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Effective number of mega-bracing, in order to minimize shear lag

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Abstract. In this paper, influence of geometric configurations of multi-story bracing on shear lag behaviour of braced tube structures is investigated. The shear lag of 24-, 36- and 72-story braced tube structures are assessed considering all possible configurations of overall *X* and Chevron bracing types. Based on the analytical results, empirical equations, useful for the preliminary design phase, are proposed to provide the optimum number of stories that braced, in order to exert minimum shear lag on structures. Studying the interaction behaviour of a tube and different bracing types along with paying attention to the shear lag behaviour, a better explanation about the reasons behind the efficiency of a specific bracing module in decreasing the shear lag is developed. The analytical results show that there are distinct differences between the anatomy of braced tube structures with *X* and Chevron bracing regarding the shear lag behaviour.

Keywords: tall buildings; framed tube; braced tube; shear lag; multi-story bracing

1. Introduction

Among all structural forms, tube systems which have been invented by Fazlur R. Khan, offer superior and desirable structural properties in tall buildings. Many tall buildings were built using tubular systems such as the 110-story World Trade Center towers in New York, and the 100-story John Hancock Building in Chicago.Owing to the fact that the perimeter columns which are spaced closely and interconnected by deep spandrels resist the entire lateral loads, considerable freedom in architectural planning of interior space as well as great structural stiffness is achievable (Kim 2010). The main weakness of tubular structures is a phenomenon called shear lag. That is explained following on.

Bending stress distribution in a cross-section under flexural moment is linear based on the simple beam theory assumption. According to this theory, a plane remains a plane after bending. This assumption, especially in box sections, could be valid if the shear stiffness of the section is infinite or when there is no shear force. For practical elements dimensions, this is not the case, especially when a box section is subjected to lateral loading (Fig. 1), the stress values at the flange-

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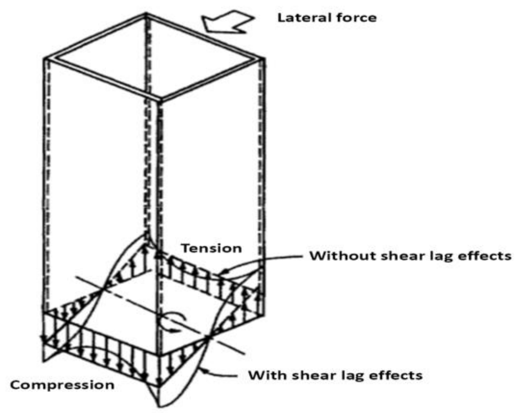


Fig. 1 Axial stress distribution of a box section structure under lateral loading (Taranath 1998)

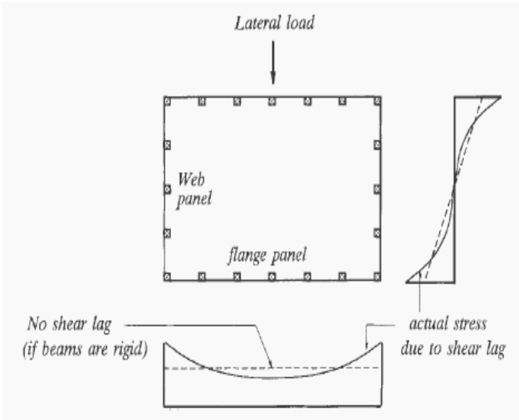


Fig. 2 Shear lag in a framed tube (Haji-Kazemi 2002)

web conjunction are underestimated by simple beam theory and overestimated in the middle of the flange. The difference between actual stress distribution due to flexural moment and that predicted by simple beam theory is called the shear lag effect.

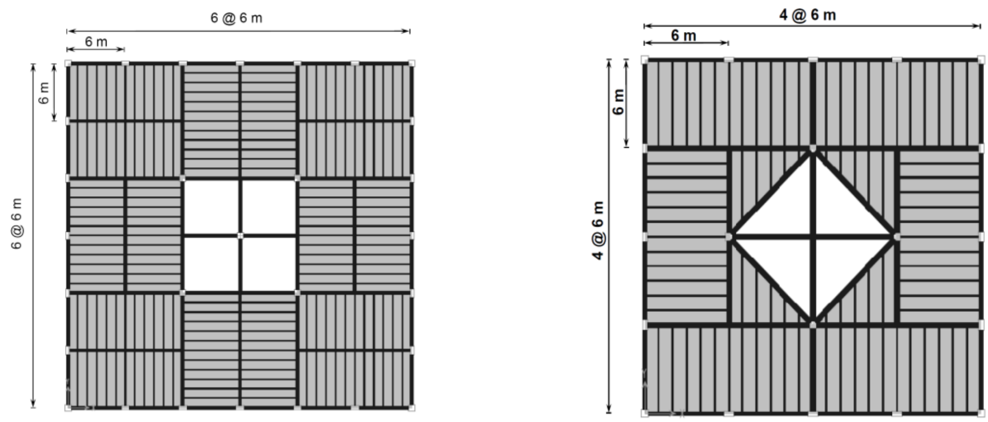
Simplistically the framed tube behaves like a hollow box and similar to what mentioned about box section, shear lag effect also exists in framed tube structures. As shown in Fig. 2, shear lag causes axial forces to be distributed differently in columns comparing to ideal distribution. This main weakness of tubular structures decreases their economically efficiency due to decreasing the resistance moment of the whole structure (Stafford Smith and Coull 1991, Shin 2010).

In order to decrease the shear lag in framed tube structures, rigidity of the spandrels beams should be increased by decreasing the columns' spacing. Contrary to what one may expect, even for a solid-wall tube, the distribution of axial forces is not uniform over the windward and leeward walls. This fact is the result of the relatively thin walls which cause the shear deformation of the tube walls; (their thickness is very small when compared to the height and plan dimensions of the building (Taranath 1998).This drawback can be solved to some extent by using other system types such as bundled tube or braced tube systems.

One of the best known ways to decrease the shear lag in framed tube structures is to apply diagonal members in the perimeter frames. Implementing multi-story bracing leads the structure to behave more like a hollow tube in the bending mode as well as increasing the strength and stiffness (Kim *et al*. 2007). The term, Multi-story bracing, here refers to a brace that covers multi bays and multi stories as a one component; for example *X* bracing. Different configurations and consequently, the number of this overall bracing can affect the structural responses such as, lateral displacement, seismic behaviour and shear lag (Zhang 2010).

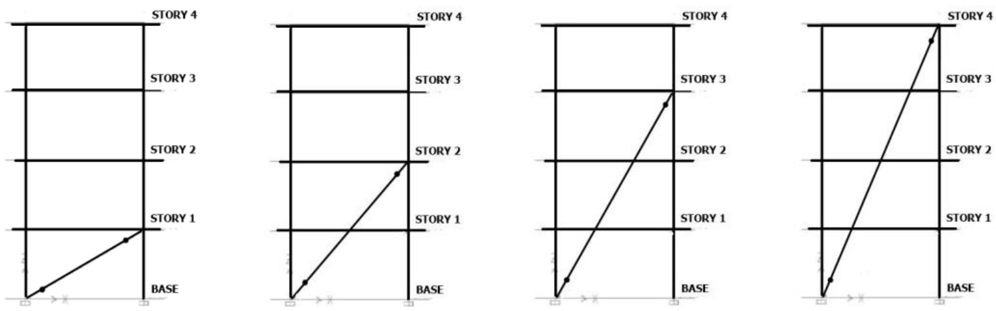
The main goal of this paper is to propose some equations to estimate optimum number of multi-story *X* and Chevron bracing for a structure with a given aspect ratio (H/B), in order to minimize the effect of shear lag for preliminary design phase. In this regard, aesthetic aspects are neglected. Furthermore, Braced Tube Structure (BTS) and Framed Tube Structure (FTS) systems are investigated in order to compare the shear lag behaviour of each system.

In this paper 24-, 36- and 72-story framed tube structures with all feasible configurations of overall *X* and Chevron bracing were designed and the shear lag behaviour of each of them were



(a) 36m × 36m (6 bays) (b) 24m × 24m (4 bays)

Fig. 3 Structural plans of tubular structures



Case (1) Case (2) Case (3) Case (4)

Fig. 4 Different cases of a bracing member

calculated. Determination of the interaction between tube and different bracing configurations could help us to have better understanding and explanation about the reasons of efficiency of a specific bracing configuration in decreasing shear lag.

2.Specifications of analysis models

The analysis models are 24-, 36- and 72-storyframed and braced tube structures. Structures are 24m × 24m (4 bays) and 36m × 36m (6 bays) in plan and the story height is 3.5 meters; therefore, aspect ratio of structures has wide range, from 2.3 to 10.5. Fig. 3 plots the structural plans of these buildings. The spacing of external columns is 6 meter; meanwhile, it must be mentioned that this spacing is not economical in the case of high stiffness demand of framed tube structures due to lateral loading. However, in this study this spacing is used to show the efficiency of different bracing configurations in decreasing shear lag effect.

Every framed tube structure is braced in 4 cases with *X* and Chevron bracing configurations to form braced tube structure. As shown in Fig. 4, case (1) is denoting a bracing member that locates between two consecutive columns with height equal to one story (Zahiri-Hashemi 2008). Further,

Fig. 5 100-story John Hancock Building, Chicago, 1969

case (2) is indicating a bracing member that locates between two consecutive columns with height equal to two-story and similarly for case (3) and (4). Bracing members of each bracing case jointly form a single level of diagonals that extend over ‘n’ stories as a *n*-story bracing module which distributed along the height by repetitive diagonal pattern. For example, the *X* bracing in John Hancock Building consists of members with case (3) and thereforea18-story braced tube module is made. It is necessary to mention that limiting the number of cases to four is related to constructional restriction in length of a bracing member according to similar existing buildings in the world (Kheyroddin and Zahiri-Hashemi 2008).

The case of bracing has been affected the number of overall bracing (Multi-story bracing) on the building façade, so that, all of the possible configurations of bracing could form. For convenience in representing the analytical results, the structures that braced with bracing cases 1, 2, 3 and 4 are called BTS-1, -2 , -3 and -4, respectively.

Gravity loads comprise a 4.4 KN/m^2 dead load and a 3.4 KN/m^2 live load. Although base shear which result from wind load in tall buildings is usually greater than base shear result from seismic load; in this paper earthquake ground motion is considered as affecting lateral force, due to lack of research in this field. Regarding the concept of tubular structures, major part of lateral loading due to earthquake were carried out by perimeter frames and the internal frames were designed mainly for gravity loadings, as a predominant force exerted on it. Despite this fact, in order to increase the ability of comparison between results of two plans, we seekthe ratio of total shear that act on internal frame due to lateral loading to be approximately equal in each plans directions.

The design seismic load was computed using the Iranian Code of Practice for Seismic Resistant Design of Buildings (BHRC 2005). According to this code requirements, dynamic analysis using design spectrum of this code is necessary for buildings which are over 50 meters height; In addition, base shear due to dynamic analysis must scale to base shear results from static analysis if that is smaller than the static base shear. Following factors were chosen for all structures to compute base shear force V=(ABI/R). Importance factor in these structures was considered as I=1. Structures were located in a site with soil classification of type II (375<Vs<750 m/s) and in a

Table 1 Member sections of framed and braced tubular structures (unit: mm)

(a) Plan 24m × 24m							
Height	Stories	Perimeter Columns □	Spandral Beams **I**	Bracing Case (1) □	Bracing Case (2) □	Bracing Case (3) □	Bracing Case (4) □
24-Story	1-24	600x300x35	600x200x20x20	200x200x11	280x280x13	400x300x24	550x350x28
48-Story	1-24 25-48	800x400x45 700x350x40	800x300x20x25 700x250x20x20	220x220x10	300x300x15	400x400x20	600x400x30
72-Story	1-40 41-72	900x450x65 900x450x60	900x350x20x38 900x350x20x35	220x220x11	320x320x15	400x400x20	500x500x30
(b) Plan 36m × 36m							
Height	Stories	Perimeter Columns □	Spandral Beams **I**	Bracing Case (1) □	Bracing Case (2) □	Bracing Case (3) □	Bracing Case (4) □
24-Story	1-4 5-10 11-24	800x400x45 700x350x40 600x300x35	800x300x20x25 700x250x20x20 600x200x20x20	260x260x13	300x300x25	300x300x25	700x300x35
48-Story	1-8 9-32 33-48	900x450x50 800x400x45 700x350x40	900x350x20x30 800x300x20x25 700x250x20x20	300x300x15	350x350x30	700x400x35	800x450x50
72-Story	1-36 37-72	900x450x65 900x450x60	900x350x20x38 900x350x20x35	340x340x15	400x400x30	800x400x40	900x500x50

region with very high level of seismicity risk (A=0.35g). Owing to this fact that, response modification factor for tubular structures has not been defined in this code, in this study R (response modification factor) set equal to 7 which is response modification factor of ordinary steel moment-resisting frame. It must be mentioned that based on Kim *et al*. (2007) research the response modification factor of these structures obtained around 10.

In design of low- and mid-rise structures, size of structural members are mostly controlled by strength, however in high-rise structures, stiffness in addition to strength, affected member's size. Therefore the design procedure of structures was started by satisfying AISC-ASD (1989) requirements and followed by checking satisfaction of maximum allowable in elastic drift by Iranian earthquake code criterion (0.02H). Because the main purpose of this paper is to investigate the effect of bracing type on shear lag behaviour, other influential parameters like stiffness of beams and columns should stand constant. Hence, size of column and beam members in braced tube structures with same height remain constant, equal to an identical framed tube structure.

The yield stress of all members is 350 MPa and the ultimate strength is 450 MPa. Box Sections were used for columns and bracing members, and I sections were used for spandrels. Table 1 summarizes the size of members in framed tube and braced tube structures.

3. Flange shear lag factor

In general case, shear lag causes the axial stress in corner columns increase and also decreases

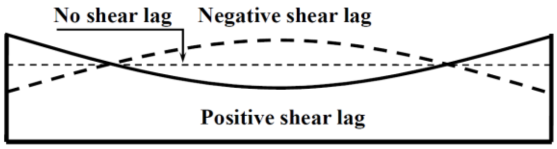


Fig. 6 Positive and negative shear lag phenomenon

those in the middle columns of flange frame. However, previous researches represented an anomaly in the shear lag behaviour (Foutch and Chang 1982, Lee 2002). In the region beyond about a-quarter of the length of the frame from the built-in end, the bending stress near the web is smaller than that near the center of the flange frame (Singh and Nagpal 1994). This phenomenon called “negative shear lag” is shown in Fig. 6.

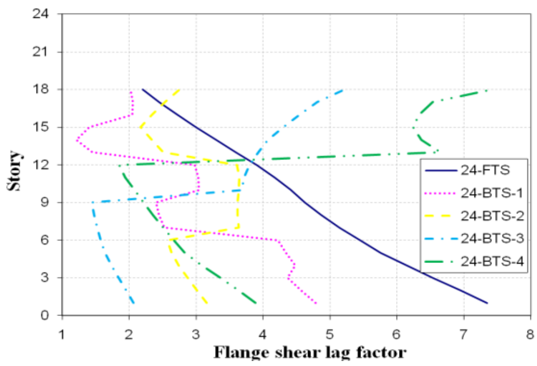
In most of previous literature “f” Factor, the ratio of axial force in corner column to that in the central column of the flange, was used as the measure of shear lag. Wherever this factor is greater than unity it will represent positive shear lag and if less than unity it will represent negative shear lag. Unit amount of “f” represent level that at which, positive shear lag change to negative and called shear lag reversal point.Singh and Nagpal (1994) concluded that negative shear lag, which occur in the top portions of the structure despite of positive shear lag in the bottom portions, originate from positive shear lag and counteracts it; furthermore whenever positive shear lag is great, negative shear lag is remarkable too.

To investigate and measure the effects of each bracing configuration in reducing shear lag of a framed tube structure, a new parameter that called “flange shear lag factor” has been introduce by authors (Zahiri-Hashemi 2008). This parameter is calculated for every story of each structure and is the ratio of maximum axial force to minimum axial force of columns in the flange of structure. This parameter is always equal or greater than unit. In order to compare the efficiency of each bracing configuration in decreasing shear lag, average of flange shear lag factor along the height of every above structures were calculated and consider as amount of shear lag for that structure; consequently in a braced tube structure with specific height and plan dimension, effective number of multi-story bracing that exert minimum shear lag on structure among all possible numbers of bracing would be determined.

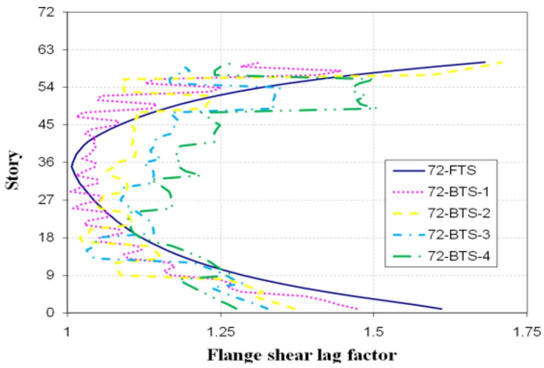
Singh and Nagpal (1994) observed that near the top of the building, the effect of negative shear lag is so predominant that the columns near the corner develop forces opposite to those in the other columns. In this study, linear static analysis of structures confirmed that approximately in a-quarter of height of the buildings from top of them, axial force of corner columns in compression flange changed into tension force and conversely, axial force change into compress force in tension flange. Calculation of flange shear lag factor for these levels is meaningless, because of its negative sign; furthermore spectral dynamic analysis couldn’t show levels which signifies the changing axial force (because of SRSS method of modal combination), so according to the number of these levels based on linear static analysis, a top quarter of the height were neglected from inferences and diagrams.

4. Proposed equation for X bracing configuration

According to what discussed in previous section, flange shear lag factor of 24-, 36- and 72-

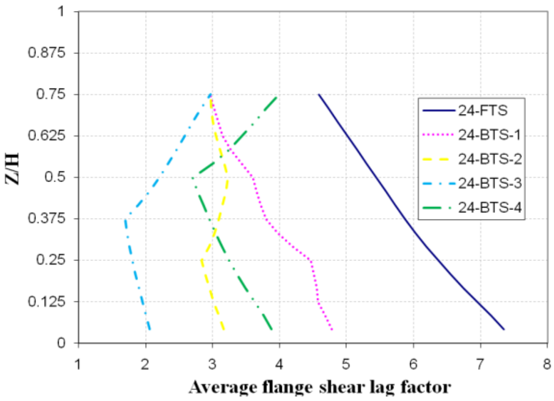


(a) 24-story structures with 6 bays

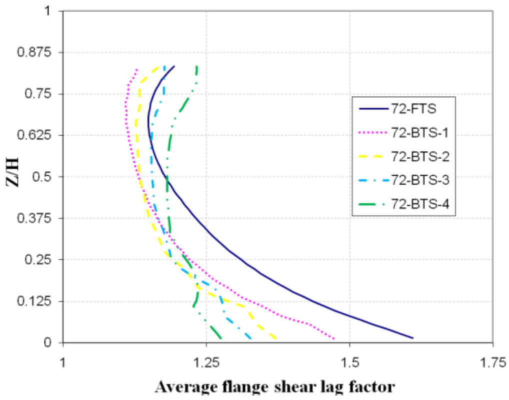


(b) 72-story structures with 4 bays

Fig. 7 Flange shear lag factor diagram (X type bracing)



(a) 24-story structures with 6 bays



(b) 72-story structures with 4 bays

Fig. 8 Average flange shear lag factor diagram

story framed tube structures and all identical braced tube structures, with four different before mentioned configurations of *X* bracing and two different plan dimensions, totally equal to 30 structures, were calculated for every story. Fig. 9 display the result for 24- and 72-story FTS and BTS with *X* bracing and plan dimension and aspect ratio of, 36m × 36m, 2.3 and 24m × 24m, 10.5 respectively.

As shown in Fig. 7, trend of changing in shear lag of FTS is steady whereas in BTS there are number of jumping in different levels along the height of the structures. The reason of this behaviour is changing in flexural stiffness of each story due to variable distance of bracings from neutral axis. Concerning the existence of numerous jumping in shear lag behaviour of BTS, making the inference about the most efficient number of multi-story bracing in decreasing shear lagis difficult. So, for convenience in determination of the optimum bracing module, average of flange shear lag factor was used as shown in Fig. 8. In these diagrams, vertical axis is dimensionless height Z/H (elevation of each story from the base to overall height of the structure) and horizontal axis for “I” level represents the average of flange shear lag factors from base to “I” level. As presented in these figures, for example, the most efficient configuration of bracing for a

24-story structure with 6 bays in plan is BTS-1, equal to a 6-story braced tube module (n-story braced module indicates to a multi-story bracing that extended over n stories). For a 72-story structure with 4 bays in plan, BTS-1 equal to a 4-story module is the best configuration in decreasing shear lag totally.

Calculated average flange shear lag factor of above structures for 0.75H is presented in Table 2.

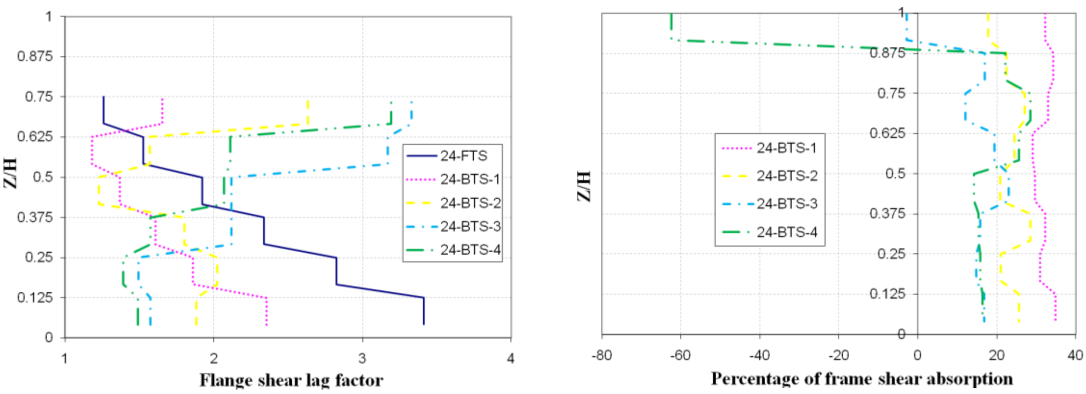
Average flange shear lag factor diagram can be exploited for determining the optimum module of bracing due to shear lag effect, and also to demonstrate the effect of different bracing configurations in general trend of changing in shear lag along the height. For example, as shown in Fig. 10 (b), although BTS-4 in lower level (approximately from base to 0.125H) has the least amount of shear lag, by ascending along the height, efficiency of this bracing is decreased and finally BTS-1 has minimum shear lag in comparison with other structures.

Better understanding of changing pattern of the flange shear lag factor of all these structures along the height is feasible by studying the interaction behaviour of tube and bracing; for this purpose, flange shear lag factor diagrams were smoothed for every 0.125 height of structure with getting average between shear lag factor magnitudes in this region. Similarly, diagrams of tube (frame) shear absorption percentage were smoothed for every 0.125 height. The comparison between these two diagrams could show that how each bracing configuration regarding its interaction behaviour with frame, could affect the shear lag behaviour of a braced tube structure versus other configurations. In Figs. 9-11 these two parameters were plotted against Z/H for structures with plan dimension of 24m × 24m.

As shown in Fig. 9, from structure supports to 0.375H, BTS-4 with minimum percentage of frame shear absorption has maximum effect on decreasing shear lag. From 0.375H to 0.5H, BTS-2byincreasing in percentage of frame shear absorption in comparison with BTS-4has minimum flange shear lag factor; then from 0.5H to 0.625H, BTS-1 with maximum portion of frame

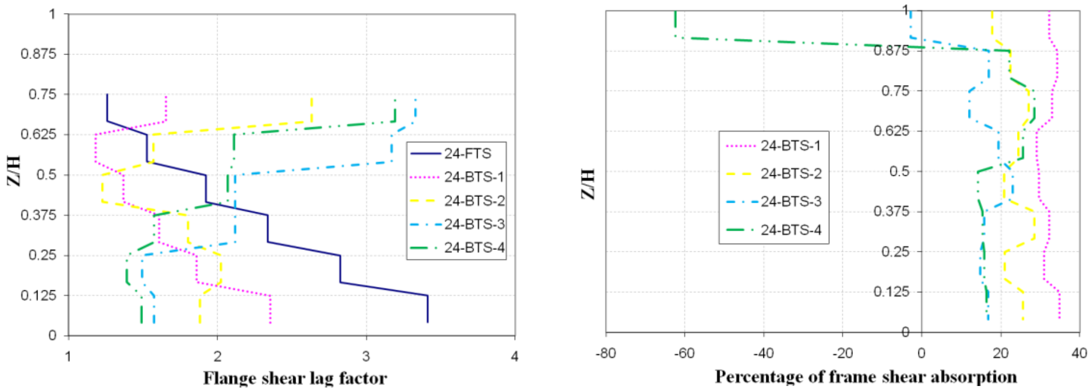
Table 2 Average flange shear lag factor of 24- and 72-story FTS and X bracing BTS for 0.75H

	FTS	BTS-1	BTS-2	BTS-3	BTS-4
24-story structure (6 ×6 bays)	4.5868	2.962	2.968	2.9662	3.9966
72-story structure (4 ×4 bays)	1.1584	1.1135	1.1335	1.1762	1.2229



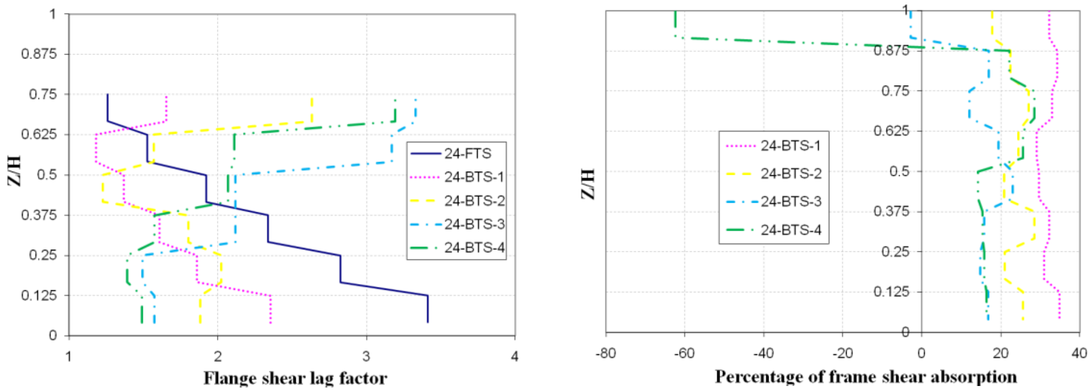
(a) flange shear lag factor (b) percentage of frame shear absorption

Fig. 9 Smoothed diagrams for every 0.125H of 24-story FTS and BTS with *X* bracing and 4 bay



(a) flange shear lag factor (b) percentage of frame shear absorption

Fig. 10 Smoothed diagrams for every 0.125H of 48-story FTS and BTS with *X* bracing and 4 bays



(a) flange shear lag factor (b) percentage of frame shear absorption

Fig. 11 Smoothed diagrams for every 0.125H of 72-story FTS and BTS with *X* bracing and 4 bays

shear absorption has maximum effect on decreasing shear lag versus other structures. Finally framed tube structure from 0.625H to 0.75H with one hundred percent of frame shear absorption has minimum shear lag when compared by all braced tube structures.

Similar to what explained above, smoothed diagrams of flange shear lag factor and percentage of frame shear absorption of 48-story framed and braced tube structures, could help to realize the exact shear lag behavior of braced tube structures with different configurations of *X* bracing along the height (Fig. 10).

As shown in Fig. 10 from *Z*=0 to 0.125H, BTS-4has minimum percentage of frame shear absorption in comparison with other braced tube structures in this region, also has maximum effect on decreasing flange shear lag factor. From 0.125H to 0.25H, BTS-2 with increasing in percentage of frame shear absorption beside case BTS-4 has minimum shear lag; then from 0.25H to 0.625H, BTS-1 with maximum percentage of frame shear absorption among all bracing configurations have most efficiency in decreasing shear lag factor. Finally from 0.625H to 0.75H, framed tube structure has one hundred percent of frame shear absorption and minimum flange shear lag factor comparing with all braced tube structures.

For 72-story structures with4 bays in plan as shown in Fig. 11, again from structure supports to 0.125H, BTS-4 with maximum percentage of brace shear absorption among all bracing modules(due to its major stiffness) has maximum effect on decreasing shear lag. From 0.125H to 0.25H, BTS-2 with increasing in portion of frame shear absorption in this region has a comparative success in decreasing flange shear lag factor. Finally from 0.25H to 0.75H, BTS-1with maximum percentage of frame shear absorption has maximum effect in decreasing shear lag.

It must be notified that similar to what explained about structures with 24m × 24m plan dimension, same results were observed for structures with 36m × 36m plan dimension. Regarding to obtained results for braced tube structures with X bracing, using bracing cases with major stiffness like case (4) and (3) are more appropriate in lowest levels of structure. However, these cases are not only incapable to decrease the effect of shear lag factor in top levels, conversely cause shear lag factor of a braced tube structure become even about two times more than a framed tube structure. Comparison between flange shear lag factor of 72-story FTS with 24- and 48-storyFTSdemonstrate that, in 24- and 48-story FTS flange shear lag factor is decreasing from bottom toward top of the structure regularly, but in 72-story FTS, it is decreasing from Z=0 to 0.5H and then it is increasing by ascending along the height. The main reason of this behaviour is effect of boundary conditions (the origin of occurrence of positive shear lag is structural supports). By increasing the height gradually, the effect of boundary conditions in occurrence the positive shear lag is becoming smaller, and finally it is changing into negative shear lag and as mentioned before, it's increasing toward the top of the structure. Due to this reason, the effect of boundary conditions in 24-and 48-story FTS is so predominant so that, negative shear lag didn’t occurred in 0.75H of this structures on the contrary to what occurred in 72-story framed tube structure.

Finally based on the results of this section, an empirical equation is proposed which can be used to determine the number of story which braced as a *X* braced tube module (N_b) to exert minimum shear lag on structure with given number of story (N), height (H), number of bays and plan dimension (B).

$$\frac{N_b}{N} = 0.57 \times \left(\frac{H}{B}\right)^{-0.99} \quad \text{For } \mathbf{3} \leq \frac{\boldsymbol{H}}{\boldsymbol{B}} \leq \mathbf{10} \text{and rectangular plan} \tag{1}$$

For this purpose, ratio of N_b to N for structures that summarized in Table 3 was calculated and drew versus aspect ratio (H/B) of these structures, then by calculatingregression between results, the best curve were fitted among them (Fig. 12).

Bracing module that exert minimum shear lag on braced tube structure with *X* bracing configuration by rectangular plan and aspect ratio between about 3 and 10 is computed from Eq. 1 by multiplying its ratio to number of story and then rounding the result to the nearest multiple

Table 3 Data summary of BTS with optimum number of Multi-story *X* bracing

Plan Dimensions (Number of bays)	**Number of Stories (N)**	**Number of Story Module (N_b)**	**(N_b/N)**	**Aspect Ratio (H/B)**
4 × **4**	24	4	4/24	3.5
4 × **4**	48	4	4/48	7
4 × **4**	72	4	4/72	10.5
6 × **6**	24	6	6/24	2.3
6 × **6**	48	6	6/48	4.6

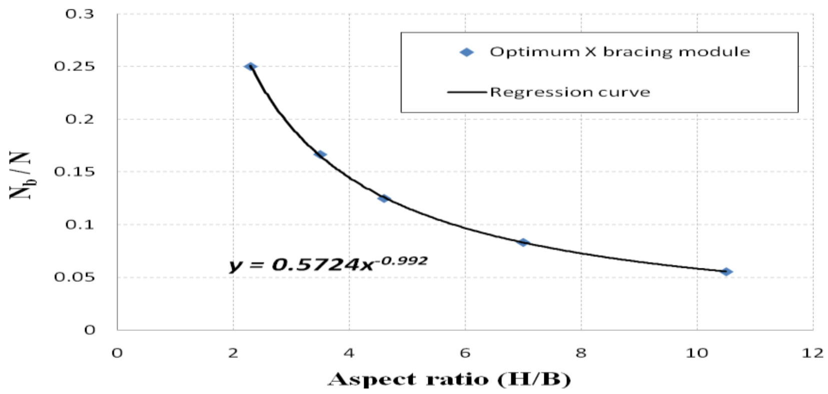


Fig. 12 Regression curve of the best modules of *X* bracing in studied BTS concerning shear lag

of number of bays. As an example for a structure with 110 stories and a story height of 3.9 meter with plan dimension of 60 m × 60 mwith 6 bays, N_b which calculated from equation 1 is equal to 8.94; then by rounding this number to the nearest multiple of 6 (number of bays), a 6-story bracing module withdiagonal angle of 21.3 degreesas a optimum module of *X* bracing regard to minimum shear lag effect was suggested.

For assessing the ability of proposed equation, similar study as what mentioned before carried out on a 72-story structure models with 36m × 36m (6 bays) plan dimension to define the best module of *X* bracing among all possible configurations, to exert minimum shear lag on braced tube structure. Average flange shear lag factor diagram of these structures is represented in Fig. 13 and result of calculating average flange shear lag factor of these structures for 0.75H is presented in Table 4. As shown in Fig. 15 the most efficient configuration of *X* bracing for a 72-story braced tube structure is BTS-1 equal to a 6-story bracing module.

Table 4Average flange shear lag factor of 72-story FTS and X bracing BTS for 0.75H

	FTS	BTS-1	BTS-2	BTS-3	BTS-4
72-story structure (6 ×6 bays)	1.3315	1.2198	1.278	1.3245	1.4543

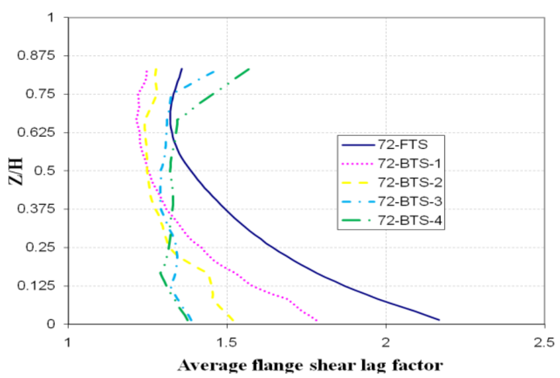
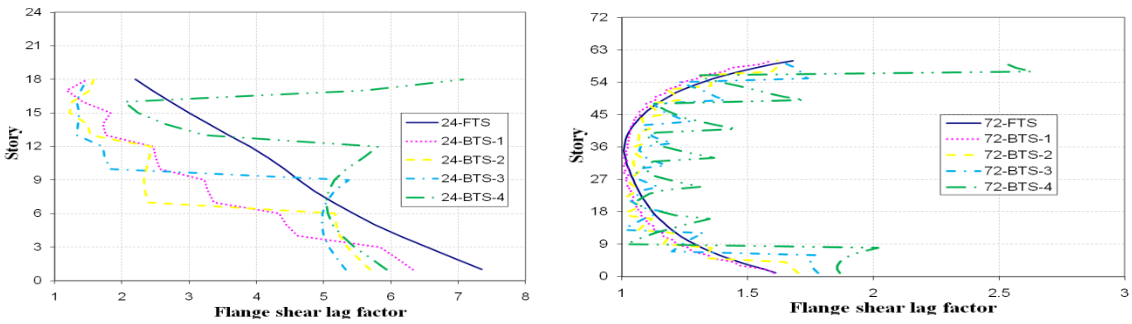


Fig. 13 Average flange shear lag factor diagram of 72-story structures with 6 bays (*X* bracing)



(a) 24-story structures with 6 bays; (b) (b) 72-story structures with 4 bays

Fig. 14 Flange shear lag factor diagram (Chevron type bracing)

Using equation 1 for a 72-story structure with 36m × 36m plan dimension and 6 bays result in N_b which is equal to 5.98; then by rounding this number to the nearest multiple of 6 (number of bays), a 6-story bracing module with diagonal angle of 30.3 degrees as a optimum module of *X* bracing regard to minimum shear lag effect was suggested. This angle is close to 35 degrees which produces the maximum shear rigidity (Moon *et al*. 2007). There is good agreement between results of proposed equation and obtained from model analysis.

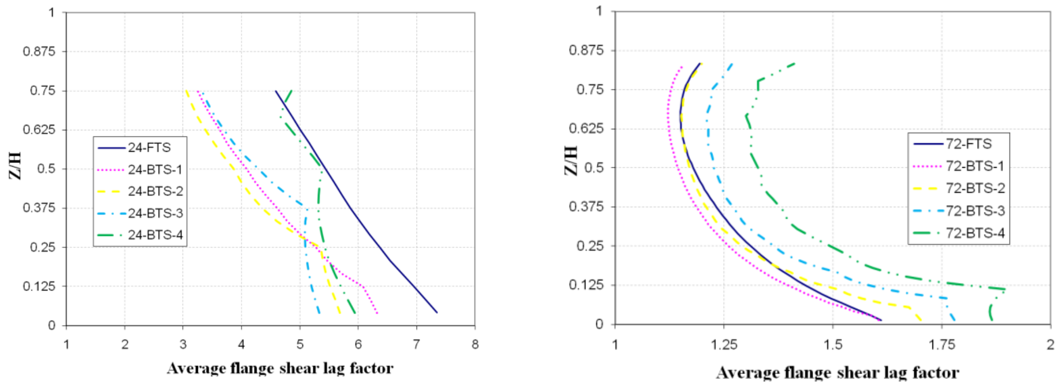
It should be mentioned that based on Moon (2005, 2008) research, the angle of about 45 degrees, which is typically used for tall buildings to satisfy both architectural and structural requirements in practice, is close to optimal angle in terms of minimum material usage, whereas this angle doesn’t guarantee minimum shear lag effect on structure. In the same way, Kim and Lee (2010) concluded that for diagrid structures the shear lag effect increased, as the slope of braces increased.

5. Proposed equation for Chevron bracing configuration

Similar to whatexplained for structures with *X* bracing, flange shear lag factor used to measure shear lag of braced tube structures with Chevron bracing, too. Again smoothed flange shear lag factor diagram for every 0.125 Height, as well as smoothed percentage of frame shear absorption could help to investigate the effect of different configurations of Chevron bracing in the shear lag behaviour of braced tube structures along the height. Fig. 14 displayed the result for 24- and 72-story FTS and BTS with Chevron bracing and plan dimension and aspect ratio of, 36m × 36m, 2.3 and 24m × 24m, 10.5, respectively.

As shown in Fig. 16 like as *X* bracing configuration, there are numbers of jumping in shear lag behaviour of BTS at different levels along the height of structures. This behaviour is due to changing in flexural stiffness of each story regarding to variable distance of bracing from neutral axis. Again for convenience in determination of the optimum bracing module, average of flange shear lag factor was used as shown in Fig. 15.

As represented in these figures, the most efficient configuration of bracing for 24-story structure with 6 bays in plan is BTS-2 equal to 6-story braced tube module andfor 72-story structure with 4 bays in planBTS-1 equal to 3-story module is the best configuration in decreasing

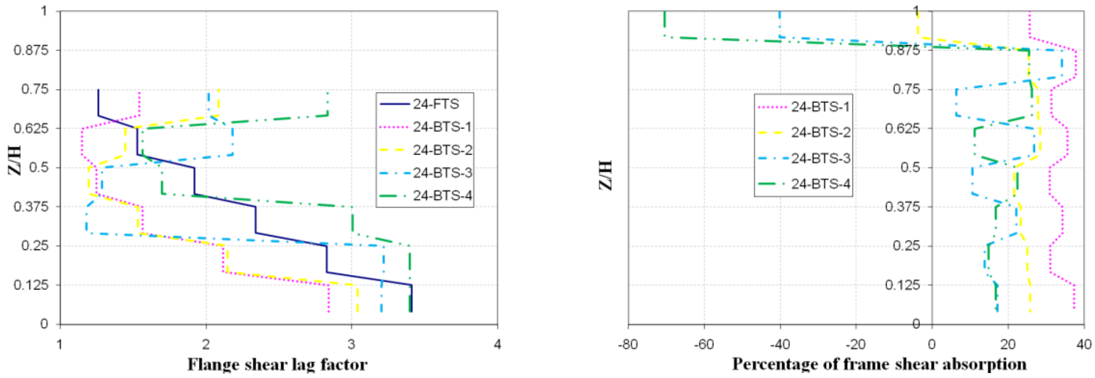


(a) 24-story structures with 6 bays (b) 72-story structures with 4 bays

Fig. 15 Average flange shear lag factor diagram (Chevron type bracing)

Table 5 Mean flange shear lag factor of 24- and 72-story FTS and Chevron bracing BTS for 0.75H

	FTS	BTS-1	BTS-2	BTS-3	BTS-4
24-story structure (6 ×6 bays)	4.5868	3.2478	3.0569	3.3147	4.8583
72-story structure (4 ×4 bays)	1.1584	1.1272	1.1631	1.2211	1.3281

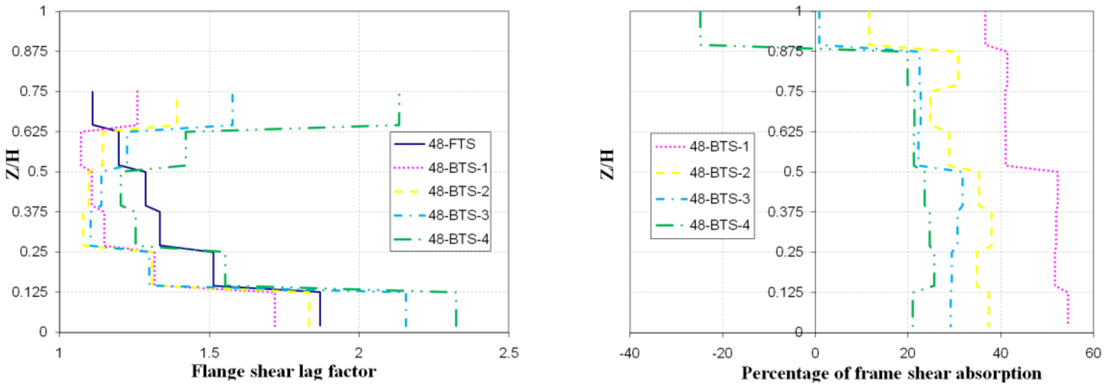


(a) flange shear lag factor (b) percentage of frame shear absorption

Fig. 16 Smoothed diagrams for every 0.125H of 24-story FTS and BTS with Chevron bracing and 4 bays

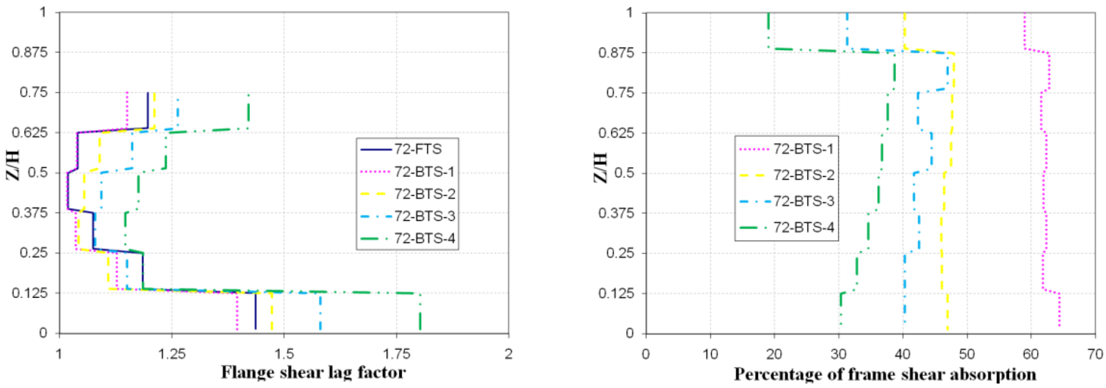
shear lag, totally. Result of calculating average flange shear lag factor of these structures for 0.75H is presented in Table 5.

Analytical results displayed that in all structures with Chevron bracing (except for 24-story BTS with 6 bays) BTS-1 in lower level has a greater effect on decreasing shear lag while in structures with *X* bracing, BTS-4 generally (except for 24-story BTS with 6 bays) has a greater effect on decreasing the shear lag in this region.Better understandingfrom the shear lag behaviour of each Chevron bracing configuration along the height is possible by means of investigating the interaction between bracing and tube.Smoothed diagrams of flange shear lag factor and frame shear absorption of structures with plan dimension of 24m × 24m andChevron bracing configurations represented in Figs. 16-18.



(a) flange shear lag factor (b) percentage of frame shear absorption

Fig. 17 Smoothed diagrams for every 0.125H of 48-story FTS and BTS with Chevron bracing and 4 bays



(a) flange shear lag factor (b) percentage of frame shear absorption

Fig. 18 Smoothed diagrams for every 0.125H of 72-story FTS and BTS with Chevron bracing and 4 bays

As shown in Fig. 16, on the contrary to what structures with BTS-1 *X* bracing behave from *Z*=0 to 0.25H (Fig. 11), BTS-1 with maximum percentage of frame shear absorption has major effect on decreasing flange shear lag factor in this region of Chevron bracing structures. From 0.25H to 0.5H, BTS-3 by decreasing in percentage of frame shear absorption in comparison with BTS-1 causes to minimize shear lag in this region. Again by increasing in the portion of frame shear absorption, BTS-1 with maximum of it, has minimum shear lag from 0.5H to 0.625H and finally from 0.625H to 0.75H framed tube structure, has least amount of flange shear lag factor among all braced tube structures; while in same structures with *X* bracing, by ascending along the height the bracing modules that gradually increasethe shear absorption of the tube have maximum effect on decreasing shear lag.

Fig. 17 display smoothed flange shear lag factor and percentage of frame shear absorption of 48-story FTS and BTS with Chevron bracing. As shown in this figure, from structure supports to 0.125H, BTS-1 with maximum percentage of frame shear absorption has maximum effect on decreasing flange shear lag factor. From 0.125H to 0.5H, BTS-2 by decreasing in percentage of frame shear absorption compare with BTS-1 has minimum shear lag factor; then in the region of

0.5H to 0.625H again BTS-1 with major percentage of frame shear absorption has maximum effect on decreasing shear lag. Finally from 0.625H to 0.75H framed tube structure with one hundred percent of frame shear absorption, has minimum flange shear lag factor. Again it could be observed that, unlike structures with *X* bracing configuration; Chevron bracing modules with minimum shear stiffness have major effect on decreasing shear lag in the lowest levels.

Smoothed flange shear lag factor and percentage of frame shear absorption for 72-story FTS and BTS with Chevron bracing were plotted in Fig. 18. Regarding to this figure similar to all other structures with Chevron bracing, from *Z*=0 to 0.125H,BTS-1 with maximum percentage of frame shear absorption has maximum effect on decreasing flange shear lag factor by comparing with other bracing configurations. From 0.125H to 0.25H, BTS-2 by decreasing in percent of frame shear absorption versus BTS-1 has minimum shear lag factor. Finally from 0.25H to 0.75H again, BTS-1 with maximum percentage of frame shear absorption has major effect on decreasing shear lag. As seen in this figure and Fig. 15(b), BTS-4causes shear lag effect being increased in BTS versus FTS and despite of structures with *X* bracing, this module doesn't have any influence in decreasing shear lag even in the lower levels.

Based on results of this section, an empirical equation is proposed which can be used to determine the number of story which braced as a Chevron braced tube module (*Nb*) to exert minimum shear lag on structure with given number of story (*N*), height (*H*), number of bays and plan dimension (*B*). For this purpose, ratio of Nb to N for structures that summarized in Table 6 was calculated and drew versus aspect ratio (H/B) of these structures, then by regression between results, the best curve were fitted among them (Fig. 19).

Table 6 Data summary of BTS with optimum number of multi-story Chevron bracing

Plan Dimensions (Number of bays)	**Number of Stories (N)**	**Number of Story Module (N_b)**	**(N_b/N)**	**Aspect Ratio (H/B)**
4 × 4	24	2	2/24	3.5
4 × 4	48	2	2/48	7
4 × 4	72	2	2/72	10.5
6 × 6	24	6	6/24	2.3
6 × 6	48	3	3/48	4.6

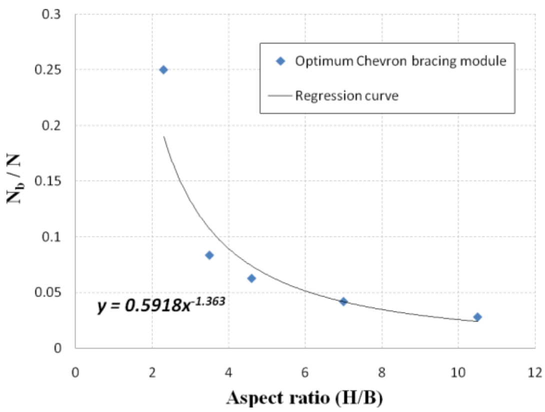


Fig. 19 Regression curve of the best modules of Chevron bracing in studied BTS concerning shear lag

Table 7 Average flange shear lag factor of 72-story FTS and BTS with Chevron bracing and 6 bays for 0.75H

	FTS	BTS-1	BTS-2	BTS-3	BTS-4
72-story structure (6 ×6 bays)	1.3315	1.264	1.3253	1.4733	1.7066

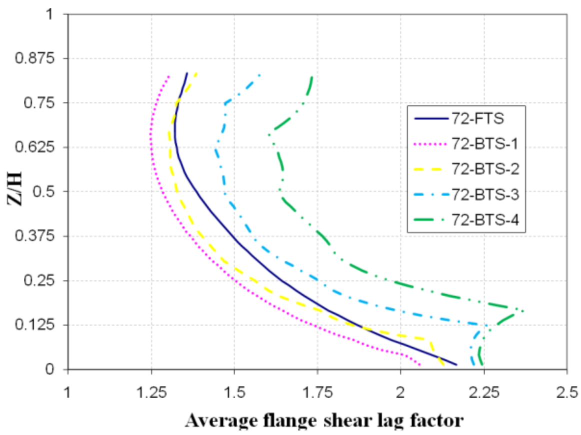


Fig. 20 Average flange shear lag factor diagram of 72-story structures with 6 bays, Chevron type bracing

$$\frac{N_b}{N} = 0.59 \times \left(\frac{H}{B}\right)^{-1.36} \quad \text{For } \mathbf{3} \leq \frac{\boldsymbol{H}}{\boldsymbol{B}} \leq \mathbf{10} \text{and rectangular plan} \tag{2}$$

Bracing module that exert minimum shear lag on braced tube structure with Chevron bracing configuration by rectangular plan and aspect ratio between around 3 and 10 is computed from equation 2 by multiplying its ratio to number of story and rounding the result to the nearest multiple of half number of bays. As an example for a structure with 80 stories and a story height of 3.9 meter with plan dimension of 60 m × 60 m with 6 bays, N_b which calculated from equation 2 is equal to 5.01; then by rounding this number to nearest multiple of 3 (half number of bays), a6-story bracing module with a diagonal angle of 37.95 degrees as a optimum module of bracing which result to minimum shear lag effect, is suggested.

For assessing the ability of proposed equation, similar study as what mentioned before carried out on a 72-story structure models with 36m × 36m (6 bays) plan dimension to define the best module of Chevron bracing among all possible configurations, to exert minimum shear lag on braced tube structure. Average flange shear lag factor diagram of these structures is represented in Fig. 20 and result of calculating average flange shear lag factor of these structures for 0.75H is presented in Table 7. As shown in Fig. 20 the most efficient configuration of Chevron bracing for a 72-story braced tube structure is BTS-1 equal to a 3-story bracing module.

Using equation 2 for a 72-story structure with 36m × 36m plan dimension and 6 bays result in N_b which is equal to 3.01; then by rounding this number to nearest multiple of 3 (half number of bays), a 3-story bracing module with diagonal angle of 30.25 degrees as a optimum module of Chevron bracing regard to minimum shear lag effect is suggested. This angle is close to 35 degrees which produces the maximum shear rigidity (Moon *et al*. 2007). There is good agreement between results of proposed equation and what obtained from model analysis. Also, it could be observed that the optimum angle of chevron bracing module concerning minimum shear lag occurrence is far from the angle of about 45 degrees, which is typically used for tall buildings to satisfy both architectural and structural requirements in practice.

6. Comparing shear lag behaviour of FTS and BTS

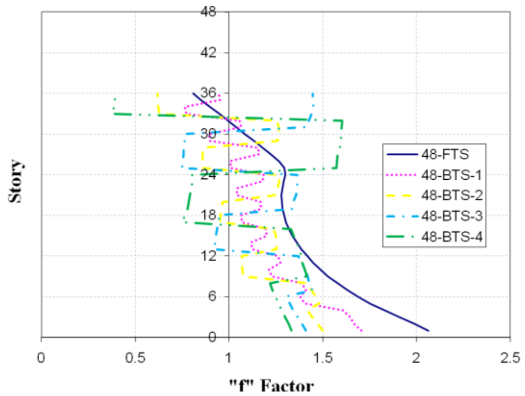
As mentioned before in majority of previous literature, the ratio of axial force in corner column to that in the central column of the flange was use as the measure of shear lag and named "f" Factor. Wherever this factor is greater than unity it will represent positive shear lag and if less than unity it will represent negative shear lag. Herein, to compare and contrast shear lag behaviour of FTS and BTS we used this factor too.

6.1 Distinction between shear lag behaviour of FTS and BTS

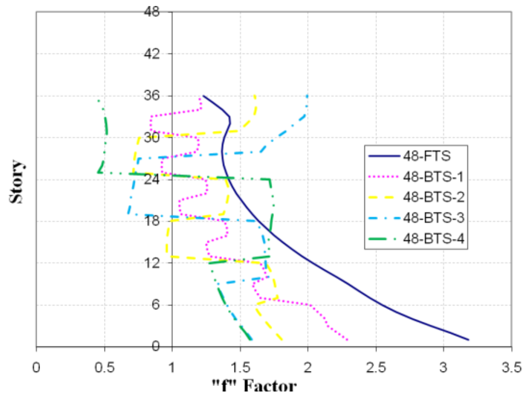
According to Singh and Nagpal (1994) research the overall behaviour of the framed tube building is the net effect of two modes" (1) the portion of the building above the j^{th} floor level is subjected to the loading with joints at the j^{th} floor level assumed to be fixed and (2) the upper portion is subjected to joint displacements at j^{th} floor level". The first mode contributes to positive shear lag while another mode contributes to negative shear lag. By ascending along the height regard to reduction of story shear force, effect of first mode is decreased and then "f" Factor decreased too. At the Level in which "f" is equal to unit quantity, positive shear lag change into negative and called shear lag reversal point.

Shear lag behaviour of 48-story FTS and *X* bracing BTS is presented for two plan dimension of 24m × 24m and 36m × 36min Fig. 21. As shown and what mentioned in previous sections, "f" Factor of FTS steadily decreases along the height while in all BTS there are numbers of jumping in diagrams because of variable flexural stiffness of each story due to inconstant distance of bracing from neutral axis (same results observed for Chevron bracing BTS).

Another point that should be noticed in all BTS with *X* bracing is existence of two types of jumping in "f" Factor diagrams, namely, reducer and amplifier jumping. Generally amplifier

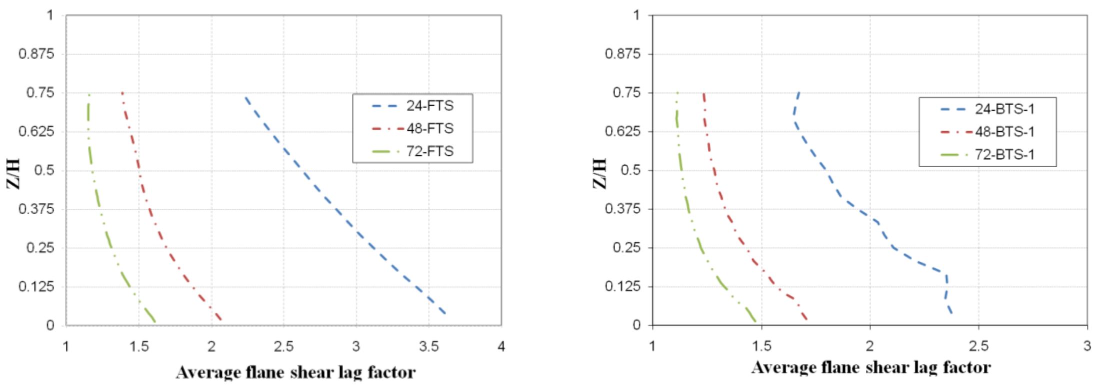


(a) 48-story structures with 4 bays



(b) 48-story structures with 6 bays

Fig. 21 "f" Factor diagram (*X* type bracing)



(a)FTS with 4 bays (b)BTS-1 with X-bracing and 4 bays

Fig. 22 Average flange shear lag factor diagram

jumps increased shear lag and reducer jumps decreased the amount of shear lag. The main reason of these jumps is configuration of bracing elements in a module of bracing. Reducer jumps observed at a story exactly above crest level of a braced module and amplifier jumps observed at a story exactly above middle level of a braced module. For example, as seen in Fig. 21(a) for BTS-3 in structure with 4 bays, a multi-story bracing is completed in every 12-story module, therefore first amplifier jump is occurred after level 6 and first reducer jump observed above level 12. However, in BTS with chevron bracing only reducer jumps were observed and like as *X* bracing these jumps observed at a story exactly above crest level of a Chevron braced module.

Another distinction between FTS and BTS in case of shear lag behavior is due to number and position of shear lag reversal point. According to previous researches and as shown in Fig. 21, FTS have only one reversal point and whenever positive shear lag of the base level increases, level of reversal point shifts upward. On the other hand, in BTS there are more than one reversal points. Also, first reversal point in BTS was occurred in the lower level in comparison with reversal point of FTS. As mentioned before in the lower levels of FTS due to the effect of base restrain, contribution of first shear lag mode in behaviourof structure is so predominant that positive shear lag occurs in this region. Bracing elements cause to constrain the columns together and therefore the effect of first shear lag mode in the overall behaviour of structure decreased, So that, reversal point shift downward. In BTS, above first reversal point, in lower half levels of a bracing module, by increasing the effect of second shear lag mode due to increasing the concavity of the vertical deformation profile, negative shear lag was occurred. In upper half levels of a bracing module by decreasing the effect of second mode, negative shear lag change into positive and another reversal point was formed.

6.2 Similarity between shear lag behaviour of FTS and BTS

Among all parameters that affecting shear lag behaviour of tubular structures, plan dimension and height of the structure chose to investigate the effect of them on shear lag behaviour of BTS and compare with FTS. Analytical results illustrated that for all BTS as well as FTS by increasing the height of structures, shear lag amount decreased, and generally this reduction in FTS is greater than BTS. Reduction in shear lag of a taller structure is due to greater cumulative shear stiffness of

Table 8 Data summary of FTS and BTS with Chevron bracing and plan dimension of 4 bays

	Average flange shear lag factor for 0.75H				
	FTS	**BTS-1**	**BTS-2**	**BTS-3**	**BTS-4**
24-story	2.214	1.744	1.908	2.181	2.649
48-Story	1.385	1.27	1.309	1.417	1.648
72-story	1.158	1.127	1.163	1.221	1.328
Ratio of reduction in shear lag from 24- to 48-story	37.428	27.163	31.414	35.027	37.776
Ratio of reduction in shear lag from 24- to 72-story	47.679	35.355	39.04	44.002	49.867

Table 9 Data summary of 48-story FTS and BTS with *X* bracing

	Average flange shear lag factor for 0.75H				
	FTS	**BTS-1**	**BTS-2**	**BTS-3**	**BTS-4**
4 × 4 bays	1.3854	1.2325	1.2945	1.3429	1.5351
6 × 6 bays	1.8528	1.4495	1.5026	1.6202	1.9367
Ratio of increasing in shear lag due to increase of plan dimension	33.7387	17.6065	16.0757	20.6483	26.1603

this structure than smaller one. Average flange shear lag factor of FTS and BTS-1 with *X* bracing and plan dimension of 24m×24m were represented in Fig. 22. Variation of shear lag by height in FTS and BTS with Chevron configurations and 4 bays were summarized in Table 8.

As shown in Fig. 21, by increasing the plan dimension (number of bays), the shear lag of FTS was increased as well as BTS, because the number of steps in which axial forces in columns are reduced from corner column to center column is larger; due to this fact the shear lag in larger plan is greater than smaller one. It should be noticed that generally increasing in shear lag due to increase of plan dimension in FTS is greater than BTS. Variations of shear lag by increasing plan dimension from 4 bays to 6 bays in 48-story FTS and BTS with *X* configuration were summarized in Table 9.

7. Conclusions

In this study, influence of the geometric configurations of multi-story bracing on shear lag behaviour of braced tube structuresis investigated. For this purpose 24- , 36- and 72-story framed tube structures were braced with *X* and Chevron multi-story bracing with all possible configurations. Shear lag behaviour of these structures were calculated using flange shear lag factor definition. Based on analytical results, empirical equations for assessing the optimal number of overall bracing regard to minimizing shear lag effects were derived. With these formulas for a structure with given geometric specification like as aspect ratio, independent from aesthetic aspect, number of story that braced as a module of overall *X* or Chevron bracing to exert minimum shear lag on structure could be defined for preliminary design phase.It was observed that the angle of about 45 degrees, which is typically used for tall buildings in practice, generally doesn’t guarantee minimum shear lag effect on structure.

Studying the interaction behaviour of tube and bracing could be useful to show what relationship exists between a bracing module that has least shear lag in a specific height region and the percentage of frame shear absorption of this bracing module in that region. Analytical results show that in a braced tube structure with *X* bracing, bracing cases with major stiffness like case (4) has maximum effect on decreasing shear lag in the lowest levels (approximately from structure supports to 0.25H). By ascending alongthe height, bracing modules which cause greater percentage of frame shear absorption in comparison with efficient bracing module in lowest levels have more efficiency in decreasing shear lag factor. On the other hand, in a braced tube structure with chevron bracing, using bracing cases with minimum stiffness like case (1) in the lowest levels (approximately from structure supports to 0.125H), have major effect on decreasing shear lag. Gradually by ascending along the height, decreasing in the percentage of frame shear absorption in the region about 0.25H to 0.5H by using stiffer bracing cases like case (2), have maximum effect on decreasing shear lag. Finally, beyond this region, again increasing in percentage of frame shear absorption by using bracing cases with minor stiffness, result in the minimum shear lag compared with other bracing cases.

Compare and contrast between FTS and BTS in the case of shear lag behavior demonstrate that despite of FTS, there are numbers of jumping in shear lag diagrams of BTS. These jumping are due to variable flexural stiffness of each story due to inconstant distance of bracing from neutral axis. Existence of more than one shear lag reversal point in BTS is another distinction between FTS and BTS in case of shear lag behaviour. It was also observed that shear lag behaviour of both FTS and BTS from changing in number of story or number of bays are similar. By increasing the height of structures (number of story), shear lag amount decreased but generally this reduction in FTS is larger than BTS also by increasing the plan dimension (number of bays), the shear lag of FTS was increased as well as BTS but this increase in shear lag of FTS generally is greater than BTS.

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