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Inelastic response of code-designed eccentric structures subject to bi-directional loading

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Abstract. The influence of bi-directional earthquake-induced loading on eccentric (plan-asymmetric) building systems has been investigated. In the first part of the study, comparisons have been made with equivalent results from uni-directional studies. The results are important in developing analytical models appropriate to the formulation of design recommendations. It is concluded that for valid comparisons, both perpendicular horizontal earthquake components must be considered when using models with transversely-orientated elements. In the second part of the study, an assessment has been made of a simplified, uni-directional (lateral) design approach. For stiffness-eccentric systems, the latter approach gives accurate and reasonably conservative estimates of the critical flexible-edge deformation, but may under estimate the stiff-edge element ductility demand by a factor of two in the short-period range.

Key words: eccentric systems; inelastic torsional effects; bi-directional loading.

1. Introduction and objectives

A significant aspect of the analysis of inelastic torsional effects using simplified stiffness-eccentric building models is the number of structural elements (typically frames and/or shear walls) used to resist the earthquake-induced loadings. Most buildings have load-resisting elements aligned along two orthogonal (principal) directions. These principal directions are herein defined as lateral (y) and transverse (x), as in Fig. 1. In design, the seismic forces in the two directions are assumed to act separately along each of these two directions (uni-directional or planar approach). Correspondingly, in most seismic torsional analyses (Chandler, *et al.* 1994, Chopra and Goel 1991, Goel and Chopra 1994, Tso and Zhu 1992), a uni-directional approach has been adopted. In such analyses, the inclusion or otherwise of the transverse elements numbered 4 and 5 in Fig. 1(a) is a controversial subject first addressed by the authors in Correnza, *et al.* 1994, and is an issue that has yet to be fully resolved.

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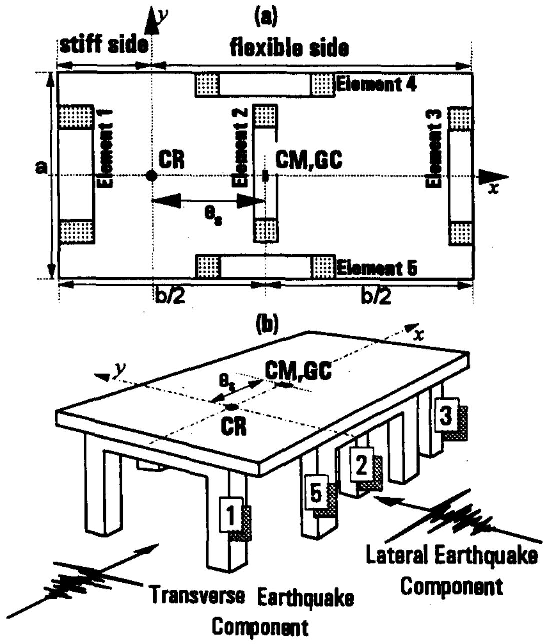

Fig. 1 (a) Plan configuration of 5-element stiffness-eccentric model, and (b) 3-D representation of model subjected to bi-directional ground motions.

In Chopra and Goel, 1991 and Goel and Chopra, 1994, transverse elements were included based on arguments of increased realism, but given that loading was considered to be uni-directional, this mixed approach may not be appropriate. In particular, it has been shown (Correnza, *et al.* 1994) that in seismic response to severe ground motions, the transverse elements remain elastic for long intervals during the response history, and hence provide significant elastic torsional bracing to the structure. The mixed approach has recently been justified (De La Llera and Chopra 1994) on the basis that the transverse earthquake component is of much lower strength than the primary, lateral component. Evidence from past earthquakes, however, does not strongly support this argument (Mohraz and Elghadamsi 1989).

Equally, the fully uni-directional approach adopted in Tso and Zhu (1992) and Chandler, *et al.* (1994) (and in many other studies of the inelastic torsional problem) may not, in certain circumstances, accurately model the dynamic torsional effects arising in realistic, fully bi-directional systems. Whilst it is possible to overcome this problem by adopting the fully bi-directional approach with respect to both structural modelling and ground motion input (Wong and Tso 1994), such analyses are disadvantaged both by the increased computational effort and, perhaps more importantly, by an increase in the number of parameters needed to define the key properties of the system.

From the foregoing review, it is clear that if the fully uni-directional or planar approach is to be utilised in the continued evaluation of code torsional provisions, a rigorous comparison must be made of the dynamic inelastic response of eccentric systems defined and loaded by the foregoing approaches. This paper provides such a comparison, and draws conclusions regarding the range of applicability of the simplified, uni-directional approach. The concept of a planar approach, in which torsional effects are considered for seismic loading acting in each of the principal directions separately, is also incorporated into the design recommendations of the draft European seismic code EC8 (European Committee for Standardisation 1994), and furthermore this approach is not specifically excluded in North American standards such as the U.S. code UBC (International Conference of Building Officials 1991) and the Canadian seismic code NBCC (Associate Committee 1995). Hence, the research presented in this paper has direct relevance not only to the evaluation of inelastic torsional response effects, but also to the development of appropriate methods of analysis for design of eccentric building structures according to seismic building codes.

2. Definition of eccentric structural models

Fig. 1(a) shows the plan configuration of the single-storey, mono-symmetric, torsionally unbalanced (TU) structural models adopted herein. The lateral (y-direction) earthquake resistance is provided by three beam-column elements (1, 2, 3), which along with two transverse elements (4 and 5) support a rigid floor diaphragm, of dimensions b and a. Unless otherwise stated, the plan aspect ratio $\lambda(=b/a)$ has been assigned the typical value 2.0. The centre of resistance (stiffness), CR, centre of mass, CM, and geometric centroid, GC, are noted along with the static eccentricity in the x-direction, e_S (normalised value $\grave{e}_S = e_S/b$, taken over the range 0~0.4). The stiff and flexible sides of the building are defined. The lateral period, T_y, has been varied in the range 0.2~2.5 sec, to represent a wide range of prototypical low to medium-rise buildings.

The model configuration studied herein has moderate torsional stiffness provided by the lateral elements; this is achieved by assigning equal lateral stiffnesses to elements 2 and 3. With the exception of element 2, all elements are located at the extreme edges of the floor plan. The transverse torsional stiffness ratio, γ_t, is defined as the ratio of torsional stiffness (about CR) provided by the transverse elements alone, to the total torsional stiffness of the system; γ_t is varied by altering the ratio of the transverse stiffness provided by the identical elements 4 and 5, to the lateral stiffness of elements 1-3.

The total nominal design strengths of resisting elements in the lateral and transverse directions have been obtained directly from the median Newmark Hall elastic 5% damped design spectrum for hard soil sites, reduced by a factor of 5 to allow for the structure's capacity for inelastic (ductile) response. The lateral and transverse design strengths are equal, in cases where the transverse lateral stiffness ratio is unity. The transverse strength is distributed equally to elements 4 and 5, and is then increased appropriately by a factor of 1.1~1.2, to allow for codified accidental torsional provisions (Chandler, *et al.* 1994). Hence in the transverse direction, the structure is torsionally balanced (TB). The static torsional provisions of five leading national/regional seismic design standards have been utilised to distribute the lateral strength amongst elements 1, 2 and 3, as described fully in Chandler, *et al.* 1994. These comprise the U.S. code UBC (International Conference of Building Officials, 1991), referred to herein as UBC 91, as well as corresponding provisions of the Canadian (NBCC), New Zealand (NZS), Australian (AS) and European (EC8) codes.

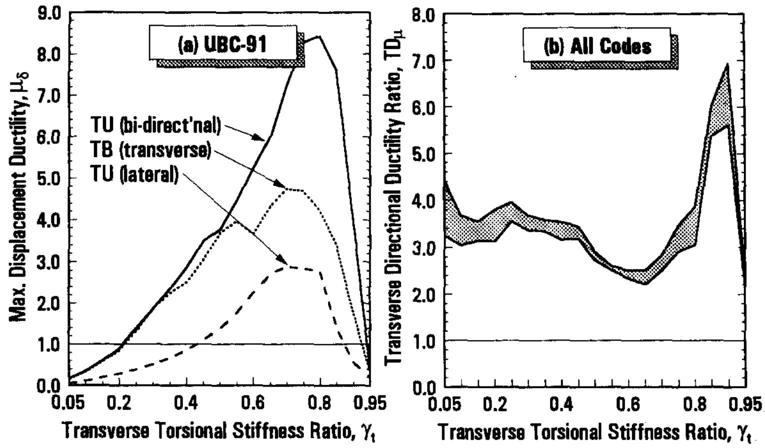

Fig. 2 (a) Maximum displacement ductility of transverse elements [UBC-designed systems, $T_y=0.5$ sec, $\grave{e}_S=0.4$, $\lambda=1.0$], and (b) Transverse directional ductility ratio for all codes.

3. Effect of including transverse earthquake ground motion

The performance of edge elements in TU structural models subjected to both uni and bi-directional ground motions has been evaluated in this first part of the study. For a building of fixed plan aspect ratio, λ, as the ratio of transverse to lateral stiffness increases there is a corresponding increase in γ_t. For buildings with square plan shape ($\lambda=1$), the maximum realistic value of γ_t is in the order of 0.9 (Correnza 1994), representing a system in which the transverse elements provide 90% of the total torsional stiffness. Typical responses of the transverse elements in such systems have been plotted in Fig. 2(a) using the UBC 91 design torsional provisions. The maximum ductility demand, μ_δ, has been computed for models with a range of γ_t, subjected to an ensemble of six spectrum-compatible strong-motion western U.S. earthquake records (as described in Chandler, *et al.* 1994). The larger of the two horizontal ground motion components has been applied in the lateral direction.

The TU models have large stiffness eccentricity in the x-direction [$\grave{e}_S=0.4$], and hence the transverse elements respond significantly as a result of the rotational floor response, even when the loading is lateral only (dashed curve). Maximum response ($\mu_\delta \approx 3$) occurs for systems with γ_t in the range 0.6~0.8, a range which according to the study by Wong and Tso (1994) may commonly be found in prototypical multistorey buildings.

The transverse element response shows a significant further increase when the transverse earthquake component is also applied (bi-directional loading, solid curve). For $\gamma_t>0.5$, the transverse element response for the bi-directional case exceeds the reference case (dotted curve), consisting of the transverse ground motion applied independently (TB in this direction). This illustrates clearly the effect of torsion on the magnitude of the transverse element seismic response. This, in turn, depends significantly on the torsional resistance provided by the transverse elements, with lower resistance in systems having larger plan aspect ratio, λ.

The ratio of transverse element responses in structures subjected to bi-directional and uni-directional (lateral) loadings (TD_μ), has been summarised in Fig. 2(b), which shows the range of values obtained for systems designed for torsion according to all five codes referred to above.

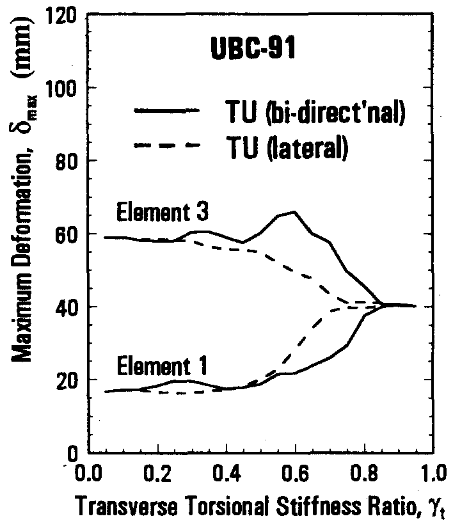

Fig. 3 Maximum element deformations in systems subjected to uni- and bi-directional ground motions [$\grave{e}_S$=0.2, T_y=0.5 sec, λ=1.0].

The increase arising from bi-directional loading is in the order of 300~400% (and in some cases even higher), throughout the range of γ_t. The primary cause of this increase is the reduction of torsional resistance, arising when the transverse earthquake component causes elements 4 and 5 to yield. Whilst the lateral earthquake component alone may, for some systems, cause slight yielding Fig. 2(a), this is increased dramatically by the application of the orthogonal earthquake component.

The difference between the responses of the edge lateral elements 1 and 3 (Fig. 1) is a measure of the torsional system response. Fig. 3(a) shows clearly that this response increases significantly when bi-directional loading is applied. Systems with very high values of γ_t are torsionally stiff with respect to the transverse elements, and as expected these behave as SDOF systems with little or no torsional response arising due to the asymmetry.

4. Applicability of uni-directional model for torsional response

The second part of the study assesses a simplified, uni-directional (lateral) design approach by investigating the inelastic response of the key lateral edge elements (1 and 3 in Fig. 1). Results in Fig. 4 relate to a structure having total transverse stiffness equal to the total lateral stiffness, with typical plan aspect ratio λ=2.0. For this case, γ_t varies between about 0.3 and 0.5, for $\grave{e}_S$ increasing from 0.1 to 0.4 (Correnza 1994). The results presented represent the range of responses determined for structures that were designed according to all five code torsional provisions listed above. The lateral directional ductility ratio, D_μ, is defined (for a given element) as the ratio of ductility demand arising in a system subject to bi-directional loading, to that of the same system loaded by uni-directional ground motion.

The results in Fig. 4 ($\grave{e}_S$=0.4) demonstrate that the transverse earthquake component has a significant effect on lateral element response for short to medium-period eccentric systems (T_y<1.2 sec). For such systems, large torsional deformations coupled with transverse loading cause the transverse elements to respond well into the inelastic range, and hence they provide little elastic

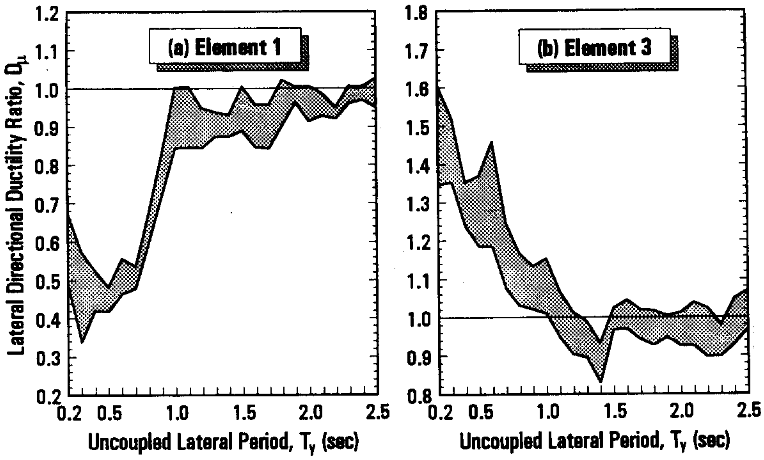

Fig. 4 Effect of bi-directional loading on the ductility ratios of the key lateraledge elements [$\lambda=2.0$, $\grave{e}_S=0.4$].

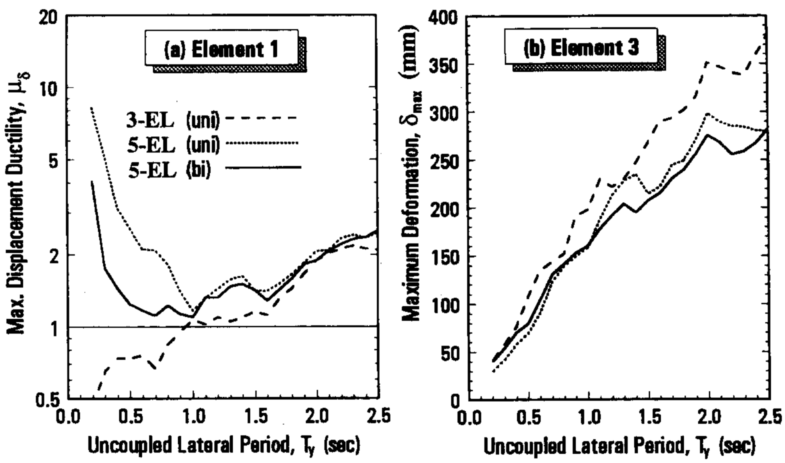

Fig. 5 Effect of various modelling and loading combinations on the inelastic response of the key lateral edge elements [$\lambda=2.0$, $\grave{e}_S=0.4$]

bracing. This is represented in Fig. 4 by an increase in D_μ for element 3, and a corresponding reduction in D_μ for element 1.

The additional response of the flexible-edge element 3 due to bi-directional loading may be in the order of 30~50%, for short-period systems. Results given in Correnza (1994) for moderately eccentric systems show that increases of similar magnitude occur, even when $\grave{e}_S$ is reduced to 0.2. In contrast, the response of the stiff-edge element 1 decreases by up to 60% due to the application of bi-directional seismic loading. For long-period systems, the bi-directional loading effect is negligible. It may be deduced that these systems are much less sensitive to the loss of elastic torsional stiffness due to increased yielding in the transverse elements.

From the results given in Fig. 4, it is concluded that for short- to medium- period systems, accurate representation of inelastic dynamic torsional effects in models incorporating transverse elements (Fig. 1) requires the application of the transverse earthquake component simultaneously with the lateral component (such as in Wong and Tso 1994). Given the increased computational effort involved in this procedure, it is important to evaluate differences between this method, and the fully uni-directional approach discussed in Section 1 above.

Typical results illustrating this comparison have been presented in Fig. 5, in which the dashed curves correspond to a 3-element model incorporating no transverse elements (fully uni-directional), as employed in a number of inelastic torsional research studies (for example Tso and Zhu 1992 and Chandler, *et al.* 1994). The solid curves correspond to the realistic, fully bi-directional approach, and the dotted curves to the 5-element model subjected to uni-directional loading only, as in Chopra and Goel (1991) and Goel and Chopra (1994). As in previous studies (Tso and Zhu 1992 and Chandler, *et al.* 1994), the critical inelastic response parameter for the stiff-edge element 1 is the maximum ductility demand, μ_δ in Fig. 5(a). For the flexible-edge element 3, the critical parameter is the maximum inelastic deformation demand, δ_{max} in Fig. 5(b).

Fig. 5(a) shows that all three approaches give similar results for medium to long period systems. In the short period range, the fully uni-directional model underestimates the stiff-edge element ductility response, compared to the fully bi-directional case. The underestimation may be significant in the context of design recommendations based on the uni-directional response of 3-element structural systems, such as those made by Tso and Zhu (1992) and by Chandler, *et al.* (1994).

For the flexible-edge element 3 in Fig. 5(b), the 3-element uni-directional model gives acceptably conservative results throughout the period range considered, and is a particularly good match to the bi-directional results (solid curve) in the short period range. As expected, a comparison of the dotted and solid curves in Fig. 5 reveals the same trends, for both elements 1 and 3, as previously observed in Fig. 4. The difference between uni- and bi-directional results is not large for element 3, although in percentage terms it may be considered significant in the short period range.

For element 1 at the stiff-edge of the building, unlike the fully uni-directional approach (3-element model), applying uni-directional loading to the 5-element model leads to conservative estimates of peak ductility demand, but this approach may be overly conservative in the short period range. All three approaches give very similar results for systems having fundamental lateral periods $T_y > 1.5$ sec.

5. Conclusions

Accurate evaluations of inelastic dynamic torsional effects in eccentric structures require both perpendicular horizontal earthquake components to be considered when adopting models with transversely-orientated elements. A fully uni-directional approach neglecting transverse elements and loading, has been found to give accurate and reasonably conservative estimates of the critical flexible-edge deformation, but may underestimate the stiff-edge element ductility demand by a factor of two in the short-period range.

Based on these results, it is considered that a re-evaluation should be made of the analytical approach utilised for inelastic torsional response studies, and that some degree of caution be applied to the interpretation of results from the many previous studies of this problem that have adopted a purely uni-directional approach to structural modelling, and to the definition of seismic ground motion input. The results summarised in this study also have important implications for the formulation of analytical models for eccentric building systems as stipulated in seismic design codes, several of which permit a planar approach to the analysis of torsional effects. This approach, whilst reasonably accurate and conservative for medium to long-period systems, may not be appropriate for analyzing the response of the stiff-edge element in short-period systems.

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