Structural Engineering and Mechanics, Vol. 5, No. 1 (1997) 59-68
DOI: http://dx.doi.org/10.12989/sem.1997.5.1.059

A two-step method for the optimum design of trusses with commercially available sections

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Abstract. A two-step method is presented for the optimum design of trusses with available sections under stress and Euler buckling constraints. The shape design of the truss is used as a means to convert the discrete solution into a continuous one. In the first step of the method, a continuous solution is obtained by sizing and shape design using an approximate polynomial expression for the buckling coefficients. In the second step, the member sizes obtained are changed to the nearest available sections and the truss is reconfigured by using the exact values for the buckling coefficients. The optimizer used is based on the sequential quadratic programming and the gradients are evaluated in closed form. The method is illustrated by two numerical examples.

Key words: truss; sizing; shape design; optimization; available sections; sensitivity analysis.

1. Introduction

The truss design has a discrete nature since the selection of the members has to be done from a set of commercially available fabricated sections. A few attempts have been made to solve this problem by discrete programming directly. In an early effort, Toakley (1968) designed determinate trusses with available sections by using a zero-one programming with Gomory's algorithm. Reinschmidt (1971) solved small scale problems by discrete programming with a modified version of Geoffrion's implicit enumeration. Cella and Logcher (1971) suggested a branch and bound algorithm for the nonlinear programming problem. Templeman and Yates (1983) proposed a linear programming method which has been modified by Duan Ming-Zhu (1986). Schmit and Fleury (1980) used a dual method. John, Ramakrishnan and Sharma (1988) merged the improved move limit method of sequential linear programming with Land and Doig's branch and bound algorithm . They also suggested approximate procedures in which either the continuous solution is converted into a discrete one by a heuristic method or the continuous solution is taken as the starting point for the discrete programming.

The discrete methods can be used with success for the design of trusses with available sections.

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However, they converge very slowly and hence are limited to small or medium size problems. John, Ramakrishnan and Sharma (1988) reported that the time taken by Land and Doig's branch and bound algorithm was large for a 25 bar truss for each iteration and was 20 to 32 times that of a continuous solution. In the present study, it is intended to overcome these difficulties by replacing the discrete solution with a continuous solution regarding both the sizing of the members and changing the geometry of the truss. The shape design is not the primary goal of the proposed method, but it is used as a means to change the discrete nature of the problem to a continuous one. It should be noted that the present approach is not applicable in the cases where the truss geometry is to remain fixed. The sectional properties of the available sections are considered throughout the process. The optimum design is obtained in two steps. In the first step, the truss geometry and member sizes are determined by employing an approximate expression for the buckling coefficients. The optimized member sizes obtained are changed to the nearest available section and are kept fixed in the second step in which the truss geometry is changed by using the exact values of the buckling coefficients.

The optimizer used in the present work is based on the sequential quadratic programming as implemented by Schittkowski (1985). The gradients of the objective function and the constraints are evaluated in closed form. In the numerical examples, the standard 13-bar and 18-bar design problems are solved with the present approach.

2. The design problem

The problem considered is to design a minimum weight truss with available sections. In the first step of the method, continuous size and shape variables are considered. The objective is to minimize the weight of the truss

$$f=\sum_{n=1}^{N}\rho_n A_n L_n \tag{1}$$

where A_n is the cross-sectional area and ρ_n and L_n are the mass density and length of the nth member, respectively. In addition to the side constraints on the design variables, the objective function is also subjected to stress and local buckling constraints as

$$g_{si}=1-\frac{\sigma_i}{\bar{\sigma}_i}\geq 0 \tag{2a}$$

$$g_{bi}=1-\frac{\sigma_i}{\tilde{\sigma}_i}\geq 0 \tag{2b}$$

where the barred quantities denote the allowables. $\tilde{\sigma}_i$ is the Euler buckling stress for the ith member which can be defined as

$$\tilde{\sigma}_i=-\frac{\kappa_i E_i A_i}{L_i^2} \tag{3}$$

where E_i is the modulus of elasticity and κ_i is the buckling coefficient of the ith member. κ_i is a geometrical property of the section and can be written as

$$\kappa_i = \frac{\pi^2 I_i}{A_i^2} \tag{4}$$

where I_i is the least area moment of inertia of the section. In commercial sections, the buckling coefficient does not remain constant as the size changes. In the majority of the previous studies, the buckling coefficient has taken to be constant (Felix 1981, Hansen and Vanderplaats 1990, Imai 1978). This is only possible if the size is changed by uniform expansions or contractions on all cross-sectional dimensions of the member which is not the case for commercial sections. Hence, the designs based on constant buckling coefficients are unrealistic if available sections are to be used in constructing the truss. In the first step of this study, the buckling coefficient κ_i is approximated as a polynomial function of A_i and the unknown coefficients in the polynomial are determined by a curve fitting process using the manufacturer's data for the available sections.

In the second step, the optimum member sizes obtained by the above process are changed to the nearest available sections and the exact values of the buckling coefficients are employed. In this step, the selected sections are kept fixed and the truss geometry is further perturbed to obtain the optimum configuration for the selected sections.

3. Gradients of the objective function and constraints

The structural variables in the problem are the cross-sectional areas of the members, A_i, and a chosen set of the joint coordinates, X_i. These variables can be expressed in terms of the area design variables a_j and configuration design variables $\boldsymbol{x}_j$ through variable linking as

$$\begin{aligned} A_i &= P_{ij}a_j + R_i \qquad (i=1, \cdots, N;\ j=1, \cdots, M) \\ X_i &= S_{ij}\boldsymbol{x}_j + T_i \qquad (i=1, \cdots, 3\times J;\ j=1, \cdots, K) \end{aligned} \tag{5}$$

where P_{ij} and S_{ij} are the elements of the area and configuration linking matrices, respectively. R_i and T_i are the linking constants. N and J are the number of members and joints in the truss. M and K are the number of area and configuration design variables, respectively. The linking of design variables is often required in the truss design to satisfy some symmetry requirements, and furthermore it helps to reduce the number of design variables, since M and K are much smaller than N and $3\times J$, respectively.

The gradients of the objective function Eq. (1) and the constraints Eqs. (2a), (2b) are evaluated in closed form to speed up the optimization process. The side constraints which are the upper and lower bounds on the design variables are handled by the optimizer.

3.1. Gradients of the objective function

The gradients of the objective function f with respect to the area and configuration design variables can be evaluated as

$$\frac{\partial f}{\partial a_k} = \sum_{i=1}^{N} \rho_i L_i P_{ik} \tag{6}$$

$$\frac{\partial f}{\partial x_k} = \sum_{i=1}^{N}\sum_{j=1}^{M} \rho_i (P_{ij}a_j + R_i) \sum_{m=1}^{3\times J} S_{mk} \frac{\partial L_i}{\partial X_m} \tag{7}$$

where $\partial L_i/\partial X_m$ are given in the Appendix.

3.2. Gradients of the constraints

The gradients of the system displacements $\{U\}$ can be calculated as

$$\frac{\partial\{U\}}{\partial a_k}=-[K]^{-1}\sum_{m=1}^{N}P_{mk}\frac{\partial[K]}{\partial A_m}\{U\} \tag{8a}$$

$$\frac{\partial\{U\}}{\partial x_k}=-[K]^{-1}\sum_{m=1}^{3\times J}S_{mk}\frac{\partial[K]}{\partial X_m}\{U\} \tag{8b}$$

where $[K]$ is the system stiffness matrix. $\partial[K]/\partial X_m$ can be evaluated by using the derivatives given in the Appendix. The evaluation of $\partial[K]/\partial A_m$ is straightforward. Having determined the gradients of the system displacements, one can readily find the gradients of element displacements $\{u_i\}$ for the ith element.

Noting that the stress in the ith member can be written as

$$\sigma_i=[G_i]\{u_i\} \tag{9}$$

where $[G_i]$ is the stress-displacement relation, one can express the stress gradients as

$$\frac{\partial\sigma_i}{\partial a_k}=[G_i]\sum_{m=1}^{N}P_{mk}\frac{\partial\{u_i\}}{\partial A_m} \tag{10a}$$

$$\frac{\partial\sigma_i}{\partial x_k}=\sum_{m=1}^{3\times J}S_{mk}\left(\frac{\partial[G_i]}{\partial X_m}\{u_i\}+[G_i]\frac{\partial\{u_i\}}{\partial X_m}\right) \tag{10b}$$

where $\partial\{u_i\}/\partial A_m$ and $\partial\{u_i\}/\partial X_m$ can be obtained from Eqs. (8a), (8b) and $\partial\{G_i\}/\partial A_m$ can be evaluated by using the derivatives given in the Appendix.

The buckling coefficients need not be approximated with high order polynomials since the design in the first step is only approximate. Assuming κ_i as a cubic polynomial in terms of A_i as

$$\kappa_i=c_0+c_1A_i+c_2A_i^2+c_3A_i^3 \tag{11}$$

the gradients $\partial\tilde{\sigma}_i/\partial a_k$ and $\partial\tilde{\sigma}_i/\partial x_k$ can be calculated as

$$\frac{\partial\tilde{\sigma}_i}{\partial a_k}=-\frac{E_i}{L_i^2}\sum_{i=1}^{N}P_{ik}(c_0+2c_1A_i+3c_2A_i^2+4c_3A_i^3) \tag{12a}$$

$$\frac{\partial\tilde{\sigma}_i}{\partial x_k}=-\kappa_iE_iA_i\sum_{m=1}^{3\times J}S_{mk}\frac{\partial L_i^{-2}}{\partial X_m} \tag{12b}$$

where $\partial L_i^{-2}/\partial X_m$ are given in the Appendix.

4. Numerical examples

In the following examples, the commercially available Schedule-40 aluminum pipes are used. The sectional properties of the pipes (Weidlinger 1956) are tabulated in the Table 1. The unknown

Table 1 Sectional properties of Schedule 40 aluminum pipes (Weidlinger 1956)

Nominal Size	Sectional Area, A (in^2)	Buckling Coefficient κ
1/8	0.0720	2.0254
1/4	0.1250	2.0935
3/8	0.1670	2.5814
1/2	0.2503	2.6922
3/4	0.3326	3.3036
1	0.4939	3.5342
$1^{1/4}$	0.6685	4.2998
$1^{1/2}$	0.8000	4.7855
2	1.075	5.6908
$2^{1/2}$	1.704	5.1988
3	2.228	5.9963
$3^{1/2}$	2.680	6.5812
4	3.174	7.0854
$4^{1/2}$	3.688	7.5771
5	4.300	8.0938
6	5.581	8.9162
7	6.926	9.5714
8	8.399	10.141
10	11.91	11.187
12	15.74	11.953

coefficients in the assumed polynomial for the buckling coefficients are determined by a curve-fitting process as

$$\kappa_i = 2.5413 + 1.9380A_i - 0.1655A_i^2 + 0.0051A_i^3$$

where the cross-sectional areas are in in^2. The buckling coefficients versus cross-sectional areas are plotted in the Fig. 1. The fitted curve is also shown. The elasticity modulus is 10^7 psi and the allowable stresses are $\pm 20{,}000$ psi. The lower and upper bounds on all design variables are 0.1 and 10 times the initial design values, respectively.

4.1. 13-bar truss with stress and Euler buckling constraints

The initial geometry and the loading are shown in the Fig. 2. The initial sizing is 1 in^2 for all members. The sizing variables are linked as

$$a_1 = A_1 = A_{12},\ a_2 = A_2 = A_{13},\ a_3 = A_3 = A_{11}$$
$$a_4 = A_4 = A_8,\ a_5 = A_5 = A_9,\ a_6 = A_6 = A_{10}$$

The coordinates that are varied are X_2, X_6, Y_2, Y_4 and Y_6 with a linking $x_1 = X_2 = -X_6$ and $x_2 = Y_2 = Y_6$. The design obtained is given in Table 2 where the first column shows the continuous design obtained at the end of the first step and the second column shows the final design with available sections obtained at the end of second step.The final configuration is shown in Fig. 3. In the present solution, the compression members are at the Euler buckling limit and the members

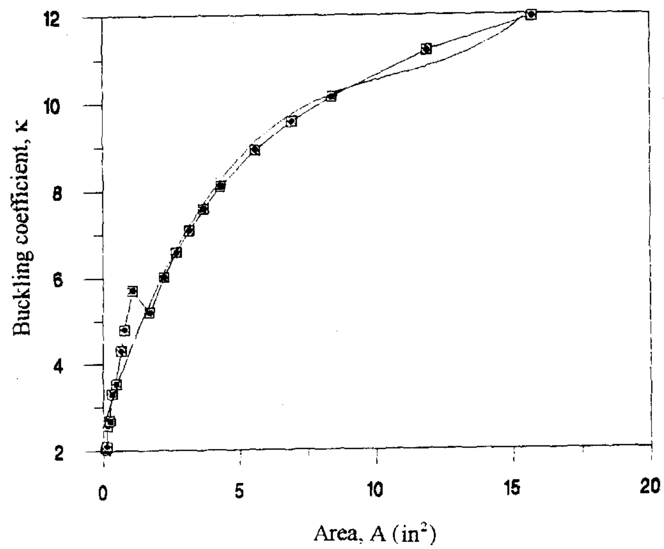

Fig. 1 Buckling coefficient vs. cross-sectional area for Schedule-40 aluminum pipes.

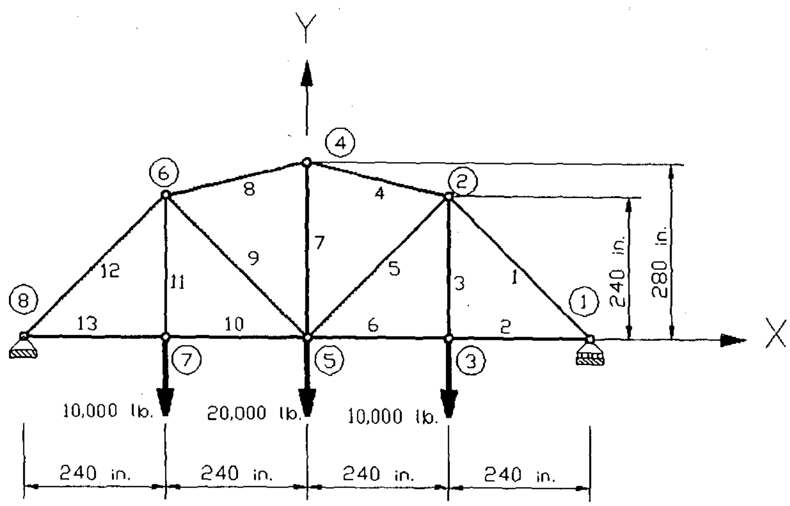

Fig. 2 13-bar truss, loading and initial geometry.

5, 6, 9 and 10 are fully stressed. The convergence of the design for both steps is shown in Fig. 4. It is seen in this figure that the design starts from the infeasible domain in the first step but approaches the optimum quickly, and the second step does not change the configuration considerably but serves only as a correction for the replacement of the continuous design of the first step by the available cross-sections.

4.2. 18-bar truss with stress and Euler buckling constraints

The initial geometry and the loading are shown in the Fig. 5. The initial sizes are

Table 2 13-bar truss with stress and Euler buckling constraints

	1st step	2nd step
a_1 (in^2)	4.923	5.581
a_2 (in^2)	1.106	1.704
a_3 (in^2)	0.514	0.669
a_4 (in^2)	5.295	5.581
a_5 (in^2)	0.183	0.250
a_6 (in^2)	1.227	1.704
A_7 (in^2)	0.805	0.799
x_1 (in)	283.0	282.8
x_2 (in)	178.1	178.3
Y_4 (in)	260.0	260.5
Optimal weight (lb)	737.7	795.2
# iterations	15	5

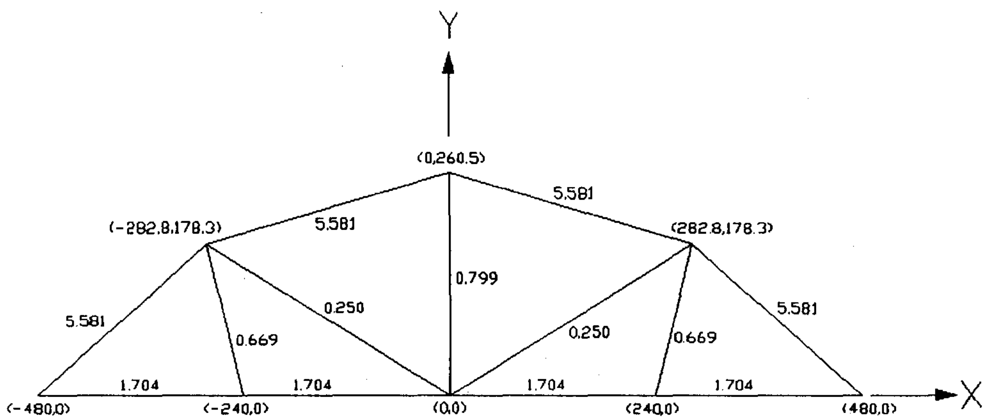

Fig. 3 Final geometry of 13-bar truss, coordinates are in inches, cross-sectional areas are in in^2.

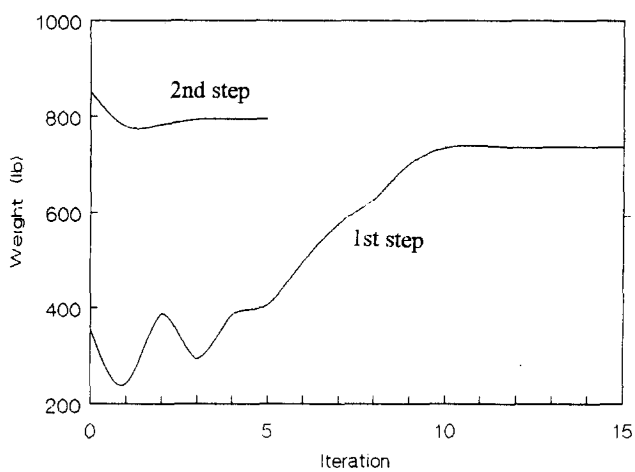

Fig. 4 Convergence of design in 13-bar truss.

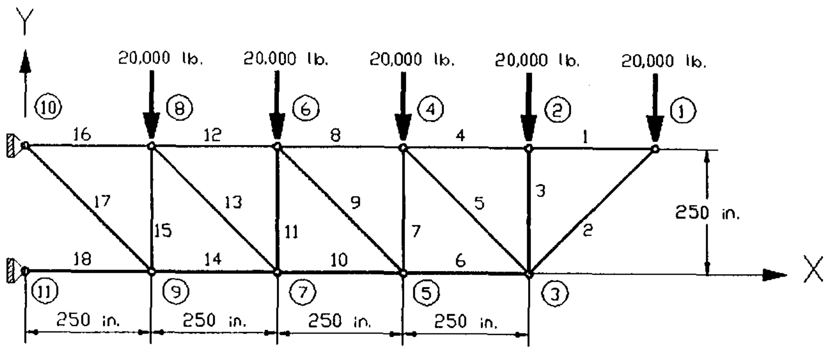

Fig. 5 18-bar truss, loading and initial geometry.

Table 3 18-bar truss with stress and Euler buckling constraints

	1st step	2nd step
a_1 (in^2)	13.06	15.74
a_2 (in^2)	15.46	15.74
a_3 (in^2)	3.201	3.174
a_4 (in^2)	2.305	2.228
X_3 (in)	968.3	985.9
Y_3 (in)	225.0	231.0
X_5 (in)	717.0	719.6
Y_5 (in)	185.0	193.4
X_7 (in)	477.7	468.3
Y_7 (in)	128.3	140.5
X_9 (in)	279.3	235.6
Y_9 (in)	69.80	75.2
Optimal weight (lb)	3981.9	4325.5
# iterations	26	15

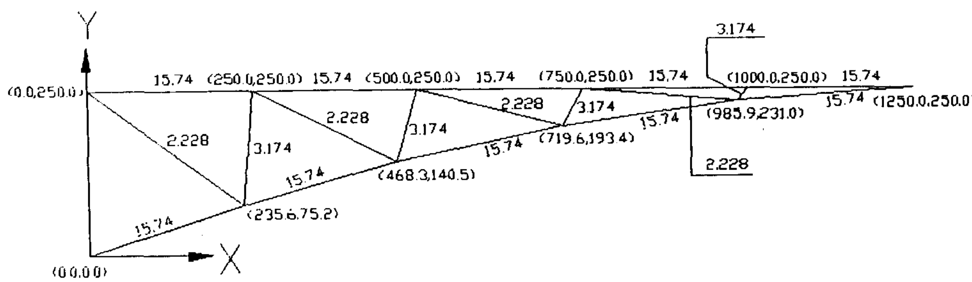

Fig. 6 Final geometry of 18-bar truss, coordinates are in inches, cross-sectional areas are in in^2.

$$a_1 = A_1 = A_4 = A_8 = A_{12} = A_{16} = 10 \text{ in}^2,$$
$$a_2 = A_2 = A_6 = A_{10} = A_{14} = A_{18} = 21.65 \text{ in}^2$$
$$a_3 = A_3 = A_7 = A_{11} = A_{15} = 12.5 \text{ in}^2$$

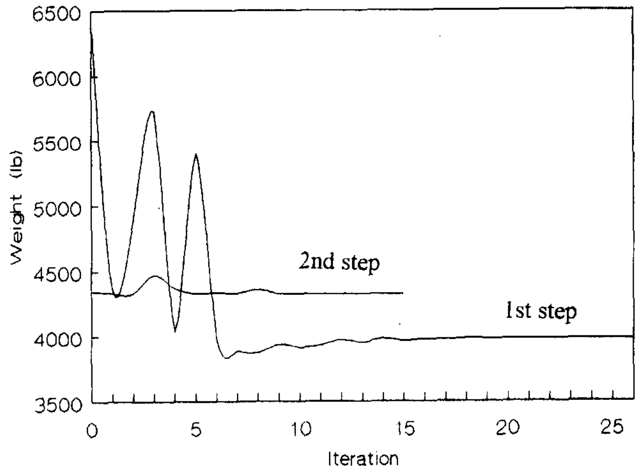

Fig. 7 Convergence of design in 18-bar truss.

$$a_4 = A_5 = A_9 = A_{13} = A_{17} = 7.07 \text{ in}^2$$

The coordinates X_3, Y_3, X_5, Y_5, X_7, Y_7, X_9 and Y_9 are varied. The result obtained is given in Table 3. In Fig. 6, the final configuration is shown. The convergence of the design is shown in Fig. 7. It is seen that the effect of replacing the continuous solution with available sections is more pronounced in this case as the second step changes the design more considerably compared with the previous example.

5. Conclusions

A two-step method has been presented for the optimum design of trusses with available sections with constraints on strength and local stability. In the first step, the member sizes and truss geometry are optimized by a continuous solution considering the properties of commercially available sections in an approximate manner. In the second step, the member sizes determined in the first step are fixed to the nearest available section and the geometry of the truss is further changed to reach the optimum with available sections. The design is improved significantly by considering the sectional properties of the available cross-sections throughout the optimization process rather than heuristically converting a continuous solution to a discrete one at the end. The method presented is also much faster than the discrete methods, therefore it is better suited to the practical applications, however it cannot be applied to trusses for which the geometry is to remain fixed.

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Appendix

Consider a truss member of length L with a nodal connectivity of m and n such that the origin of the local coordinate is at m. The global coordinate system is $X_1X_2X_3$ where $X_1=X$, $X_2=Y$, $X_3=Z$. Let the direction cosines of the member be λ_1, λ_2, λ_3. Then,

$$\frac{\partial L}{\partial X_i^{(m)}} = -\lambda_i$$

$$\frac{\partial(1/L^2)}{\partial X_i^{(m)}} = -\frac{2\lambda_i}{L^3}$$

$$\frac{\partial(\lambda_j^2/L)}{\partial X_i^{(m)}} = 3\lambda_i\lambda_j^2 - 2\delta_{ij}\lambda_j$$

$$\frac{\partial(\lambda_j\lambda_k/L)}{\partial X_i^{(m)}} = \delta_{ij}\lambda_k(3\lambda_i^2-1)+\delta_{ki}\lambda_j(3\lambda_i^2-1)+3(1-\delta_{ij})(1-\delta_{ki})\,\lambda_i\lambda_j\lambda_k \qquad (j\neq k)$$

where δ_{ij} is the Kronecker delta. Note that

$$\frac{\partial(\ )}{\partial X_i^{(n)}} = -\frac{\partial(\ )}{\partial X_i^{(m)}} \text{ and } i,\ j,\ k=1,\ 2,\ 3.$$