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Damage assessment of frame structure using quadratic time-frequency distributions

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Abstract. This paper presents the processing of nonlinear features associated with a damage event by quadratic time-frequency distributions for damage identification in a frame structure. A time-frequency distribution is a function which distributes the total energy of a signal at a particular time and frequency point. As the occurrence of damage often gives rise to non-stationary, nonlinear structural behavior, simultaneous representation of the dynamic response in the time-frequency plane offers valuable insight for damage detection. The applicability of the bilinear time-frequency distributions of the Cohen class is examined for the damage assessment of a frame structure from the simulated acceleration data. It is shown that the changes in instantaneous energy of the dynamic response could be a good damage indicator. Presence and location of damage can be identified using Choi-Williams distribution when damping is ignored. However, in the presence of damping the Page distribution is more effective and offers better readability for structural damage detection.

Keywords: damage detection; nonlinear features; frame structure; time-frequency distributions; location

1. Introduction

Damage assessment of the engineering infrastructures is necessary to ensure their regular safe operation. Condition evaluation of structures, such as buildings, bridges, nuclear or aerospace structures, becomes a critical concern while decision making after disasters. Traditionally, damage detection of structure is carried out either by visual inspection or localized experimental techniques, such as acoustic or ultrasound waves, radiography, eddy current, thermal or magnetic field methods. However, these techniques require the location of damage to be known a *priori* and also the portion of the structure being inspected should be readily accessible. Such limitations have necessitated the search for reliable nondestructive damage evaluation (NDE) techniques that can provide global and rapid detection of damage (Aktan *et al.* 2000, Van der Auweraer and Peeters 2003, Chang *et al.* 2003). Global methods aim to utilize the changes in overall response characteristics of the structure as indicators of damage.

Among the global methods, vibration-based damage detection (VBDD) from the changes in the

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dynamic properties (e.g., natural frequencies, mode shapes) of a structure has received much attention from the research community (Doebling *et al.* 1996, Sohn *et al.* 2003, Doebling *et al.* 1998). Many damage indices have been proposed using the changes in modal parameters due to damage (Catbas and Aktan 2002). Assessment of some VBDD techniques and the related reviews of literature are presented by various researchers (Ndambi *et al.* 2002, Carden and Fanning 2004, Alvandi and Cremona 2006, Zhou *et al.* 2007, Zhao and DeWolf 2007, Fan and Qiao 2011).

Many VBDD methods employ an analytical model (such as FE model) of the structure and the modal parameters of the undamaged structure are utilized to update the model. Subsequently, the refined model is used in damage detection algorithm. However, there are always errors associated with the formulation of a theoretical model and this leads to uncertain accuracy of the predicted response. Therefore, as pointed out by many researchers, the dependence on the prior analytical model should be minimized or if possible eliminated for the development of a successful damage detection technique.

Some VBDD techniques do not require an analytical model and only few natural frequencies and mode shapes, before and after damage are required. However, the use of modal parameters for structural integrity assessment has a number of shortcomings. Only first few lower vibration modes of a structure can be excited and subsequently extracted for large structures in real-life test conditions. Hence, both the dynamically measured stiffness and flexibility matrices are rank-deficient. Global vibration modes are insensitive to localize small damage. To estimate the mode shapes with sufficient spatial accuracy a number of measurement points are required on the test structure, which might be impractical. Also, modal data shows significant variations due to variability of the environmental factors, such as temperature, humidity. This presents problem in discriminating between the changes due to damage and that due to environmental variability. Further, time-invariant modal parameters cannot capture complex phenomena like crack propagation, opening and closing of fatigue crack or the nonlinear, non-stationary nature of the dynamic response associated with a damage event (e.g. sudden damage). Therefore, the processing of the non-stationary, nonlinear features of the measured response offers valuable alternative for structural damage detection.

Recent developments in wavelet theory have shown some promises for structural damage detection by overcoming many of the limitations of the frequency domain (Staszewski 1998, Rucka and Wlide 2006, Reda Taha *et al.* 2006, Bagheri *et al.* 2009). Researchers have applied short-time Fourier transform (STFT) for fault diagnosis of mechanical components (Wang and McFadden 1993). However, due to the fixed time-window width spectrogram can be applied successfully only when the spectral content of the signal vary slowly.

The application of quadratic time-frequency distributions makes it possible to localize a non-stationary, nonlinear signal in the time-frequency plane with arbitrary time-frequency resolution. Several authors have applied the joint time-frequency distributions for the fault diagnosis of mechanical components (Staszewski *et al.* 1997, Peng *et al.* 2002).

Bonato *et al.* (1997) utilized various time-frequency transforms for structural damage assessment using synthetic and real acceleration signals recorded during the impulsive loading of damaged aluminum beams. Different position and damage levels were considered on the beams for damage identification purpose. They also used neural classification techniques for the diagnostic interpretation of time-frequency maps based on invariant moments. Based on the results they concluded that the Choi-Williams distribution is the best choice for such applications.

Staszewski and Robertson (2007) discussed the two main approaches, the time-frequency and time-scale analysis, for damage detection applied to the time-variant properties of structures. A number of examples are presented to discuss the applicability of the various approaches. Finally, the authors concluded that most of these applications are largely limited to academic research due to the complexity of the mathematical background and algorithms connected with the analysis, and toolboxes of disparate techniques are needed to facilitate further progress in structural health monitoring.

This paper examines notable time-frequency distributions of the generalized Cohen class for damage identification of a two-story frame structure using simulated acceleration data. Structural sudden damage is introduced by removing different bracing member from the first storey at a certain time instant. Subsequently, the nodal accelerations are considered as signal for extracting sensitive features for damage identification.

2. Theoretical background

Simultaneous representation of a signal in the time-frequency plane offers important advantage for the analysis of general non-stationary, nonlinear signal and has been examined extensively in the past (Priestley 1967, Cohen 1989, Hammond and White 1996, Cohen 1995). The fundamental aim of a quadratic time-frequency distribution is to devise a joint function of time and frequency that represents the energy of a signal per unit time per unit frequency. For a joint time-frequency distribution, $P(t, \omega)$, we have

$P(t,\omega)$ = intensity at time t and frequency ω

or $P(t,\omega)\Delta t.\Delta\omega$ = energy in the time-frequency cell '$\Delta t\, \Delta\, \omega$' at t, ω.

Therefore, summing up of the energy distribution for all frequencies at a particular time would give the instantaneous energy and summing up over all times at a particular frequency would give the energy density spectrum

$$\int_{-\infty}^{+\infty} P(t,\omega)d\omega = |s(t)|^2 \tag{1}$$

$$\int_{-\infty}^{+\infty} P(t,\omega)dt = |S(\omega)|^2 \tag{2}$$

where $S(\omega)$ is the Fourier transform of the signal $s(t)$ given by

$$S(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} s(t)e^{-j\omega t}dt \tag{3}$$

The total energy E, expressed in terms of the joint distribution is given by

$$E = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(t,\omega)d\omega dt \tag{4}$$

and will be equal to the total energy of the signal if the marginals, given by the Eqs. (1)-(2), are satisfied.

2.1 Time-frequency distributions of the Cohen class

Among several desirable properties of a joint time-frequency distribution the members of the generalized Cohen class are characterized by two important properties: time and frequency covariance. These two properties guaranty that if the signal is delayed in time and modulated, its time-frequency distribution is translated by the same quantities in the time-frequency plane. It is possible to construct every member of the bilinear time-frequency distributions, which satisfies the shift invariance in time and frequency, by the following formula

$$P(t,\omega)=\frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} s\left(u+\frac{\tau}{2}\right)s^*\left(u-\frac{\tau}{2}\right)\phi(\theta,\tau)e^{(-j\theta t-j\tau\omega+j\theta u)}du\,d\tau\,d\theta \tag{5}$$

where $s(t)$ = signal; $s^*(t)$ = complex conjugate of the signal; t = time; ω = angular frequency; τ = time delay; θ =frequency delay. And $\phi(\theta, \tau)$ is the kernel of the distribution.

The Wigner-Ville distribution (WVD) can be obtained by substituting the kernel $\phi(\theta, \tau) = 1$ in Eq. (5). Although WVD satisfies many desirable properties it suffers from serious interference terms which reduce its applicability for the useful feature extraction.

The Choi-Williams distribution (CWD) can be obtained by introducing an exponential kernel, $\phi(\theta,\tau)=e^{-\theta^2\tau^2/\sigma}$. Substituting this kernel in Eq. (5) and integrating over θ we obtain

$$P_{CW}(t,\omega)=\frac{1}{4\pi^{3/2}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{\tau^2/\sigma}}\,s\left(u+\frac{\tau}{2}\right)s^*\left(u-\frac{\tau}{2}\right)\exp\left[-\frac{(u-t)^2}{\left(4\tau^2/\sigma\right)}-j\tau\omega\right]du\,d\tau \tag{6}$$

The ability to suppress the cross terms of the CWD is controlled by the choice of the parameter σ. When $\sigma \to +\infty$, the kernel is equal to 1 everywhere in the θ, t plane (referred as the plane of the ambiguity function) thus leading to WVD. Inversely for better reduction of the cross terms a smaller σ value must be chosen. The energy localization characteristic of the CWD also depends on the nature of the analyzed signal.

The Page distribution of a discrete time signal $s(t)$ is defined as

$$P(t,\omega)=2\,\mathrm{Re}\left\{\frac{1}{\sqrt{2\pi}}s^*(t)S(\omega)e^{j\omega t}\right\} \tag{7}$$

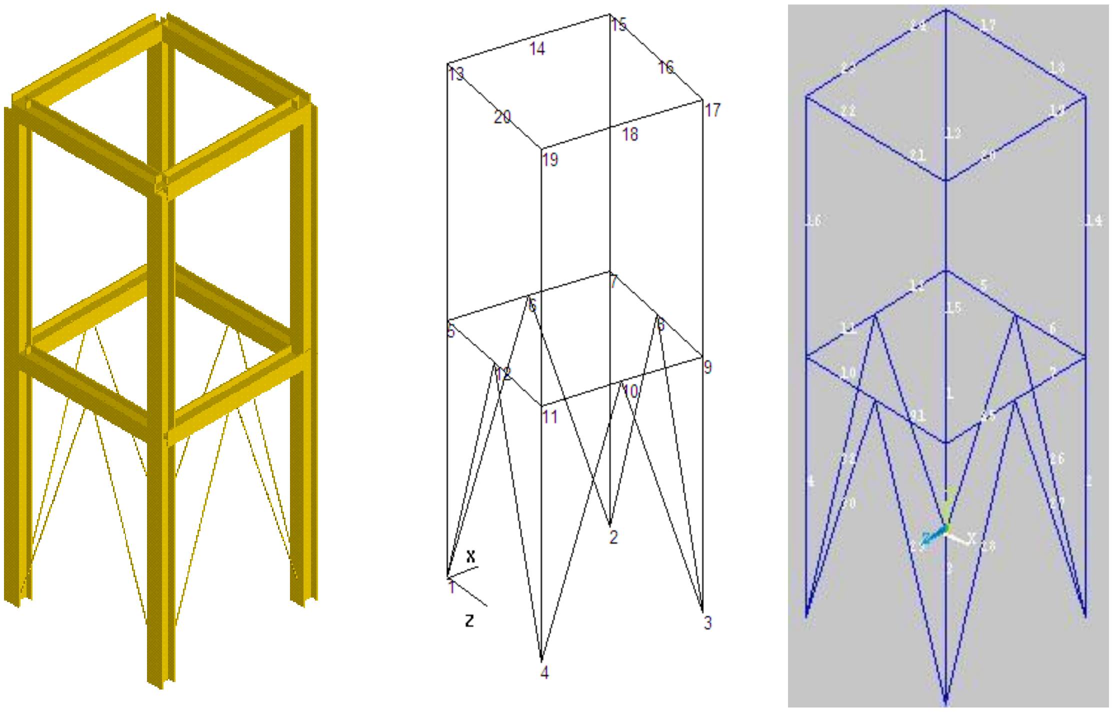
where, Re denotes the real part, $s^*(t)$ is the complex conjugate signal, $S(\omega)$ is the Fourier transform of the signal taken up to the present time which is called the running spectrum.

$$S(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}s(\tau)e^{-j\omega\tau}d\tau$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}s(\tau)e^{-j\omega\tau}d\tau \tag{8}$$

where, τ is the running time and t is the present time instant.

Page distribution can be obtained from the Eq. (5) by taking the kernel as $\phi(\theta,\tau)=e^{\frac{j\theta|\tau|}{2}}$. It is the only distribution of the Cohen class that is casual which means it depends on the observed signal up to the current time. The general behavior of Page distribution is that the longer a particular frequency is observed the larger the intensity is at that frequency.



(a) 3-D frame structure (b) Node numbers (c) Element numbers

Fig. 1 Frame structure with node and element numbers

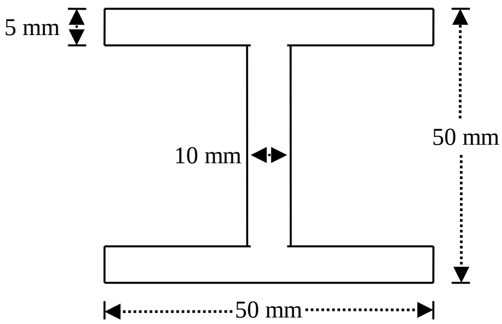


Fig. 2 Cross section of main member

3. Numerical investigation

In the present study, the applicability of quadratic time-frequency distributions for structural damage detection is demonstrated using simulated acceleration time-histories of a 3-D two-storey steel frame structure having plan dimension 1 m × 1 m and each storey height 1.5 m, as shown in Fig. 1. The cross section diagram of the main member is shown in Fig. 2. Eight bracing members, each having cross sectional dimension of 4 mm × 2 mm, are located at the first storey. All the

Table 1 Material constants and member properties

Modulus of elasticity, E	210 GPa
Passion ratio, *v*	0.28
Density	7800 kg/ m^3
Area (Main member)	0.9 10^{-3} m^2
Area (Bracing)	0.8 10^{-5} m^2

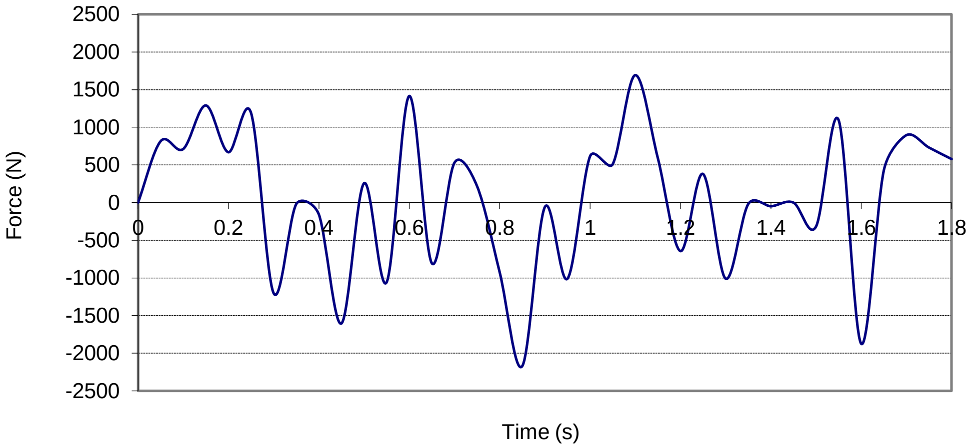


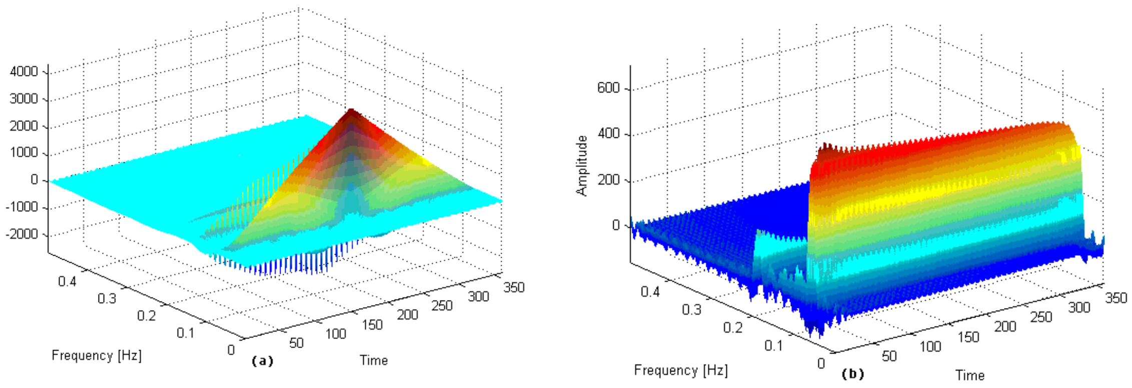
Fig. 3 Applied dynamic loading at node 14

Table 2 Damage cases for the frame structure

Damage Case	Description	Elements	Connecting Nodes
	No damage, damping ignored.	-	-
1	One brace is removed, damping ignored.	31	1-12
2	Two braces are removed, damping ignored.	31, 32	1-12, 4-12
3	All braces are removed, damping ignored.	25-32	1-12, 4-12, 1-6, 2-6, 2-8, 3-8, 3-10, 4-10
4	One brace is removed, mild damping.	31	1-12
5	Two braces are removed, mild damping.	31, 32	1-12, 4-12

members are modeled by the Timoshenko beam element having six degrees of freedom at each node. The material constants and member properties are given in Table 1.

The frame structure is subjected to a Gaussian loading at node 14 along *z*-axis for a duration of 1.8 s, as shown in Fig. 3. The acceleration time histories at various degrees of freedom of the structure, in the undamaged as well as damaged state, are evaluated by finite element analysis using ANSYS. Structural sudden damage is introduced by removing different bracing member at a time instant, t = 0.6 s for various damage cases by using the element birth/ death concept of the analysis software. For each damage case, a nonlinear analysis with large deflection effect is performed. A small time step size, δt = 0.005 s, is used in the dynamic analysis. The various damage cases considered in the present study are given in Table 2. The acceleration time histories



(a) Wigner-Ville distribution (b) Choi-Williams distribution

Fig. 4 Undamaged state: time-frequency representations of the acceleration at node 12

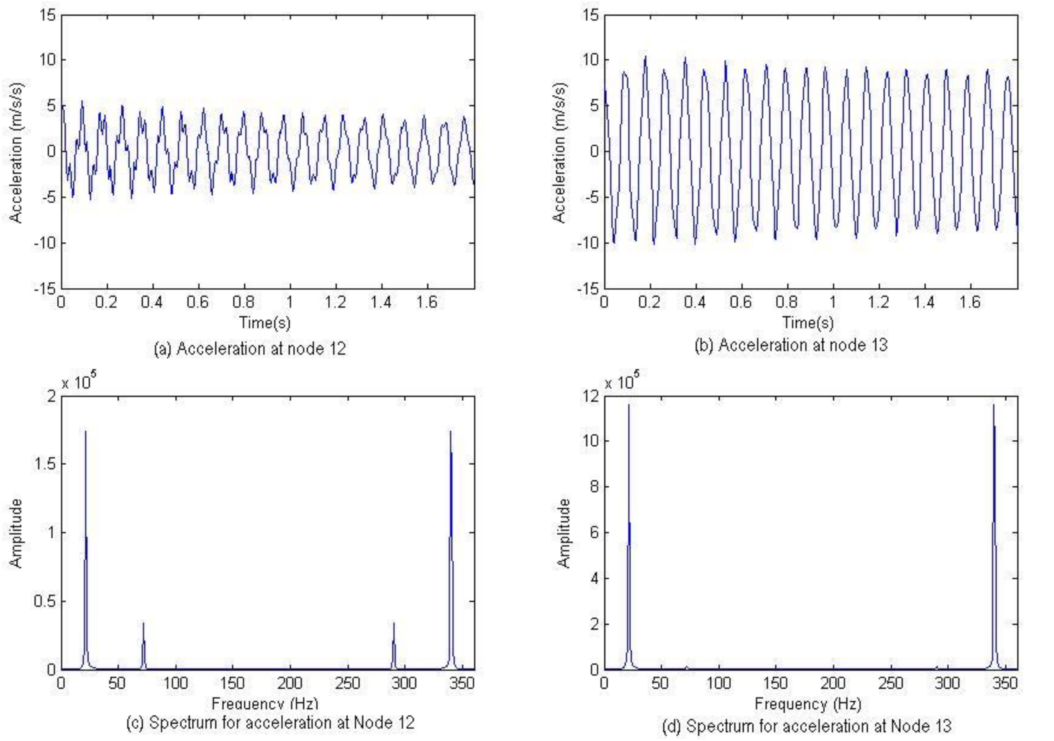


Fig. 5 Undamaged state: (a) Accelerations at node 12 (b) Accelerations at node 13 (c) Spectrum for acceleration at node 12 (d) Spectrum for acceleration at node 13

at different nodal position of the frame structure are subsequently used as signal for obtaining various time-frequency representations using time-frequency toolbox (Auger *et al.* 2005) in MATLAB.

Figs. 4(a) and (b) show the 3-D energy-normalized frequency-time plots of the acceleration at node 12 by the WVD and CWD, respectively, in the undamaged state of the frame structure. It is

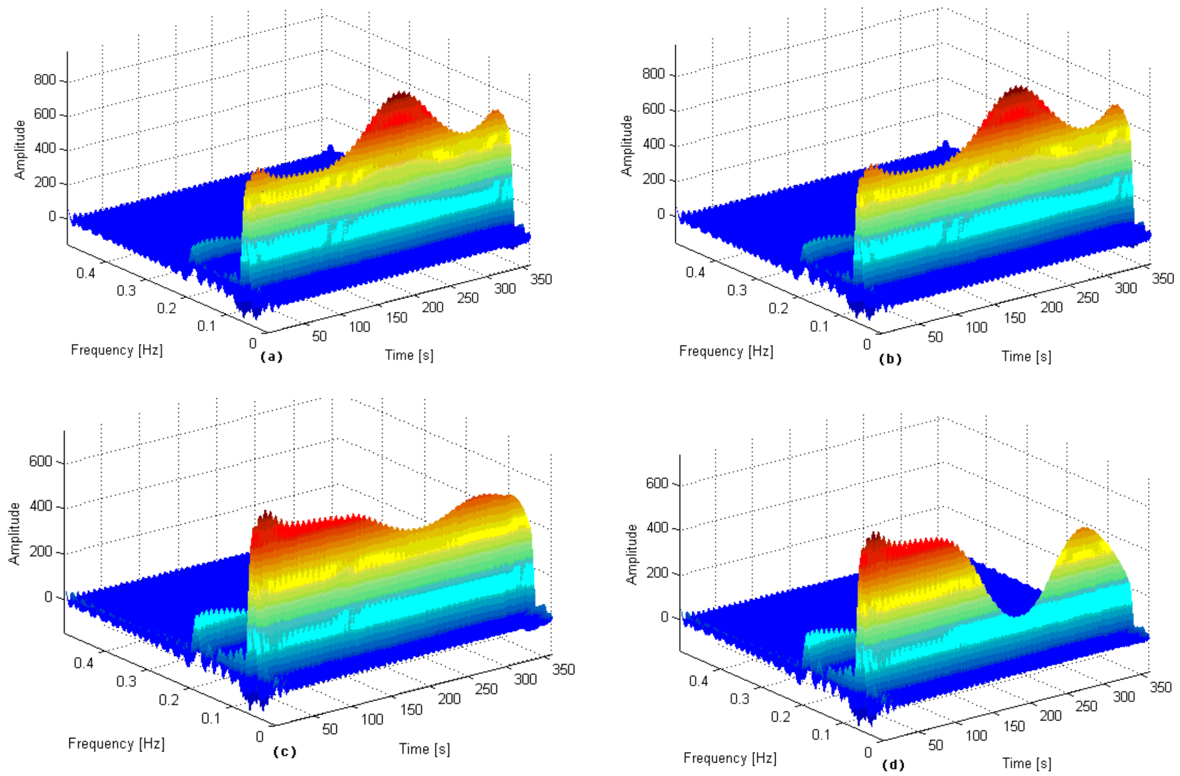


Fig. 6 Damage Case-1: CWD of the accelerations at (a) node 12 (b) node 5 (c) node 6 (d) node 7

seen that the WVD peaks at the middle of the acceleration response. Further, it contains significant negative values and also suffers from the interference terms at positions in the time-frequency plane where the signal energy should be zero. These factors reduce the readability of the WVD for extracting damage information in a reliable manner. On the other hand, the CWD is almost flat and contains less cross terms.

Fig. 5 shows the acceleration time-histories along *z*-axis at node 12 (1^{st} floor) and node 13 (2^{nd} floor), respectively, and the corresponding power spectrums in the undamaged state. It can be observed that acceleration at first floor nodal positions have two frequency components: 22 Hz and 72 Hz. On the other hand, second floor nodal accelerations have, predominantly, a single frequency component: 22 Hz.

<u>*Damage Case-1*</u>

Damage Case-1 is simulated by removing a single bracing member, element 31 connecting nodes 1 and 12, at the time instant t = 0.6 s. Fig. 6 shows the CWD of the acceleration at different first floor nodal positions of the frame structure. From visual comparison between Fig. 4(b) with Figs. 6(a) and (b), it is seen that in the vicinity of damage (node 12 or 5) the instantaneous energy of the acceleration increases sharply after the damage event. Therefore, the presence and time instant of damage can be determined. However, for locations away from the damage region, such as at node 6 and 7, the energy content of the accelerations show a decline as seen in Figs. 6(c) and (d), respectively. The magnitude of decrease is more at node 7 than node 6. Therefore, the

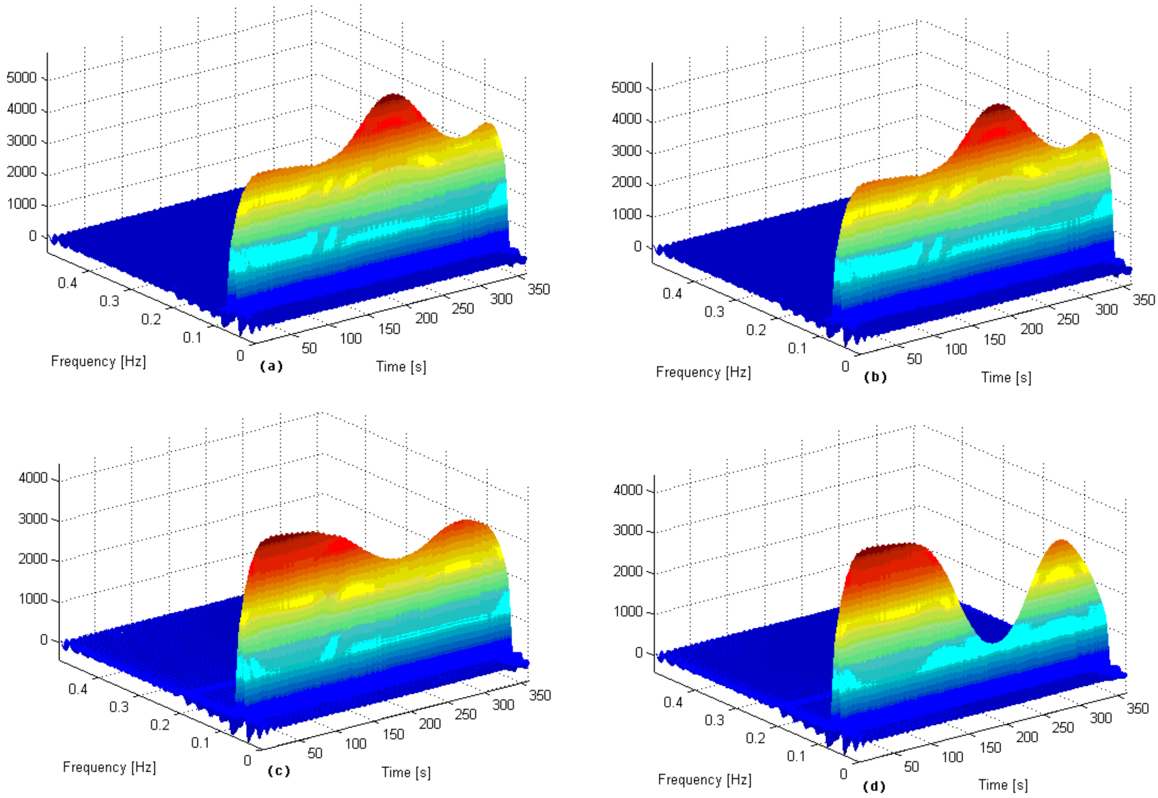


Fig. 7 Damage Case-1: CWD of the accelerations at (a) node 20 (b) node 19 (c) node 18 (d) node 17

geometric location of damage can be also estimated by comparing the CWD of the acceleration at various nodes along the frame structure. Similar results are obtained from the CWD of the acceleration at various second floor nodal positions of the frame structure, as shown in Fig. 7.

Damage Case-2

In Damage Case-2, two bracing members, element 31 and 32, are removed simultaneously at time instant t = 0.6 s. Fig. 8 shows the CWD of the acceleration at various nodes of the first floor. The presence and location of the damage can be determined similarly as discussed for Damage Case-1. Further, by comparing the CWD of the acceleration response at a particular node of the frame from Figs. 6 and 8, it is observed that the rate of increase in instantaneous energy near the damage region, i.e. at node 12 and 5, is more for Damage Case-2. Similarly the rate of decrease in instantaneous energy away from the damage region, i.e., at node 6 and 7, is higher for Damage Case-2 than Case-1. Hence, the CWD can correctly represent the severity of damage.

Damage Case-3

In Damage Case-3, all the bracing members from the first storey are removed at t = 0.6 s. It is found that the changes in instantaneous energy of the acceleration do not represent the damage event clearly for identification purpose in this severe damage scenario. However, the frequency content shows a significant shift after the damage instant. Fig. 9 shows the CWD of the acceleration at node 12 in the time-frequency plane for various damage cases considered. It is seen that, before the damage event the normalized frequency of the acceleration lies around 0.05 for all the damage cases. For Damage Case-1, when single bracing member is removed, there is no shift in the frequency content. For Damage Case-3, the normalized frequency of the acceleration is reduced by 12.2% after the damage instant, t = 0.6 s. The percentage reduction in frequency for various damage cases is given in Table 3.

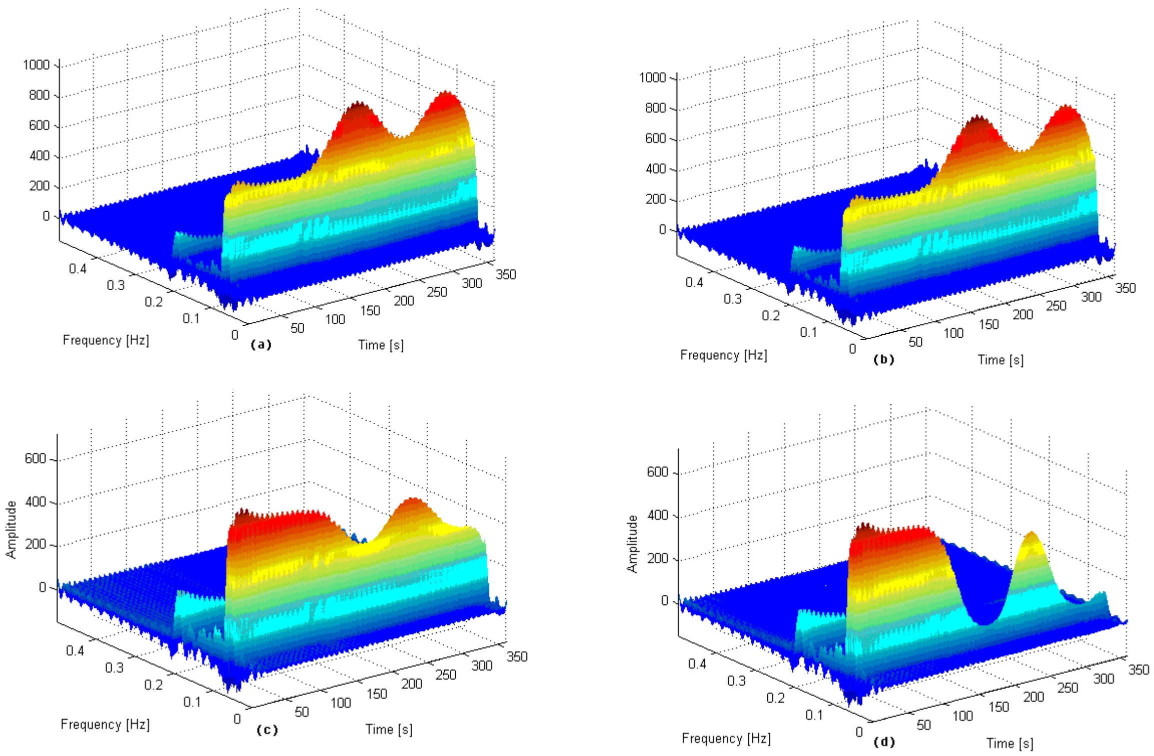


Fig. 8 Damage Case-2: CWD of the accelerations at (a) node 12 (b) node 5 (c) node 6 (d) node 7

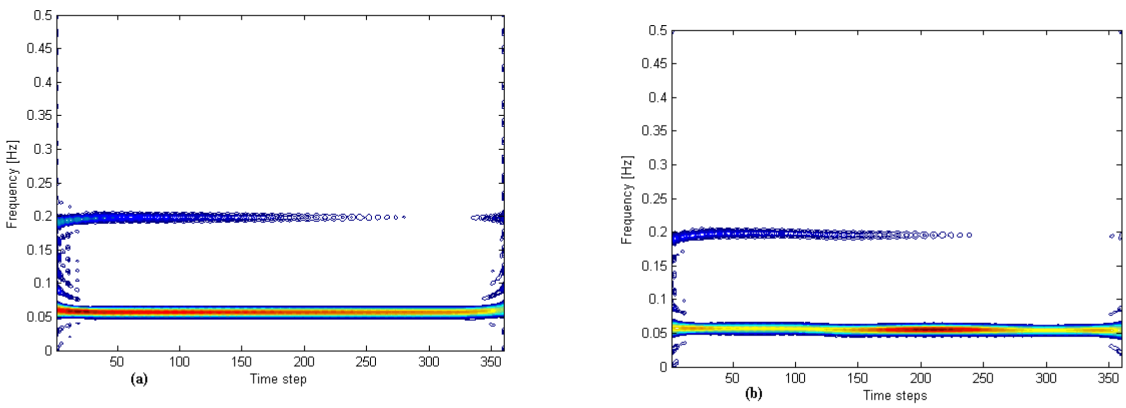


Fig. 9 2-D time-frequency plot of the acceleration at node 12 using CWD for (a) Undamaged state (b) Damage Case-1 (c) Damage Case-2 (d) Damage Case-3

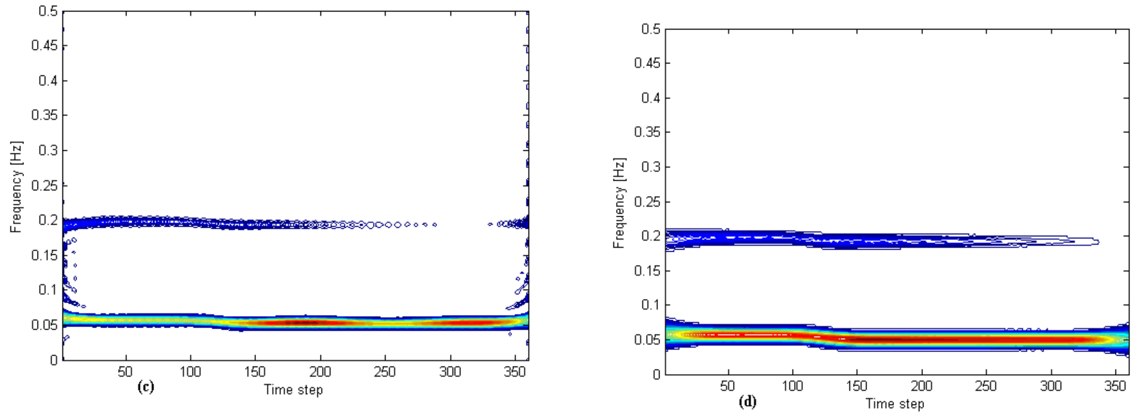
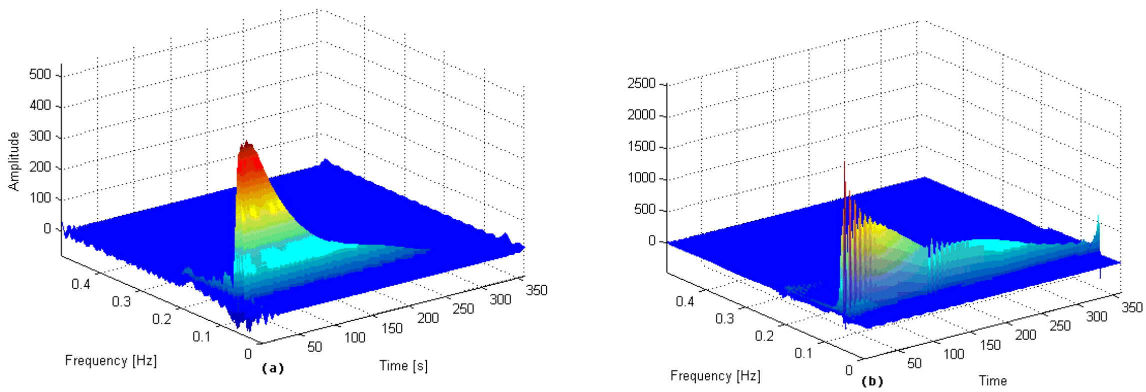


Fig. 9 Continued

Table 3 Percentage reduction of frequency for various damage cases

Damage case	Description	Reduction in Normalized Frequency (%)
	Undamaged state	-
1	Single bracing is removed	0
2	Two bracings are removed	7.3
3	All bracings are removed	12.2



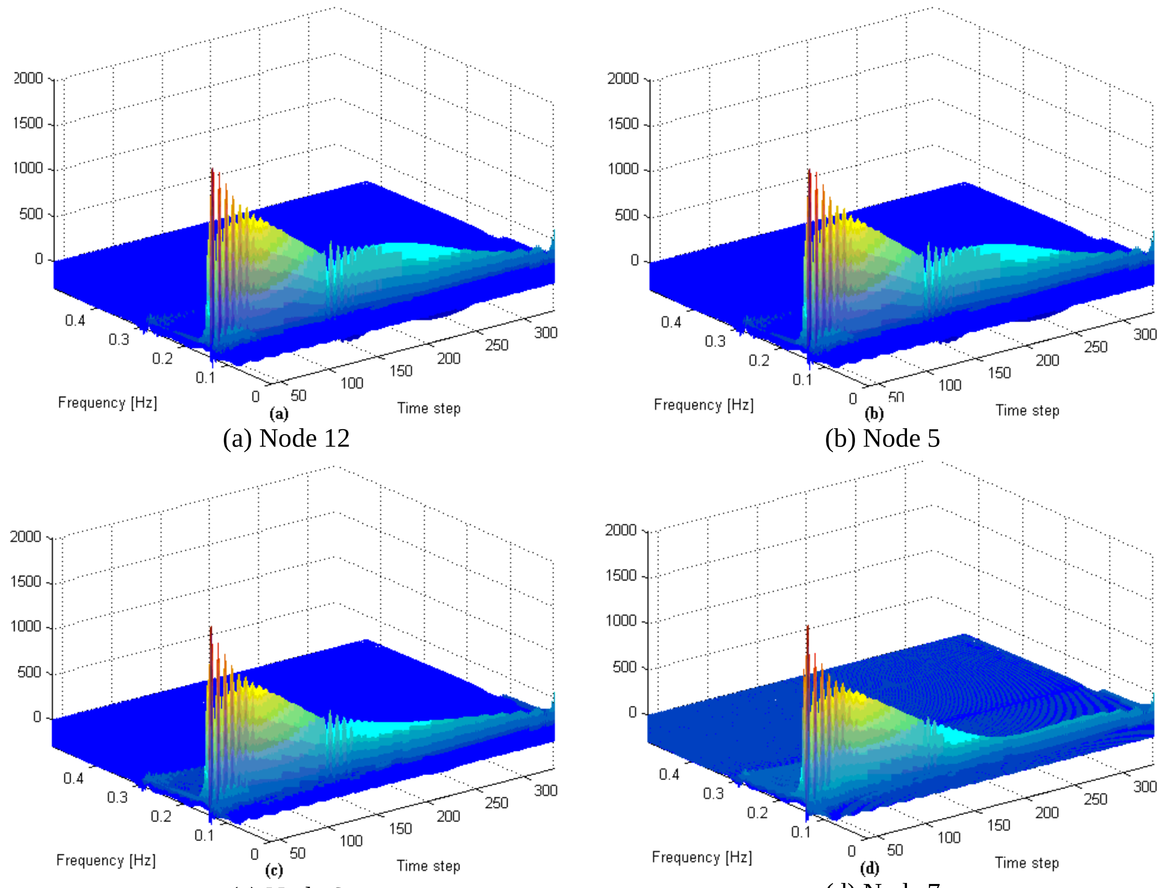
(a) Choi-Williams distribution (b) Page distribution

Fig. 10 Damage Case-4: time-frequency representation of acceleration at node 12

Damage Case-4

To examine the effect of damping, a mild damping factor (0.0005) is considered and damage is introduced by removing a single bracing member in Damage Case-4. The CWD doesn't represent the damage event clearly, as shown in Fig. 10(a). However, Page distribution shows sharp spikes just after the damage instant, as seen in Fig. 10(b).

Fig. 11 shows the Page distribution of the acceleration at various nodal position of the frame structure for Damage Case-4. The occurrence of damage can be confirmed from the presence of spike in the 3-D energy-normalized frequency-time plots. It is also seen that the spike length is



(a) Node 12 (b) Node 5

(c) Node 6 (d) Node 7

Fig. 11 Damage Case-4: Page distribution of the acceleration at different nodal position

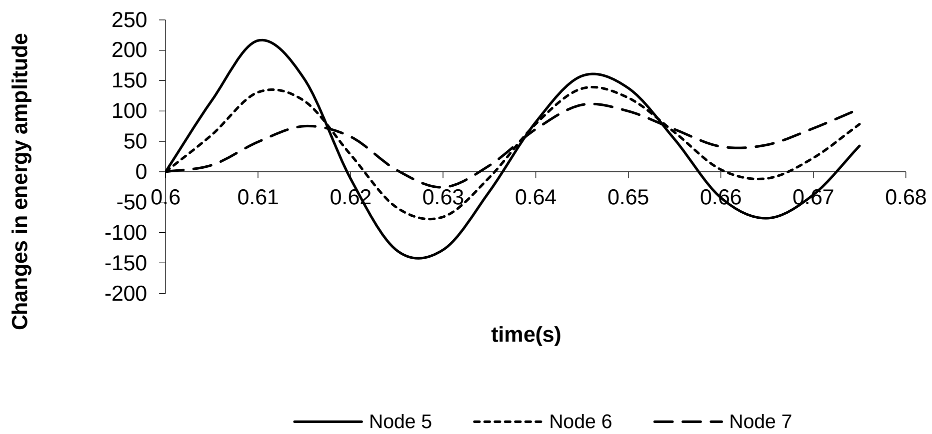


Fig. 12 Damage Case-4: Changes in the instantaneous energy of the acceleration at different nodal positions by Page distribution

maximum near the damage region, i.e., for node 12 and 5 as shown in Figs. 11(a) and (b) respectively. Hence, location of damage can be estimated by comparing the Page distribution of the acceleration at various nodes along the frame structure. Similar results were obtained for Damage Case-5.

Fig. 12 shows the relative changes in instantaneous energy of the acceleration at nodes 5, 6 and 7 for a short time span after the damage instant using Page distribution. The base value of the energy is taken corresponding to time instant, t = 0.6 s. It is seen that the rate of increase of the energy amplitude is maximum near the vicinity of damage region, i.e., at node 5 or 12, and reduces steadily for nodes 6 and 7 which are located away from the damage region. Hence, the location of damage can be estimated. It is also seen that the rate of increase in instantaneous energy is maximum just after the damage instant and reduces with the time. Therefore, for the success of the presented approach, the accelerometers should be placed in the vicinity of the damage event to capture the nonlinear features because a sudden damage is often a low energy phenomena and its wave propagation decays fast.

4. Conclusions

The quadratic time-frequency distributions have been used in this study for damage assessment of a frame structure from the simulated acceleration time-histories. Based on the results the following conclusions are summarized:

• Wigner-Ville distribution (WVD) suffers from negative values and significant interference terms and its readability is found unsuitable for extracting damage sensitive features.

• Choi-Williams distribution (CWD) can correctly localize the signal energy in the time-frequency plane and offer valuable insight for damage identification. The changes in the instantaneous energy of the acceleration are good damage indicators.

• From the changes in instantaneous energy, the presence and time instant of damage can be determined.

• Location of damage can be determined by comparing the changes in instantaneous energy of the acceleration at few representative nodal positions along the frame structure.

• It is shown that for small/ moderate damage the instantaneous energy is a good damage indicator, however for severe damage changes in instantaneous energy do not represent the damage event clearly and the frequency content of the acceleration shifts significantly after the damage event.

• In the presence of damping, the Page distribution, due to its casual structure, represents the damage event more effectively. Occurrence of damage can be confirmed from the presence of spikes in the time-frequency plane. Location of damage can be estimated by comparing the magnitude of spike at different nodal positions of the structure.

The following observations are further noted:

◦ In the present study, sudden damage was introduced by removing bracing members at a pre-selected moment. However, the conclusions mentioned above should apply for a general case that is when damage occurs at an unknown moment due to excessive response.

◦ The proposed approach has the limitation that measured response data must include the moment when damage occurred.

◦ As the quadratic time-frequency distributions are naturally bilinear transform of the signal they are suitable for the analysis of “mild” nonlinearity (i.e. quadratic nonlinearity) associated with a damage event.

As final remarks it is pointed out that this paper provided a detailed analysis on the applicability of quadratic time-frequency distributions (TFDs) for damage detection in a frame structure using simulated acceleration data. However, there are significant scientific objectives and critical issues for a successful application of the presented damage detection algorithm to laboratory or in-situ experimental test. As described in this paper the approach can determine the presence and location of damage with reasonable accuracy when noiseless and sufficient numbers of acceleration data at critical locations of the frame structure are available. However, when the acceleration data are contaminated by random noise in an experimental setting the readability of the TFDs will be seriously affected as the instantaneous energies of the response will be polluted by that of the noise. Since noise is an important part of experimental test, an investigation on the applicability of the proposed approach to laboratory test is of critical importance and authors look forward to carry out such a study in future research.

It is further pointed out that the acceleration data with additive white noise can be analyzed by different bilinear time-frequency distributions of Cohen class to evaluate the performance of the presented damage detection algorithm. As a pure white noise has a constant power spectral density or in other words contains equal power for all frequency bands the effectiveness of the presented approach will be seriously challenged for a high value of the signal-to-noise-ratio (SNR). As shown in the paper the frequency (or normalized frequency) content of the acceleration responses for a particular excitation remains the same even in the presence of mild to moderate damage and changes significantly for a severe damage level. So, for a low value of SNR the presence of damage can be still detected from the changes in instantaneous energy along the dominant frequency of the noisy acceleration data in the time-frequency plane when the damage is of mild to moderate level.

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