformed configuration C in which the extensibility of the cable should be accounted for in determining the profile. This configuration is static equilibrium under all loads.

It is assumed that the displacement is large, but the strain is small in the problem.

3. Equilibrium condition and the formula of vertical displacement

Vertical equilibrium of the cable requires that

$$\frac{d}{d^{o}s}\left[{}^{K}T\frac{dy}{d^{o}s}\right]+mg+q_{i}=0, \qquad {}^{o}s_{i-1}<{}^{o}s<{}^{o}s_{i}, \ (i=1,\ 2,\ \cdots,\ n), \tag{3}$$

where

$$\left[{}^{K}T\frac{dy}{d^{o}s}\right]_{^{o}s_{i}^{-}}^{^{o}s_{i}^{+}}=-P_{i}, \qquad (i=1,\ 2,\ \cdots,\ n-1), \tag{4}$$

where ^{K}T is the cable tension in terms of Kirchhoff stress, which is related to the actual cable tension in C^{o} by the equation, we have

$$^{K}T=T\frac{d^{o}s}{ds}\ . \tag{5}$$

and T is the cable tension in C: Let

$$\begin{cases} {}^{o}s_{i}^{+}=\lim\limits_{\delta\to0}\ ({}^{o}s_{i}+\delta); \\ {}^{o}s_{i}^{-}=\lim\limits_{\delta\to0}\ ({}^{o}s_{i}-\delta). \end{cases} \tag{6}$$

From Eq. (3), we can obtain

$$\sum_{j=1}^{i-1}\int_{^{o}s_{j-1}^{+}}^{^{o}s_{j}^{-}}\left[\frac{d}{d^{o}s}\left({}^{K}T\frac{dy}{d^{o}s}\right)+mg+q\right]d^{o}s+\int_{^{o}s_{i-1}^{+}}^{^{o}s}\left[\frac{d}{d^{o}s}\left({}^{K}T\frac{dy}{d^{o}s}\right)+mg+q\right]d^{o}s=0, \tag{7}$$

where $^{o}s_{i-1}<{}^{o}s<{}^{o}s_{i}$. From Eqs. (7) and (4), we have

$$({}^{o}H+h)\frac{d}{dx}({}^{o}y+v)=V_{i-1,i}+\frac{1}{2}mgl\left(1-\frac{2x}{l}\right), \tag{8}$$

where

$$^{o}H+h=T\frac{dx}{ds} \tag{9}$$

is the horizontal component of cable tension T, and

$$V_{i-1,i}=R_A-\sum_{j=1}^{i-1}(P_j+q_j(x_j-x_{j-1})]-q_i(x-x_{i-1}), \tag{10}$$

$$R_A=\sum_{i=1}^{n}\left[P_i\left(1-\frac{x_i}{l}\right)+q_i(x_i-x_{i-1})\left(1-\frac{x_i+x_{i-1}}{2l}\right)\right]. \tag{11}$$

From Eq. (8), we have

$$\frac{dv}{dx}=\frac{1}{{}^oH+h}\left[V_{i-1,i},\ \frac{mgl}{2}\ \frac{h}{{}^oH}\left(1-\frac{2x}{l}\right)\right]. \tag{12}$$

Eq. (12) may be integrated to yield

$$v(x)=v_{i-1}-\frac{1}{2}q_i\frac{(x-x_{i-1})^2}{{}^oH+h}+\frac{x-x_{i-1}}{{}^oH+h}\left\{R_A-\sum_{j=1}^{i-1}[P_j+q_j(x_j-x_{j-1})]\right\}$$
$$-\frac{1}{2}\ \frac{mgl}{{}^oH+h}\ \frac{h}{{}^oH}\left[x-\frac{x^2}{l}-\left(x_{i-1}-\frac{x_{i-1}^2}{l}\right)\right],\qquad (x_{i-1}\leq x\leq x_i). \tag{13}$$

Therefore we have

$$v_i=v_{i-1}-\frac{1}{2}q_i\frac{(x_i-x_{i-1})^2}{{}^oH+h}+\frac{x_i-x_{i-1}}{{}^oH+h}\left\{R_A-\sum_{j=1}^{i-1}[P_j+q_j(x_j-x_{j-1})]\right\}$$
$$-\frac{1}{2}\ \frac{mgl}{{}^oH+h}\ \frac{h}{{}^oH}\left[x_i-\frac{x_i^2}{l}-\left(x_{i-1}-\frac{x_{i-1}^2}{l}\right)\right],\qquad (i=1,\ 2,\ \cdots,\ n). \tag{14}$$

where $v_{i-1}=v(x_{i-1})$. According to Eqs. (13) and (14), we have the following general analytical formula of vertical displacement $v(x)$.

$$({}^oH+h)\,v(x)=\left(1-\frac{x}{l}\right)\sum_{j=1}^{i-1}\left[p_jx_j+\frac{1}{2}q_j(x_j^2-x_{j-1}^2)\right]-\frac{mgl}{2}\ \frac{h}{{}^oH}\left(x-\frac{x^2}{l}\right)$$
$$+x\left\{\sum_{j=i}^{n}P_j\left(1-\frac{x_j}{l}\right)+\sum_{j=i+1}^{n}q_j\left[x_j-x_{j-1}-\frac{1}{2l}(x_j^2-x_{j-1}^2)\right]\right\}$$
$$+q_i\left\{\left[x_i-\frac{1}{2l}(x_i^2-x_{i-1}^2)\right]x-\frac{1}{2}(x^2+x_{i-1}^2)\right\}.\qquad (x_{i-1}\leq x\leq x_i). \tag{15}$$

In Eq. (15), $v(x)$ has been expressed as the function of P_i, q_i, x, x_i and h, however, h is unknown. To get the formula about h for our problem, we have to discuss the constitutive equation, geometric equation and compatibility condition of the cable.

4. Constitutive equation, geometric equation and compatibility condition

Because it is assumed that the strain is small, we have following constitutive equation

$$\frac{{}^KT}{EA}+\alpha\Delta t\frac{d^os}{ds}=\varepsilon\frac{d^os}{ds} \tag{16}$$

For ε, we have following geometric equation

$$\varepsilon = \frac{dx}{d^os}\frac{du}{d^os} + \frac{d^oy}{d^os}\frac{dv}{d^os} + \frac{1}{2}\left(\frac{dv}{d^os}\right)^2. \tag{17}$$

The superscript 0 is omitted for ^ox, because of the fact that $x \approx {}^ox$. As it is assumed that the strain in C^o can be ignored, from Eqs. (16) and (17), we have

$$\frac{du}{dx} = \frac{h}{EA}\left(\frac{d^os}{dx}\right)^3 + \alpha\Delta t\left(\frac{d^os}{dx}\right)^2 - \frac{dv}{dx}\frac{d^oy}{dx} - \frac{1}{2}\left(\frac{dv}{dx}\right)^2. \tag{18}$$

Eq. (18) is then integrated and we have

$$u_i = u_{i-1} + \frac{hl}{EA}\left[\frac{1}{l}\int_{x_{i-1}}^{x_i}\left(\frac{d^os}{dx}\right)^3 dx\right] + \alpha\Delta tl\left[\frac{1}{l}\int_{x_{i-1}}^{x_i}\left(\frac{d^os}{dx}\right)^3 dx\right]$$
$$-\int_{x_{i-1}}^{x_i}\frac{d^oy}{dx}dv - \frac{1}{2}\int_{x_{i-1}}^{x_i}\frac{dv}{dx}dv. \tag{19}$$

From Eqs. (19), (1), (12) and (10), we have

$$u_i = u_{i-1} + \frac{hl}{EA}\left[\frac{1}{l}\int_{x_{i-1}}^{x_i}\left(\frac{d^os}{dx}\right)^3 dx\right] + \alpha\Delta tl\left[\frac{1}{l}\int_{x_{i-1}}^{x_i}\left(\frac{d^os}{dx}\right)^3 dx\right] - \frac{1}{2}\frac{q_i}{^oH+h}\int_{x_{i-1}}^{x_i} vdx$$
$$-\frac{mg}{^oH+h}\left[1+\frac{h}{2^oH}\right]\int_{x_{i-1}}^{x_i} vdx - \left[\frac{d^oy}{dx}v\,\Big|_{x_{i-1}}^{x_i} + \frac{1}{2}\frac{dv}{dx}v\,\Big|_{x_{i-1}^+}^{x_i^-}\right], \quad (i=1,\ 2,\ \cdots,\ n), \tag{20}$$

where

$$\begin{cases} x_i^+ = \lim\limits_{\delta\to 0}(x_i+\delta); \\ x_i^- = \lim\limits_{\delta\to 0}(x_i-\delta). \end{cases} \tag{21}$$

From (20), we can obtain

$$\frac{1}{2}\sum_{i=1}^{n-1}\left[\frac{dv}{dx}v\right]_{x_i^-}^{x_i^+} - \frac{1}{2(^oH+h)}\sum_{i=1}^{n} q_i\int_{x_{i-1}}^{x_i} vdx - \frac{mg}{2(^oH+h)}\left[1+\frac{h}{2^oH}\right]\sum_{i=1}^{n}\int_{x_{i-1}}^{x_i} vdx \tag{22}$$
$$+\psi_1\frac{hl}{EA} + \psi_2\alpha\Delta tl = 0,$$

where

$$\begin{cases} \psi_1 = \frac{1}{l}\int_0^l\left(\frac{d^os}{dx}\right)^3 dx = 1 + \frac{1}{8}\left(\frac{^od}{l}\right)^2; \\ \psi_2 = \frac{1}{l}\int_0^l\left(\frac{d^os}{dx}\right)^2 dx = 1 + \frac{16}{3}\left(\frac{^od}{l}\right)^2. \end{cases} \tag{23}$$

From Eqs. (8) and (10), and

$$\left[\frac{dv}{dx}v\right]_{x_i^-}^{x_i^+} = -\frac{P_i v_i}{^oH+h}. \quad (i=1,\ 2,\ \cdots,\ n-1) \tag{24}$$

Substitution of Eq. (24) into (22) gives

$$({}^{o}H+h)\left[\psi_1\frac{hl}{EA}+\psi_2\alpha\Delta tl\right]=\frac{1}{2}\sum_{i=1}^{n}\left[P_iv_i+q_i\int_{x_{i-1}}^{x_i}vdx\right]+mg\left[1+\frac{h}{2^oH}\right]\sum_{i=1}^{n}\int_{x_{i-1}}^{x_i}vdx, \tag{25}$$

where h is described by Eq. (9), it is the increment of horizontal component of cable tension. Eq. (25) is called the compatibility condition of cable (Lu 1994). In the next section, we will get the dimensionless equation of h from Eq. (25).

5. Calculation of horizontal component of cable tension

From Eq. (15), we can obtain

$$\begin{aligned}({}^{o}H+h)v_i=&\left(1-\frac{x_i}{l}\right)\sum_{j=1}^{i}\left[P_jx_j+\frac{1}{2}q_j(x_j^2-x_{j-1}^2)\right]-\frac{mgl}{2}\frac{h}{{}^oH}\left(x_i-\frac{x_i^2}{l}\right)\\&+x_i\sum_{j=i+1}^{n}\left\{P_j\left(1-\frac{x_j}{l}\right)+q_j\left[(x_j-x_{j-1})-\frac{1}{2l}(x_j^2-x_{j-1}^2)\right]\right\}\end{aligned} \tag{26}$$

From Eqs. (15) and (26), we have

$$\begin{aligned}({}^{o}H+h)\sum_{i=1}^{n}\int_{x_{i-1}}^{x_i}vdx=&\frac{l}{2}\sum_{i=1}^{n}\left\{P_ix_i\left(1-\frac{x_i}{l}\right)+q_i\left[\frac{1}{2}(x_i^2-x_{i-1}^2)-\frac{1}{3l}(x_i^3-x_{j-1}^3)\right]\right\}\\&-\frac{l^2}{12}mgl\frac{h}{{}^oH}\end{aligned} \tag{27}$$

$$\begin{aligned}({}^{o}H+h)\sum_{i=1}^{n}q_i\int_{x_{i-1}}^{x_i}vdx=&\sum_{i=1}^{n}q_i^2\left[\frac{1}{3}(x_i^3-x_{i-1}^2)-x_{i-1}^2(x_i-x_{i-1})-\frac{1}{4l}(x_i^2-x_{i-1}^2)^2\right]\\&+\sum_{i=1}^{n-1}\left\{\sum_{j=i+1}^{n}q_iq_j(x_i^2-x_{i-1}^2)\left[x_j-x_{j-1}-\frac{1}{2l}(x_j^2-x_{j-1}^2)\right]\right\}\\&+\frac{1}{2}\sum_{i=1}^{n}\left[\sum_{j=1}^{n}q_i(x_i^2-x_{i-1}^2)P_j\left(1-\frac{x_j}{l}\right)\right]\\&+\sum_{i=2}^{n}\left\{\sum_{j=1}^{i-1}q_i[x_i-x_{i-1}-\frac{1}{2l}(x_i^2-x_{i-1}^2)]P_jx_j\right\}\\&-\frac{1}{2}mgl\frac{h}{{}^oH}\sum_{i=1}^{n}q_i\left[\frac{1}{2}(x_i^2-x_{i-1}^2)-\frac{1}{3l}(x_i^3-x_{i-1}^3)\right]\end{aligned} \tag{28}$$

$$\begin{aligned}({}^{o}H+h)\sum_{i=1}^{n}P_iv_i=&\sum_{i=1}^{n}x_i\left(1-\frac{x_i}{l}\right)P_i\left(P_i-\frac{mglh}{2^oH}\right)+2\sum_{i=1}^{n-1}\left[\sum_{j=i+1}^{n}P_iP_jx_i\left(1-\frac{x_j}{l}\right)\right]\\&+\frac{1}{2}\sum_{i=1}^{n}\left[\sum_{j=i}^{n}q_i(x_i^2-x_{i-1}^2)P_j\left(1-\frac{x_j}{l}\right)\right]\\&+\sum_{i=2}^{n}\left\{\sum_{j=1}^{i-1}q_i\left[x_i-x_{i-1}-\frac{1}{2l}(x_i^2-x_{i-1}^2)\right]P_jx_j\right\}.\end{aligned} \tag{29}$$

Substitution of Eqs. (27), (28) and (29) into Eq. (25) gives

$$\bar{h}^3+\left(2+\frac{\lambda^2}{24}+\zeta\right)\bar{h}^2+\left(1+\frac{\lambda^2}{12}+2\zeta\right)\bar{h}=\lambda^2[f_n(\bar{P})+f_n(\bar{q})+f_n(\bar{P},\bar{q})]-\zeta \tag{30}$$

where

$$f_n(\bar{P})=\frac{1}{2}\sum_{i=1}^{n}\bar{P}_i(1+\bar{P}_i)\bar{x}_i(1-\bar{x}_i)+\sum_{i=1}^{n-1}\left[\sum_{j=i+1}^{n}\bar{P}_i\bar{P}_j\bar{x}_i(1-\bar{x}_j)\right], \tag{31}$$

$$\begin{aligned}f_n(\bar{q})=&\frac{1}{2}\sum_{i=1}^{n}\bar{q}_i\left[\frac{1}{2}(\bar{x}_i^2-\bar{x}_{i-1}^2)-\frac{1}{3}(\bar{x}_i^3-\bar{x}_{i-1}^3)\right]\\&+\frac{1}{2}\sum_{i=1}^{n}\bar{q}_i^2\left[\frac{1}{3}(\bar{x}_i^3-\bar{x}_{i-1}^3)-\bar{x}_{i-1}^2(\bar{x}_i-\bar{x}_{i-1})-\frac{1}{4}(\bar{x}_i^2-\bar{x}_{i-1}^2)^2\right]\\&+\frac{1}{2}\sum_{i=1}^{n-1}\left\{\sum_{j=i+1}^{n}\bar{q}_i\bar{q}_j(\bar{x}_i^2-\bar{x}_{i-1}^2)\left[\bar{x}_j-\bar{x}_{j-1}-\frac{1}{2}(\bar{x}_j^2-\bar{x}_{j-1}^2)\right]\right\}\end{aligned} \tag{32}$$

$$\begin{aligned}f_n(\bar{P},\bar{q})=&\frac{1}{2}\sum_{i=1}^{n}\left[\sum_{j=i}^{n}\bar{q}_i(\bar{x}_i^2-\bar{x}_{i-1}^2)\bar{P}_j(1-\bar{x}_j)\right]\\&+\sum_{i=2}^{n}\left\{\bar{q}_i\left[\bar{x}_i-\bar{x}_{i-1}-\frac{1}{2}(\bar{x}_i^2-\bar{x}_{i-1}^2)\right]\sum_{j=i}^{i-1}\bar{P}_j\bar{x}_j\right\},\end{aligned} \tag{33}$$

$$\lambda^2=\frac{EA}{\psi_1{}^oH}\left(\frac{mgl}{{}^oH}\right)^2, \tag{34}$$

$$\zeta=\frac{EA}{\psi_1{}^oH}\alpha\Delta t\psi_2, \tag{35}$$

$$\bar{h}=\frac{h}{{}^oH}, \tag{36}$$

$$\bar{x}_i=\frac{x_i}{l}, \tag{37}$$

$$\bar{P}_i=\frac{P_i}{mgl}, \tag{38}$$

$$\bar{q}_i=\frac{q_i}{mg}. \tag{39}$$

The cubic Eq. (30) is the dimensionless equation of the horizontal component increment of cable tension.

6. Example and conclusions

A new general analytical procedure for nonlinear analysis of cable has been derived with the intention of being a practical design analytical method. When we use this procedure to analyse the nonlinear response of a cable under complex loads, such as the problem shown in Fig. 1, we should take the following steps:

1. divide the cable into several sections so that the load applied on each section is constant;
2. calculate the coefficients of Eq. (30) from (31)-(39);
3. solve the cubic Eq. (30) using the requisite form of Cardan's equations;
4. calculate the horizontal component of cable tension by following equation

$$H = {}^oH + h = {}^oH(1+\bar{h});$$

5. calculate the vertical displacement $v(x)$ by Eq. (15);
6. calculate the cable tension T by equation

$$T = ({}^oH + h)\frac{ds}{dx} = ({}^oH + h)\sqrt{1+\left(\frac{dy}{dx}\right)^2},$$

where $\frac{dy}{dx}$ can be determined by Eq. (8).

Here, it should be emphasized that there is not relationship between the solution accuracy and the number of sections in first step.

To illustrate the use of such analytical approach, we give the following example. Consider the cable shown in Fig. 2. It is divided into seven sections so that the load applied on each section is constant. To get more accurate results, the mass per unit length of this heavy cable can be divided into two parts $\bar{m}$ and m_a, where $\bar{m}=0.1$ kg and $m_a g=3.5$ kN. The natural configuration C^o is the static equilibrium under $\bar{m}g$, and the deformed configuration C is the static equilibrium under all loads. The physical and geometric parameters are listed in table 1. The loads applied on every section and every node are listed in table 2. The temperature change is $\Delta t=30°$C. Such complex cable problem is solved by using the steps from 1 to 6. The horizontal component of the cable is 3.48×10^4 kN. The vertical displacement $v(x)$ is shown in Fig. 3. The Cable tension is illustrated in Fig. 4. The deformed configuration C shown in Fig. 2 is obtained from the calculation.

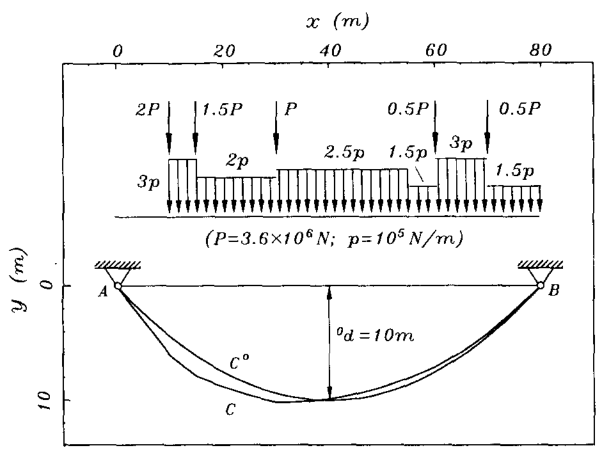

Fig. 2 Cable configurations and loads.

Table 1 The parameters of the cable

$E(Pa)$	$A(m^2)$	α(°C)	$l(m)$	${}^od(m)$
3.5×10^{10}	0.2	2.4×10^{-5}	80	10

Table 2 The nodal coordinates and the loads

i	$x_i(m)$	q_i(kN/m)	P_i(kN)
1	10.0	3.5	7200.0
2	15.0	303.5	5400.0
3	30.0	203.5	3600.0
4	55.0	253.5	0.0
5	60.0	153.5	1800.0
6	70.0	303.5	1800.0
7	80.0	153.5	–

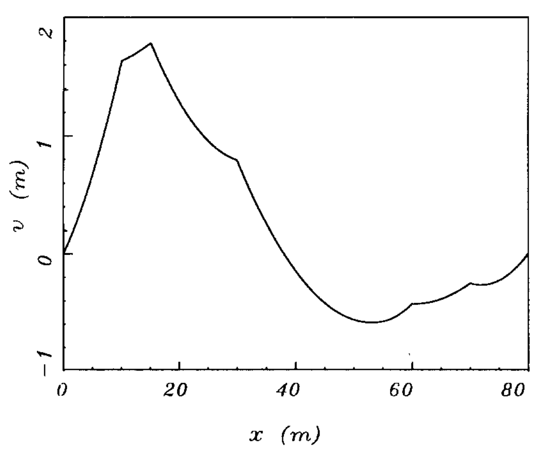

Fig. 3 Vertical displacement of cable.

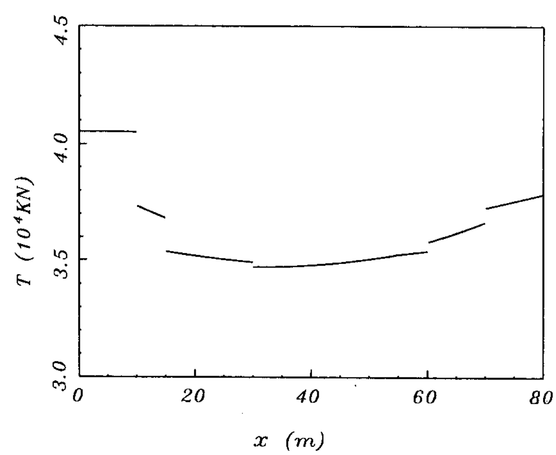

Fig. 4 Cable tension.

In summary, an analytical approach is proposed in this paper. By using such procedure, the exact solution of the nonlinear response of the cable under complex loads can be obtained easily. The effect of temperature change on the cable is considered. This procedure can be easily implemented by a general computer program for various cable problems and there is not the superposition problem which there has always been in traditional analytical analysis.

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