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Wave propagation in a microbeam based on the modified couple stress theory

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Abstract. This paper presents responses of the free end of a cantilever micro beam under the effect of an impact force based on the modified couple stress theory. The beam is excited by a transverse triangular force impulse modulated by a harmonic motion. The Kelvin–Voigt model for the material of the beam is used. The considered problem is investigated within the Bernoulli-Euler beam theory by using energy based finite element method. The system of equations of motion is derived by using Lagrange's equations. The obtained system of linear differential equations is reduced to a linear algebraic equation system and solved in the time domain by using Newmark average acceleration method. In the study, the difference of the modified couple stress theory and the classical beam theory is investigated for the wave propagation. A few of the obtained results are compared with the previously published results. The influences of the material length scale parameter on the wave propagation are investigated in detail. It is clearly seen from the results that the classical beam theory based on the modified couple stress theory must be used instead of the classical theory for small values of beam height.

Keywords: wave propagation; modified couple stress theory; microbeam

1. Introduction

With the great advances in technology in recent years, micro and nano structures have found many applications. In these structures, microbeams and microtubes are widely used in micro- and nanoelectromechanical systems (MEMS and NEMS) such as sensors (Zook *et al*. 1992, Pei *et al*. 2004), actuators (Senturia 1998, Rezazadeh *et al*. 2006). In investigation of micro and nano structures, the classical continuum mechanics which is scale independent theories, are not capable of explanation of the size-dependent behaviors. Nonclassical continuum theories such as higher-order gradient theories and the couple stress theory are capable of explanation of the size-dependent behaviors which occur in micro-scale structures. Microbeams may be subjected to impact forces at any point and this leads to wave propagation in them. Therefore, understanding of the mechanism of wave propagation in the microbeams - nanoparticles is important for their desings. In recent years, many researchers have been interested in the study of micro structures. The classical couple stress elasticity theory is introduced by Mindlin and Tiersten (1962), Mindlin (1963), Toupin (1964), Koiter, (1964), included four material constants (two classical and two

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additional) for isotropic elastic materials. Eringen (1993) initiated size-dependent theory which contains two additional material constants. Rayleigh waves in a linear elastic couple-stress medium are investigated by Ottosen *et al*. (2000). Using Eringen's nonlocal continuum theory, Peddieson *et al*. (2003) investigated the non-local Bernoulli–Euler beam model. Yang *et al*. (2002) proposed the modified couple stress theory in which the strain energy has been shown to be a quadratic function of the strain tensor and the symmetric part of the curvature tensor, and only one length scale parameter is included. A new model for the bending of a Bernoulli–Euler beam is developed using a modified couple stress theory by Park and Gao (2006). Wang and Liew (2007) investigated static analysis of micro- and nano-structures based on the nonlocal continuum theory. Nonlocal theories for bending, buckling and vibration of beams are investigated by Reddy (2007). Aydoğdu (2009) investigated a generalized nonlocal beam theory based on Euler–Bernoulli, Timoshenko, parabolic shear deformation and general exponential shear deformation theory to study bending, buckling and free vibration of nanobeams. Simsek (2010a) investigated analytical and numerical solution procedures for vibration of an embedded microbeam under action of a moving microparticle based on the modified couple stress theory within the framework of Euler–Bernoulli beam theory. Simsek (2010b) studied forced vibration of a simply supported single-walled carbon nanotube subjected to a moving harmonic load by using nonlocal Euler–Bernoulli beam theory. Yang *et al*. (2011) investigated the characteristics of wave propagation in double-walled carbon nanotubes by using nonlocal Timoshenko beam theory.

There are many studies related to wave propagation in macroscale beams (For example Ostachowicz *et al*. (2004), Palacz and Krawczuk (2002), Palacz *et al*. (2005a), Palacz *et al*. (2005b), Kocatürk *et al*. (2011)). However, as far as the authors know, there is no study on the wave propagation in micro beams investigated by using finite element method. The primary purpose of this study is to fill this gap.

In this study, wave propagation in a cantilever micro beam under the effect of an impact force is studied based on the modified couple stress theory. The considered problem is investigated within the Bernoulli-Euler beam theory by using energy based finite element method. The system of equations of motion is derived by using Lagrange's equations. The obtained system of linear differential equations is reduced to a linear algebraic equation system and solved in the time domain by using Newmark average acceleration method. The influence of the material length scale parameter is investgated in detail. Some of the special results of the present study are compared with the results of previously published studies.

2. Theory and formulations

2.1 The modified couple stress theory

The strain energy density for a linear elastic material which is a function of both strain tensor and curvature tensor is introduced by Yang *et al*. (2002) for the modified couple stress theory

$$U = \int_V (\sigma : \varepsilon + m : \chi)\, \mathrm{d}V \tag{1}$$

where σ is the stress tensor, ε is the strain tensor, m is the deviatoric part of the couple stress tensor, χ is the symmetric curvature tensor, defined by

$$\sigma = \lambda \, tr\,(\varepsilon)\, \mathrm{I} + 2\,\mu\,\varepsilon \tag{2}$$

$$\varepsilon = \frac{1}{2}[\nabla u + (\nabla u)^T] \tag{3}$$

$$m = 2l^2\,\mu\,\chi \tag{4}$$

$$\chi = \frac{1}{2}[\nabla \theta + (\nabla \theta)^T] \tag{5}$$

where λ and μ are Lame's constants, l is a material length scale parameter which is regarded as a material property characterizing the effect of couple stress, u is the displacement vector and θ is the rotation vector, given by

$$\theta = \frac{1}{2}\,\mathrm{c\,u\,r\,l}\,u \tag{6}$$

Láme's constants are given by

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \tag{7}$$

2.2 Governing equation

A cantilever microbeam with coordinate system x, z having the origin O *is* shown in Fig. 1. One of the supports of the beam is assumed to be fixed and the other free. The beam is subjected to an impact force in the transverse direction as seen from Fig. 1. The length of the beam is L.

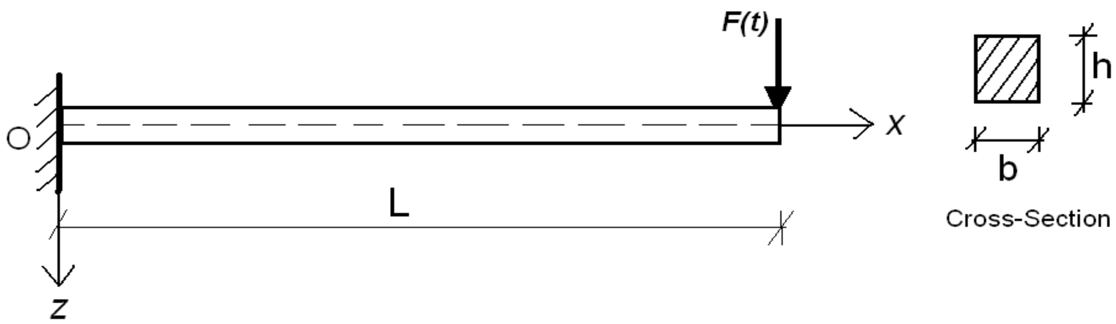

Fig. 1 Cantilever microbeam subjected to an impact force

According to the coordinate system *(x, z)* shown in Fig. 1, the displacement components in a Bernoulli– Euler beam theory can be given as follows

$$u\,(x,z,t) = -z\frac{\partial w\,(x,t)}{\partial x} \tag{8}$$

$$v\,(x,z,\mathrm{t}) = 0 \tag{9}$$

$$w(x, z, t) = w(x, t) \tag{10}$$

where u, v, w are x, y and z components of the displacement vector u, respectively, and t denotes time.

Because the transversal surfaces of the beam is free of stress, then

$$\sigma_{zz} = \sigma_{yy} = 0 \tag{11}$$

By using Eqs. (3), (8) and (10) and from the physics of the problem

$$\varepsilon_{xx} = -z\frac{\partial^2 w(x,t)}{\partial x^2} \tag{12a}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = z\frac{\nu\partial^2 w(x,t)}{\partial x^2} \tag{12b}$$

$$\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{xy} = 0 \tag{12c}$$

By using Eqs. (6), (8), (9) and (10)

$$\theta_y = -\frac{\partial w(x,t)}{\partial x}, \quad \theta_x = \theta_z = 0 \tag{13}$$

Substituting Eq. (13) into Eq. (5), the curvature tensor χ can be obtained as follows

$$\chi_{xy} = -\frac{1}{2}\frac{\partial^2 w(x,t)}{\partial x^2}, \quad \chi_{xx} = \chi_{xz} = \chi_{yy} = \chi_{yz} = \chi_{zz} = 0 \tag{14}$$

Substituting Eq. (12a) into Eq. (2), σ_{xx} is obtained as follows

$$\sigma_{xx} = E(-z\frac{\partial^2 w(x,t)}{\partial x^2}) \tag{15}$$

where E is Young's modulus. Its relationship with Lamê's constants is given by Eq. (7).

Substituting Eq. (14) into Eq. (4), the couple stress tensor can be obtained as follows

$$m_{xy} = -\mu l^2 \frac{\partial^2 w(x,t)}{\partial x^2} \tag{16a}$$

$$m_{xx} = m_{xz} = m_{yy} = m_{yz} = m_{zz} = 0 \tag{16b}$$

where μ is shear modulus which is defined by Eq. (7).

The potential energy of the microbeam is as follows

$$U_i = -\frac{1}{2}\int_{x=0}^{L}(M_X + Y_{XY})\frac{\partial^2 w(x,t)}{\partial x^2}\mathrm{d}x \tag{17}$$

where M_X is the resultant moment and Y_{XY} is the couple moment, given as follows

$$M_x = \int_A \sigma_{xx} z \mathrm{d}A \tag{18}$$

$$Y_{xy} = \int_A m_{xy} \mathrm{d}A \tag{19}$$

where σ_{xx} and m_{xy} are defined by Eqs. (15) and (16a), respectively.

Substituting Eqs. (15), (16a) into Eqs. (18), (19), the resultant moment M_X and the couple moment Y_{XY} can be obtained as follows

$$M_x = -EI \frac{\partial^2 w(x,t)}{\partial x^2} \tag{20}$$

$$Y_{xy} = -\mu A l^2 \frac{\partial^2 w(x)}{\partial x^2} \tag{21}$$

Substituting Eqs. (20), (21) into Eq. (17), the potential energy of the microbeam is obtained as follows

$$U_i = \frac{1}{2} \int_{x=0}^{L} (EI + \mu A l^2)(\frac{\partial^2 w(x,t)}{\partial x^2})^2 \mathrm{d}x \tag{22}$$

The kinetic energy of the microbeam at any instant t is

$$T = \frac{1}{2} \int_0^L \rho A (\frac{\partial w(x,t)}{\partial t})^2 \mathrm{d}x \tag{23}$$

where ρ is the mass of the beam per unit volume.

The potential energy of the external load can be written as

$$U_e = - \int_{x=0}^{L} F(x,t) w(x,t) \mathrm{d}x \tag{24}$$

The Kelvin–Voigt model is used for the material. The constitutive relations for the Kelvin–Voigt model between the stresses and strains become

$$\sigma = E(\varepsilon + \eta_1 \dot{\varepsilon}) \tag{25}$$

$$m = 2l^2 \mu (\chi + \eta_2 \dot{\chi}) \tag{26}$$

where η_1 and η_2 are the damping ratios, as follows

$$\eta_1 = \frac{c}{E}, \quad \eta_2 \frac{c}{2l^2 \mu} \tag{27}$$

where c is the coefficient of damping of the beam.

In this case, the dissipation function of the beam at any instant t is

$$R = \frac{1}{2} \int_0^L (\eta_1 EI + \eta_2 \mu A l^2)(\frac{\partial^2 \dot{w}(x,t)}{\partial x^2})^2 \mathrm{d}x \tag{28}$$

Lagrangian functional of the problem is given as follows

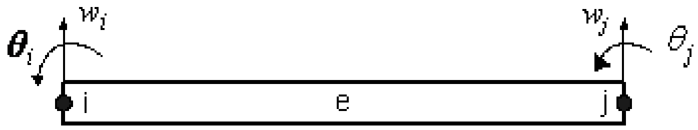

Fig. 2 A two-node beam element

$$I = T - (U_i + U_e) \tag{29}$$

2.3 Solution method of the problem

The problem is solved by using Lagrange's equations and time integration method of Newmark (1959). In order to apply the Lagrange's equations, the displacements of nodes of the unknown functions $w(x, t)$ which is written for a two-node beam element shown in Fig. 2 are defined as follows

$$\{w(t)\}(e) = [w_i^{(e)}(t) \quad \theta_i^{(e)}(t) \quad w_j^{(e)}(t) \quad \theta_j^{(e)}(t)]^T \tag{30}$$

The displacement field of the finite element is expressed in terms of nodal displacements as follows

$$w^{(e)}(x, t) = N_1(x)w_1^{(e)}(t) + N_2(x)\,\theta_1^{(e)}(t) + N_3(x)w_2^{(e)}(t) + N_4(x)\theta_2^{(e)}(t) \tag{31}$$

where N_1, N_2, N_3 and N_4 are interpolation functions and given as follows

$$N_1(x) = 1 - 3\,(x/L_e)^2 + 2(x/L_e)^3$$

$$N_2(x) = L\,(-(x/L_e) + 2(x/L_e)^2 - (x/L_e)^3)$$

$$N_3(x) = 3\,(x/L_e)^2 - 2(x/L_e)^3$$

$$N_4(x) = L\,((x/L_e)^2 - (x/L_e)^3) \tag{32}$$

where L_e is the length of the beam element.

After substituting Eq. (31) into Eq. (29) and then using the Lagrange's equations gives the following equation;

$$\frac{\partial I}{\partial w_k^{(e)}} - \frac{d}{dt}\frac{\partial I}{\partial \dot{w}_k^{(e)}} + Q_{D_k} = 0, \qquad k = 1, 2, 3, \ldots \tag{33}$$

where

$$Q_{D_k} = -\frac{\partial R}{\partial \dot{w}_k^{(e)}}, \quad k = 1, 2, 3, \ldots \tag{34}$$

Q_{D_k} is the generalized damping load which can be obtained from the dissipation function by differentiating R with respect to $\dot{w}_k^{(e)}$.

The Lagrange's equations yield the system of equations of motion for the finite element and by use of usual assemblage procedure the following system of equations of motion for the whole system can be obtained as follows

$$[K]\{w(t)\}+[D]\{\dot{w}(t)\}+[M]\{\ddot{w}(t)\}=\{F(t)\} \tag{35}$$

where

$$[M]=\int_{x=0}^{L}\rho A\{N(x)\}^{T}\{N(x)\}\,\mathrm{d}x \tag{36}$$

$$[D]=\int_{x=0}^{L}\overline{\eta}(EI+\mu A l^{2})\{\ddot{N}(x)\}^{T}\{\ddot{N}(x)\}\mathrm{d}x \tag{37}$$

$$\{F(t)\}=\int_{x=0}^{L}\{N(x)\}^{T}F(x,t)\,\mathrm{d}x. \tag{38}$$

$$[K]=\int_{x=0}^{L}\{\ddot{N}(x)\}^{T}\left(EI+\mu A l^{2}\right)\{\ddot{N}(x)\}\,\mathrm{d}x \tag{39}$$

By making necessary integrations in Eq. (39), [K] is obtained as follows

$$[K]=(EI+\mu A l^{2})\begin{bmatrix}\frac{12}{L_e^2} & \frac{-6}{L_e^2} & \frac{-12}{L_e^3} & \frac{-6}{L_e^2}\\ \frac{-6}{L_e^2} & \frac{4}{L_e} & \frac{6}{L_e^2} & \frac{2}{L_e}\\ \frac{-12}{L_e^3} & \frac{6}{L_e^2} & \frac{12}{L_e^3} & \frac{6}{L_e^2}\\ \frac{-6}{L_e^2} & \frac{2}{L_e} & \frac{6}{L_e^2} & \frac{4}{L_e}\end{bmatrix} \tag{40}$$

where, [K] is the stiffness matrix, [D] is the damping matrix, [M] is mass matrix and {$F(t)$} is the load vector. The motion equations which is given by Eq. (35), are solved in the time domain by using Newmark average acceleration method (Newmark 1959). The differential equations for classical Bernoulli-Euler beam theory and for the modified couple stress theory in the undamped case are

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}=q(x,t) \tag{41}$$

$$\left(EI+\mu A l^2\right)\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}=q(x,t) \tag{42}$$

The phase speeds for classical Bernoulli-Euler beam theory and for the modified couple stress theory are

$$c = \sqrt{\omega}\left(\frac{EI}{\rho A}\right)^{1/4} \tag{43}$$

$$c = \sqrt{\omega}\left(\frac{EI + \mu A l^2}{\rho A}\right)^{1/4} \tag{44}$$

3. Numerical results

In the numerical results, the material of the beam considered here is taken to be made of epoxy and material properties used in the calculations are taken to be E = 1.44 GPa, ν = 0.38, c = 14.4 Pa/s and material density assumed as ρ = 1220 kg/m^3 (Lam *et al.* 2003). The material length scale parameter is kept constant as l = 17.6 μm.

In order to establish the accuracy of the present formulation and the computer program developed by the authors, the results obtained from the present study are compared with the available results in the literature. For this purpose, static deflections of a cantilever beam which is subjected to a point load are calculated for modified couple stress theory and compared with those of Park and Gao (2006). It is clearly seen that the curves of Fig. 3 of the present study are very close to those of Fig. 3 of Park and Gao (2006).

Dispersion relations for classical Bernoulli-Euler beam theory and for the modified couple stress theory are given in the Fig. 4 by using Eqs. (43), (44). As seen from Fig. 4, increase in the height of the beam causes decrease in the difference between the phase speeds of the Bernoulli-Euler beam theory and the modified couple stress theory results. Although the difference is very significant for the parameters given in Fig. 4(a), it is almost negligible for the parameters given in Fig. 4(b).

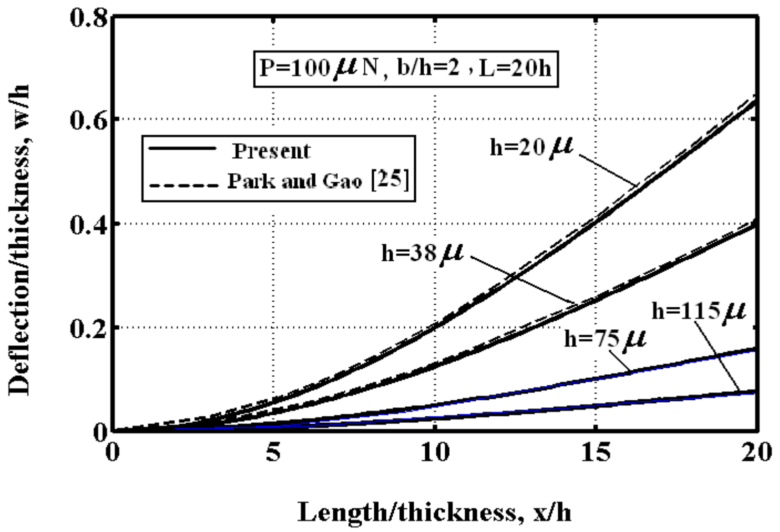

Fig. 3 Deflections of the cantilever beam based on the modified coupled stress theory

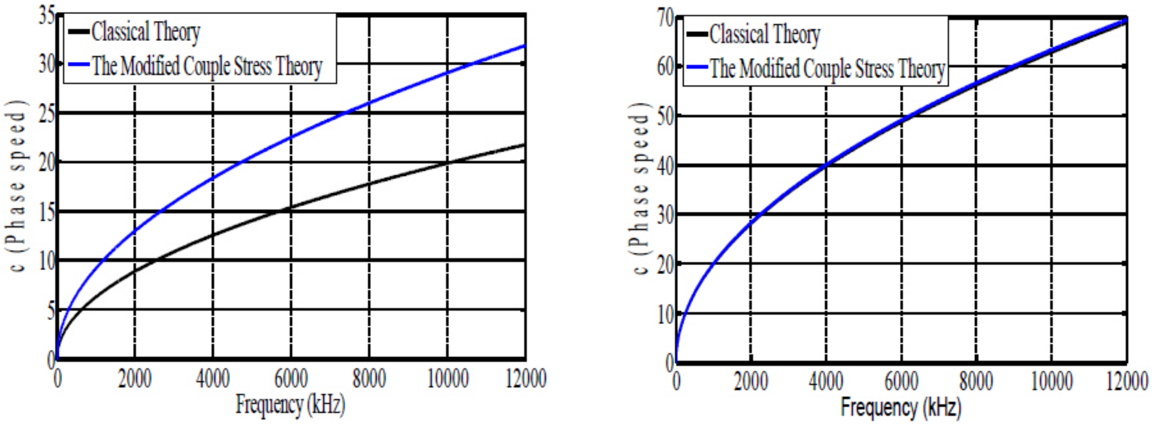

Fig. 4 Dispersion relations for E = 1.44 GPa, v = 0.38, ρ = 1220 kg/m^3, l = 17.6 μm, for (a) h = 20 μm, b = 2h, (b) h = 200 μm, b = 2h

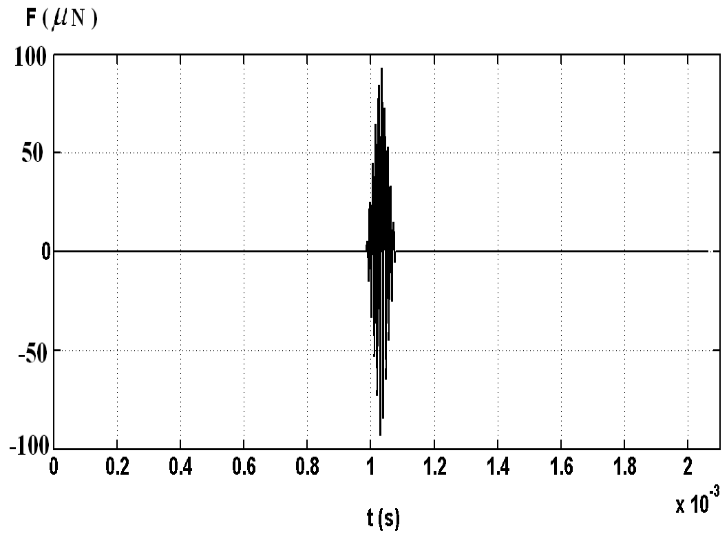

Fig. 5 Excitation triangle signal for $0.0925\mu s$ multiplied by sinusoidal signal whose period is 1/10 of the duration of the triangle signal and lasts for $0.0925\mu s$

The problem is analyzed within the framework of the Bernoulli–Euler beam theory. Numerical calculations in the time domain are performed by using Newmark average acceleration method. The system of linear differential equations which are given by Eq. (35), is reduced to a linear algebraic system of equations by using average acceleration method. In the numerical calculations, the number of finite elements is taken as n = 100. The beam is excited by a transverse triangular force impulse (with a peak value 100 μN) modulated by a harmonic function (Fig. 5). Excitation force is obtained by multiplying a triangle signal lasting $0.0925\mu s$ with a sinusoidal signal whose period is 1/10 of the duration of the triangle signal and lasts for $0.0925\mu s$.

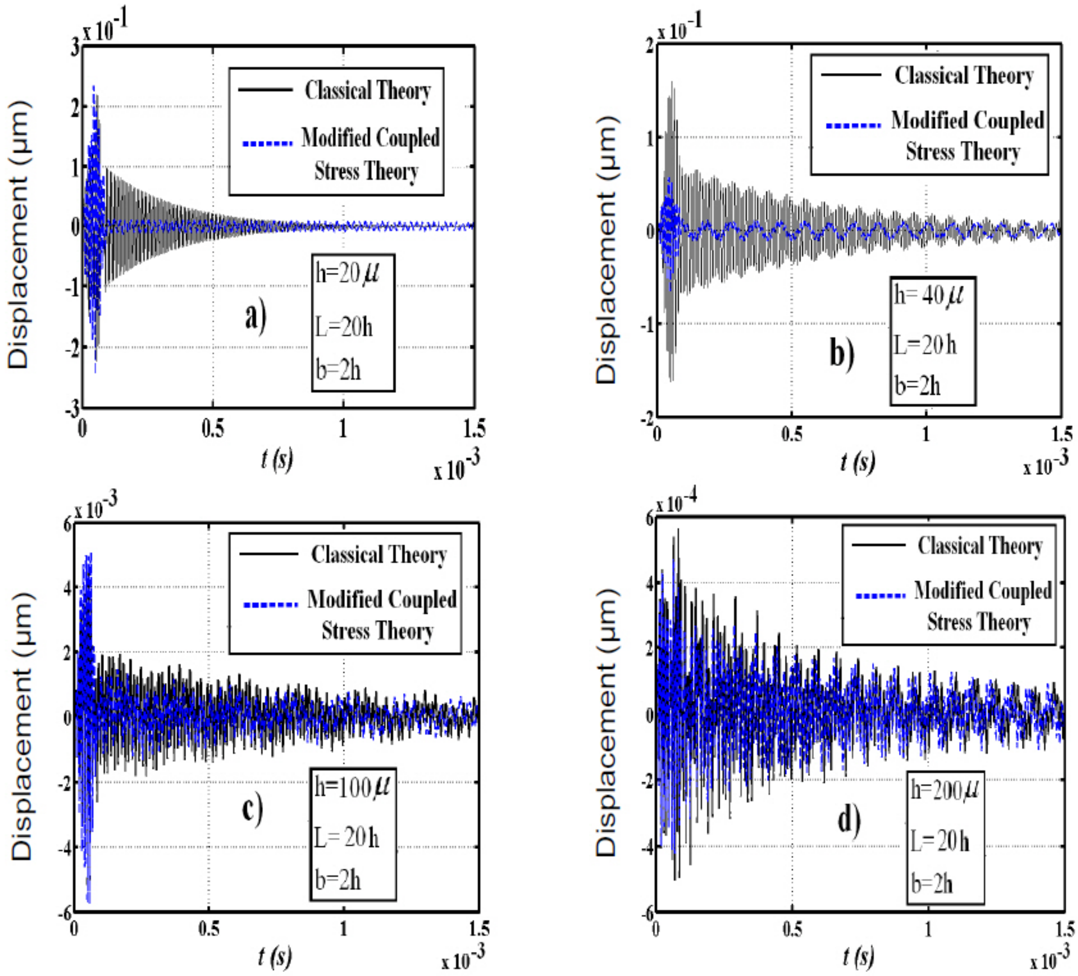

Fig. 6 Transverse displacement at the free end of the cantilever beam based on modified couple stress theory and classical theory, respectively, for various values of the height h. (a) h = 20 μm, (b) h = 40 μm, (c) h = 100 μm and (d) h = 200 μm for the length of the beam L = 20h and width of the beam b = 2h

Figs. 6, 7, 8, 9 and 10 shows the transverse displacement at the free end of the cantilever beam for various lengths of the beam L = 20h, L = 50h, L = 200h, L = 500h and L = 1000h with various beam heights based on modified couple stress theory and classical beam theory.

It is seen from figures 6-10 that the waves which appear after the time interval of the applied impact force are reflecting waves from the fixed end support of the beam. These waves increase in the the case of classical beam theory based on the modified couple stress theory compared to the classical beam theory for the same time interval.

It is clearly seen from Figs. 6, 7, 8, 9 and 10 that, with increase in the height h, the difference between the results of the classical beam theory based on modified couple stress theory and classical beam theory decrease considerably. Decrease of the beam height h causes increase of the number of the waves for the number of the same time interval and this increase is greater for the

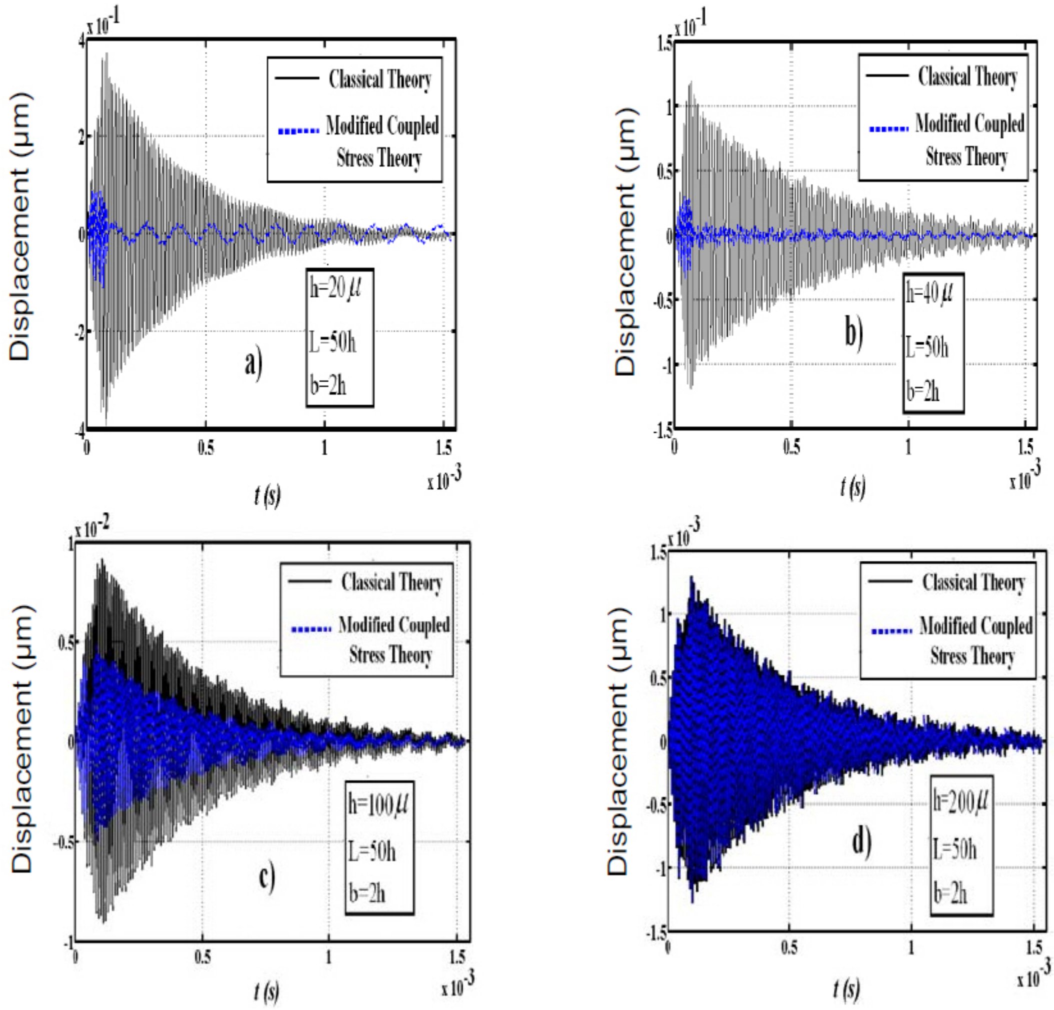

Fig. 7 Transverse displacement at the free end of the cantilever beam based on modified couple stress theory and classical theory, respectively, for various values of the height h. (a) h = 20 μm, (b) h = 40 μm, (c) h = 100 μm and (d) h = 200 μm for the length of the beam L = 50h and width of the beam b = 2h

classical beam theory based on modified couple stress theory compared to the classical beam theory. The number of waves of the classical beam theory based on modified couple stress theory and classical beam theory closes to each other with increase in the height of the beam,

Increase in the ratio of L/h causes decrease in the number of waves for both of modified couple stress theory and the classical beam theory for the considered time interval.

Decrease in the beam height h causes increase in the number of the waves in the classical beam theory based on the modified couple stress theory compared to the classical beam theory for the same time interval. In the case of increase in the beam height h, the number of waves of the classical beam theory based on modified stress theory and classical beam theory closes to each other.

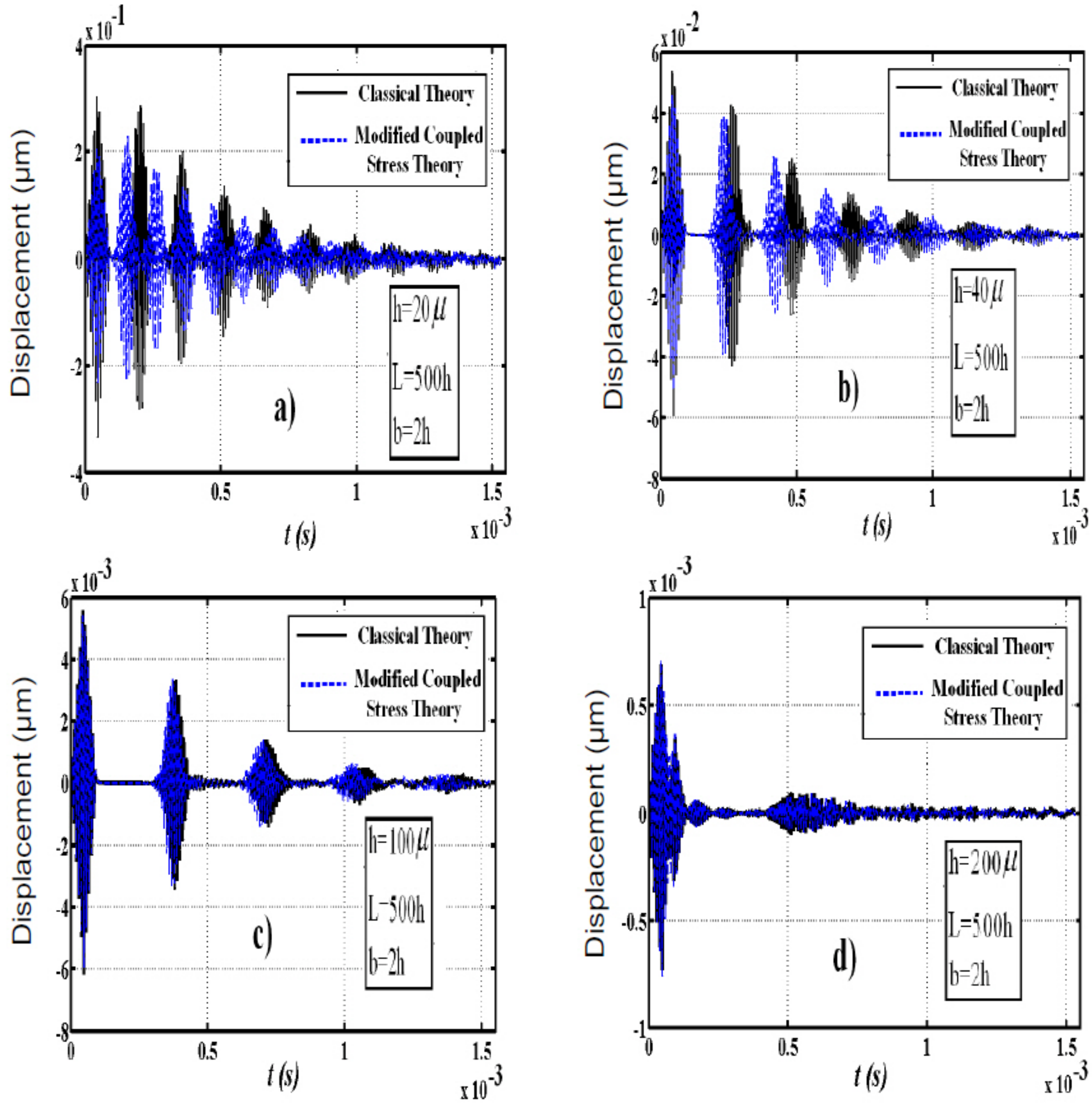

Fig. 9 Transverse displacement at the free end of the cantilever beam based on modified couple stress theory and classical theory, respectively, for various values of the height h. (a) $h = 20\ \mu m$, (b) $h = 40\ \mu m$ (c) $h = 100\ \mu m$ and (d) $h = 200\ \mu m$ for the length of the beam $L = 500h$ and width of the beam $b = 2h$

In the case of the classical beam theory based on modified couple stress theory, the displacements of the free end of the cantilever beam decreases compared to the classical beam theory for the considered time interval.

Therefore, for the smaller values of h, the classical beam theory based on the modified couple stress theory must be used instead of the classical beam theory.

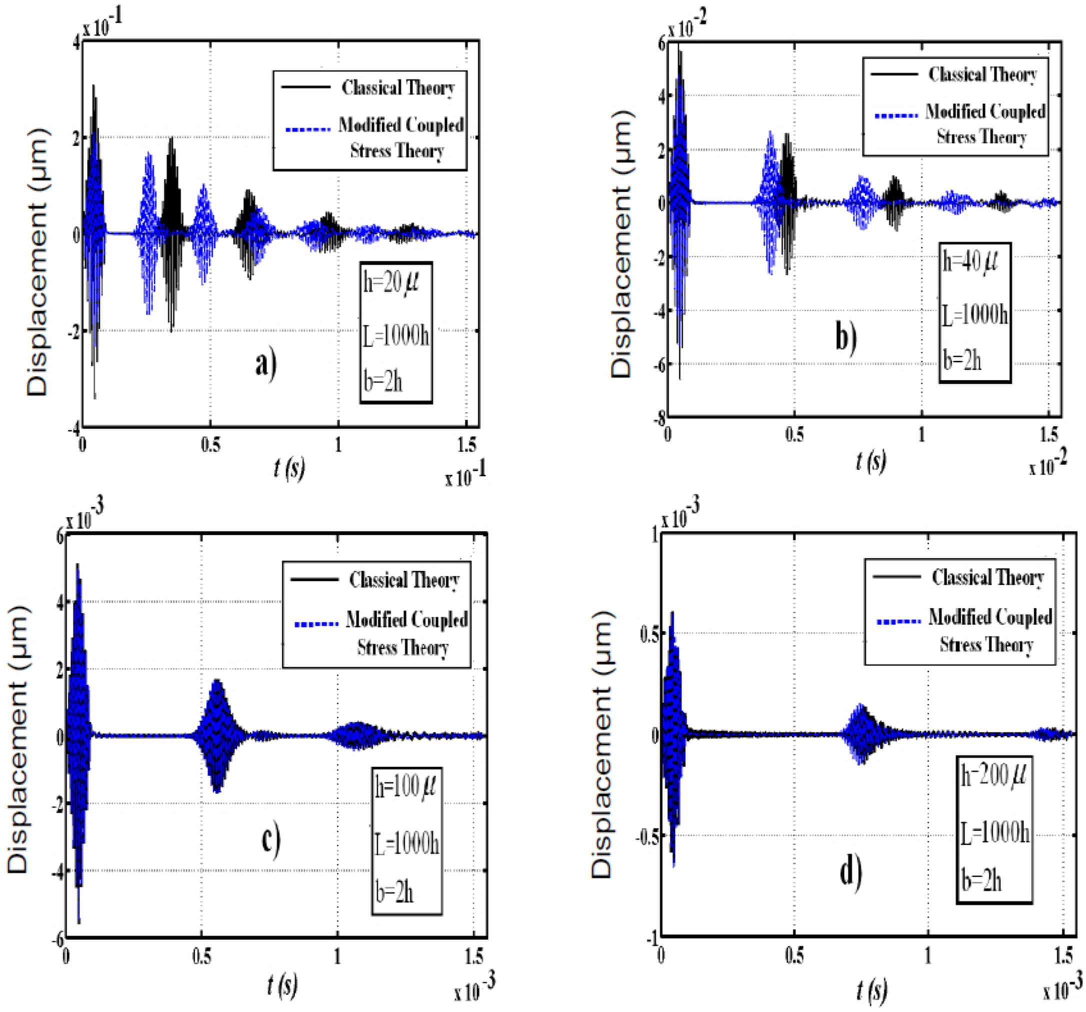

Fig. 10 Transverse displacement at the free end of the cantilever beam based on modified couple stress theory and classical theory, respectively, for various values of the height h. (a) $h = 20\ \mu m$, (b) $h = 40\ \mu m$, (c) $h = 100\ \mu m$ and (d) $h = 200\ \mu m$ for the length of the beam $L = 1000h$ and width of the beam $b = 2h$

4. Conclusions

Wave propagation in a cantilever micro beam under the effect of an impact force is investigated based on the modified coupled stress theory. The considered problem is investigated within the Bernoulli-Euler beam theory by using energy based finite element method. The system of equations of motion is derived by using Lagrange's equations. The obtained system of linear differential equations is reduced to a linear algebraic equation system and solved in the time domain by using Newmark average acceleration method. It is observed from the investigations that the number of waves of the microbeam predicted by the modified couple stress theory is greater than that of the classical beam theory but the displacements predicted by modified couple stress theory are smaller than that of classical beam theory. It is clearly seen from the results that

Bernoulli-Euler beam based on modified couple stress theory must be used instead of the classical beam theory for small values of beam height *h*.

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