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A hybrid imperialist competitive ant colony algorithm for optimum geometry design of frame structures

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Abstract. This paper describes new optimization strategy that offers significant improvements in performance over existing methods for geometry design of frame structures. In this study, an imperialist competitive algorithm (ICA) and ant colony optimization (ACO) are combined to reach to an efficient algorithm, called Imperialist Competitive Ant Colony Optimization (ICACO). The ICACO applies the ICA for global optimization and the ACO for local search. The results of optimal geometry for three benchmark examples of frame structures, demonstrate the effectiveness and robustness of the new method presented in this work. The results indicate that the new technique has a powerful search strategies due to the modifications made in search module of ICACO. Higher rate of convergence is the superiority of the presented algorithm in comparison with the conventional mathematical methods and non hybrid heuristic methods such as ICA and particle swarm optimization (PSO).

Keywords: geometry optimization; hybrid optimization method; frame structure; imperialist competitive algorithm; ant colony optimization

1. Introduction

Looking for the geometry of a structure that minimizes an objective function, like mass or compliance, subject to mechanical constraints is called the geometry optimization. It is a traditional field in structural design, and there are many books and papers dealing with it and related fields (Sokolowski and Zolesio 1992, Bendsoe 1995, Choi and Kim 2004, Van Keulen *et al*. 2005, Pedersen 2000).

In recent years, heuristic algorithms (HAs) such as genetic algorithm (Rahami *et al*. 2008, Tang *et al.* 2005), simulated annealing (Hasancebi and Erbatur 2002), particle swarm optimization (Guan and Chun 2011, Ghoddosian and Sheikhi 2011), ant colony optimization (Luh and Lin 2008), imperialist competitive algorithm (Kaveh and Talatahari 2010, Sheikhi *et al*. 2012), charged system search (Kaveh and Talatahari 2010), water cycle algorithm (Eskandar *et al.* 2012), Mine blast algorithm (Sadollah *et al*. 2012) and hybrid method (Ferhat *et al.* 2011, Kaveh and Talatahari 2009, Eskandar *et al*. 2011, Kaveh and Talatahari 2012) have attracted much attention for structural optimization problems due to their superior advantages. HAs do not require the objective function to be derivable or even continuous, and in many cases HAs perform as global

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optimization techniques due to the appropriate balance between the exploration and exploitation of the whole search space.

Compared to other evolutionary algorithms, the advantages of ICA is its easy implementation, smaller number of parameters to be adjusted, high ability to deal with nonlinear optimization problems and fast convergence speed (Eskandar *et al.* 2011). However, it is known that the original ICA had difficulties in controlling the balance between exploration (global investigation of the search place) and exploitation (the fine search around a local optimum) (Sheikhi *et al*. 2012). In order to improve this character of ICA, it is hybridized with ACO. Imperialist competitive ant colony optimization is based on the standard imperialist competitive algorithm that is one of the newest algorithms in optimization field (Atashpaz-Gargari and Lucas 2007) and the Ant Colony Optimization scheme. To show the robustness of the ICACO method, it is employed for three benchmark examples of frame structural geometry optimization and the results are presented.

2. Imperialist Competitive Algorithm (ICA)

ICA was proposed by Atashpaz-Gargari and Locus (Atashpaz-Gargari and Lucas 2007) which develops a strong optimization strategy using socio-political evolution of human as a source of inspiration. Like other heuristic optimization algorithms, this algorithm starts with an initial population. Each individual of the population is called a 'country'. Countries are divided into two groups: the imperialists and colonies of these imperialists. Colonies are under the possession of an imperialist. In fact, each imperialist represents the local or global optimization minimum. In this algorithm, first N countries are chosen randomly. Then, the countries with much power (the more optimized ones) are chosen as imperialists and the rest are considered as colonies.

Based on the power of imperialists, all the colonies of initial population are divided among them. Each imperialist together with its colonies form an Empire. When all colonies are divided among imperialists, they begin to approach their associated imperialist country. In this process, if a colony in an empire has a lower cost than that of imperialist, the position of the colony and its relevant imperialist is exchanged. To model the total power of an empire in the proposed algorithm, the power of an imperialist is summed with the percentage of the mean power of its colonies. The empires which are unable to increase their total power in the imperialistic competition, will gradually become weaker and will finally collapse. Accordingly, their colonies will join other empires and make those empires stronger. As the empires collapse in the competition among them, there remains just one empire in the world. Eventually, all the colonies in this empire will reach to the same position and power as the imperialist. Fig. 1 shows a typical example of the movement of a colony toward the imperialist. θ and x are random numbers which are considered to define the movement of the colony toward the imperialist.

2.1 The implementation of ICA

The ICA is implemented as follows:

1-Selection of some random points to create initial empires

Initial country locations are defined as

$$x_{i,j}^{(0)} = Ubx_i + r.(Ubx_i - Lbx_i) \tag{1}$$

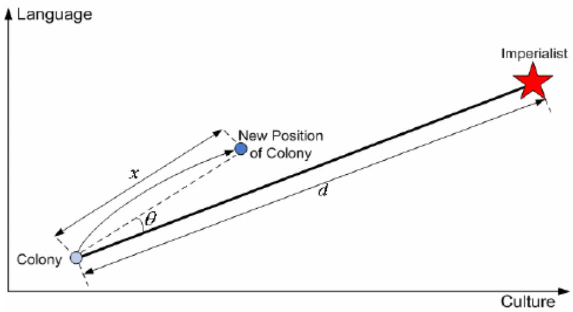

Fig. 1 The movement of the colony toward the imperialist (Sheikhi *et al.* 2012)

Where $x_{i,j}^{(0)}$ determines the initial value of the ith variable for the jth country; Lbx_i and Ubx_i are the lower and upper bound values for the ith variable; r is a random number in the interval [0, 1]. When the cost values for initial countries are calculated, some of the countries with the lower costs will be chosen as the imperialist states while the other countries will form the colonies. According to power, all the colonies of initial countries are divided among the imperialists.

2-Movement of the colonies toward their associated imperialists

The movement of the colony towards the imperialist is described as followed

$$\{x\}_{new} = \{x\}_{old} + U(0, \beta \times d) \times \{V_1\} \tag{2}$$

Where U defines a random value which is distributed evenly between 0 and $\beta \times d$ (Atashpaz-Gargari *et al*. 2008); β is a parameter with a value greater than one, and d is the distance between colony and imperialist. $\{V_1\}$is a vector which starts from the previous location of the colony and directs toward the imperialist location. The vector's length is considered equal to unity.

To extend the searching domain around the imperialist, an amount of deviation (θ) is randomly added to the direction of movement where θ is defined as

$$\theta = U(-\gamma, +\gamma) \tag{3}$$

Where γ is a parameter that adjusts the deviation from the original direction.

3- Exchange of the position of the colony and its relevant imperialist, if a colony exists with lower cost than that of the imperialist in its empire. Based on both powers of the imperialist and its colonies the total power of an empire is calculated. A mathematical model for this fact is defined by the total cost as:

$$TC_j = f_{\cos t}^{(imp,j)} + \xi . \frac{\sum_{i=1}^{NC_j} f_{\cos t}^{(col,j)}}{NC_j} \tag{4}$$

Where TC_j is the total cost of the jth *Empire* and ξ is positive number which is considered to be less than 1. The value of 0.1 for ξ is found to be a suitable value in most of the implementations (Atashpaz-Gargari *et al*. 2008).

4-Imperialistic Competition: choosing the weakest colony in the weakest empire and giving it to the most powerful empire.

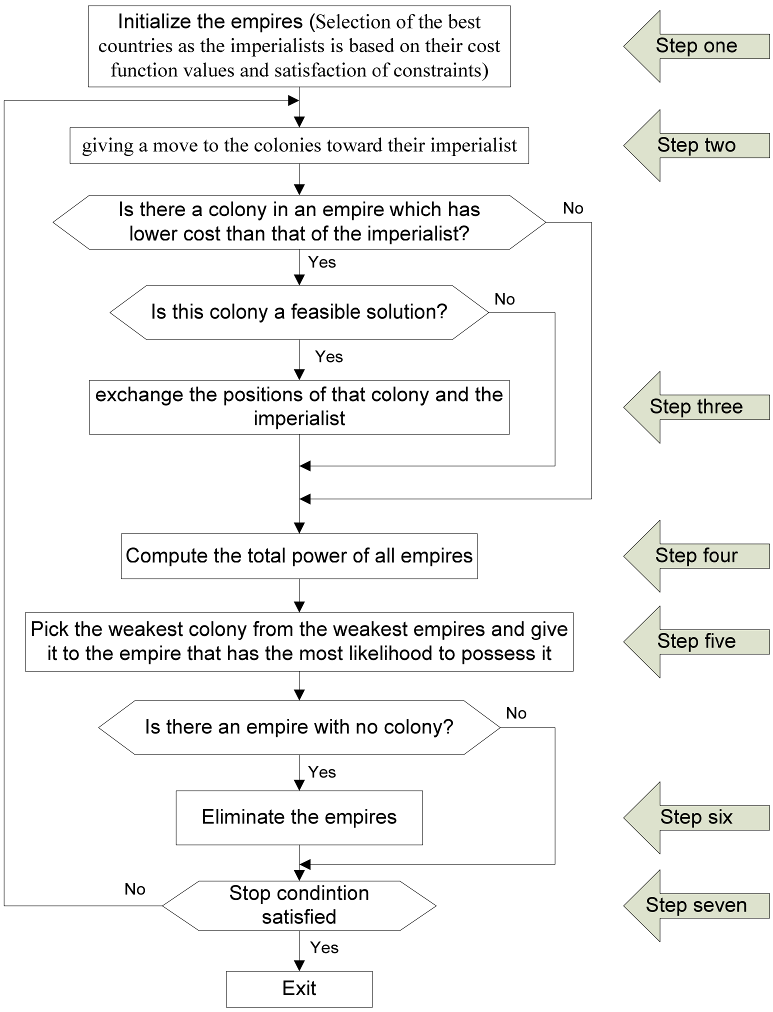

Fig. 2 Flowchart of the ICA (Eskandar *et al*. 2011)

5- Elimination of the empires with no colonies.

6- Stopping the algorithm if the number of iterations reaches to a pre-defined value, or there is just one unique empire and the amount of improvement in the best result reduces to a pre-defined value (Kaveh and Talatahari 2010), otherwise going back to step 2.

The flowchart of Imperialist Competitive Algorithm is illustrated in Fig. 2 (Eskandar *et al.* 2011).

3. Ant colony optimization

Ant colony optimization was first proposed by Dorigo (1992) and population-based methodology applied to numerous NP-hard combinatorial optimization problems. They have been inspired by the behaviour of real ant colonies especially by their foraging behaviour. Ants can find the shortest path to food by laying a pheromone (chemical) trail as they walk. Other ants follow the pheromone trail to food. Ants that happen to pick the shorter path will create a strong trail of pheromone faster than the ones choosing a longer path. Since stronger pheromone attracts ants better, more and more ants choose the shorter path until eventually all ants have found the shortest path. Consider the case of three possible paths to the food source with one longer than the others. Ants choose each path with equal probability. Ants that went and returned on the shortest path will cause it to have the most pheromone soonest. Consequently new ants will select that path first and further reinforce the pheromone level on that path. Eventually all the ants will follow the shortest path to the food. One problem is premature convergence to a less than optimal solution because too much virtual pheromone was laid quickly. To avoid this stagnation, the pheromone associated with a solution disappears after a period of time. The ACO procedure is illustrated in Fig. 3 (Kaveh and Talatahari 2009).

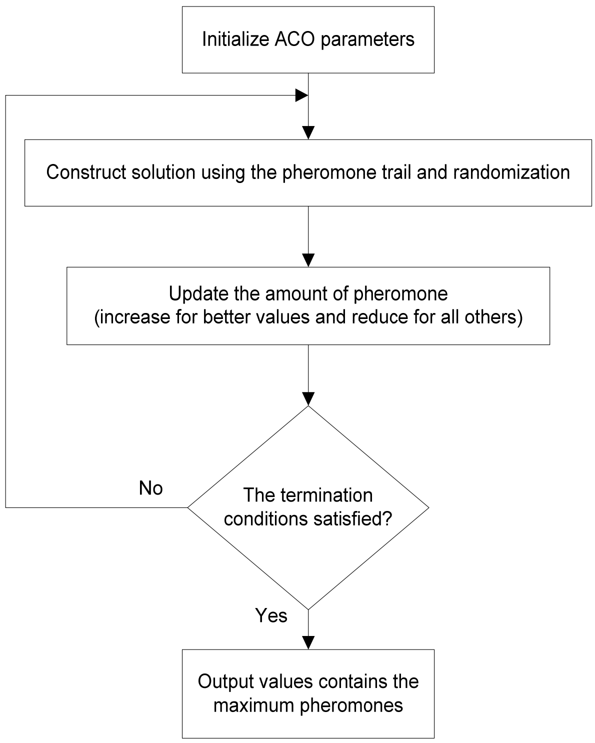

Fig. 3 The flow chart for ACO (Kaveh and Talatahari 2009)

4. Imperialist competitive ant colony optimization

The Imperialist Competitive Ant Colony Optimization (ICACO) algorithm applies the ICA for searching global optimization, while ACO works as a local search, wherein ants apply a pheromone-guided mechanism to refine the positions found by countries in the ICA.

In ACO stage, first of all, initial ants of size N_{Col} are produced. These ants generate solutions around their relevant imperialist country which can be expressed as

$$Ant_{j,n}^{k} = N(imperialist_{n}, \sigma),\ j = 1,2,\ldots,N.C_{n}, n = 1,2,\ldots,N_{imp} \tag{5}$$

In the Eq. (5), $N.C_n$ is the number of colonies of the nth empire so

$$Ant^{k} = \begin{bmatrix} Ant_{1,1} \\ \vdots \\ Ant_{N.C_1,1} \\ Ant_{1,2} \\ \vdots \\ Ant_{N.C_1,2} \\ \vdots \\ Ant_{N.C_{imp},Nimp} \end{bmatrix},\ N.C_1 + NC_2 + \ldots + N.C_{N_{imp}} = N_{col} \tag{6}$$

Therefore, $Ant_{j,n}^{k}$ is the solution constructed by ant jth in empire nth in the stage k; $N(imperialist_n, \sigma)$ denotes a random number normally distributed with mean value imperialist nth and variance σ, where

$$\sigma = (Ub - Lb) \times \eta \tag{7}$$

In the Eq. (7), Ub and Lb are the upper and lower bound respectively. Also, η is used to control the step size which in first trial is equal to 1 and by approaching to optimal point, reduces gradually and at the end tends to zero. The ACO stage in the ICACO algorithm works as a helping factor to guide the exploration and to increase the control in the exploitation.

After generating Ants, the value of the objective function for each ant ($f(Ant_{j,n}^{k})$) is computed and the current position of ant jth in empire nth ($Ant_{j,n}^{k}$) is replaced with the position $Colony_{j,n}^{k}$ (the current position of colony jth in empire nth), if $f(Colony_{j,n}^{k})$ is bigger than $f(Ant_{j,n}^{k})$ and current ant is in the feasible space. The flowchart of Imperialist Competitive Ant Colony Optimization (ICACO) algorithm is illustrated in Fig. 4.

5. Geometry structural optimization

Since in frame structures developed maximum bending moment is one of the main criterions to evaluate the efficiency of the design, the goal of the optimization in the following examples is to minimize the absolute value of maximum bending moment. For generality structures are subjected to multi load cases. Thus, the objective function can be stated mathematically as Eq. (8).

$$Minimize\ Max\big|_{l=1}^{L} \{|M|\} \tag{8}$$

Where $|M|$ and L are the maximum absolute value of the bending moment and the total number of load cases respectively. In many cases, constraints on design variables, which directly specify the bounds of the nodal position, are often imposed in a geometry optimization process. Also due to retain the structural symmetry or to decrease the number of design variables, some nodal coordinates are linked to each other by defining additional constraints. In Eqs. (9) and (10) the constraints of optimization problem are presented.

$$S_{Li} \leq S_i \leq S_{Ui},\ (i=1,\ldots,n) \tag{9}$$

$$S_j = f(S_i),\ (j=1,\ldots,m) \tag{10}$$

Where S_i and S_j are the coordinates of independent and dependent nodal coordinates; S_{Li} and S_{Ui} are the lower and upper bounds on the independent coordinates respectively. n and m are the number

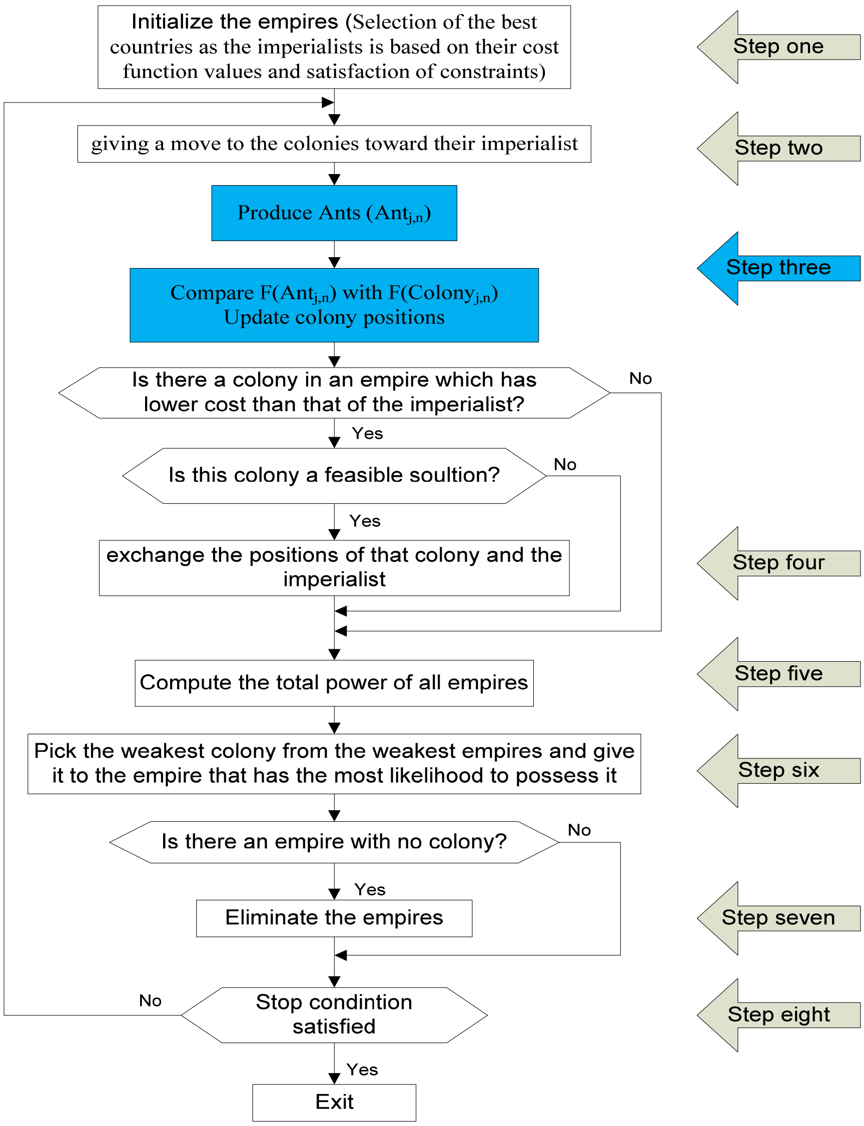

Fig. 4 Flowchart of the ICACO

of independent and dependent nodal coordinates.

The maximum and absolute bending moment in Eq. (8) does not refer to the response measured at a single point; the maximum bending moment may frequently transfer from one point to another in the solution process. Consequently, abrupt changes may often occur in the objective function as well as in its derivative while the optimization is progressing, which then brings a practical obstacle into first or higher order optimization algorithm and deteriorates the convergence of the solution.

6. Design examples

To illustrate the efficacy, validity and capability of the presented method, the geometry of three typical frame structures as benchmark problems are optimized. In the design process, the layout of the structure is initially determined and remains invariable. The algorithms are coded in MATLAB and structures are analyzed using the finite element method. In Table 1, the parameters that are used in each of the optimization algorithms (ICA, ICACO and PSO Clerc and Kennedy 2002) are presented.

6.1 Two-member planar frame structure

A two-member frame structure, initially designed as shown in Fig. 5, is loaded at Node 2 with two load cases of 20 kN downwards and horizontally, respectively (Wang 2007). The cross sectional area and Young's modulus of all members are A = 7.26 cm^2 and E = 210 GPa respectively.

In this example, it is assumed that the position of node 2 is fixed and the other two nodal coordinates can be relocated symmetrically ($y_1 = y_3 = y$) for minimizing the absolute value of maximum bending moment ($|M|$). It is evident that the individual optimal configurations for each load case is when the both members are vertical or horizontal ($y = \infty$ or $y = 0$). But when both load cases are considered together, the optimum configuration seems not so evident.

Table 1 The parameters of optimization algorithms

	C_1	C_2	χ	Ψ
PSO (Clerc and Kennedy 2002)	1.0	1.0	0.729	1.0
	β	γ	ξ	
ICA	2.0	0.3	0.1	
ICACO	2.0	0.3	0.1	

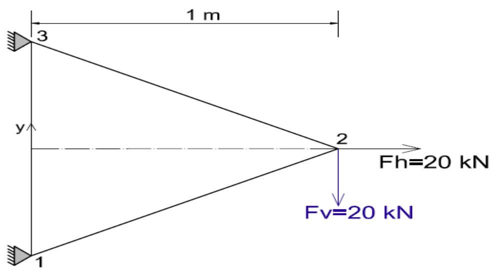

Fig. 5 A two-member planar frame structure (Wang 2007)

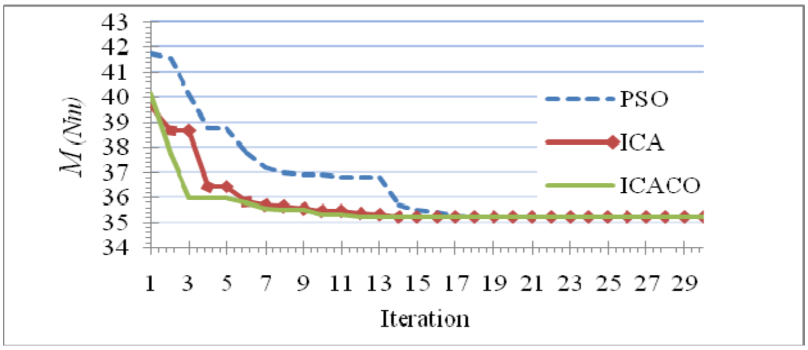

Fig. 6 Comparison of the convergence rates of the three algorithms for the two-element planar Frame structure

Table 2 Best optimal design comparison for the two-member planner frame structure

	Optimal nodal coordinates (*m*)			
	(Wang 2007)	PSO	ICA	ICACO
y (*m*)	0.794	0.79404	0.79404	0.79404
|*M*| (*Nm*)	35.24	35.2417	35.2417	35.2417

Table 3 Statistical results of different methods for the optimal geometry of two-member planner frame structure

	PSO *(Nm)*	ICA *(Nm)*	ICACO *(Nm)*
Best	35.2417	35.2417	35.2417
Mean	40.2246	40.7928	35.2418
Worst	102.8435	109.2610	35.2444
Standard deviation	12.5136	11.4538	3.7936e-4

In order to demonstrate the ability of the proposed algorithm, this example is run with the other methods such as PSO algorithm with 20 particles, ICA and ICACO algorithms with 20 countries where 5 of them are selected as the imperialists. Fig. 6 provides a comparison of the convergence rates of the three algorithms. The PSO and ICA algorithms achieve the best solutions after 20 and 17 iterations respectively. However, the ICACO algorithm finds the best solution after about 13 iterations. Table 2 compares the best obtained results by different methods.

The statistical simulation results are summarized in Table 3. From Table 3, it can be seen that the standard deviation of the results by ICACO in 100 independent runs is very small.

6.2 Thirteen-member planar frame structure

A thirteen-member frame structure (Michell type structure) shown in Fig. 7, is a standard problem for evaluating the efficiency and validity of the structural optimization methods. Recently, geometry optimization of this structure is performed while minimizing the weight (Wang *et al.* 2002, Isenberg *et al.* 2002) and also the maximum bending moment is minimized in the work of Wang (2007).

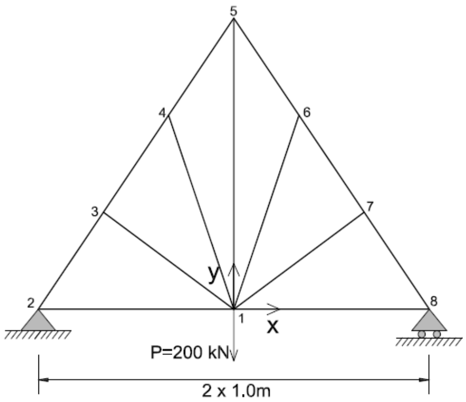

Fig. 7 Thirteen-member frame structure

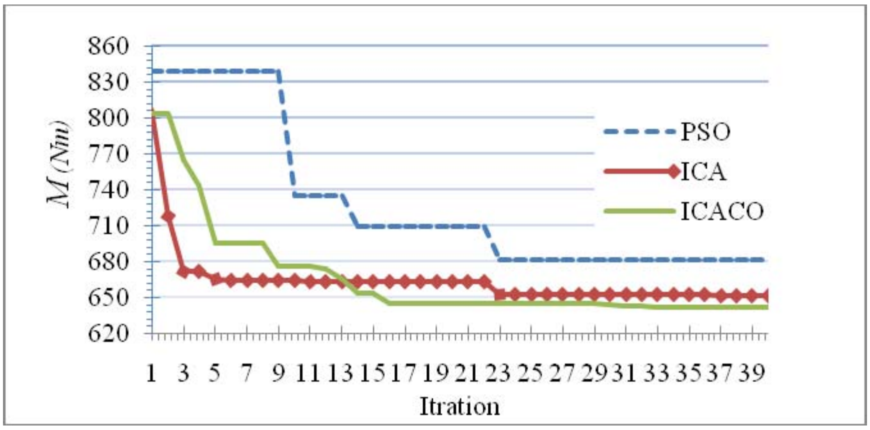

Fig. 8 Convergence rate comparison for the three algorithms for thirteen-member frame structure

Suppose the cross sectional area and Young's modulus of all members are A = 4.9 cm^2 and E = 210 GPa. Assume the coordinates of nodes 1, 2 and 8 are fixed while the coordinates of nodes 3, 4, 5, 6 and 7 can shift in both horizontal and vertical directions. During the optimization process, the symmetry of the structure is maintained. Therefore, only five nodal coordinates need to be redesigned independently for minimizing of maximal bending moment (|M|).

In this example, the size population of PSO, ICA and ICACO algorithms are 50 particles (50 countries), which in ICA and ICACO, 8 countries of them are selected as the imperialists.

In Fig. 8 the convergence rate of the three algorithms are compared. In Table 4 the best optimal values of the nodal coordinates obtained by PSO, ICA and ICACO are listed.

It can be seen that the ICACO method reaches to optimal point with higher convergence rate in comparison with other optimization methods. In Table 5, the reduction percentage of the maximum bending moment of the best optimal design of three algorithms respect to evolutionary shift method (Wang 2007) for the thirteen member frame structure is illustrated.

The statistical simulation results are summarized in Table 6. It can be seen from Table 6 that the best, mean, worst and standard deviation of solutions found by ICACO are better than the solutions of other algorithms.

Table 4 Best optimal design comparison for the thirteen-member frame structure

	Optimal nodal coordinates (m)			
	(Wang 2007)	PSO	ICA	ICACO
X_3	-0.7925	-0.6011	-0.7473	-0.9062
Y_3	0.3703	0.7790	0.6512	0.1920
X_4	-0.5314	-0.4225	-0.5071	-0.4457
Y_4	0.6071	0.9620	0.9607	0.8159
Y_5	0.6959	1.1044	1.1095	1.0218
X_6	0.5314	0.4225	0.5071	0.4457
Y_6	0.6071	0.9620	0.9607	0.8159
X_7	0.7925	0.6011	0.7473	0.9062
Y_7	0.3703	0.7790	0.6512	0.1920
$\|M\|$ (Nm)	807.0	679.89	650.50	638.75

Table 5 The reduction percentage of the maximum bending moment of the best optimal design of three algorithms respect to the result of evolutionary shift method (Wang 2007)

	PSO	ICA	ICACO
Percent of reduction (%)	15.8	19.4	20.8

Table 6 Statistical results of different methods for the optimal geometry of Thirteen-member frame structure

	PSO (Nm)	ICA (Nm)	ICACO (Nm)
Best	679.89	650.50	638.75
Mean	864.38	665.06	662.02
Worst	1316.79	751.95	681.87
Standard deviation	162.15	19.82	13.45

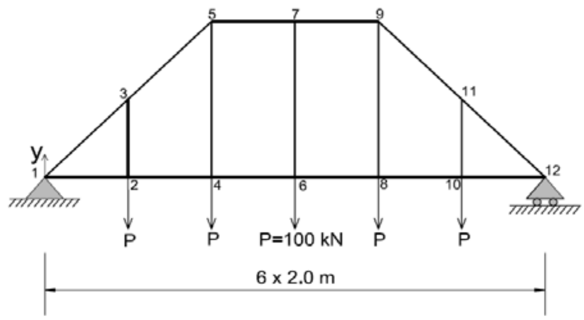

Fig.9 The topology of Seventeen-member frame structure

6.3 Seventeen-member planar frame structure

Fig. 9 shows the topology of a seventeen-member planar frame. It loaded by five concentrated forces together. Young's modulus is E = 210 GPa. The members are categorized into three groups, as follows: The cross sectional area on the upper chord is A_u = 27.49 cm^2, the lower chord is A_l = 150 cm^2 and the five columns are A_c = 19.63 cm^2. An external force is applied downward at each node on the lower chord. During the optimization process, the positions of nodes at the lower chord (1, 2, 4, 6, 8, 10, and 12) remain fixed while the positions of nodes at the upper chord (3, 5, 7, 9, and 11) are allowed to move vertically. To maintain symmetry of the structure, only three independent coordinate variables need to be redesigned. In this example, population size is similar to the pervious example.

In Fig. 10, the convergence rate for the seventeen-member frame structure is shown. In Table 7 the best optimal design variables of this structure are listed. In this example, the convergence rate of ICA and ICACO are approximately equal but the convergence rate of these methods is better than PSO method. In Table 8, the percent of reduction of the objective functions for optimum designs of three algorithms respect to the result of evolutionary shift method (Wang 2007) for the seventeen member frame structure are illustrated.

The statistical simulation results of this example are shown in Table 9. From Table 9, it can be seen that the best, mean and worst solutions found by ICACO are better than the best, mean and worst solutions found by other techniques respectively. In addition, it can be found from Table 9 that the worst solution found by ICACO is better than the best solution found with PSO.

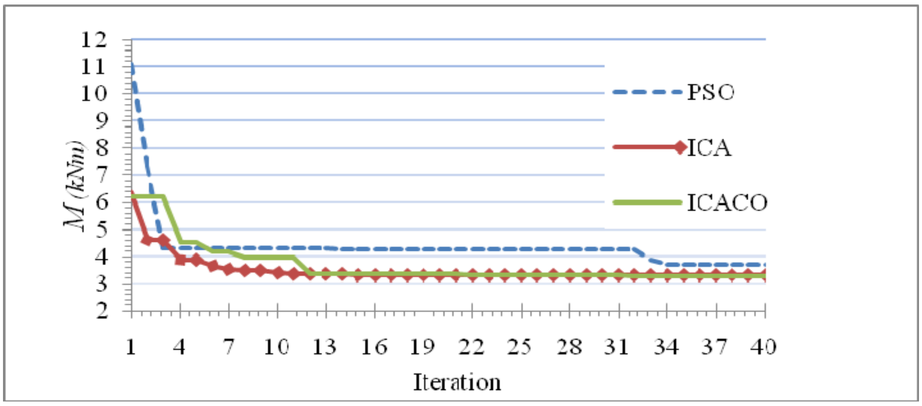

Fig. 10 Convergence rate comparison for the three algorithms for the seventeen-member frame structure

Table 7 Best optimal design comparison for the seventeen-member frame structure

Optimal nodal coordinates (*m*)				
	(Wang 2007)	PSO	ICA	ICACO
Y_3	1.3272	2.8524	1.6422	1.7504
Y_5	2.1252	4.5821	2.6274	2.8000
Y_7	2.3934	5.1334	2.9581	3.1523
Y_9	2.1252	4.5821	2.6274	2.8000
Y_{11}	1.3272	2.8524	1.6422	1.7504
$\|M\|$ (kNm)	4.35	3.713	3.3187	3.1092

Table 8 The percent of reduction of the objective functions for optimum designs of three algorithms respect to the result of evolutionary shift method (Wang 2007) for the seventeen member frame structure

	PSO	ICA	ICACO
Percent of reduction (%)	14.6	23.7	28.5

Table 9 Statistical results of different methods for the optimal geometry of Seventeen-member planar frame structure

	PSO (*kNm*)	ICA (*kNm*)	ICACO (*kNm*)
Best	3.713	3.3187	3.1092
Mean	6.6895	3.4236	3.2693
Worst	11.4913	3.6545	3.7042
Standard deviation	1.9537	0.0954	0.1585

7. Conclusions

In this paper ICACO a hybrid method, based on ICA and ACO, is employed for optimizing geometry of the frame structures. In this method, ACO helps ICA process not only to efficiently perform the global exploration for rapidly attaining the feasible solution space but also effectively helps to reach optimal or near optimal solution. The comparisons based on several well-studied benchmark frame structures demonstrate the effectiveness, efficiency and robustness of the proposed method. The results show that not only the optimal geometry design is achieved but also this goal is attained faster. In other words the convergence rate of the proposed method is higher than PSO and ICA.

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