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Exact solution for transverse bending analysis of embedded laminated Mindlin plate

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Abstract. Laminated Rectangular plates embedded in elastic foundations are used in many mechanical structures. This study presents an analytical approach for transverse bending analysis of an embedded symmetric laminated rectangular plate using Mindlin plate theory. The surrounding elastic medium is simulated using Pasternak foundation. Adopting the Mindlin plate theory, the governing equations are derived based on strain-displacement relation, energy method and Hamilton's principle. The exact analysis is performed for this case when all four ends are simply supported. The effects of the plate length, elastic medium and applied force on the plate transverse bending are shown. Results indicate that the maximum deflection of the laminated plate decreases when considering an elastic medium. In addition, the deflection of the laminated plate increases with increasing the plate width and length.

Keywords: transverse bending; mindlin plate theory; energy method; pasternak model

1. Introduction

Laminated Rectangular plates resting on elastic foundations occupy a prominent place for many practical problems in contemporary structural mechanics. Examples are operational activities of large transportation aircraft on runways, footings, mat foundations, foundation of spillway dam, floor system, civil building in cold regions, foundations of deep wells, ship and bridge structures. Knowledge of the bending characteristics of these structures is important. The problem of bending of thick plates has attracted considerable attention in recent years.

In the classical plate theory (CPT), it is assumed that plane sections initially normal to the mid surface before deformation remain plane and normal to that surface after deformation. This is the result of neglecting transverse shear strains. However, non-negligible shear deformations occur in thick and moderately thick plates and the theory gives inaccurate results for laminated plates. So, it is obvious that transverse shear deformations have to be taken into account in the analysis. One of the well known plate theories is the Mindlin model Mindlin (1951) which takes the displacement field as linear variations of midplane displacements. In this theory, the relation between the

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resultant shear forces and the shear strains is affected by the shear correction factors. This method has some advantages due to its simplicity and low computational cost.

The effect of transverse shear deformation on the bending of elastic plates was studied by Reissner (1945). Norman *et al*. (1997) proposed first-order shear deformation theory for bending analysis of laminated plates. Nonlinear bending analysis of composite laminated plates was done by Fares (1999) using a refined first-order theory. Swaminathan and Ragounadin (2004) applied an analytical solution for static analyzing of antisymmetric angle-ply composite and sandwich plates based on a higher-order refined theory. Analysis of composite plates using higher-order shear deformation theory and a finite point formulation based on the multiquadric radial basis function method was presented by Ferreira *et al*. (2003). Zenkour (2003) proposed an exact mixed-classical solution for the bending analysis of shear deformable rectangular plates. A simple higher order theory for laminated composite plates was presented by Reddy (1984). A new boundary element method is used by Bezine (1988) for Bending of plates on elastic foundations. El-Zafrany *et al*. (1995) Analyzed thin plates on Winkler foundation using a new fundamental solution for boundary element. Choudhary and Tungikar (2011) analyzed the geometrically nonlinear behavior of laminated composite plates using the finite element analysis. They studied the effect of number of layers, effect of degree of orthotropy (both symmetric and antisymmetric) and different fibre orientations on central deflections. The wave propagation in a generalized thermo elastic plate embedded in an elastic medium (Winkler model) is studied by Ponnusamy and Selvamani (2012). Reddy *et al*. (2012) studied the effect of transverse shear deformation on deflection and stresses of laminated composite plates subjected to uniformly distributed load using finite element analyses. They observed that, the deflections are larger for smaller modulus ratios and aspect ratios, the degree of orthotropy has less influence on the deflections for large ratios of E_1/E_2, the effect of shear deformation is to decrease the deflections and increase the stresses with the increase of modulus ratios and side-to-thickness ratios. To analyze the complex nonlinear dynamic behaviour of the laminated composite piezoelectric rectangular thin plate they used phase portraits and Lyapunov exponents. Transverse bending of shear deformable laminated composite plates in Green-Lagrange sense accounting for the transverse shear and large rotations are presented by Dash and Singh (2010).

All these researchers have considered Winkler model for simulation of elastic medium. In this simplified model, a proportional interaction between pressure and deflection of plate is assumed, which is carried out in the form of discrete and independent vertical springs. Whereas, Pasternak (1954) suggested considering not only the normal stresses but also the transverse shear deformation and continuity among the spring elements, and its subsequent applications for developing the model for buckling analysis, which proved to be more accurate than the Winkler model. Buczkowski and Torbacki (2001) used finite element for modelling of thick plates on two-parameter elastic foundation. Based on the boundary element method, analysis of plates on two-parameter elastic foundations with nonlinear boundary conditions was studied by Chucheepsakul and Chinnaboon (2003). Sladek *et al*. (2002) investigated meshless local boundary integral equation method for simply supported and clamped plates resting on elastic foundation. Akhavan *et al*. (2009a, b) introduced exact solutions for the buckling analysis of rectangular Mindlin plates subjected to uniformly and linearly distributed in-plane loading on two opposite edges simply supported resting on elastic foundation. Baltacıoğlu *et al*. (2011) presented the nonlinear static analysis of a rectangular laminated composite thick plate resting on nonlinear two-parameter elastic foundation with cubic nonlinearity. They used the first-order shear deformation theory (FSDT) for plate formulation and investigated the effects of foundation and geometric parameters

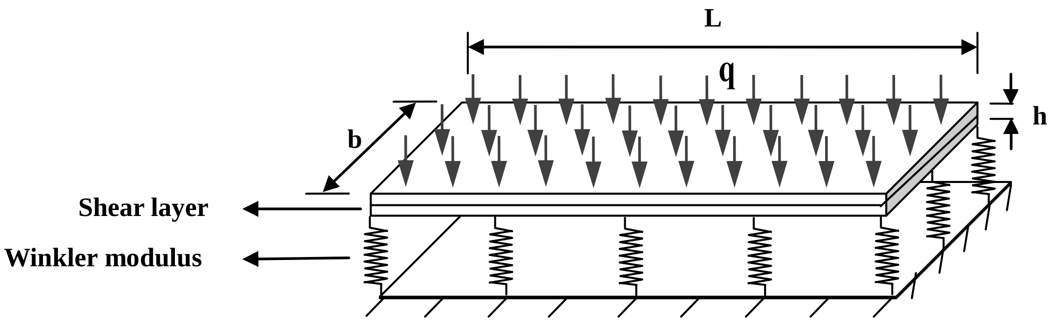

Fig. 1 The Laminated plate embedded in elastic foundations with external loads

of plates on nonlinear deflections.

However, in the present study, the Mindlin plate theory is used for transverse bending behavior of laminated plates resting on Pasternak foundation. The governing equations are obtained based on energy method and Hamilton's principal. An exact solution is applied for transverse bending deflections of symmetric laminated plate. The effects of the plate width, plate length, elastic medium constants and applied force on the deflection of the laminated plate are disused in detail.

2. Modelling of the plate

A laminated rectangular plate as depicted in Fig. 1 with length L, width b and thickness h is considered. The plate is surrounded by an elastic medium which is simulated using Pasternak foundation. As is well known, this foundation model is both capable of transverse shear loads (k_g) and normal loads (k_w).

3. Constitutive relations for plate

The constitutive equation for stresses $\boldsymbol{\sigma}$ and strains $\boldsymbol{\varepsilon}$ matrix may be written as follows (Yas and Sobhani 2010)

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}, \tag{1}$$

where $C_{ij}(i, j=1,2,\ldots,6)$ denotes elastic coefficients. For solving the above equation, the strain components are calculated using Mindlin plate theory.

3.1 Mindlin plate theory

Based on Mindlin plate theory, the displacement field can be expressed as (Samaei *et al*. 2011)

$$u_x(x,y,z,t) = z\psi_x(x,y,t),$$

$$u_y(x,y,z,t) = z\psi_y(x,y,t),$$

$$u_z(x,y,z,t) = w(x,y,t), \tag{2}$$

where $\psi_x(x, y)$ and $\psi_y(x, y)$ are the rotations of the normal to the mid-plane about x- and y-directions, respectively.

The von Kármán strains associated with the above displacement field can be expressed in the following form

$$\varepsilon_{xx} = z\frac{\partial\psi_x}{\partial x} \tag{3}$$

$$\varepsilon_{yy} = z\frac{\partial\psi_y}{\partial y} \tag{4}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \psi_y \tag{5}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \psi_x \tag{6}$$

$$\gamma_{xy} = z(\frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_y}{\partial x}), \tag{7}$$

where (ε_{xx}, ε_{yy}) are the normal strain components and (γ_{yz}, γ_{xz}, γ_{xy}) are the shear strain components.

3.2. Energy method

The total potential energy, V of the laminated rectangular plate is the sum of strain energy, U and the work done by the elastic medium, W.

The strain energy can be written as

$$U = \frac{1}{2}\int_{\Omega_0}\int_{-h/2}^{h/2}\left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{xy}\gamma_{xy} + \sigma_{xz}\gamma_{xz} + \sigma_{yz}\gamma_{yz}\right)dV\,, \tag{8}$$

Combining of Eqs. (2)-(8) yields

$$U = \frac{1}{2}\int_{\Omega_0}\left(M_{xx}\frac{\partial\psi_x}{\partial x} + M_{yy}\frac{\partial\psi_y}{\partial x} + M_{xy}\left(\frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_y}{\partial x}\right) + Q_{xz}\left(\frac{\partial w_0}{\partial x} + \psi_x\right) + Q_{yz}\left(\frac{\partial w_0}{\partial y} + \psi_y\right)\right)dxdy \tag{9}$$

where the stress resultant-displacement relations can be written as

$$\{(N_{xx}, N_{yy}, N_{xy}), (M_{xx}, M_{yy}, M_{xy})\} = \int_{-h/2}^{h/2} \{\sigma_{xx}, \sigma_{yy}, \tau_{xy}\}(1, z)dz, \tag{10}$$

$$\{Q_{xx}, Q_{yy}\} = K\int_{-h/2}^{h/2} \{\tau_{xz}, \tau_{yz}\}dz, \tag{11}$$

in which K is shear correction coefficient.

The external work due to surrounding elastic medium can be written as

$$W = \frac{1}{2}\int_0^L q\, w dx, \tag{12}$$

where q is related to Pasternak foundation and distributed load on upper surface of the plate.

Pasternak foundation can be expressed as Khajeansari *et al*. (2012)

$$q = -k_w w + k_g \nabla^2 w, \tag{13}$$

where k_w and k_g are spring constant of Winkler type and shear constant of Pasternak type, respectively.

The governing equations can be derived by Hamilton's principal as follows

$$\delta\int_0^t (-U + W)dt = 0 \Rightarrow \int_0^t (-\delta U + \delta W)dt = 0 \tag{14}$$

Substituting Eqs. (9) and (12) into Eq. (14) yields the following governing equations

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_{xx} = 0, \tag{15}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_{yy} = 0, \tag{16}$$

$$\frac{\partial Q_{xx}}{\partial x} + \frac{\partial Q_{yy}}{\partial y} + q = 0, \tag{17}$$

Substituting Eq. (1) into Eqs. (10) and (11), the stress resultant-displacement relations can be obtained as follow

$$M_{xx} = \frac{h^3}{12}\left(C_{11}\frac{\partial \psi_{xx}}{\partial x} + C_{12}\frac{\partial \psi_{yy}}{\partial y}\right), \tag{18}$$

$$M_{yy} = \frac{h^3}{12}\left(C_{11}\frac{\partial \psi_{yy}}{\partial y} + C_{12}\frac{\partial \psi_{xx}}{\partial x}\right), \tag{19}$$

$$M_{xy}=\frac{h^3C_{66}}{12}(\frac{\partial\psi_{xx}}{\partial y}+\frac{\partial\psi_{yy}}{\partial x}), \tag{20}$$

$$Q_{xx}=KhC_{55}(\psi_{xx}+\frac{\partial w}{\partial y}), \tag{21}$$

$$Q_{yy}=KhC_{44}(\psi_{yy}+\frac{\partial w}{\partial y}), \tag{22}$$

4. Solution procedure

Steady state solutions to the governing equations of the plate which relate to the simply supported boundary conditions can be assumed as Pietrzakowski (2008)

$$\begin{Bmatrix} w(x,y,t) \\ \psi_x(x,y,t) \\ \psi_y(x,y,t) \end{Bmatrix}=\begin{Bmatrix} \bar{w}\sin(\frac{m\pi x}{L})\sin(\frac{n\pi y}{b}) \\ \bar{\psi}_x\cos(\frac{m\pi x}{L})\sin(\frac{n\pi y}{b}) \\ \bar{\psi}_y\sin(\frac{m\pi x}{L})\cos(\frac{n\pi y}{b}) \end{Bmatrix}. \tag{23}$$

where m and n are the integrals denoting the half wave numbers for the x and y directions. It should be noted, that the plate is assumed to be subjected to a uniform load which can be expressed as Akavci *et al*. (2007)

$$q=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Q_{mn}\sin(\frac{m\pi x}{L})\sin(\frac{n\pi y}{b}) \tag{24}$$

where

$$Q_{mn}=\frac{16q_0}{mn\pi^2} \tag{25}$$

Substituting Eqs. (18)- (24) into (15)-(17) yields

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}\begin{bmatrix} \bar{w} \\ \bar{\psi}_x \\ \bar{\psi}_y \end{bmatrix}=\begin{bmatrix} Q \\ 0 \\ 0 \end{bmatrix}, \tag{26}$$

where

$$A_{11}=-\frac{KC_{55}m^2\pi^2h}{L^2}-\frac{KC_{44}n^2\pi^2h}{b^2}-k_w-k_g\left(\frac{m^2\pi^2}{L^2}+\frac{n^2\pi^2}{b^2}\right), \tag{27}$$

$$A_{12} = -\frac{KC_{55} m\pi h}{L}, \tag{28}$$

$$A_{13} = -\frac{KC_{44} n\pi h}{b}, \tag{29}$$

$$A_{22} = -\frac{C_{11} m^2 \pi^2 h^3}{12L^2} - \frac{C_{66} n^2 \pi^2 h^3}{12b^2} - KC_{55} h, \tag{30}$$

$$A_{23} = -\frac{m\pi^2 n h^3}{12Lb}(C_{12} + C_{66}), \tag{31}$$

$$A_{33} = -\frac{C_{66} m^2 \pi^2 h^3}{12L^2} - \frac{C_{11} n^2 \pi^2 h^3}{12b^2} - KC_{44} h. \tag{32}$$

$$Q = \frac{16 q_0}{mn\pi^2} \tag{33}$$

Finally, using Eq. (26) the following displacement and rotations can be obtain

$$\overline{w} = -\frac{(A_{12}A_{33} - A_{13}A_{23})Q}{2A_{13}A_{12}A_{23} - A_{13}^2 A_{22} - A_{12}^2 A_{33} + A_{11}A_{22}A_{33} - A_{23}^2 A_{11}}, \tag{34}$$

$$\overline{\psi}_x = \frac{(-A_{13}^2 + A_{11}A_{33})Q}{2A_{13}A_{12}A_{23} - A_{13}^2 A_{22} - A_{12}^2 A_{33} + A_{11}A_{22}A_{33} - A_{23}^2 A_{11}}, \tag{35}$$

$$\overline{\psi}_y = -\frac{(A_{11}A_{23} - A_{13}A_{12})Q}{2A_{13}A_{12}A_{23} - A_{13}^2 A_{22} - A_{12}^2 A_{33} + A_{11}A_{22}A_{33} - A_{23}^2 A_{11}}, \tag{36}$$

5. Numerical results

A computer program is written and used for solving analytically the transverse bending of laminated plate resting on an elastic foundation. Here, a symmetrically laminated simply supported plate is considered with E_1=181 GPa, E_2=10.3 GPa, G_{12}=7.17 GPa, υ_{12}=0.27, h=2 mm, L=10 m and b=5 m (Akavci *et al*. 2007).

The effect of the plate width on the maximum deflection of the laminated plate with respect to applied force is demonstrated in Fig. 2. It is shown that the applied force can increase the deflection of the laminated plate. This is because the imposed force generates the transverse compressive force in the plate. It is also concluded that increasing the plate width increases the deflection of the laminated plate. This is due to the fact that the increase of plate width leads to a softer structure. Meanwhile, the effect of plate width becomes more prominent at higher applied force.

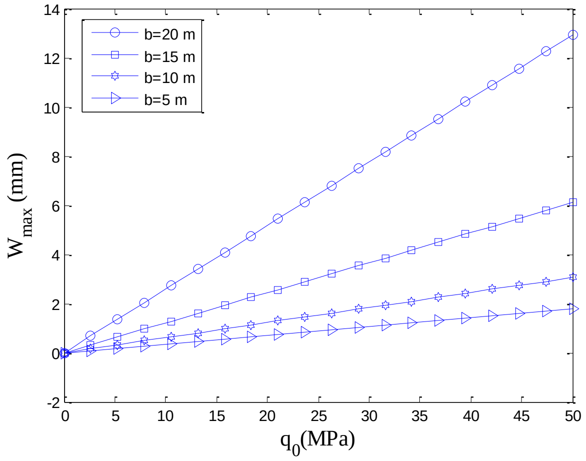

Fig. 2 The effect of the plate width on the maximum deflection of the laminated plate with respect to applied force

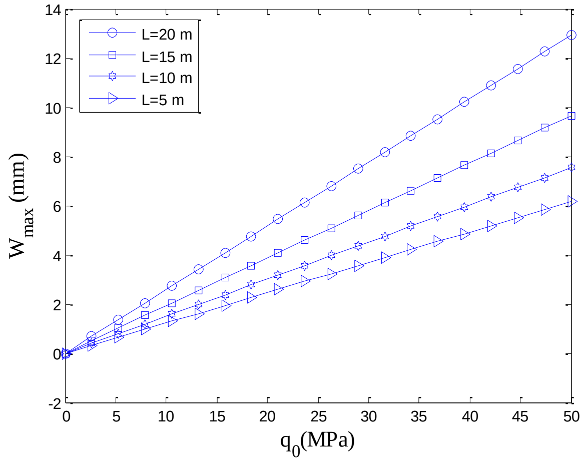

Fig. 3 The effect of the plate length on the maximum deflection of the laminated plate with respect to applied force

The effect of the plate length on the maximum deflection of the laminated plate with respect to applied force is shown in Fig. 3. It can be found that applied force increases the deflection of the laminated plate. It is also observed that increasing the plate length increases the deflection of the laminated plate. This is due to the fact that the increase of plate length leads to a softer structure. Meanwhile, the effect of plate length becomes more considerable at higher applied force.

The effect of the elastic medium on the maximum deflection of the laminated plate with respect to applied force is illustrated in Fig. 4. In this figure three cases are considered which are without elastic medium, Winkler medium and Pasternak medium. As can be seen, considering elastic medium decreases maximum deflection of the laminated plate. It is due to the fact that considering elastic medium leads to stiffer structure. Furthermore, the effect of the Pasternak-type is higher

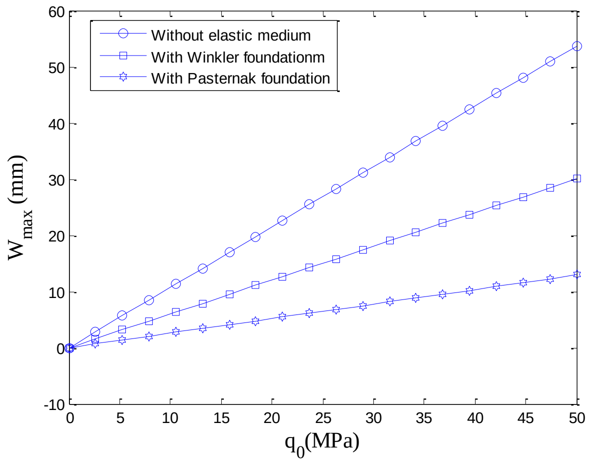

Fig. 4 The effect of the elastic medium on the maximum deflection of the laminated plate with respect to applied force

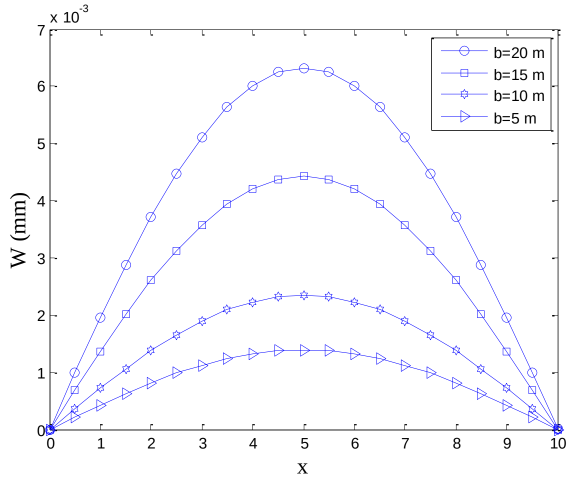

Fig. 5 The deflection of the laminated plate versus plate length for different plate width

than the Winkler-type on the maximum deflection of the laminated plate. It is perhaps due to the fact that the Winkler-type is capable to describe just normal load of the elastic medium while the Pasternak-type describes both transverse shear and normal loads of the elastic medium.

Fig. 5 shows the deflection of the laminated plate versus plate length for different plate width. It is obvious that the simply supported boundary condition at edges of the plate is satisfied. It is also concluded that increasing the plate width increases the deflection of the laminated plate. This is due to the fact that the increase of plate width leads to a softer structure. Meanwhile, the effect of plate width is higher at center of the plate.

Fig. 6 demonstrates the deflection of the laminated plate versus plate length for different applied force. As can be seen increasing the applied force increases the deflection of the laminated plate. Furthermore, the effect of applied force is higher at center of the plate.

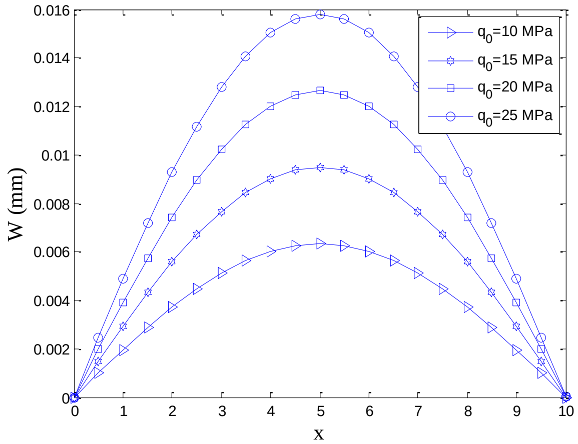

Fig. 6 The deflection of the laminated plate versus plate length for different applied force

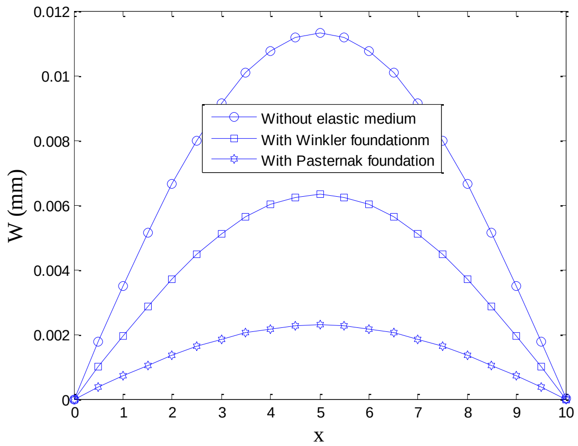

Fig. 7 The effect of elastic medium on the deflection of the laminated plate versus plate length

The effect of the elastic medium on the deflection of the laminated plate with respect to length is depicted in Fig. 7. In this figure three cases are considered which are without elastic medium, Winkler medium and Pasternak medium. As can be seen, considering elastic medium decreases deflection of the laminated plate. It is due to the fact that considering elastic medium leads to stiffer structure. Furthermore, the effect of the Pasternak-type is higher than the Winkler-type on the maximum deflection of the laminated plate. It is perhaps due to the fact that the Winkler-type is capable to describe just normal load of the elastic medium while the Pasternak-type describes both transverse shear and normal loads of the elastic medium.

6. Conclusions

Bending analysis of rectangular plates has applications in designing many mechanical system devices such as strain sensor, mass and pressure sensors and atomic dust detectors. Transverse bending analysis of an embedded symmetric laminated rectangular plate based on Mindlin plate theory is the main contribution of the present paper which has been studied for the first time. The laminated rectangular plate was surrounded by an elastic medium which was simulated Pasternak foundation. Using strain-displacement relation, energy method and Hamilton's principle, the governing equations were derived. In order to obtain the deflection of the plate, an exact analysis was performed for the case when all four ends were simply supported. The effects of the plate width, plate length, elastic medium constants and applied force on the deflection of the laminated plate were studied. Results indicated that considering elastic medium decreases maximum deflection of the laminated plate. In addition, the effect of the Pasternak-type was higher than the Winkler-type on the maximum deflection of the laminated plate. It was also found that applied force increases the deflection of the laminated plate. Furthermore, increasing the plate width and length increases the deflection of the laminated plate. Meanwhile, the effect of plate length becomes more considerable at higher applied force.

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