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Energy based procedure to obtain target displacement of reinforced concrete structures

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Abstract. Performance-based seismic design allows a structure to develop inelastic response during earthquakes. This modern seismic design requires more clearly defined levels of inelastic response. The ultimate deformation of a structure without total collapse (target displacement) is used to obtain the inelastic deformation capacity (inelastic performance). The inelastic performance of a structure indicates its performance under excitation. In this study, a new energy-based method to obtain the target displacement for reinforced concrete frames under cyclic loading is proposed. Concrete structures were analyzed using nonlinear static (pushover) analysis and cyclic loading. Failure of structures under cyclic loading was controlled and the new method was tested to obtain target displacement. In this method, the capacity energy absorption of the structures for both pushover and cyclic analyses were considered to be equal. The results were compared with FEMA-356, which confirmed the accuracy of the proposed method.

Keywords: target displacement; reinforced concrete frame; pushover; cyclic loading; FEMA-356; energy based procedure

1. Introduction

Most structures are seismically designed using the equivalent static method. In this method, design forces are obtained from elastic spectra and reduced using a response modification factor. This factor represents inelastic performance and indicates the strength and hidden ductility of structures in the inelastic phase. The larger the factor, the higher the level of energy absorption and formation of plastic hinges.

Accurate determination of the yielding point and ultimate displacement are required to calculate the inelastic deformation capacity of a structure. Nonlinear static analysis is a simple technique which can be used to estimate the dynamic demands of structures under seismic excitation. Hysteretic energy and seismic input energy are among the most important areas of concern for structural design under earthquake loading. Hysteretic energy is the energy dissipated by the structure through the inelastic deformation of its components. Seismic input energy has a direct bearing on the survival of a structure. A structure can survive an earthquake if its structural energy absorption capacity is greater than the input seismic energy (Zahrah and Hall 1984, Leger and Dussault 1992).

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The seismic energy imparted to a building dissipates through the movements and deformations of the structural members in the form of kinetic energy, damping energy, elastic strain energy, and inelastic hysteretic energy. Higazy and Elnashai (1997) have investigated energy-based excitation. They provided a measure for investigating the level of shear deformation with an acceptable limit of energy dissipation capacity. Gong *et al.* (2012) used an energy-based procedure to find a method to minimize structural cost and earthquake input energy and to maximize absorbed hysteresis energy of the structure.

Cyclic loading analysis is used to investigate dissipated energy in structures under excitation. For example, Sadeghi and Nouban (2010) proposed a simplified damage index based on an energy analysis method for both concrete and steel structures. They used cyclic loading analysis and investigated input energy to the structures to propose a new damage index. Yuchuan *et al.* (2011) used cyclic loading and an energy based procedure to develop a damage model for concrete structures. Uang and Bondad (1998), Han *et al.* (2009) also studied cyclic loading to investigate input energy in structures.

Pushover analysis is a nonlinear static method widely used in civil engineering to evaluate the seismic performance of existing and new structures. It provides reliable information on the seismic demands imposed by design ground motion on a structural system and its components. A number of investigators have estimated the demands of buildings using pushover analysis (Krawinkler and Seneviratna 1997, Gupta and Krawinkler 2000, Rofooei *et al.* 2006, Shakeri *et al.* 2010, Jiang *et al.* 2010, Goel 2011). Shakeri *et al.* (2010) used nonlinear static analysis to investigate the effects of higher modes in the seismic demands of buildings. Gupta and Krawinkler (2000) estimated seismic drift demands for frame structures from spectral displacement demand at the first mode period. They concluded that roof drift demands are related to story drift demands and are strongly dependent on the number of stories and ground motion characteristics. Jiang *et al.* (2010) estimated seismic demands on buildings and developed an energy-based method using multimode pushover analysis in which higher vibration modes were computed by assuming the buildings to be linearly elastic. They proposed their procedure to be an alternative for nonlinear response history analysis. Monavari and Massumi (2010) used failure criteria and pushover analysis to estimate the seismic demands of structures and proposed a simple equation to estimate target displacement. Ucar *et al.* (2012) used an energy based procedure that uses equality of energy demand (by using seven strong ground motions) and energy capacity (pushover curves) of the structure. They assumed that the displacements, obtained from the energy capacity diagrams that fit to the energy demand values of the RC structures, are equal to the energy-based performance point of the structures. Their results showed that the target displacements of RC frame structures obtained from proposed method were very close to the values calculated by the approach given in the Turkish Seismic Design Code.

It should be noted that target displacement may be affected by soil-structure interaction phenomenon, which is beyond the scope of this work. Soil-structure interaction may relieve the demands on the structure (e.g., the structure can reach its target displacement with less distress due to base rotation, uplift limits forces on building, and soil acts as a damper that reduces demand on the structure) (Comartin *et al.* 2000, Tabatabaiefar and Massumi 2010, Massumi and Tabatabaiefar 2007).

In the present study, failure criteria and both pushover and cyclic loading analyses were used to develop an energy-based procedure to obtain target displacement. To accurately calculate the target displacement, nonlinearity of the structures and all vibration modes were considered. The structures were investigated using pushover and cyclic analysis and the amount of energy applied

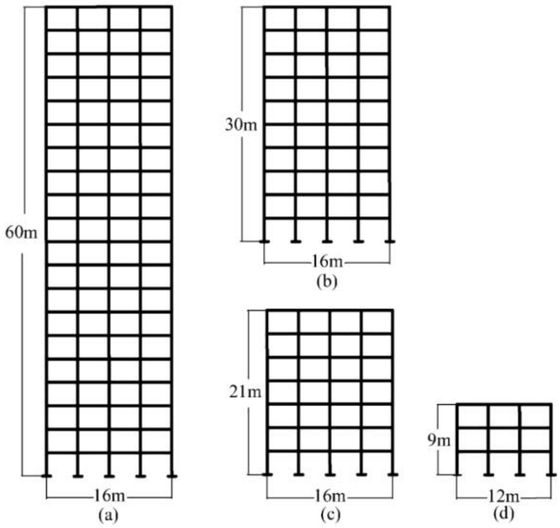

Fig. 1 Sample frames: (a) 20 story, (b) 10 story, (c) 7 story, (d) 3 story

to structures in both analyses were calculated. Maximum drift without total collapse was estimated by pushover analysis using the amount of energy applied to the structure under cyclic loading. The results of the proposed method were compared to the procedure to estimate target displacement as defined by FEMA-356 (ASCE/FEMA 2000) for a nonlinear static procedure.

2. Method

Thirteen reinforced concrete (RC) frame buildings with 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 16 and 20 stories and having 3 or 4 bays were designed using seismic force levels obtained from the Iranian Seismic Code (BHRC 2005). They were then proportioned using ACI318-99 (ACI 1999) and modeled using IDARC (Valles *et al.* 1996).

The configuration was regular in elevation. The floor elevation was 3 m and the span of the frames was 4 m. Fig. 1 depicts the 3, 7, 10 and 20 story frames. The criteria investigated were:

- Structural instability (SI) caused by hinge formation and mechanisms, and
- Exceeding the Park-Ang damage index (DI) from unit.

2.1 Structural instability

SI occurs in all or part of a structure as a result of hinge formation and mechanisms. Fig. 2 shows the four types of total instability caused by structural geometry (Massumi 2004, Tasnimi and Massumi 2006).

2.2 Park-Ang damage index

A number of researchers have investigated damage indices (Park *et al.* 1988, Ladjinovic and Folic 2004, de Guzman and Ishiyama 2004, Sadeghi and Nouban 2010, Ghosh *et al.* 2011,

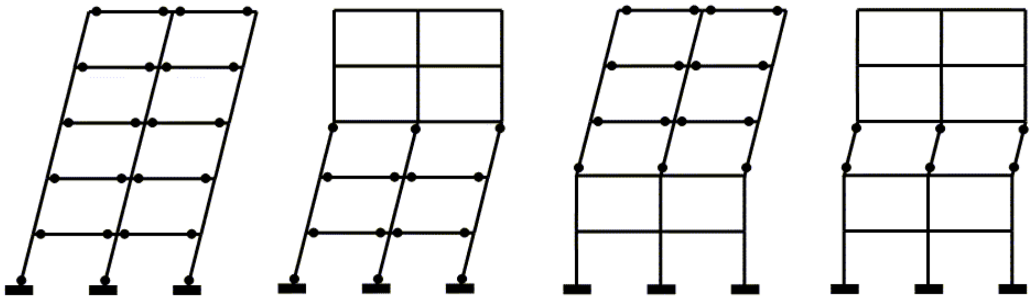

Fig. 2 Four types of total instability

Massumi and Moshtagh 2013). The Park-Ang damage index is based on experimental studies and actual damage observed in buildings as defined by Park and Ang (1985). The work of Park *et al.* (1988) and de Guzman and Ishiyama (2004) verify these parameters

$$DI_{P\&A} = \frac{\delta_m}{\delta_u} + \frac{\beta \int dE_h}{\delta_u P_y} \tag{1}$$

where

$$\begin{cases} \delta_m \text{: Maximum experienced deformation,} \\ \delta_u \text{: Ultimate deformation of the element,} \\ P_y \text{: Yield strength of the element,} \\ dE_h \text{: Incremental dissipated hysteretic energy,} \\ \beta \text{: Model constant parameter,} \end{cases}$$

and

$$\begin{cases} \mathrm{DI} < 0.4 & \text{: Repairable damage,} \\ 0.4 \le \mathrm{DI} \le 1 & \text{: Damage beyond repair,} \\ \mathrm{DI} \ge 1 & \text{: Loss of building or story.} \end{cases}$$

2.3 Calculating target displacement

Fig. 3 illustrates the procedure for obtaining the target displacement used in this study. Since the specific patterns of the cyclic load may affect the results, cyclic loading was applied as pushover analysis, following a prescribed height-wise distribution of lateral forces. Pushover analysis with increasing triangular loading was used. The cyclic loading for each structure was calculated using a capacity curve and ATC-24 (Partridge *et al.* 2003). Each structure was analyzed under cyclic loading and its total collapse was checked using DI and SI. If failure did not occur, the cyclic loading was revised by keeping the number of cycles constant and increasing only the peaks of the cycles; the structure was then analyzed again. Each cyclic loading had 24 cycles. The capacity and bilinear curves (idealized curve) are shown in Fig. 4 and cyclic loading for a 3-story frame is shown in Fig. 5.

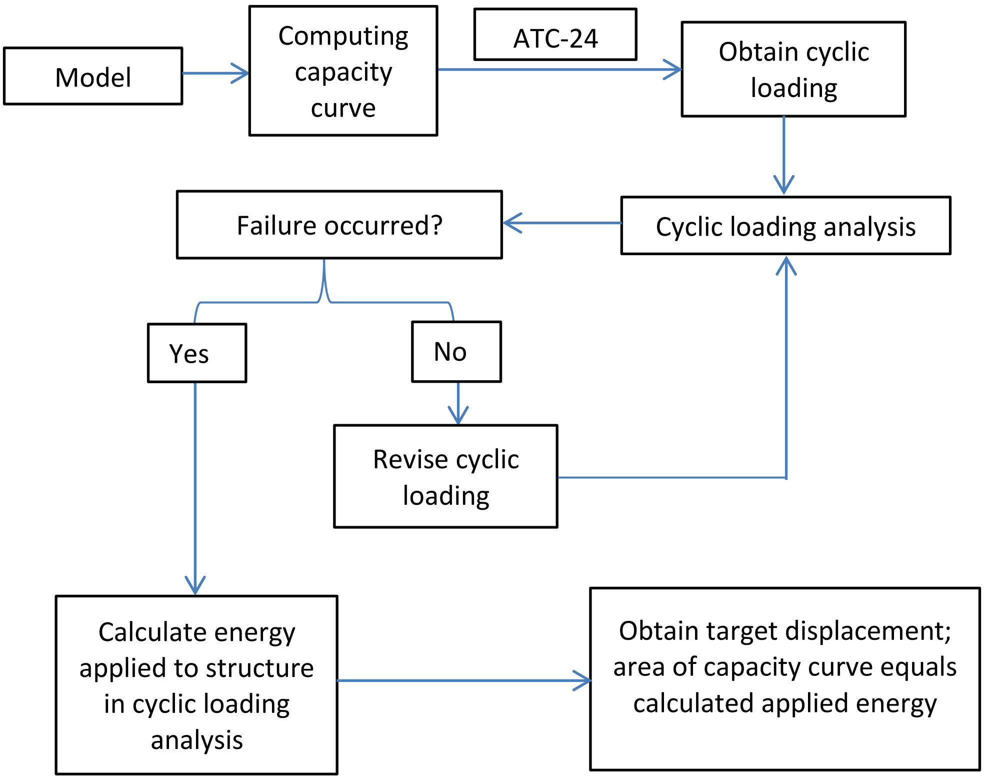

Fig. 3 Procedure for target displacement

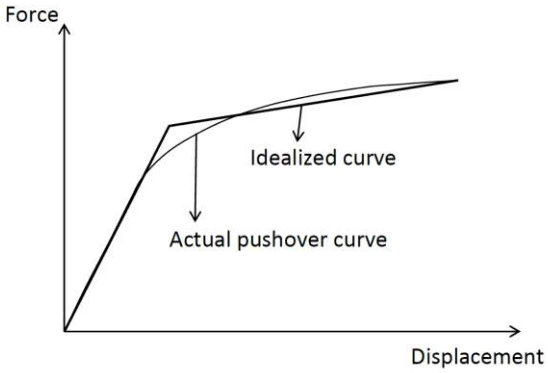

Fig. 4 Capacity and bilinear curves

2.4 Estimating target displacement

Deformation capacity is the ultimate displacement or rotation that corresponds to failure of the structure. Hysteretic energy capacity is the total area under all hysteretic loops that a structural element undergoes during cyclically-changing lateral loading to the point in the force-displacement history where the structure fails. Naeim (2000) states that damage is related to a

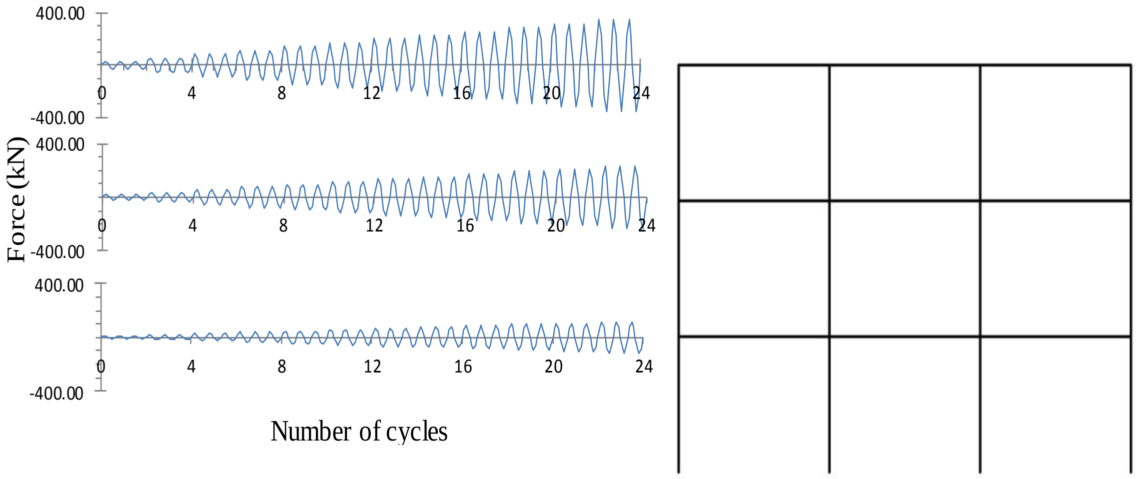

Fig. 5 Cyclic loading for 3-story frame

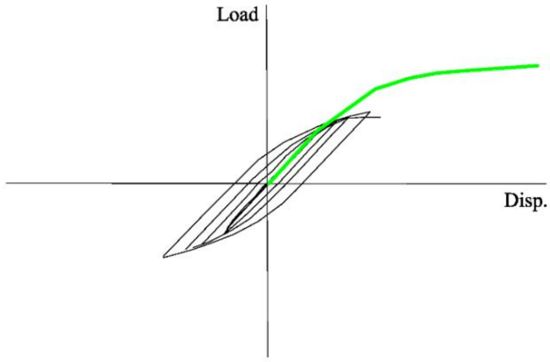

Fig. 6 Cyclic analysis and pushover curves for a structure

structure's ability to dissipate energy and Poljansek and Fajfar (2008) said that energy dissipation capacity depends on the load history. Some studies, however, indicate that the absorbed energy calculated for cyclic and monotonic loadings are very similar. The small difference between total absorbed energy to failure in cyclic and monotonic loading can be explained by the different types of loading employed (Sadeghi 2011). In this study, the small difference was negligible and the absorbed energy from both monotonic and cyclic loading assumed to be equal.

The structures were analyzed using cyclic loading and the applied energy (area on the force-displacement curve) was calculated for each cyclic analysis. The estimated target displacements for pushover analysis were obtained assuming that structures with the same applied energy will fail. For each structure, the applied energy for the pushover was assumed to be equal to the applied energy for cyclic analysis. For example, Fig. 6 shows the results of one cyclic analysis and pushover. As Fig. 7 shows, the total dissipated energy caused by cyclic loading (E) is calculated as the sum of the separate areas in cyclic loading analysis (A_i). The area for each cycle was obtained separately and the applied energy in the cyclic analysis was equal to the sum of all areas. The last

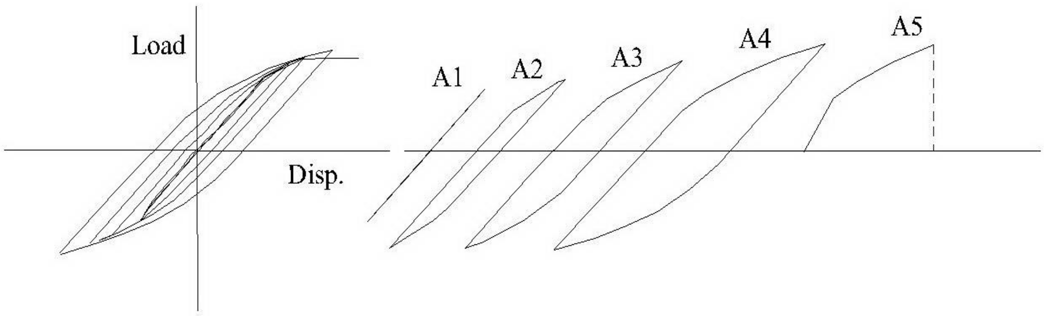

Fig. 7 Applied energy in cyclic analysis

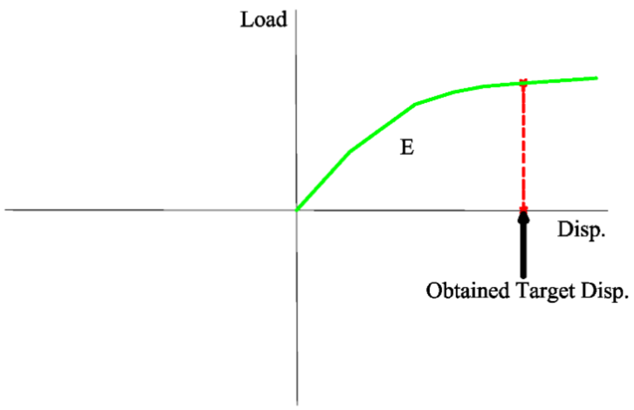

Fig. 8 Obtaining target displacement

cycle in each was considered complete at failure and its area was obtained at failure of the structure (A_5 in Fig. 7).

$$E = A_1 + A_2 + A_3 + A_4 + A_5 \tag{2}$$

As Fig. 8 shows, the target displacement was obtained by assuming the area on the pushover equals *E*. Fig. 9 shows the target displacements obtained by the proposed method. The estimated target displacements are illustrated in Fig. 10.

2.5 Target displacement from FEMA-356

FEMA-356 (ASCE/FEMA 2000) recommends the following equation to compute target displacement

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} g \tag{3}$$

The target displacements for each structure calculated using FEMA-356 are shown in Fig. 12 (ASCE/FEMA 2000).

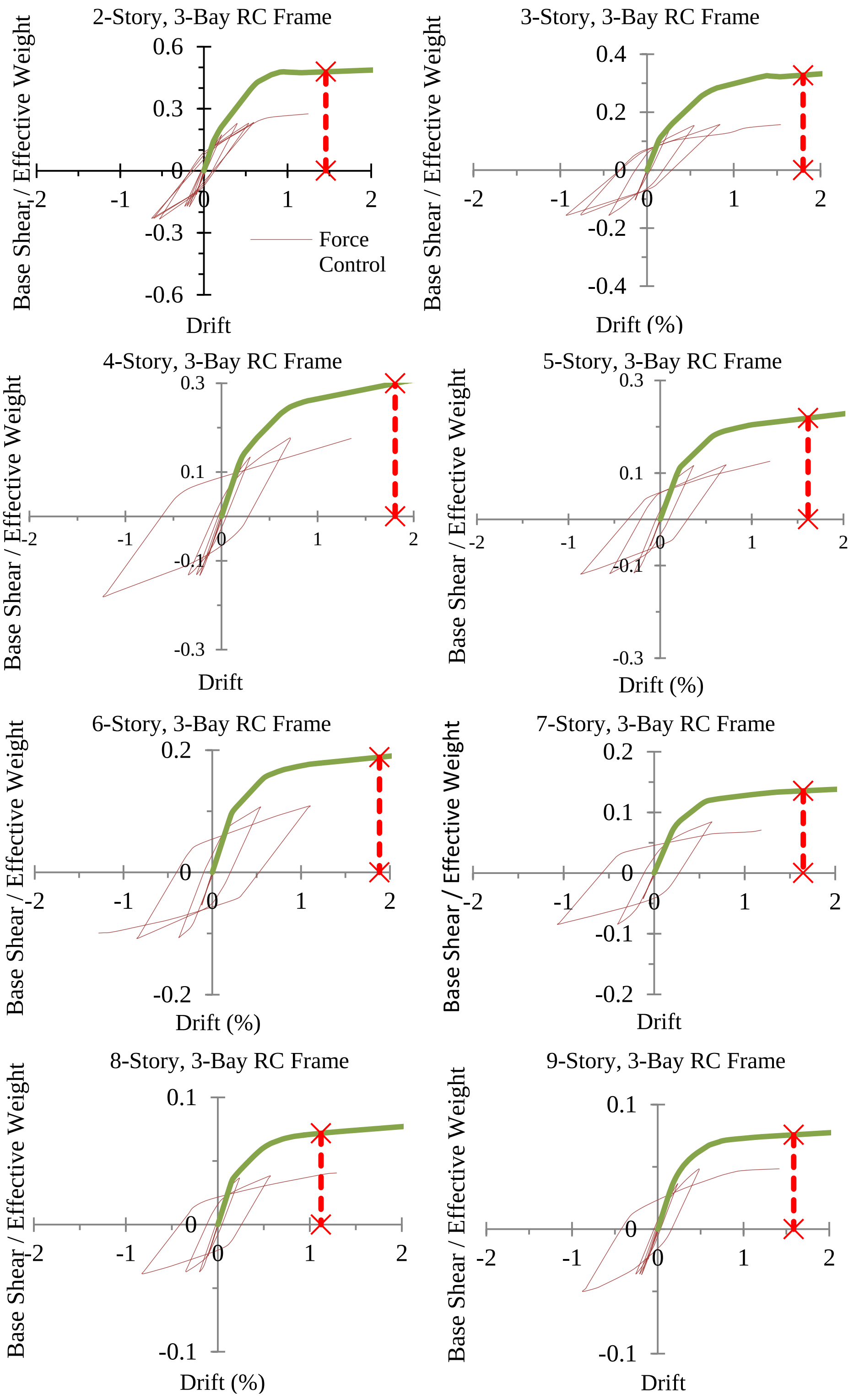

Fig. 9 Target displacements for investigated structures by story

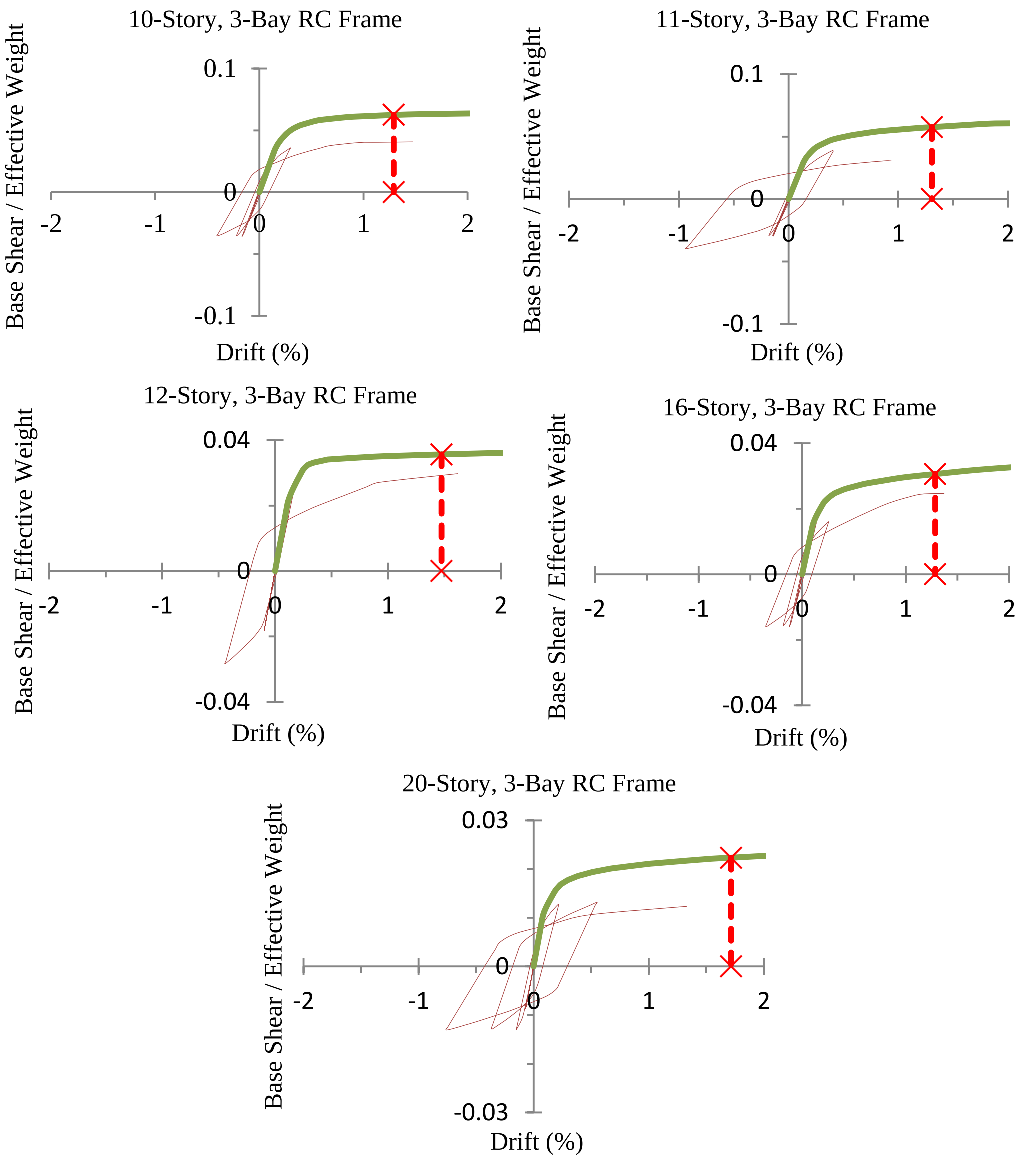

Fig. 9 Continued

3. Results and discussion

In the proposed method, the target displacements were estimated using pushover and cyclic loading analysis. A comparison of the results of this study and FEMA-356 is shown in Fig. 11. This figure shows that the results of the new energy-based method are similar to the FEMA-356 target displacements, indicating that the proposed method estimated target displacement correctly. The difference between the results of the energy-based method and FEMA-356 reflect the different methods used. The target displacement can be approximated using the following equation

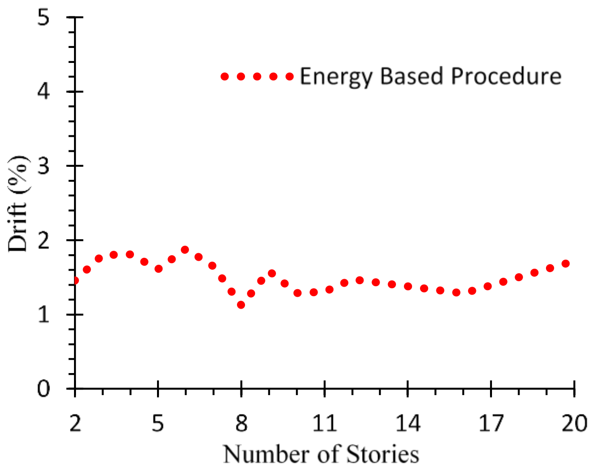

Fig. 10 Estimated target displacement for each structure

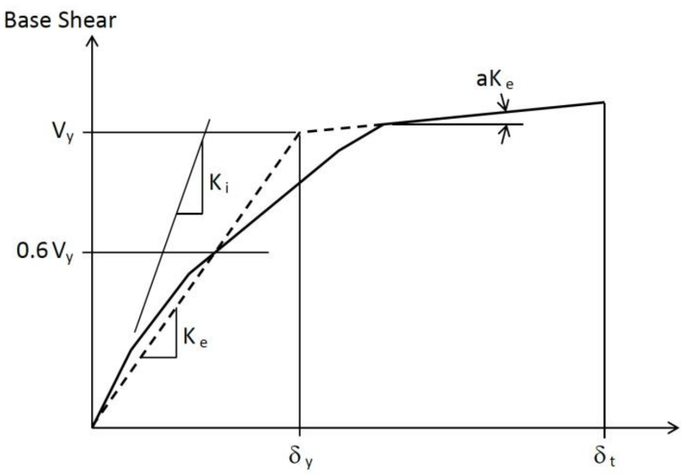

Fig. 11 Bilinear relationship of base shear versus roof displacement

$$D_t = \frac{4\pi^2 m}{N}\left(\frac{T_e^2}{T^{0.5}}\right) \tag{4}$$

where T is the approximate period of the structure (BHRC 2005), N is the number of stories, m is the proposed period dependent parameter, T_e is the effective fundamental period of the structure, and T_i is the elastic fundamental period of the structure calculated by elastic dynamic analysis. This can be taken as the secant stiffness calculated at a base shear force equal to 60% of the yield strength (ASCE/FEMA 2000) (Fig. 13)

$$T = 0.07H^{3/4} \tag{5}$$

$$\begin{cases} m = 0.71 & \text{for} \quad T \leq 0.7 \\ m = 0.38 & \text{for} \quad T > 0.7 \end{cases}$$

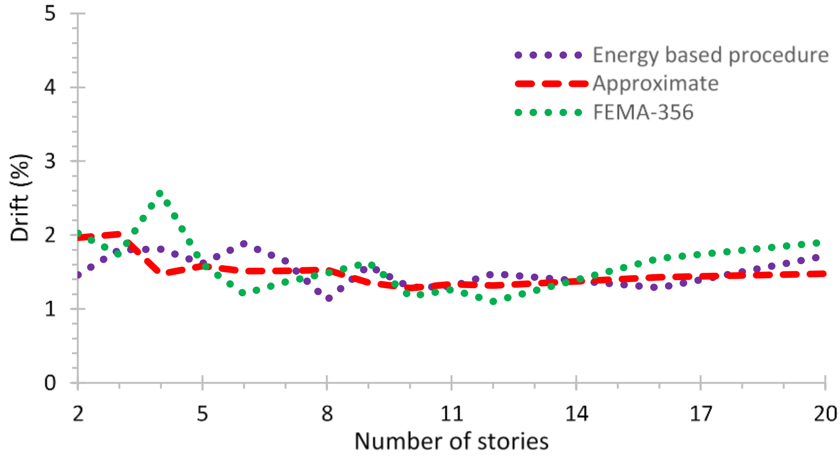

Fig. 12 Results of present study versus FEMA-356

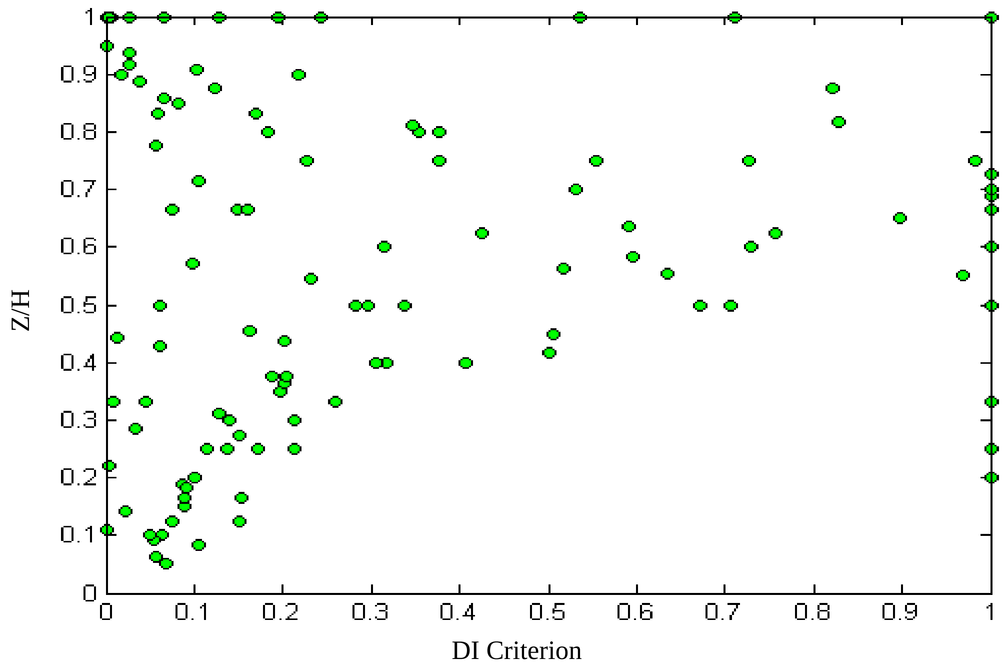

Fig. 13 Damage observed in structures

T_e was calculated as

$$T_e = T_i \sqrt{\frac{K_i}{K_e}} \tag{6}$$

where K_i is the elastic lateral stiffness of the building (initial stiffness of the non-linear base shear versus the roof displacement curve), and K_e is the effective lateral stiffness of the building.

Eq. (4) depends on the number of stories, height of the structure, elastic fundamental period,

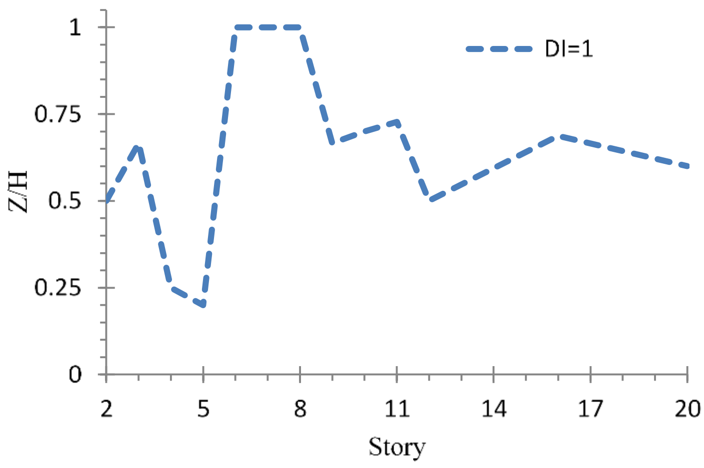

Fig. 14 Total failure in structures

and effective fundamental period of the structure. Fig. 12 shows the target displacements from the energy-based method, their approximate values and the target displacement calculated using FEMA-356.

The structures were analyzed using cyclic loading and the DI was investigated. The most damage occurred at 50%-80% of the height of the structure (Fig. 13). Total failure (DI = 1) occurred in 23% of structures at roof level, in 62% of structures at 50%-75% of the height of the structure, and in 15% of structures at 20%-40% the height of the structure (Fig. 14).

4. Conclusions

In this study, 13 structures were investigated and their target displacements estimated using a new energy-based procedure. FEMA-356 was used to approximate the estimated target displacement. As there is not any exact analytical or numerical method to obtain the target displacement, and also FEMA-356 method (based on experimental and field observations) was widely used and relied by civil engineers, the two values were then compared. As shown, the estimated target displacements were close to target displacements obtained from FEMA-356, indicating that the proposed method can be reliably used to obtain target displacement. This new approximation equation for estimating target displacement is easy to use. Being based on both structural capacity (imparted energy to the structure due to cyclic loading) and seismic structural demand (based on measured and observed seismic responses of structures by considering some limitations as failure criteria) is the merit of this method; while the FEMA-356 method is based on seismic demands. It is important to note that experimental studying on factual structures can develop the new proposed method and improve its accuracy. Considering a wide range of structures, with different numbers of floors in the current study, makes the results useful for both short and tall buildings.

DI occurred in 23% of structures at roof level, in 62% of structures at 50%-75% of the height of a structure, and in 15% of structures at 20%-40% the height of a structure. Most damage occurred at 50-80% of the height of a structure. The height at which the most damage occurred without considering maximum displacement at roof level can be called the critical height of a structure. The results of this study indicated that the critical height of the structures was at 50%-80% of height of a structure. Engineers currently consider the maximum displacement of structures to be the target displacement. In view of the results of this study, it can be concluded that the critical height is equivalent to the target displacement. This should be the focus of future study.

This is also important to note that these results were obtained using triangular pushover analysis and cyclic loading. It is strongly recommended that this research be continued using seismic loading and modal pushover analysis.

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