Structural Engineering and Mechanics, *Vol. 47, No. 1 (2013) 119-129*
DOI: http://dx.doi.org/10.12989/sem.2013.47.1.119

Optimization of modal load pattern for pushover analysis of building structures

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(Received February 17, 2012, Revised June 7, 2013, Accepted July 3, 2013)

Abstract. Nonlinear Static Procedures (NSPs) have been developed as a practical tool to estimate the seismic demand of structures. Several researches have accomplished to minimize errors of NSPs, namely pushover procedures, in the Nonlinear Time History Analysis (NTHA), as the most exact method. The most important issue in a typical pushover procedure is the pattern and technique of loading which are extracted based on structural dynamic fundamentals. In this paper, the coefficients of modal force combination is focused involving a meta-heuristic optimization algorithm to find the optimum load pattern which results in a response with minimum amount of errors in comparison to the NTHA counterpart. Other parameters of the problem are based on the FEMA recommendations for pushover analysis of building structures. The proposed approach is implemented on a high-rise 20 storey concrete moment resisting frame under three earthquake records. In order to demonstrate the effectiveness and robustness of the studied procedure the results are presented beside other well-known pushover methods such as MPA and the FEMA procedures, and the results show the efficiency of the proposed load patterns.

Keywords: nonlinear static analysis; pushover; optimization; seismic demand; concrete frame

1. Introduction

Over the last years, the nonlinear static procedure, namely pushover analysis, has been developed as a practical tool to estimate the seismic demand of structures (ATC-40 1996, FEMA-356 2000). In pushover analysis a model of structure is subjected to an invariant (or adaptive) lateral force (or displacement) pattern of loading until a certain target displacement is reached, the structure gets on threshold of collapse, or the computing tool fails to continue. Conventional pushover procedures which proposed in the guideline documents and codes accurately estimates the seismic demand of the regular and low-rise building structures (Saiidi and Sozen1981, Fajfar and Gaspersic 1996, Gupta and Krawinkler 2000), while this procedure cannot appropriately predict the seismic response of the irregular and the high-rise structures (Krawinkler and Seneviratna 2000, Kim and D'Amore 1999, Mwafy and Elnashai 2001). The reason is that taking the first vibration mode into account is sufficient to evaluate the seismic behavior of such buildings. On the other hand, the fundamental deficiency of conventional pushover procedures is ignoring the participation of other higher modes and the alteration of dynamic properties of

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

structure due to degradation and softening of its materials. Therefore it is needed to consider more or other improved load patterns for estimating the capacity of structures.

In this regard, in recent years several researches accomplished to consider the effects of the higher modes. Different types of combination are utilized for different variables in different stages of the pushover analysis. In the well known MPA procedure (Chopra and Goel 2002), structure is analyzed under several modal load patterns and then, the responses were combined based on the Square Root of Sum of Squares (*SRSS*) or Complete Quadratic Combination (*CQC*) rules. One other important idea was the method of modal combination by Kunnath (2004), in which the same modal forces were *linearly* combined to make a single load pattern for applying on the structure. The linear combination of modal forces is rationalized attractively by Kunnath (2004). The mentioned trends in achieving a proper lateral load pattern, kindled us to utilize a novel meta-heuristic optimization algorithm to obtain an optimum lateral load distribution.

2. Optimum modal combination

Based on the principles of structural dynamic (Chopra 2001), the modal forces corresponding to storey levels are computed following an Eigen-Value analysis of the structure, as stated in Eq. (1).

$$f_{ij} = \Gamma_j \phi_{ij} m_i Sa_j \tag{1}$$

where, f_{ij} is modal force of mode j and storey i, Γ is modal participation factor, ϕ represents the eigen vectors of the structure, m is the seismic mass of the storey and, S_a is spectral acceleration. The modal combination procedure involves an appropriate number of modes to include in the analysis, so that the applied load pattern will be computed by Eq. (2) (Kunnath 2004)

$$F_i = \sum_{j=1}^{n} \alpha_j f_{ij} \tag{2}$$

where F_i is the lateral force to be applied at storey level i, and j represents the mode number. The factor α that is used to control the relative effect of each mode, is focused in this paper. Referring to the original paper, the Modal Combination method (Kunnath 2004): “A default value of positive or negative unity can be assigned to this factor though the response maybe sensitive to this parameter if the mass participation of the mode is small but the spectral acceleration demand is significant for higher modes”. Accordingly, regarding that the α includes wide spectrum of values and no certain procedure is stated to compute it, the innovative idea here is to involve an optimization algorithm, with which the most optimum values for the factors would be achievable. The most optimum vector of α is the one with which the final factored combined load pattern results in the most accurate response of the structure under pushover analysis, in comparison to NTHA.

2.1 Optimization algorithm

In this paper, a meta-heuristic method named Cuckoo Search algorithm (CS) is utilized in Optimum Modal Combination method. This algorithm is based on the obligate brood parasitic behavior of some Cuckoo species in combination with Lévy flight behavior of some birds and fruit flies, which is recently developed by Yang (2008). These species lay their eggs in the nests of other host birds (almost other species) with amazing abilities such as selecting the recently spawned nests, and removing existing eggs that increase hatching probability of their eggs. On the other hand, some of host birds are able to combat this parasite behavior of Cuckoos, and throw out the discovered alien eggs or build their new nests in new locations. This algorithm contains a population of nests or eggs. For simplicity, following representations is used; each egg in a nest represents a solution and a Cuckoo egg represents a new one. If the Cuckoo egg be very similar to the host's, then this Cuckoo's egg is less likely to be discovered, thus the fitness should be related to the difference in solutions. The aim is to employ the new and potentially better solutions (Cuckoos') to replace a not-so-good solution in the nests (Yang 2010). The Lévy flight is a random process in which a series of consecutive random steps perform. From the implementation point of view, the generation of random numbers with Lévy flights includes two steps: choice of a random direction, and the generation of steps which obey the chosen Lévy distribution, while the generation of steps is quite tricky. There are a few ways to achieve this, but one of the most efficient and yet straightforward ways is to use the so-called Mantegna algorithm (Yang 2008).

The original version of the CS (Yang 2010), is a sequential version and each iterations of the algorithm consists of two main steps, but another version of the CS which is supposed to be different and more efficient, is provided by Yang and Deb (Yang and Ddeb 2010). In this study the later version of the CS algorithm is used. The pseudo code of optimum design algorithm can be summarized as follows (Kaveh and Bakhdshpoori 2011):

2.1.1 Initialize the Cuckoo search algorithm parameters

The CS parameters are set in the first step. These parameters are number of nests (n), step size parameter (α), discovering probability (pa) and maximum number of frame analyses as the stopping criterion.

2.1.2 Generate initial nests or eggs of host birds

The initial locations of the nests are determined by the set of values assigned to each decision variable randomly as

$$nest_{i,j}^{(0)} = x_{j,\min} + rand.(x_{j,\max} - x_{j,\min}) \tag{3}$$

where $nest_{i,j}^{(0)}$ determines the initial value of the jth variable for the ith nest; $x_{j,min}$ and $x_{j,max}$ are the minimum and the maximum allowable values for the jth variable; $rand$ is a random number in the interval [0, 1].

2.1.3 Generate new Cuckoos by Lévy flights

In this step all of the nests except for the best so far are replaced in order of quality by new Cuckoo eggs produced with Lévy flights from their positions as

$$nest_i^{(t+1)} = nest_i^{(t)} + \alpha\,.\,S.(nest_i^{(t)} - nest_{best}^{(t)}).r \tag{4}$$

where $nest_i^t$ is the ith nest current position, α is the step size parameter which is considered to be 0.1; S is the Lévy flights vector as in Mantegna's algorithm; r is a random number from a standard normal distribution and $nest_{best}$ is the position of best nest so far.

2.1.4 Alien eggs discovery

The alien eggs discovery is performed for all of eggs but in the term of probability matrix for each component of each solution such as

$$P_{ij} = \begin{cases} 1 & if\ rand < pa \\ 0 & if\ rand \geq pa \end{cases} \tag{5}$$

where *rand* is a random number in [0, 1] interval and P_{ij} is discovering probability for *j*th variable of *i*th nest.

Existing eggs are replaced considering quality by newly generated ones from their current position by random walks with step size such as

$$\begin{aligned} &S = rand\,.\big(nests\big(randperm(n),:\big) - nests\big(randperm(n),:\big)\big) \\ &nest^{t+1} = nest^{t} + S.*P \end{aligned} \tag{6}$$

where *randperm* is a random permutation function is used for different rows permutation applied on nests matrix and *P* is the probability matrix.

2.1.5 Termination criterion

The generating of new Cuckoos and discovering the alien eggs steps are performed alternatively until a termination criterion is satisfied. The maximum number of analyses is considered as algorithm's termination criterion.

2.2 Assumptions and analysis

As depicted in Fig. 1, α vector is being optimized to achieve the best combined load pattern, and according to the fact that the role of higher modes becomes negligible (Jan 2004), only the first three modes of vibration are considered here.

For the comparison purpose, three force distributions of FEMA-273, i.e., Uniform, ELF, and SRSS load patterns are implemented on the considered structure and the results are plotted beside the NTHA as well as the proposed procedure.

The modal combination optimization procedure is performed as following steps:

1. A model of structure is framed in a Finite Element Analysis (FEA) software.
2. The eigenvalue analysis is performed and modal characteristics are computed.

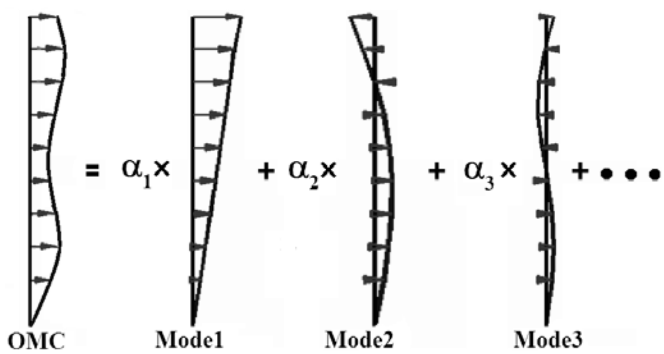


Fig. 1 Factored combination of modal forces

3. Spectral accelerations corresponding to each mode are extracted from each ground motion record.

4. Modal force patterns are calculated based on Eq. (1) for sufficient number of modes. First three modes of vibrations are considered here

5. The CS starts and the fitness function is evaluated for each generated α vector as following;

5.1. The single load pattern is calculated based on Eq. (2) and incrementally applied on the structure in the FEA software until a target displacement is reached. (Several methods are proposed for evaluating the target displacement (ATC-40 1996), (FEMA-356 2000). The method of Displacement Coefficient is utilized here.)

5.2. Inter-storey drift profile is extracted in the target step, as seismic response of the structure.

5.3. The fitness function is calculated as drift difference of each solution and of the NTHA under corresponding ground motion record (of which the S_a was extracted in Step. 3).

The CS algorithm runs on and conducts the responses toward the optimum point, where the termination criterion reaches.

Like other parameters of the problem, several options were tried out for the α range to observe the performance of the algorithm. Finally, a range of real numbers between [-3, +3] was considered. Other assumptions of pushover analysis are the same as recommended by the FEMA-273.

It is also noteworthy to mention that the simulations in this study are all carried out in a two dimensional space, both pushover and time history analysis, as well as structural model and ground motion excitations. Therefore, further researches are required for three dimensional situation and the results of this study are only applicable for two dimensional problems.

For minimizing the difference vector between the pushover analysis and its NTHA counterpart, consists of arrays corresponding to storey levels, all its components and sum of them must be minimized, so the objective function is defined as

$$E = \sqrt{\sum_{i}^{n} (R_{NTHA} - R)_i^2} \tag{7}$$

where, E is size of the difference vector, i is counter of stories, and n is number of stories, R_{NTHA} is response of the NTHA, and R is response of the proposed OMC procedure.

The OpenSEES software (PEER 1998) is utilized as FEA tool for analysis of the structure, and the optimization algorithm is coded in the MATLAB software because of its suitable environment for interfacing with the OpenSEES.

3. Structural model

The selected space frame were taken from FEMA-P695 (2009) and belongs to the class of reinforced concrete special moment resisting frame buildings with height of 20 stories, designed according to current building code provisions (ICC 2003, ASCE 7-02 2002, ACI 318-02 1996) as reported in FEMA-P695 (2009). The frame conforms to design requirements for special moment frames (SMF) according to IBC (2003) and ACI 318 (2002). Typical floor plan is shown in Fig. 2.

The frame has 20 ft (6.10 m) bays. First story and upper stories heights are 15 ft (4.57 m) and 13 ft (3.96 m), respectively. The dead and live loads are equal to 175 psf (8.38 MPa) and 50 psf (2.40 MPa). Concrete compressive strength is in range of 6 to 7 ksi (48.26 GPa), reinforcing steel

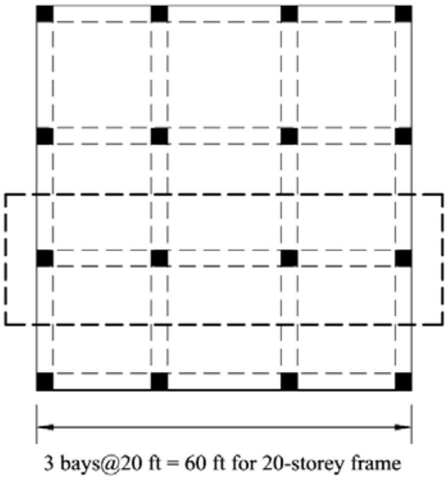


Fig. 2 Floor plan of the RC space moment frame building (1ft =0.30m)

Table 1 Ground motion characteristics

	Earthquake records	Magnitude	Distance (km)	Scale factor	PGA (g)
1	Imperial Valley, El Centro, 1940	6.9	10	2.01	0.46
2	Imperial Valley, Array #6, 1979	6.5	1.2	0.84	0.30
3	Landers, Barstow, 1992	7.3	36	3.20	0.42

yield stress and elastic modulus are 60 ksi (414 GPa) and 29000 ksi (200 GPa) respectively. First mode period is 2.36 seconds. The damping ratio is set as 6.5% for RC SMF (Miranda and Bertero 2004). P-Delta effect and strong-column/weak-beam criterion are considered. In respect to building site, a general high seismic site in Los Angeles, California is included.

The fundamental period of structures is commonly considered (FEMA P695 2009, Kafrawy *et al*. 2011, Khoshnoud and Marsono 2012) as a representative parameter for classification of structures. Moreover, the vibration period widely represents the structural geometry and slenderness. On the other hand, the contribution of higher modes, as a concern in this study, on higher periods, as of the model utilized here, is obviously vital. Therefore, it is reasonable to assume that utilizing such a well-designed RC structure could inclusively help getting ideas about many other structures with the similar configuration and in the same range of periods and damping.

4. Ground motion characteristics

Three high-cited far-fault ground motions with 10% probability of exceedance in 50 years, representing Design Basis Earthquake (DBE) in IBC (2003) are considered to evaluate the effectiveness of the proposed procedure. The ground motion records were compiled by SAC Phase II Steel Project (Somerville *et al.* 1997) for a site in Los Angeles with a stiff soil profile (site class D in IBC-2003). The ground motions characteristics are listed in Table 1.

The records are available in the Pacific Earthquake Engineering Research (PEER) site, http://peer.berkeley.edu/smcat.

Table 2 α factors for each ground motion record

	Earthquake records	α_1	α_2	α_3	E
1	Imperial Valley, El Centro, 1940	0.9377	-0.0047	0.3475	0.46
2	Imperial Valley, Array #6, 1979	0.9954	0.0925	-0.0253	0.30
3	Landers, Barstow, 1992	0.8405	-0.0892	0.5344	0.42

Regarding the importance of magnitude, distance, and peak ground acceleration of ground motion records, and also the plenty number of available seismograph data for different soil types, in this study it is tried to cover a remarkable range of distance and PGA scaled for the certain region (south of California). Considering the time-consuming procedure of the optimization algorithm for such an exact detailed structural model, which is pushover analysis for thousands of times for each ground motion, the number of ground motion records is limited to three, but the most inclusive ones.

5. Results

The optimization procedure required the 20-storey nonlinear fiber modeled frame to pushover analysis for more than five thousand times for each ground motion record. In each analysis, the pushover/capacity curve is extracted, it is idealized into bilinear curve based on FEMA recommendation, the target displacement is computed according to Displacement Coefficient method (FEMA-356 2000), and the story-drift profile of the structure is extracted in the target point and compared with its maximum due to the NTHA.

To compare the accuracy of the different NSPs, an error index defined by Eq. (4) which has been presented by Lopez-Menjivar and Pinho (2004)

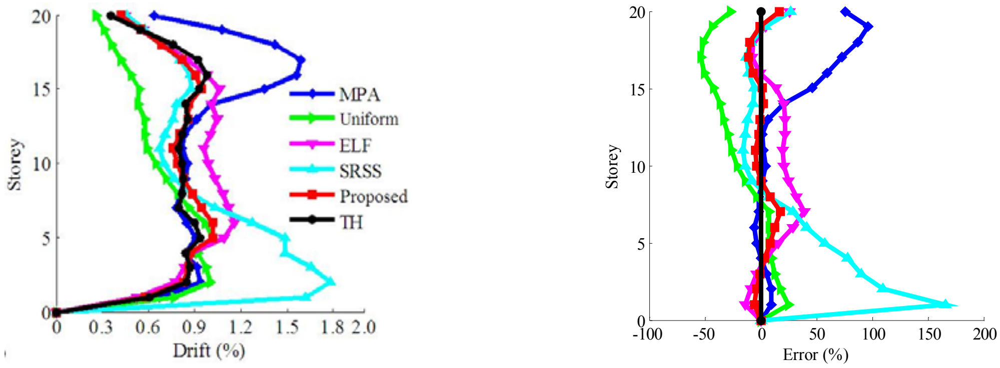
$$Error(\%) = 100 \times \frac{1}{n} \sqrt{\sum_{i=1}^{n} \left(\frac{R_{i-NTHA} - R_{i-NSP}}{R_{i-NTHA}} \right)^2} \tag{8}$$

where $R_{i\text{-}NTHA}$ and $R_{i\text{-}NSP}$ are peak response of NTHA, and pushover target response at level i, respectively. n is the number of stories.

Considering that the termination criterion for the optimization algorithm was the number of iterations, the results for each earthquake recorded as following. The variables of the problem, i.e., the α factors and error of the proposed procedure are stated in Table 2.

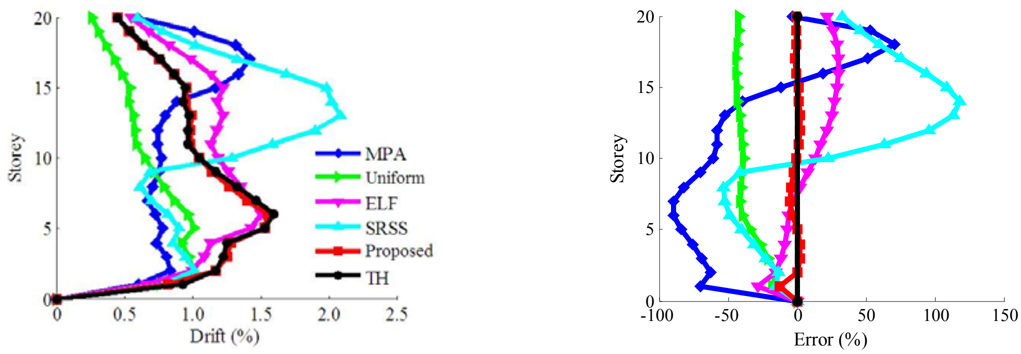
According to the Table 2, contribution of third mode is more than the second mode for the first and the last ground motion records. These earthquakes have greater Peak Ground Acceleration (PGA), and farther distance (Table 1). So the relation between the PGA (and maybe distance), and role of third mode is noteworthy. Another observation is the negligible contribution of second mode-forces with minus sign which is less than one-hundredth. For the second ground motion, with a relatively low PGA and scale factor, and a near distance of recording (Table 1), role of first mode is sensibly dominant with a lower amount of error (Table 2). Undeniably, more ground motions utilized, more reliable conclusion expected, however, standard deviations equal to 0.078, 0.091, and 0.285 for the studied parameters $\boldsymbol{\alpha_1}$, $\boldsymbol{\alpha_2}$, and $\boldsymbol{\alpha_3}$ respectively, for such different ground motion records, account for a promising sign for further researches with more ground motions and structural models.

Drift profiles for the well-known MPA (Chopra and Goel 2002) procedure, three load patterns of FEMA-273, and the proposed procedure are presented in Figs. 3-5, respectively. Maximum drift of each storey during the NTHA is extracted and plotted beside other drift profiles as benchmark.



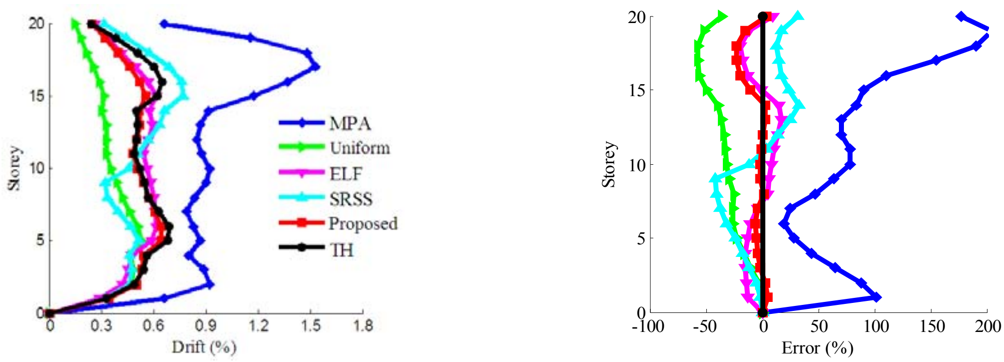
(a) (b)

Fig. 3 (a) Drift profiles, (b) Drift errors, of all considered procedures due to El Centro earthquake record



(a) (b)

Fig. 4 (a) Drift profiles, (b) Drift errors, of all considered procedures due to Imperial Valley Array #6 earthquake record



(a) (b)

Fig. 5 (a) Drift profiles, (b) Drift errors, of all considered procedures due to Barstow earthquake record

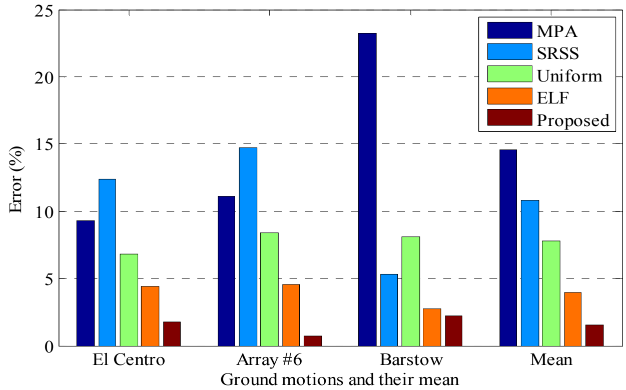


Fig. 6 Drift error for each NSP and each earthquake records and their mean value

For comparison purpose, the errors based on the Eq. (4) for all the three ground motions and the five procedures are depicted in Fig. 6 as well as their mean values. As shown in the Figures, the optimized factors have made the results remarkably efficient in all of the observed conditions.

Regarding that the pattern of the α factors are similar for two of three ground motions utilized here, the role of third mode is higher than second mode for the records with higher PGA (0.42g and 0.46g) and farther distance for the tall concrete frame. On the other hand, as observed in results of Array #6 ground motion, higher modes may have no important contribution in pushover analysis of a tall structures, despite that it is widely accepted that role of higher modes are important.

Anyhow, the main point of this work would be the innovative viewpoint of optimization in pushover analysis and performance based design. Regarding the high-level of nonlinearity in the process through implementing the load pattern until the extracting of drift profiles, it is not irrational to assume that the load pattern for each drift profile is unique. This assumption proves that such load pattern as achieved in this research would be the reference pattern in which other researches should result. Undeniably much further researches need to accomplish to bolster the confesses of this study and also to find a reasonable relation between the optimum load pattern, the structure's characteristics, as well as its region type (earthquake features in this study).

5. Conclusions

An efficient optimization algorithm utilized to study the optimum lateral load pattern which is constituted by combination of factored modal force patterns. For a 20-storey RC special moment resisting frame, and under three well-known far-fault earthquake records, efficiency of the method was validated. The results are as following:

• Involving the optimization algorithms is strongly efficient to predict an optimum load distribution for pushover analysis of structures.

• Contribution of third mode is more than second mode for regions with higher PGA (higher than 0.4g) probability, for this range of period (2D symmetric 20-story frame) and damping (RC structure).

• At least for the high-rise reinforced concrete moment resisting frames fundamental period around 2.36 seconds, the role of higher modes is negligible under pushover analysis due to ground motions with a lower PGA around 0.3g and relatively near distance.

The explained research here, is not a just pushover method, but with some reservations could be considered as an innovative viewpoint to recognize the unique optimum load pattern which results in the most exact seismic response of structures. However, in order to have a better classification of results, and to confirm the conclusions obtained in this research, supplementary researches including more ground motions, and other types of structures need to investigate which is under way by the authors.

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