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Wave propagation in a generalized thermo elastic circular plate immersed in fluid

R. Selvamani*[1] and P. Ponnusamy[2]

[1]*Department of Mathematics, Karunya University, Coimbatore,Tamil Nadu, India*
[2]*Department of Mathematics,Government Arts College (Autonomous), Coimbatore, Tamil Nadu, India*

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Abstract. In this paper, the wave propagation in generalized thermo elastic plate immersed in fluid is studied based on the Lord-Shulman (LS) and Green-Lindsay (GL) generalized two dimensional theory of thermo elasticity. Two displacement potential functions are introduced to uncouple the equations of motion. The frequency equations that include the interaction between the plate and fluid are obtained by the perfect-slip boundary conditions using the Bessel function solutions. The numerical calculations are carried out for the material Zinc and the computed non-dimensional frequency, phase velocity and attenuation coefficient are plotted as the dispersion curves for the plate with thermally insulated and isothermal boundaries. The wave characteristics are found to be more stable and realistic in the presence of thermal relaxation times and the fluid interaction.

Keywords: solid-fluid interface; wave propagation; vibration of thermal plate; plate immersed in fluid; generalized thermo elastic plate; ultrasonic transducers and resonators

1. Introduction

The circular plates are often used as structural components and their vibration characteristics are important for practical design. In view of available experimental evidence in favor of the finiteness of heat propagation speeds, the generalized thermo elasticity theories are considered to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes and/or short time intervals, such as those occurring in laser units and energy channels. Further, the interaction of liquid with varying temperature can be utilized as a thermal source in addition to normal load (hydrostatic pressure) simultaneously in many engineering applications. The analysis of thermally induced wave propagation of a cylindrical plate immersed in fluid is a problem that may be encountered in the design of structures such as atomic reactors, steam turbines, submarine structures subjected to wave loadings and other devices operating at elevated temperatures. Moreover, it is recognized that the thermal effects on the elastic wave propagation supported by fluid interaction may have implications related to many seismological applications. This study can be potentially used in applications involving nondestructive testing (NDT) and qualitative nondestructive evaluation (QNDE) using ultrasonic transducers and

*Corresponding author, Ph.D., E-mail: Selvam1729@gmail.com

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resonators.

The generalized theory of thermo elasticity was developed by Lord and Shulman (1967), which involves one relaxation time for isotropic homogeneous media, and is called the first generalization to the coupled theory of elasticity. Their equations determine the finite speed of wave propagation of heat and the displacement distributions. The corresponding equations for an isotropic case were obtained by Dhaliwal and Sherief (1980). The second generalization to the coupled theory of elasticity is known as the theory of thermo elasticity with two relaxation times, or as the theory of temperature-dependent thermoelectricity. A generalization of this inequality was proposed by Green and Laws (1972). Green and Lindsay (1972) obtained an explicit version of the constitutive equations. These equations were also obtained independently by Suhubi (1975). This theory contains two constants that act as the relaxation times and modifies not only the heat equations, but also all the equations of the coupled theory. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Erbay and Suhubi (1986) studied the longitudinal wave propagation in a generalized thermoplastic infinite cylinder and obtained the dispersion relation for the cylinder with a constant surface temperature.

Sharma and Pathania (2005) investigated the generalized wave propagation in circumferential curved plates. Modeling of circumferential waves in a cylindrical thermo elastic plate with voids was discussed by Sharma and Kaur (2010). Ashida and Tauchert (2001) presented the temperature and stress analysis of an elastic circular cylinder in contact with heated rigid stamps. Later, Ashida (2003) analyzed the thermally induced wave propagation in a piezoelectric plate. Tso and Hansen (1995) studied the wave propagation through cylinder/plate junctions. Heyliger and Ramirez (2000) analyzed the free vibration characteristics of laminated circular piezoelectric plates and discs by using a discrete-layer model of the weak form of the equations of periodic motion. The thermal deflection of an inverse thermo elastic problem in a thin isotropic circular plate was presented by Gaikward and Deshmukh (1979). The study about a plate immersed in fluid is important for design of structures such as biosensor, atomic reactors, steam turbines and submarine structures with wave loads other devices operating at elevated temperatures. Rama Rao (1999) studied the acoustic of fluid filled boreholes with pipes. Here he developed a three dimensional elasto dynamic equation for axis symmetric waves of pipes immersed inside fluid filled boreholes in infinite elastic spaces. Nagy (1995) investigated longitudinal guided wave propagation in a transversely isotropic rod immersed in fluid based on the superposition of partial waves. Ahamed (2001) discussed the guided waves in a transversely isotropic cylinder immersed in fluid. Ponnusamy (2007) has studied the wave propagation of generalized thermo elastic solid cylinder of arbitrary cross section immersed in fluid using Fourier collocation method. Ponnusamy and Selvamani (2012) have studied the dispersion analysis of generalized magneto-thermo elastic waves in a transversely isotropic cylindrical panel using the wave propagation approach. Ponnusamy and Selvamani (2012) discussed the wave propagation in a generalized thermo elastic plate embedded in elastic medium using Winkler model. Here they analyzed the influence of thermal relaxation time and foundation impact on the fundamental vibrational mode. Ahamed et.al (2002) discussed the guided waves in a transversely isotropic plate immersed in fluid. Chan (1998) studied the Lamb waves in highly attenuative plastic plate.

In this paper, the in-plane vibration of a generalized thermo elastic thin circular plate immersed in an inviscid fluid composed of homogeneous isotropic material is studied. The solutions to the equations of motion for an isotropic medium is obtained by using the two dimensional theory of generalized thermo elasticity and Bessel function solutions. The numerical calculations are carried out for the material Zinc. The computed non-dimensional frequency, phase velocity and attenuation coefficient are plotted as dispersion curves for the plate with thermally insulated and isothermal boundaries.

2. Formulation of the problem

We consider a thin homogeneous, isotropic, thermally conducting elastic thin plate of radius R with uniform thickness h and temperature T_0 in the undisturbed state initially, immersed in an in viscid fluid with density ρ_0 is shown in Fig. 1. The system displacements and stresses are defined in the polar coordinates r and θ for an arbitrary point inside the plate, with u denoting the displacement in the radial direction of r and v the displacement in the tangential direction of θ. The in-plane vibration and displacements of the plate immersed in fluid is obtained by assuming that there is no vibration and a displacement along the z axis in the cylindrical coordinate system (r, θ, z).

The two dimensional stress equations of motion and heat conduction equation in the absence of body force for a linearly elastic medium are

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho u_{,tt}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt}$$

$$k\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}\right) - \rho c_v\left(T + \tau_0 T_{,tt}\right) = \beta T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left[e_{rr} + e_{\theta\theta}\right] \tag{1}$$

where ρ is the mass density, c_v is the specific heat capacity, $\kappa = k/\rho c_v$ is the diffusivity, k is the thermal conductivity, τ_0, τ_1 are the thermal relaxation time, and T_0 is the reference temperature.

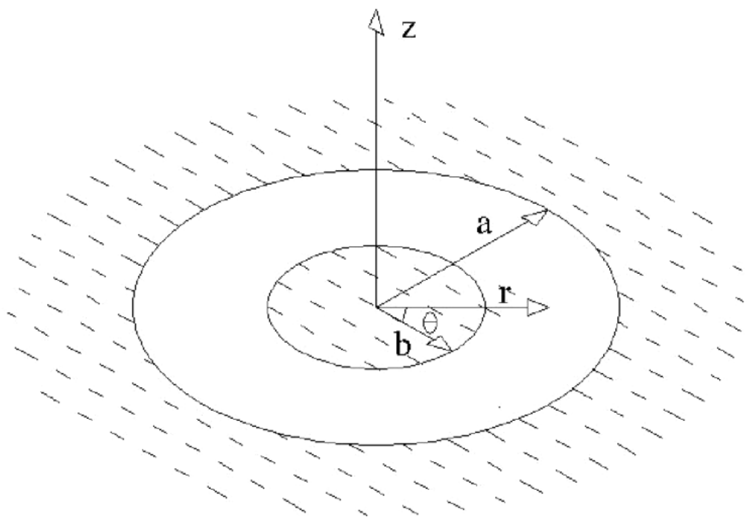

Fig. 1 Geometry of the problem

The strain-displacement relations for the isotropic plate are given by

$$\sigma_{rr} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{rr} - \beta\left(T + \delta_{2k}\tau_1 T_{,t}\right)$$

$$\sigma_{\theta\theta} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T + \delta_{2k}\tau_1 T_{,t}\right)$$

$$\sigma_{r\theta} = 2\mu e_{r\theta} \tag{2}$$

where e_{ij} are the strain components, $\beta = (3\lambda + 2\mu)\alpha_T$ is the thermal stress coefficients, α_T is the coefficient of linear thermal expansion, T is the temperature, t is time, λ and μ are Lame's constants and the comma in the subscripts denotes the partial differentiation with respect to the variable foll owing. Here δ_{ij} is the Kronecker delta function. In addition, we can replace $k = 1$ for the L-S theory and $k = 2$ for the G-L theory. The thermal relaxation times τ_0 and τ_1 satisfies the inequalities $\tau_0 \geq \tau_1 \geq 0$ for the G-L theory only.

The strain e_{ij} are related to the displacements as given by

$$e_{rr} = u_{,r}, \quad e_{\theta\theta} = r^{-1}\left(u + v_{,\theta}\right), \quad e_{r\theta} = v_{,r} - r^{-1}\left(v - u_{,\theta}\right) \tag{3}$$

in which u and v are the displacement components along the radial and circumferential directions, respectively. σ_{rr}, $\sigma_{\theta\theta}$ are the normal stress components and $\sigma_{r\theta}$ the shear stress component, e_{rr}, $e_{\theta\theta}$ th e normal strain components and $e_{r\theta}$ the shear strain component.

By substituting Eqs. (3) and (2) into Eq. (1), the following displacement equations of motions are obtained

$$\left(\lambda + 2\mu\right)\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + \mu r^{-2}u_{,\theta\theta} + r^{-1}\left(\lambda + \mu\right)v_{,r\theta}$$
$$+ r^{-2}\left(\lambda + 3\mu\right)v_{,\theta} - \beta\left(T_{,r} + T\delta_{2k}\tau_1 T_{,rt}\right) = \rho u_{,tt}$$
$$\mu\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + r^{-2}\left(\lambda + 2\mu\right)v_{,\theta\theta} + r^{-2}\left(\lambda + 3\mu\right)u_{,\theta}$$
$$+ r^{-1}\left(\lambda + \mu\right)u_{,r\theta} - -\beta\left(T_{\theta} + \eta T_{,\theta t}\right) = \rho v_{,tt}$$
$$k\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}\right) - \rho c_v\left(T + \tau_0 T_{,tt}\right) = \beta T_0\left(\frac{\partial}{\partial t} + \tau_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left[u_{,r} + r^{-1}(u + v_{,\theta})\right] \tag{4}$$

In an inviscid fluid-solid interface, the perfect-slip boundary condition allows discontinuity in planar displacement components. That is, the radial component of displacement of the fluid and solid must be equal and the circumferential, longitudinal components are discontinuous at the interface. The above coupled partial differential equations are also subjected to the following non-dimensional boundary conditions at the surfaces $r = a$, b.

(i) Stress free inner boundary conditions

$$\left(\sigma_{rr} + p_1^{\,f}\right) = \left(\sigma_{r\theta}\right) = \left(u - u^f\right) = 0 \tag{5a}$$

(ii) Stress free outer boundary conditions

$$\left(\sigma_{rr} + p_2^{\,f}\right) = \left(\sigma_{r\theta}\right) = \left(u - u^f\right) = 0 \tag{5b}$$

(iii) Thermal boundary conditions

$$T_{,r} + hT = 0 \tag{5c}$$

where h is the surface heat transfer coefficient. Here $h \rightarrow 0$ corresponds to a thermally insulated surface and $h \rightarrow \infty$ refers to an isothermal one.

2.1 Lord-Shulman (LS) theory

Based on the Lord-Shulman theory of thermo elasticity, the three dimensional rate dependent temperature with one relaxation time is obtained by replacing k=1 in the heat conduction equation of Eq. (1), namely

$$k\left(T_{,rr} + \frac{1}{r}T_{,r} + \frac{1}{r^2}T_{,\theta\theta}\right) = \rho C_v\left[T + \tau_0 T_{,tt}\right] + \beta T_0\left[\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right]\left(e_{rr} + e_{\theta\theta}\right) \tag{6a}$$

The stress-strain relation is replaced by

$$\sigma_{rr} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T\right)$$

$$\sigma_{\theta\theta} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T\right)$$

$$\sigma_{r\theta} = 2\mu e_{r\theta} \tag{6b}$$

By substituting the preceding stress-strain relations into Eq. (1), we can get the following displacement equation

$$\left(\lambda + 2\mu\right)\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + r^{-2}\mu u_{,\theta\theta} + r^{-1}\left(\lambda + \mu\right)v_{,r\theta} + r^{-2}\left(\lambda + 3\mu\right)v_{,\theta} - \beta\left(T\right) = \rho u_{,tt}$$

$$\begin{aligned}\left(\mu\right)\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + r^{-1}\left(\lambda + \mu\right)u_{,r\theta} + r^{-2}\left(\lambda + 3\mu\right)u_{,\theta} + r^{-2}\left(\lambda + 3\mu\right)v_{,\theta} \\ + r^{-2}\left(\lambda + 2\mu\right)v_{,\theta\theta} - \beta\left(T\right) = \rho v_{,tt}\end{aligned} \tag{6c}$$

The symbols and notations involved have the same meanings as defined in earlier sections. Since the heat conduction equation of this theory is of the hyperbolic wave type, it can automatically ensure the finite speeds of propagation for heat and elastic waves.

2.2 Green-Lindsay (G L) theory

The second generalization to the coupled thermo elasticity with two relaxation times called the Green-Lindsay theory of thermo elasticity is obtained by setting k=2 in the heat conduction equation of Eq. (1), namely

$$k\left(T_{,rr} + \frac{1}{r}T_{,r} + \frac{1}{r^2}T_{,\theta\theta}\right) = \rho C_v\left[T + \tau_0 T_{,tt}\right] + \beta T_0\frac{\partial}{\partial t}\left(e_{rr} + e_{\theta\theta}\right) \tag{7a}$$

The stress-strain relation is replaced by

$$\sigma_{rr} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T + \tau_1 T_{,t}\right)$$

$$\sigma_{\theta\theta} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T + \tau_1 T_{,t}\right)$$

$$\sigma_{r\theta} = 2\mu e_{r\theta} \tag{7b}$$

By substituting the preceding relations into Eq. (1), the displacement equation can be reduced as

$$\left(\lambda + 2\mu\right)\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + r^{-2}\mu u_{,\theta\theta} + r^{-1}\left(\lambda + \mu\right)v_{,r\theta} + r^{-2}\left(\lambda + 3\mu\right)v_{,\theta} - \beta\left(T_{,r} + \tau_1 T_{,rt}\right) = \rho u_{,tt}$$

$$\left(\mu\right)\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + r^{-1}\left(\lambda + \mu\right)u_{,r\theta} + r^{-2}\left(\lambda + 3\mu\right)u_{,\theta} + r^{-2}\left(\lambda + 3\mu\right)v_{,\theta}$$
$$+ r^{-2}\left(\lambda + 2\mu\right)v_{,\theta\theta} - \beta\left(T_{,\theta} + \tau_1 T_{,\theta t}\right) = \rho v_{,tt} \tag{7c}$$

where the symbols and notations have been defined in the previous sections.
To uncouple Eq. (4), the mechanical displacement u, v along the radial and circumferential directions given by Sharma (2010) is adopted as follows

$$u = \phi_{,r} + r^{-1}\psi_{,\theta}, \quad v = r^{-1}\phi_{,\theta} - \psi_{,r} \tag{8}$$

Substituting Eq. (8) in to Eq. (4) yields the following second order partial differential equation with constant coefficients

$$\left\{\left(\lambda + 2\mu\right)\nabla^2 + \rho\omega^2\right\}\phi - \beta\left(T + \delta_{2k}\tau_1 T_{,t}\right) = 0 \tag{9a}$$

$$\left\{k\nabla^2 - \rho C_\upsilon i\omega\eta_0\right\}T + \beta T_0\left(i\omega\eta_1\right)\nabla^2\phi = 0 \tag{9b}$$

$$\left(\nabla^2 + \frac{\rho}{\mu}\omega^2\right)\psi = 0 \tag{9c}$$

where $\nabla^2 \equiv \partial^2/\partial x^2 + x^{-1}\partial/\partial x + x^{-2}\partial^2/\partial\theta^2$.

3. Solutions of the solid medium

The equations are given in Eq. (9) are coupled partial differential equations with two displacements potential and heat conduction components. We assume the vibration and displacements along the axial direction z to be zero. Hence, the solutions of Eq. (9) can be presented in the following form

$$\phi(r,\theta,t) = \overline{\phi}(r)\exp\left\{i(p\theta - \omega t)\right\} \tag{10a}$$

$$\psi(r,\theta,t)=\overline{\psi}(r)\exp\{i(p\theta-\omega t)\} \tag{10b}$$

$$T(r,\theta,t)=\left(\lambda+2\mu/\beta a^2\right)\overline{T}(r)\exp\{i(p\theta-\omega t)\} \tag{10c}$$

where $i=\sqrt{-1}$, ω is the angular frequency, p is the angular wave number, ϕ (r, θ), ψ (r, θ), T (r, θ) are the displacement potentials. Substituting Eq. (10) into Eq. (9) and introducing the dimensionless quantities such as $x = r/a$, $c_1^2=(\lambda+2\mu)/\rho$, $c_2^2=\mu/\rho\Omega^2 = \rho\omega^2a^2/\mu$, $\overline{\lambda}=\lambda/\mu$, $\overline{d}=\rho c_v\mu/\beta T_0$, we can get the following partial differential equation with constant coefficients

$$\left\{\left(2+\overline{\lambda}\right)\nabla_1^2+\Omega^2\right\}\overline{\phi}-\left(2+\overline{\lambda}\right)\eta_2\overline{T}=0 \tag{11a}$$

$$\left\{k_1\nabla_1^2-i\omega\overline{d}\eta_0\right\}\overline{T}+\beta T_0\left(i\omega\eta_1\right)\nabla_1^2\overline{\phi}=0 \tag{11b}$$

$$\left(\nabla_1^2+\Omega^2\right)\psi=0 \tag{11c}$$

where $\nabla_2^2=\dfrac{\partial^2}{\partial r^2}\dfrac{1}{r}\dfrac{\partial}{\partial r}-\dfrac{p^2}{r^2}$ and $\eta_0=1+i\omega\tau_0$, $\eta_1=1+i\omega\delta_{1k}\tau_0$, $\eta_2=1+i\omega\delta_{2k}\tau_1$

Eq. (11c) in terms of ψ gives a purely transverse wave. This wave is polarized in planes perpendicular to the z-axis. We assume that the disturbance is time harmonic through the factor e^{iwt} Rewriting Eq. (11) yields the following fourth order differential equation

$$\left(A\nabla_2^4+B\nabla_2^2+C\right)\left(\overline{\phi},\overline{T}\right)=0 \tag{12}$$

where $A = (2+\overline{\lambda})k_1$, $B = \{k_1\,\Omega^2 - i\omega(2+\overline{\lambda})\overline{d}\,\eta_0 + i\omega T_0\,(2+\overline{\lambda})\,\beta\eta_1\,\eta_2\}$, $C = -(i\omega\Omega^2\,\overline{d}\,\eta_0)$.
By solving the partial differential Eq. (12), the solutions are obtained as

$$\overline{\phi}=\sum_{i=1}^{2}\left[A_iJ_n\left(\alpha_iax\right)+Y_nB_i\left(\alpha_iax\right)\right] \tag{13a}$$

$$\overline{T}=\sum_{i=1}^{2}d_i\left[A_iJ_n\left(\alpha_iax\right)+Y_nB_i\left(\alpha_iax\right)\right] \tag{13b}$$

where

$$di = \{k_1\,(\alpha_iax)^4 + (2+\overline{\lambda})\beta T_0 i\omega\eta_1\eta_2(\alpha_iax)^2 - (2+\overline{\lambda})i\omega\,\overline{d} \tag{14}$$

Eq. (11c) is a Bessel equation with possible solutions given as

$$\overline{\psi}=\begin{cases}A_3J_n(\alpha_3ax)+B_3Y_n(\alpha_3ax) & \alpha_3ax>0\\ A_3a^n+B_3a^{-n} & \alpha_3ax=0\\ A_3I_n(\alpha_3ax)+B_3K_n(\alpha_3ax) & \alpha_3ax<0\end{cases} \tag{15}$$

where J_n and Y_n are Bessel functions of the first and second kinds, respectively, while I_n and k_n are modified Bessel functions of first and second kinds, respectively. (A_i, B_i) i = 1, 2, 3 are arbitrary constants. Since $\alpha_3 ax \neq 0$, thus the condition $\alpha_3 ax \neq 0$ will not be discussed in the following. For convenience, we will pay attention only to the case of $\alpha_3 ax > 0$ in what follows. The derivation for the case of $\alpha_3 ax < 0$ is similar.

$$\overline{\psi} = \left[A_3 J_n \left(\alpha_3 ax\right) + Y_n B_3 \left(\alpha_3 ax\right)\right] \tag{16}$$

where $(\alpha_3 a)^2 = \Omega^2$.

4. Solution of fluid medium

In cylindrical coordinates, the acoustic pressure and radial displacement equation of motion for an in viscid fluid are of the form

$$p^f = -B^f \left(u^f{}_{,r} + r^{-1}\left(u^f + v^f{}_{,\theta}\right)\right) \tag{17}$$

And

$$c_f^{-2} u^f{}_{,tt} = \Delta_{,r} \tag{18}$$

respectively, where (u^f, v^f) is the displacement vector, B^f is the adiabatic bulk modulus, $c^f \sqrt{B^f / \rho^f}$ is the acoustic phase velocity of the fluid in which ρ^f is the density of the fluid and

$$\Delta = \left(u^f{}_{,r} + r^{-1}\left(u^f + v^f{}_{,\theta}\right)\right) \tag{19}$$

substituting $u^f = \phi^f{}_{,r}$ and $v^f = r^{-1}\phi^f{}_{,\theta}$ and seeking the solution of Eq. (14) in the form

$$\overline{\phi}^f \left(r, \theta, t\right) = \phi^f \cos n\theta \exp\left\{i(p\theta - \omega t)\right\} \tag{20}$$

where

$$\phi^f = A_4 J_n^1 \ (\delta ax) \tag{21}$$

for inner fluid. In Eq. (17), $(\delta a)^2 = \Omega^2 \sqrt{\rho^f B^f}$ in which $\overline{\rho} = \rho / \rho^f, \overline{B}^f = B^f / \mu, J_n^1$ is the Bessel function of the first kind. If $(\delta a)^2 < 0$, the Bessel function of first kind is to be replaced by the modified Bessel function of second kind K_n. Similarly

$$\overline{\phi}^f \left(r, \theta, t\right) = \phi^f \cos n\theta \exp\left\{i(p\theta - \omega t)\right\} \tag{22}$$

where

$$\phi^f = A_5 H_n^1 \left(\delta ax\right) \tag{23}$$

for outer fluid. In Eq. (17) $(\delta a)^2 = \Omega^2 \sqrt{\rho B^f}$. H_n^1 is the Hankel function of the first kind. If $(\delta a)^2 < 0$, then the Hankel function of first kind is to be replaced by K_n, where K_n is the modified Bessel function of second kind. By substituting the expression of the displacement vector in terms of ϕ^f and the Eqs. (17) and (19) in Eq. (13), we could express the acoustic pressure both inner and outer surface of the ring as

$$p_1^f = A_4 \Omega^2 \overline{\rho} J_n^1(\delta ax) \cos n\theta \exp\{i(p\theta - \omega t)\} \tag{24}$$

for inner fluid and

$$p_2^f = A_5 \Omega^2 \overline{\rho} H_n^1(\delta ax) \cos n\theta \exp\{i(p\theta - \omega t)\} \tag{25}$$

for outer fluid.

5. Frequency equations

In this section we shall derive the frequency equation for the two dimensional vibration of the generalized thermo elastic cylindrical plate immersed in fluid subjected to perfect slip boundary conditions at the inner and outer surfaces at $r = a$, b. Substituting the expressions in Eqs. (1)-(3) into Eq. (5), we can get the frequency equation for free vibration as follows

$$\left|E_{ij}\right| = 0 \quad i, j = 1, 2.....8 \tag{26}$$

$$\begin{aligned}
E_{11} &= (2+\overline{\lambda})(ip)\big((nJ_n(\alpha_1 ax) + (\alpha_1 ax)J_{n+1}(\alpha_1 ax)) - (ip)((\alpha_1 ax)^2 R^2 - n^2)J_n(\alpha_1 ax)\big) \\
&\quad + \overline{\lambda}\big((ip)n(n-1)\big(Jn(\alpha_1 ax) - (\alpha_1 ax)J_{\delta+1}(\alpha_1 ax)\big)\big) - \beta T(i\omega)\eta_2 d_1 (ax)^2 \\
E_{13} &= (2+\overline{\lambda})(ip)\big((nJ_n(\alpha_2 ax) + (\alpha_2 ax)J_{n+1}(\alpha_2 ax)) - (ip)((\alpha_2 ax)^2 R^2 - n^2)J_n(\alpha_2 ax)\big) \\
&\quad + \overline{\lambda}(ip)\big(n(n-1)\big(Jn(\alpha_2 ax) - (\alpha_2 ax)J_{\delta+1}(\alpha_2 ax)\big)\big) - \beta T(i\omega)\eta_2 d_2 (ax)^2 \\
E_{15} &= (2+\overline{\lambda})\big((n(n-1)J_n(\alpha_3 ax) - (\alpha_3 ax)J_{n+1}(\alpha_3 ax)\big) + \overline{\lambda}\big(n(n-1)J_n(\alpha_3 ax) - (\alpha_3 ax)J_{n+1}(\alpha_3 ax)\big) \\
E_{17} &= \Omega^2 \overline{\rho}(ax)^2 \big(nJ_n(\delta ax) - (\delta ax)J_{n+1}(\delta ax)\big), \qquad E_{18} = 0 \\
E_{21} &= nJ_n(\alpha_1 ax) - (\alpha_1 ax)J_{n+1}(\alpha_1 ax) \\
E_{23} &= nJ_n(\alpha_2 ax) - (\alpha_2 ax)J_{n+1}(\alpha_2 ax) \\
E_{25} &= nJ_n(\alpha_3 ax), \qquad E_{27} = nJ_n(\delta ax) - (\delta ax)J_{n+1}(\delta ax) \\
E_{31} &= 2n(n-1)J_n(\alpha_1 ax) - 2n(\alpha_1 ax)J_{n+1}(\alpha_1 ax) \\
E_{33} &= 2n(n-1)J_n(\alpha_2 ax) - 2n(\alpha_2 ax)J_{n+1}(\alpha_2 ax) \\
E_{35} &= (ip)\big(2n(n-1)J_n(\alpha_3 ax) - 2(\alpha_3 ax)J_{\delta+1}(\alpha_3 ax)\big) + (ip)\big((\alpha_3 ax)^2 - n^2\big)J_n(\alpha_3 ax) \\
E_{37} &= 0 \\
E_{41} &= d_1\big(nJ_n(\alpha_1 ax) - (\alpha_1 ax)J_{n+1}(\alpha_1 ax) + hJ_n(\alpha_1 ax)\big) \\
E_{43} &= d_2\big(nJ_n(\alpha_2 ax) - (\alpha_2 ax)J_{n+1}(\alpha_2 ax) + hJ_n(\alpha_2 ax)\big) \\
E_{45} &= 0 \qquad E_{47} = 0
\end{aligned}$$

$$E_{58} = \Omega^2 \overline{\rho}(bx)^2 \left(nH_n(\delta ax) - (\delta ax)H_{n+1}(\delta ax)\right)$$

Obviously E_{ij} (j = 2, 4, 6) can be obtained by just replacing the Bessel functions of the first kind in E_{ij} (i = 1, 3, 5) with those of the second kind, respectively, while E_{ij} (i = 5, 6, 7, 8) can be obtained by just replacing a in E_{ij} (i = 1, 2, 3, 4) with b.

6. Numerical results and discussion

The coupled free wave propagation in a homogenous isotropic generalized thermo elastic cylindrical plate immersed in water is numerically solved for the Zinc material. The material properties of Zinc are given as follows and for the purpose of numerical computation the liquid is taken as water.

For the solid

$$\rho = 7.14\times10^3 kgm^{-3} \qquad T_0 = 296K \qquad K = 1.24\times10^2 Wm^{-1}\deg^{-1}$$

$$\mu = 0.508\times10^{11} Nm^{-2} \qquad \beta = 5.75\times10^6 Nm^{-2}\deg^{-1} \qquad \in_1 = 0.0221$$

$$\lambda = 0.385\times10^{11} Nm^{-2} \quad \text{and} \quad C_v = 3.9\times10^2\, J\,kg^{-1}\deg^{-1}$$

For the fluid

$$p^f = 1000\,kg\,m^{-3} \qquad c_f = 1500\ ms^{-1}$$

The roots of the algebraic equation in Eq. (12) were calculated using a combination of the Birge-Vita method and Newton-Raphson method. For the present case, the simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using the Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. Such a combination can overcome the difficulties encountered in finding the roots of the algebraic equations of the governing equations. Here the values of the thermal relaxation times are calculated from Chandrasekharaiah (1986) as τ_0 = 0.05 and τ_1 = 0.75. Because the algebraic Eq. (12) contains all the information about the wave speed and angular frequency, and the roots are complex for all considered values of wave number, therefore the waves are attenuated in space.

We can write $c^{-1} = v^{-1} + i\omega^{-1}q$, so that $p = R + iq$, where $R = \omega/v$, v and q are real numbers. Upon using the above relation in Eq. (26), the values of the wave speed (v) and the attenuation coefficient (q) for different modes of wave propagation can be obtained. When a solid medium such as the circular plate is surrounded by fluid medium, guided waves are transmitted across the interface. Thus bulk waves are excited in the embedding medium, radiating away from the solid medium. In addition, there exits two independent waves such as shear harmonic waves and Lamb waves. If the elastic plate immersed in an in viscid fluid, shear harmonic waves are unaffected, however the lamb waves are affected.

A comparison is made for the non-dimensional frequencies among the Green -Lindsay Theory (GL), Lord-Shulman Theory (LS) and Classical-Theory (CT) of thermo-elasticity for the free and fluid loaded circular plate of thermally insulated and isothermal boundaries in Tables 1 and 2, respectively. From these tables, it is clear that as the sequential number of the vibration modes increases, the non dimensional frequencies also increases for both the free plate and fluid loaded plate.

Table 1 Comparison of non-dimensional frequencies among the Green Lindsay-Theory (GL), Lord-Shulman Theory (LS) and Classical-Theory (CT) of thermo-elasticities for thermally insulated boundaries of circular plate

Mode	Plate immersed in fluid			Free plate		
	LS	GL	CT	LS	GL	CT
1	0.1572	0.0865	0.0239	0.1408	0.0442	0.0115
2	0.2345	0.2019	0.0651	0.2205	0.1969	0.0554
3	0.5337	0.4977	0.1674	0.4473	0.3888	0.1444
4	0.7292	0.5385	0.1994	0.5941	0.5023	0.2487
5	0.9408	0.6952	0.3964	0.7303	0.6050	0.5504
6	1.4579	0.9714	0.4051	0.8070	0.8512	0.5551
7	1.6707	1.1350	0.7478	1.2107	1.0230	0.6038

Table 2 Comparison of non-dimensional frequencies among the Green-Lindsay Theory (GL), Lord-Shulman Theory (LS) and Classical-Theory (CT) of thermo-elasticities for isothermal boundaries of circular plate

Mode	Plate immersed in fluid			Free plate		
	LS	GL	CT	LS	GL	CT
1	0.1681	0.1236	0.0455	0.1284	0.1039	0.0209
2	0.2747	0.2636	0.1601	0.1530	0.1213	0.1702
3	0.3492	0.3928	0.2263	0.4220	0.4563	0.1950
4	0.5391	0.4727	0.3867	0.5295	0.4702	0.2837
5	0.7853	0.6036	0.6010	0.6752	0.5031	0.5029
6	1.5288	0.7308	0.6910	0.8349	0.6092	0.7727
7	1.5824	1.7015	0.9025	0.9342	0.9231	0.8081

Also, it is clear that the non dimensional frequency exhibits higher amplitudes for the LS theory compared with the GL and CT since; there exist two kinds of sub-waves for the LS and three for the GL model. One propagates at the speed of the quasi-heat wave, another at the speed of the quasi-elastic wave in LS but one propagates at the speed of the quasi-heat wave and two at the speed of the quasi-elastic wave in GL. From the classical theory of elastic waves, we know that when elastic waves are splitting, the only reason for this is that the material becomes damaged and dissipative. So, we can obtain the results such as: the LS model is suitable for elastic materials, and the GL model is more suitable for dissipative materials.

Achenbach (2005) says unlike the hyperbolic solutions, the classical solutions show no distinct wave front and therefore as expected and increase in temperature starts at the initial time. However, the difference in the predicted temperature between the two theories is small and only apparent for very small in the fundamental modes. These fundamental frequencies are large enough for the solutions given by both the theories to be numerically undistinguishable in case of many nondestructive evaluation (NDE) applications.

6.1 Dispersion curves

In Figs. 2 and 3, the dispersion of frequencies with the wave number is studied for both the thermally insulated and isothermal boundaries of the immersed cylindrical plate in different modes of vibration. From Fig. 2, it is observed that the frequency increases exponentially with increasing wave number for thermally insulated modes of vibration. But smaller dispersion exists in the frequency in the current range of wave numbers in Fig. 3 for the isothermal boundary. As the wave number increases in both Figs. 2 and 3, the effect of fluid loading becomes more pronounced in all the modes of vibration, resulting higher frequency with small oscillation and cross over points which denote the energy transformation between the two medium.

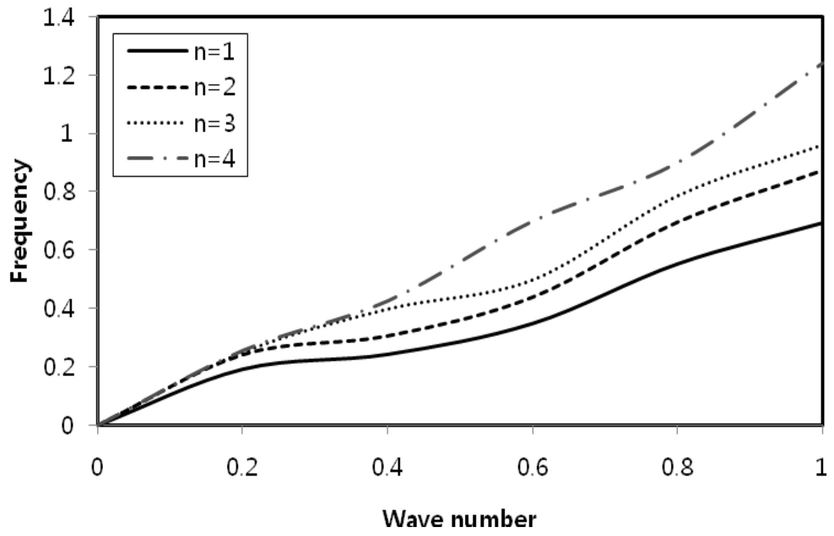

Fig. 2 Variation of frequency with wave number of thermally insulated cylindrical plate immersed in fluid

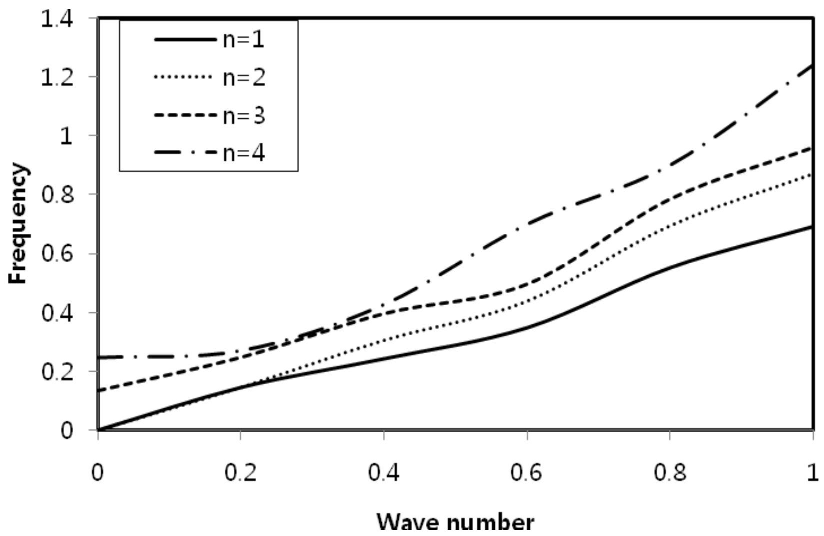

Fig. 3 Variation of frequency with wave number of isothermal cylindrical plate immersed in fluid

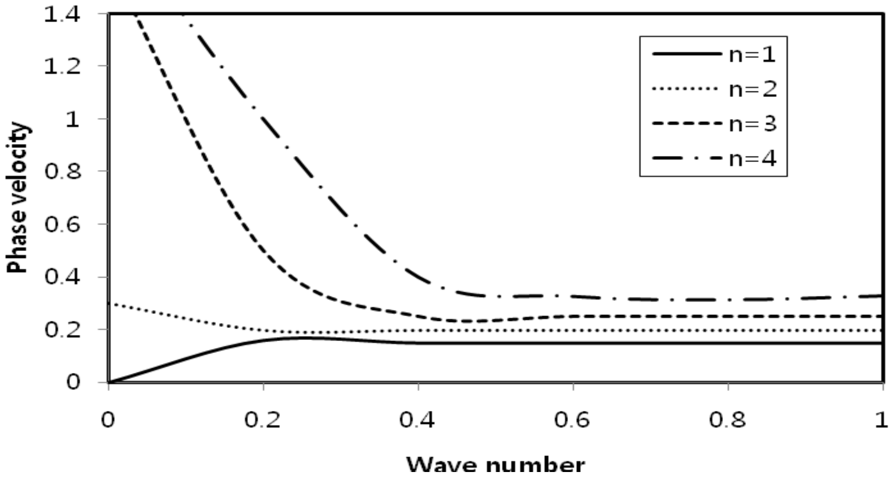

Fig. 4 Variation of phase velocity with wave number of thermally insulated cylindrical plate immersed in fluid

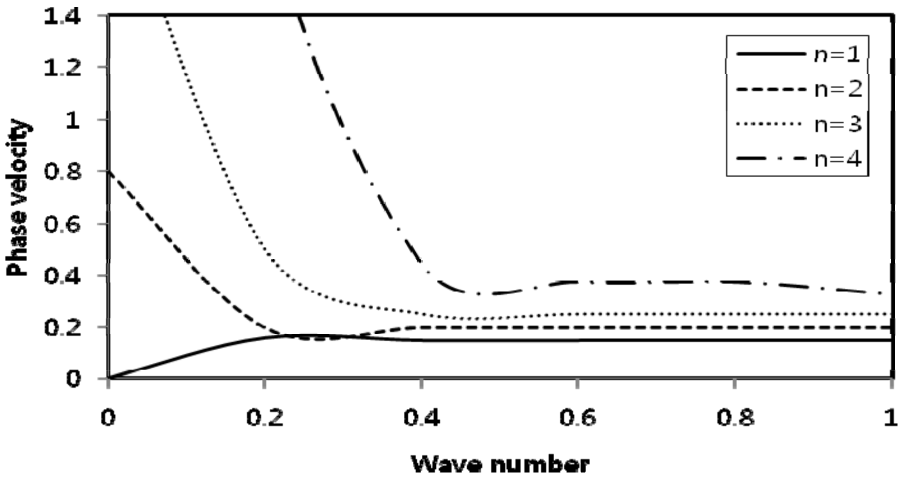

Fig. 5 Variation of phase velocity with wave number of thermally insulated cylindrical plate immersed in fluid

The variation of phase velocities with the wave number is discussed in Fig. 4 and Fig. 5 for both the thermally insulated and isothermal boundaries of the immersed cylindrical plate in different modes of vibration. In Fig. 4 the phase velocity is decreasing at small wave number between 0 and 0.4 and become steady for higher values of the wave number for thermally insulated modes of vibration. For isothermal boundary there is a small deviation on the phase velocity in Fig. 5 due to the damping effect of fluid medium and thermal relaxation times. From the Figs. 4 and 5 it is observed that the phase velocity of both thermally insulated and isothermal

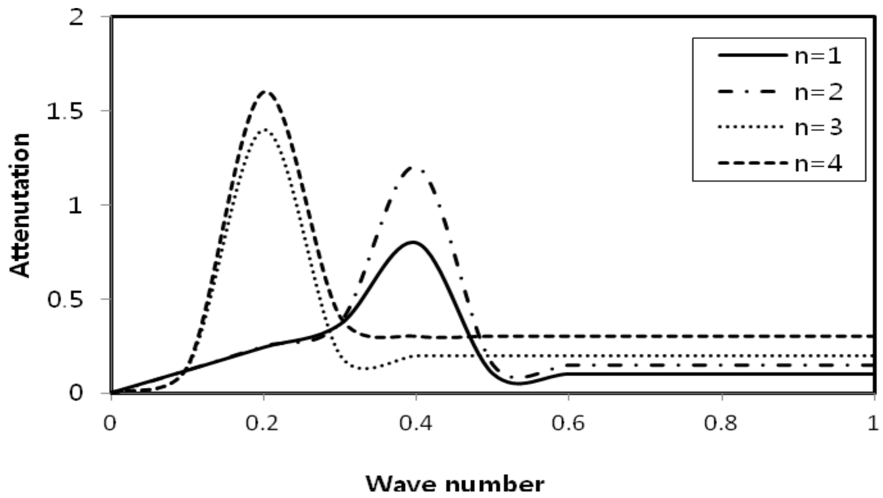

Fig. 6 Variation of attenuation with wave number of thermally insulated cylindrical plate immersed in fluid

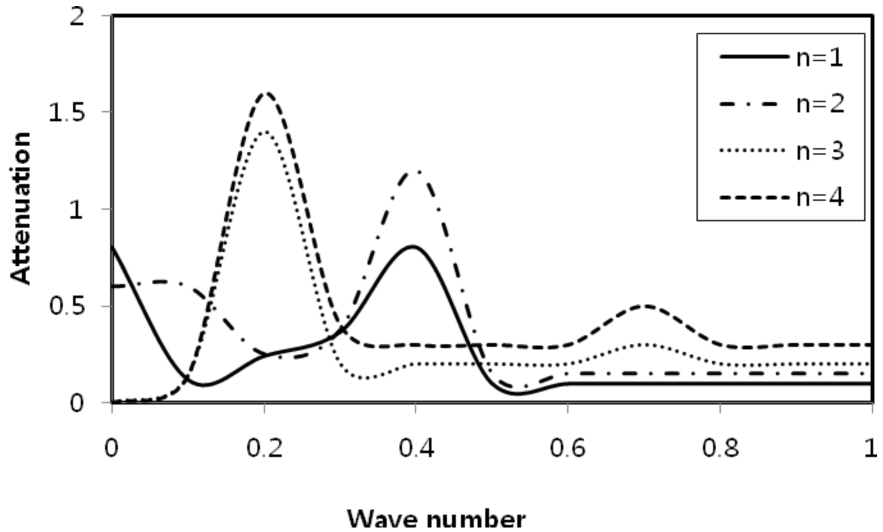

Fig. 7 Variation of attenuation with wave number of isothermal cylindrical plate immersed in fluid

cylindrical plate with fluid interaction attains quite large values at vanishing wave number which slashes down to become steady and asymptotic to the shear wave velocity with increasing wave number.

In Fig. 6, the variation of attenuation coefficients with respect to the wave number of the cylindrical plate is presented for the thermally insulated boundary. The magnitude of the attenuation coefficient increases monotonically and attaining the maximum between 0.1 and 0.3 for first four modes of vibration, and slashes down to become asymptotically linear in the remaining range of wave number. The variation of attenuation coefficients with respect to the wave number of the isothermal cylindrical plate is presented in Fig. 7, where the trend and behavior of the attenuation coefficient attains the maximum between 0.1 and 0.3 with dispersion for the small values of wave number and decreases to become steady and linear due to the relaxation times.

From Figs. 6 and 7, it is clear that the effects of stress free thermally insulated and isothermal boundaries of the plate are quite pertinent due to the combined effect of thermal relaxation times and damping effect of the fluid medium. When the ratio of the densities of the fluid and elastic material is small (0.14), then the mode spectrum of immersed plate is slightly different from that of free plate. But, when the normal component of the guided waves exceeds that of the sound in the surrounding fluid, energy is radiated from the solid plate to the fluid medium and these waves are referred to be the Leaky waves.

7. Conclusions

The two dimensional wave propagation of a homogeneous isotropic generalized thermo elastic cylindrical plate immersed in fluid was investigated in this paper. For this problem, the governing equations of two dimensional linear theory of generalized thermo elasticity have been employed in the context of the Lord-Shulman and Green-Lindsay theory and solved by the Bessel function solutions with complex arguments. The effects of the frequency, attenuation coefficient and phase velocity with respect to the wave number of a Zinc cylindrical plate was investigated, with the results presented as the dispersion curves. In addition, a comparative study is made among the LS, GL and CT theories and the frequency change is observed to be highest for the LS theory, followed by the GL and CT theories due to the thermal relaxation time factor and added mass of the surrounding fluid medium.

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