Structural Engineering and Mechanics, Vol. 46, No. 2 (2013) 231-244 231
DOI: http://dx.doi.org/10.12989/sem.2013.46.2.231

An efficient method for structural damage localization based on the concepts of flexibility matrix and strain energy of a structure

Mehdi Nobahari[1] and Seyed Mohammad Seyedpoor*[2]

[1]*Department of Civil Engineering, Islamic Azad University, Neyshabur Branch, Neyshabur, Iran*
[2]*Department of Civil Engineering, Shomal University, Amol, Iran*

(Received September 20, 2012, Revised March 4, 2013, Accepted March 31, 2013)

Abstract. An efficient method is proposed here to identify multiple damage cases in structural systems using the concepts of flexibility matrix and strain energy of a structure. The flexibility matrix of the structure is accurately estimated from the first few mode shapes and natural frequencies. Then, the change of strain energy of a structural element, due to damage, evaluated by the columnar coefficients of the flexibility matrix is used to construct a damage indicator. This new indicator is named here as flexibility strain energy based index (FSEBI). In order to assess the performance of the proposed method for structural damage detection, two benchmark structures having a number of damage scenarios are considered. Numerical results demonstrate that the method can accurately locate the structural damage induced. It is also revealed that the magnitudes of the FSEBI depend on the damage severity.

Keywords: structural damage detection; flexibility matrix; strain energy; modal information

1. Introduction

Many structural systems may experience some local damage during their lifetime. If the local damage is not identified timely, it may lead to a terrible outcome. Therefore, damage identification is an essential issue for structural engineering and it has received considerable attention during the last years. Structural damage detection consists of three different levels aiming to identify the existence, localization and quantification of the damage, respectively. After discovering the damage occurrence, damage localization is more important than damage quantification. Due to a great number of members in a structural system, properly finding the damage location has been the main concern of many studies.

In the last years, several methods have been proposed for structural damage detection (Messina *et al*. 1998, Wang *et al*. 2001, Bakhtiari-Nejad *et al*. 2005, Koh and Dyke 2007). Structural damage detection by a hybrid technique consisting of a grey relation analysis for damage localization and an adaptive real-parameter simulated annealing genetic algorithm for damage quantification has been presented by He and Hwang (2007). A two-stage method for finding the structural damage sites and extent through an evidence theory and a micro-search genetic algorithm has been introduced by Guo and Li (2009). The changes of modal flexibility matrix and

*Corresponding author, Ph.D., E-mail: s.m.seyedpoor@gmail.com

modal strain energy of flexural members before and after damage have been used by Shih *et al.* (2009) as a basis for locating the structural damage. Locating the structural damage through an adaptive neuro-fuzzy inference system (ANFIS) has been introduced by Fallahian and Seyedpoor (2010). A multi-level damage localization strategy for achieving an efficient damage detection system for structural systems based on wireless sensors has been proposed by Yan *et al*. (2010). Damage detection using an efficient correlation based index and a modified genetic algorithm has been proposed by Nobahari and Seyedpoor (2011). The change of displacement curvature derived from measured static data has been used by Abdel-Basset Abdo (2011) as a good indicator for locating the structural damage. A two-stage method for determining structural damage sites and extent using a modal strain energy based index (MSEBI) and particle swarm optimization (PSO) has been proposed by Seyedpoor (2012).The studies on the subject of introducing a proper method for damage detection are being developed.

The objective of this work is to present an efficient damage localization method based on the flexibility matrix of a structure and strain energy of structural elements. The flexibility matrix is first predicted through few lower mode shapes and frequencies of the structure. Then, the columnar coefficients of the flexibility matrix are used to evaluate the strain energy of structural members. Finally, a relative change of strain energy of structural elements is introduced to make a new indicator nominated here as flexibility strain energy based index (FSEBI) for locating structural damage. Numerical results demonstrate the effectiveness of the proposed method for structural damage localization.

2. Vibration based damage detection methods

Structural damage detection using non-destructive methods has received significant attention during the last years. The fundamental law is that damage will change the mass, stiffness and damping properties of a structure. Such a change would lead to changes in the static and dynamic characteristics of the structure. This enables us to identify the damage by comparing the response data of the structure before and after damage. Therefore, damage detection techniques have been generally classified into two main categories. They include the dynamic and static identification methods requiring the dynamic and static test data, respectively. Furthermore, the dynamic identification methods have shown their superior accuracy in comparison with the static ones. Based on this concept, various dynamic responses based methods have been introduced to identify the damage in structural systems (Alvandi and Cremona 2006, Shih *et al*. 2009, Ciambella *et al*. 2011, Seyedpoor 2012). The relative change of natural frequencies of a structure can be used as a simple tool for being aware of damage presence, i.e.

$$\Delta f_j = \frac{f_{hj} - f_{dj}}{f_{hj}} \qquad , \qquad j = 1,2,...,nd \tag{1}$$

where f_{hj} and f_{dj} denote the jth natural frequency of healthy and damaged structures, respectively and nd stands here for the total number of degrees of freedom. Theoretically, structural damage reduces the stiffness and then natural frequencies of the structure. Therefore, the reduction of natural frequencies can be used to provide an indicator for damage occurrence.

The flexibility method is another vibration based identification method. Using information of a modal analysis, the flexibility change of a structure before and after damage can be considered as an index for identifying structural damage. The modal flexibility matrix of a structure can be given by (Pandey and Biswas 1994, Alvandi and Cremona 2006, Shih *et al*. 2009)

$$[\boldsymbol{F}] = [\varphi][1/\omega^2][\varphi]^{\mathrm{T}} = \sum_{j=1}^{nd} \frac{1}{\omega_j^2} \varphi_j \, \varphi_j^{\mathrm{T}} \tag{2}$$

where [$\boldsymbol{F}$] is the modal flexibility matrix; [φ] contains the mass normalized mode shape vectors; and [$1/\omega^2$] is a diagonal matrix containing the reciprocal of the square of circular frequencies in ascending order. Also, ω_{j} and φ_{j} are jth circular frequency and mode shape of the structure, respectively. Theoretically, damage reduces the stiffness and then increases the flexibility of the structure. Increase in the structural flexibility can therefore serve as an indicator for structural damage detection.

The strain energy of a vibrating structure determined by a mode shape vector is usually referred to as modal strain energy (MSE) and can be considered as a valuable characteristic for damage identification. The modal strain energy of a structural element in vibrating mode j can be expressed as (Au *et al*. 2003, Seyedpoor 2012)

$$m\mathrm{se}_j^{\mathrm{e}} = \frac{1}{2} \varphi_j^{\mathrm{e}\,\mathrm{T}} \boldsymbol{K}^e \varphi_j^{\mathrm{e}} \quad , \quad j = 1,\ldots, nd \;\; , \;\; e = 1,\ldots, n\mathrm{e} \tag{3}$$

where $\boldsymbol{K}^e$ is the stiffness matrix of the eth element and φ_j^e is the vector of corresponding nodal displacements of the element e in mode j. Also, ne is the total number of structural elements. Hypothetically, the damage occurrence leads to increasing the MSE and therefore can be used as an efficient indicator for damage detection.

3. The proposed damage detection method

In this study, an efficient method combining the concepts of flexibility matrix and strain energy of a structure is proposed for structural damage detection. The flexibility matrix $\boldsymbol{F}$ is the inverse of stiffness matrix $\boldsymbol{K}$ and can be predicted dynamically using modal analysis information as given by Eq. (2). It can be observed from the equation that the modal contribution to the flexibility matrix decreases as the frequency increases. Therefore, using only a few vibrating modes of the structure a good estimation of the flexibility matrix can be obtained as

$$[\boldsymbol{F}] \cong \sum_{j=1}^{nm} \frac{1}{\omega_j^2} \varphi_j \, \varphi_j^{\mathrm{T}} \tag{4}$$

where nm is the number of vibrating modes considered.

Each column of the flexibility matrix represents the nodal displacement pattern of the structure when a unit force is applied to the degree of freedom corresponding to that column. Therefore, the columnar coefficients of the flexibility matrix f_{ij} (j = 1,…, nd) can be utilized to obtain the strain energy stored in structural elements due to applying the unit force to the degrees of freedom. The strain energy of a structural element calculated in this way is named here as flexibility strain energy (FSE). The FSE of eth element for jth column of the flexibility matrix can be expressed as

$$fse_j^e = \frac{1}{2} f_j^{e\,\mathrm{T}} \boldsymbol{K}^e f_j^e \quad , \quad j = 1,\ldots,nd \;,\; e = 1,\ldots,n\mathrm{e} \tag{5}$$

where $\boldsymbol{K}^e$ is the stiffness matrix of eth element of the structure and f_j^e is the vector of corresponding nodal displacements of element e for the column j. Also, ne is the total number of structural elements and nd is the total number of columns in the flexibility matrix. The nd is always equal to total degrees of freedom of the structure.

The total flexibility strain energy of jth column of the structure can also be determined by summation of FSE for all elements ne, given by

$$fse_j = fse_j^1 + fse_j^2 + \ldots + fse_j^{ne} = \sum_{e=1}^{ne} fse_j^e \quad , \quad j = 1,\ldots,nd \tag{6}$$

For computational purpose, it is better to normalize the FSE of elements with respect to the total FSE of the structure as

$$nfse_j^e = \frac{fse_j^e}{fse_j} \quad , \quad j = 1,\ldots,nd \;,\; e = 1,\ldots,n\mathrm{e} \tag{7}$$

where $nfse_j^e$ is the normalized FSE of eth element for jth column of the flexibility matrix. The mean of Eq. (7) for the nd columns can now be selected as an efficient parameter as

$$mnfse^e = \frac{\sum_{j=1}^{nd} nfse_j^e}{nd} \quad , \quad e = 1,\ldots ne \tag{8}$$

When damage occurs in a structural element, it increases the flexibility leading to increasing the FSE and consequently the efficient parameter $mnfse^e$. As a result, in this study, the efficient parameter $mnfse^e$ is evaluated twice, one for healthy structure and another for the damaged structure denoted here as $(mnfse^e)^h$ and $(mnfse^e)^d$, respectively. Therefore, by considering a relative change of the efficient parameter, a good indicator for estimating the presence, location and relative severity of the damage in an element can be defined. This indicator termed here as flexibility strain energy based index (FSEBI) and can be determined as

$$FSEBI^e = \max\left[0\,, \frac{(mnfse^e)^d - (mnfse^e)^h}{(mnfse^e)^h}\right] \quad , \quad e = 1,\ldots,ne \tag{9}$$

It should be noted that, as the damage locations are unknown for a real-world damaged structure, therefore for this case the element stiffness matrix of the healthy structure is used for estimating the parameter $(mnfse^e)^d$. According to the Eq. (9), for a healthy element the index will be equal to zero ($FSEBI^e = 0$) while for a damaged element the index will be grater than zero ($FSEBI^e > 0$).

4. Test examples

In order to show the capabilities of the proposed method for structural damage detection, two illustrative test examples are considered. The first example is a 56-element planar frame and the second example is a 31-bar planar truss. In the first example, the effects of the number of vibrating modes, considered for damage detection, on the performance of the method is studied. The efficiency of FSEBI, compared to the damage indicator MSEBI proposed by Seyedpoor (2012) is assessed in the second example. The measurement noise effect on the effectiveness of the method is also investigated for 31-bar planar truss. For modal analysis using the finite element method and damage detection a program is provided by the MATLAB (2006).

4.1 Fifty six-element planar frame

The first example considered in this work is a concrete portal frame (Gomes and Silva 2008) to show the robustness of the proposed method. The frame shown in Fig. 1 has the span L = 2.4m and height H = 1.6m with a rectangular cross sectional area having the width b = 0.14m and depth h = 0.24m. The elasticity modulus is E = 25 GPa and the mass density is ρ = 2500 kg/m^3. The 2D-beam element with three degrees of freedom per node is used for finite element discretization of the structure. Three damage cases listed in Table 1 are numerically simulated here by reducing the elasticity modulus of some elements and the method is tested.

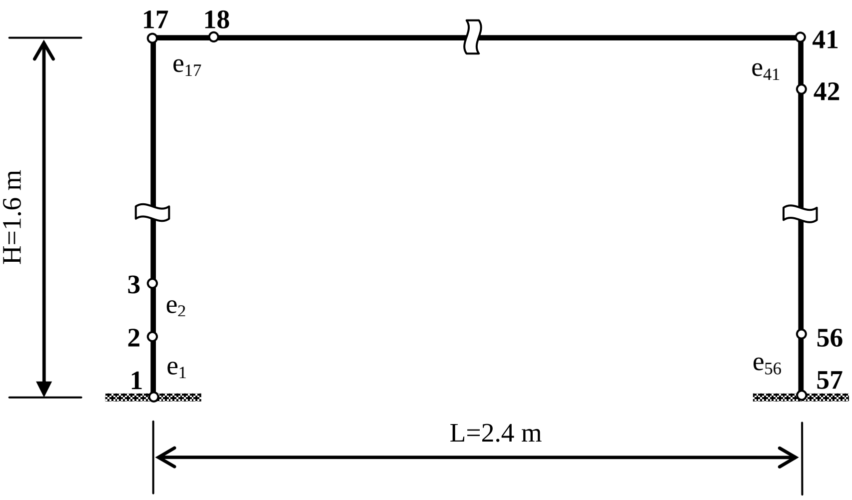


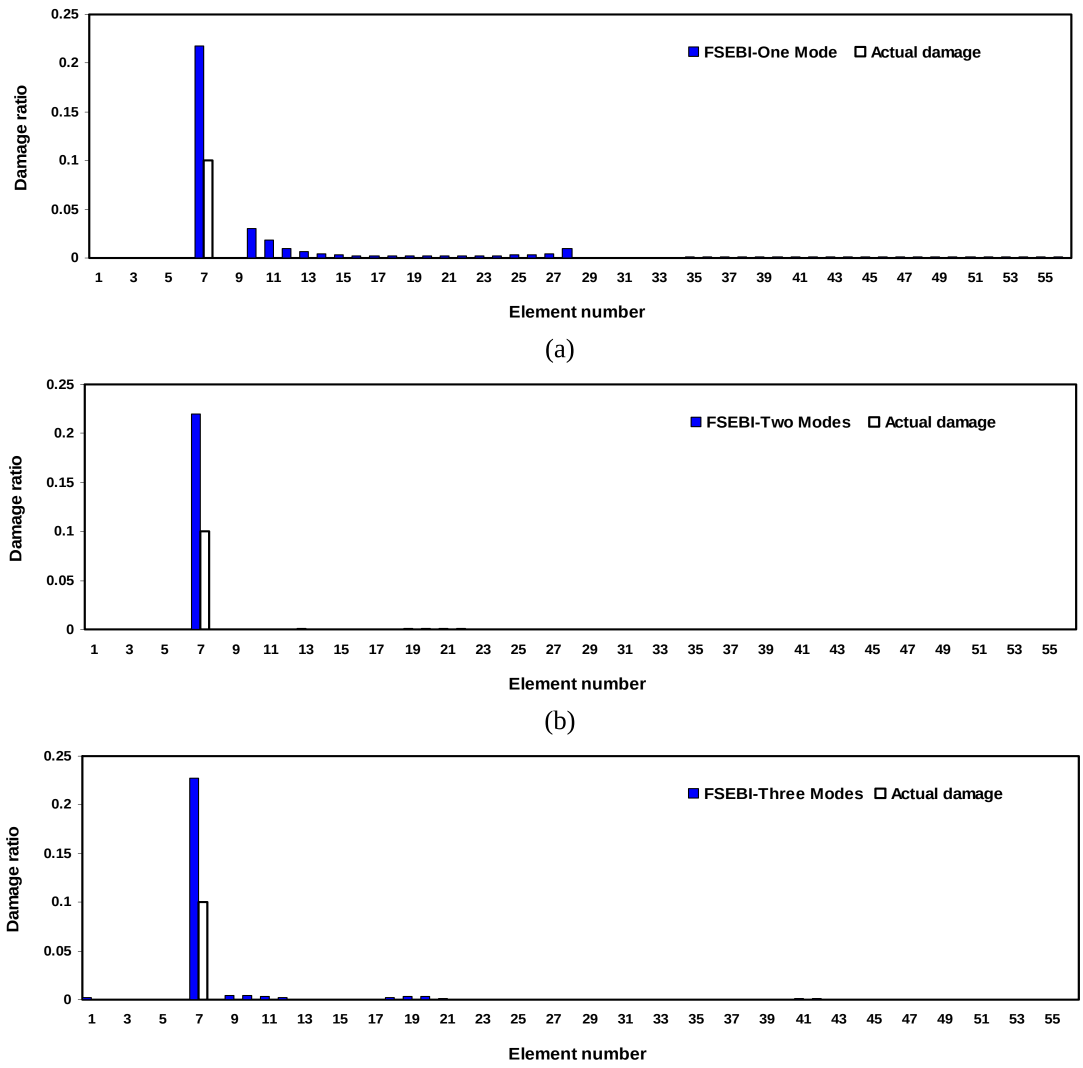
Fig. 1 A portal frame having 56 elements

Table 1 Three different damage cases induced in 56-element planar frame

Case 1		Case 2		Case 3	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
7	0.10	44	0.10	10	0.10
-	-	-	-	28	0.10
-	-	-	-	52	0.10

4.1.1 The effect of number of vibrating modes

In order to examine the effect of number of vibrating modes on the performance of the method, the proposed indicator FSEBI is evaluated when 1 to 5 modes are considered to estimate the flexibility matrix. Figs. 2-4 show the value of FSEBE versus element number for damage cases 1 to 3, respectively when the first one to five modes of the structure are considered for each damage case. As shown in the damage identification charts of cases 1 to 3, for locating the damage accurately, only 2 vibrating modes of the structure are required to be considered. Although the method can properly locate all the damage cases, however, the index gives a damage extent which is about two times the actual sizes.

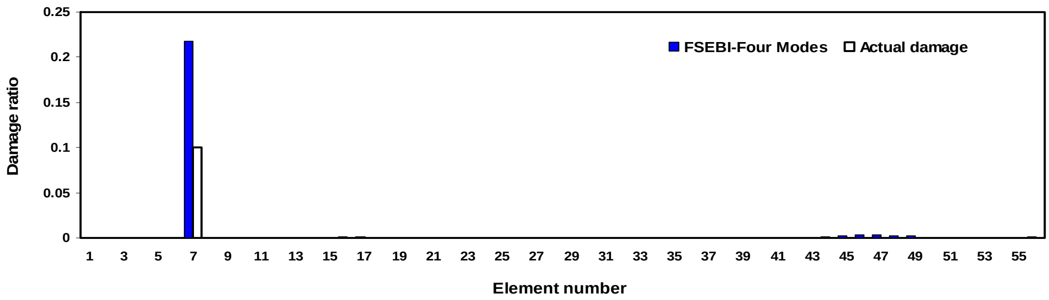


(a)

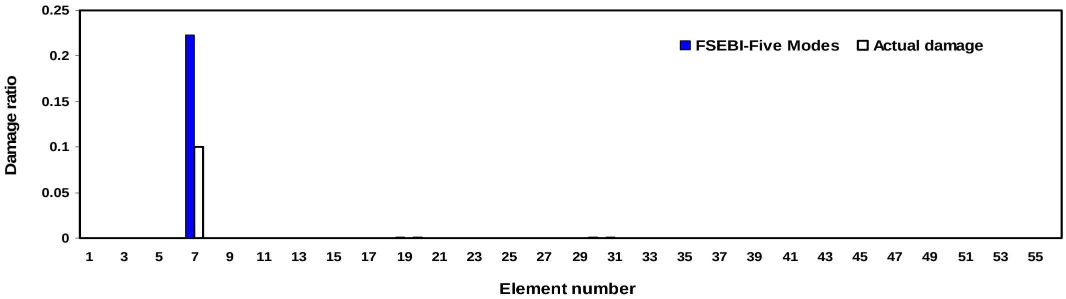
(b)

(c)

Fig. 2 Damage identification chart of 56- element planar frame for damage case 1 considering: (a) one mode, (b) two modes, (c) three modes, (d) four modes, (e) five modes

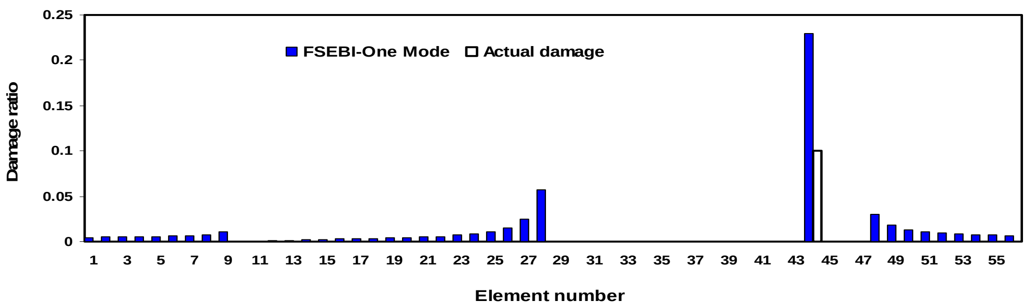


(d)

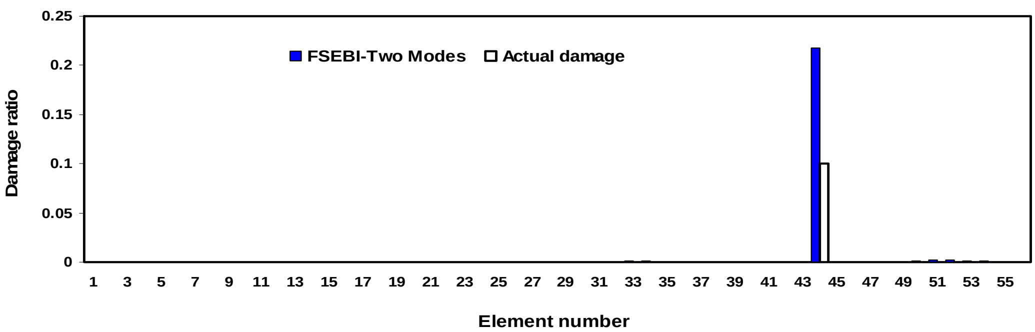


(e)

Fig. 2 Continued

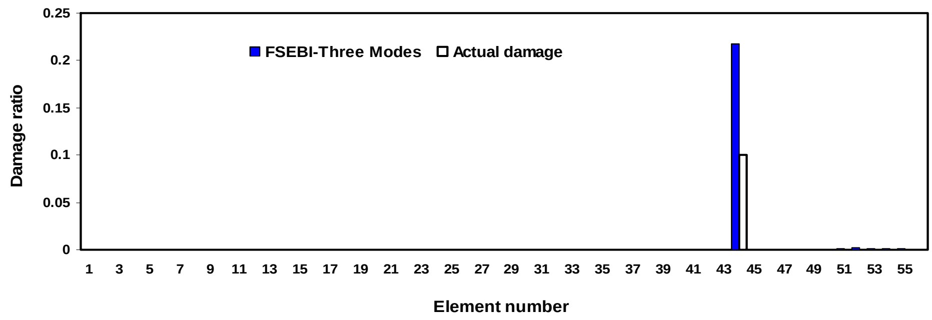


(a)

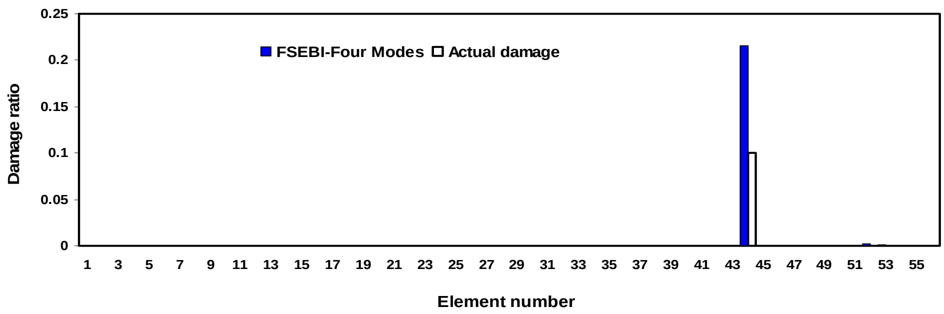


(b)

Fig. 3 Damage identification chart of 56- element planar frame for damage case 2 considering: (a) one mode, (b) two modes, (c) three modes, (d) four modes, (e) five modes



(c)



(d)

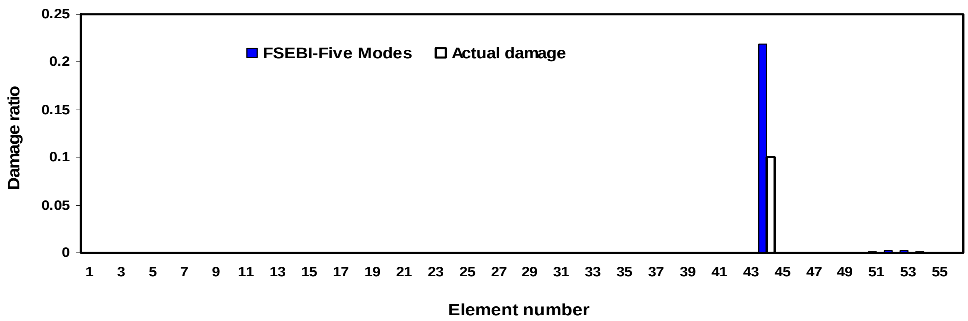
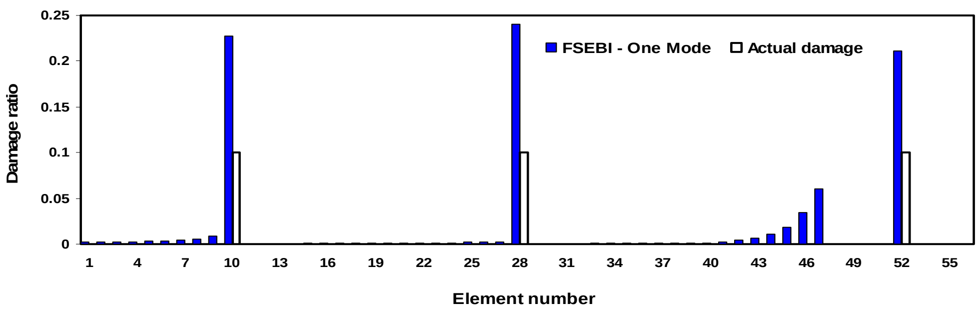
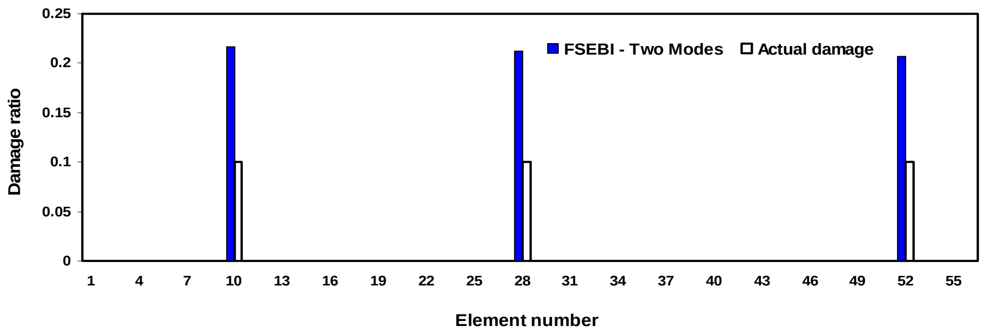


Fig. 3 Continued

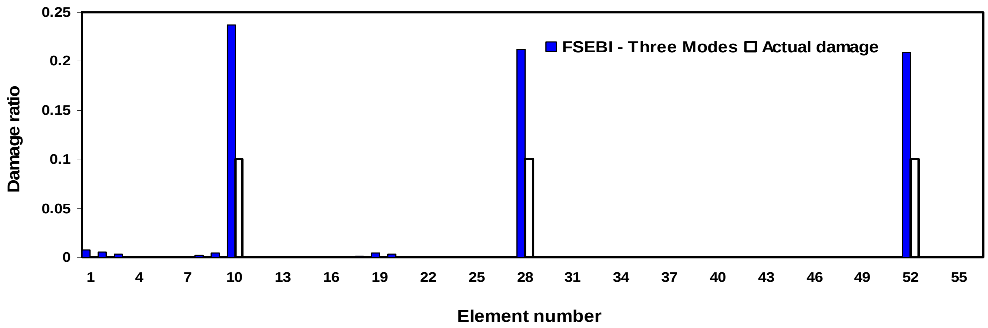


(a)

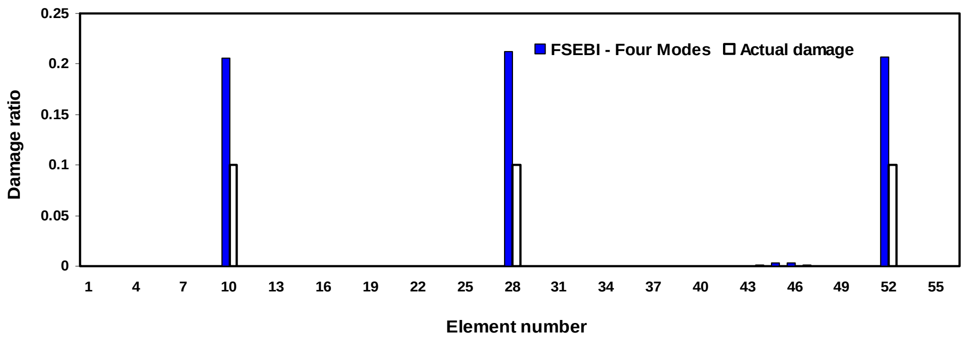
Fig. 4 Damage identification chart of 56- element planar frame for damage case 3 considering: (a) one mode, (b) two modes, (c) three modes, (d) four modes, (e) five modes



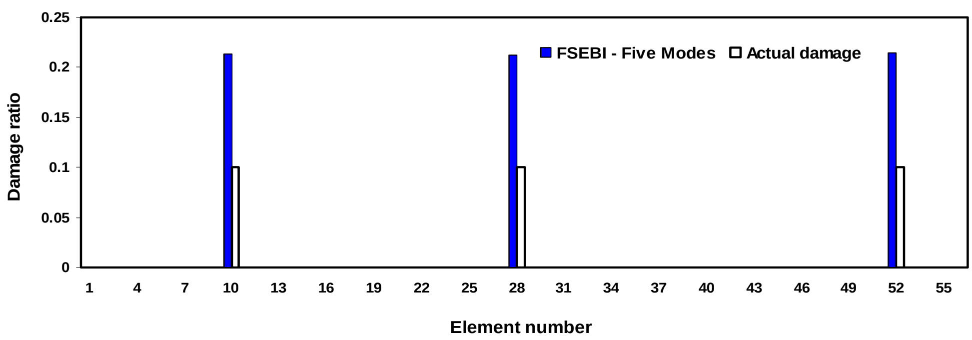
(b)



(c)



(d)



(e)

Fig. 4 Continued

4.2 Thirty one-bar planar truss

The 31-bar planar truss shown in Fig. 5 studied by Messina *et al.* (1998) and Seyedpoor (2012) is modeled using the conventional finite element method without internal nodes leading to 25 degrees of freedom. The material density and elasticity modulus of aluminum truss are 2770 kg/m^3 and 70 GPa, respectively. For this example, three different damage cases given in Table 2 are induced in the structure and the proposed method is used for damage detection.

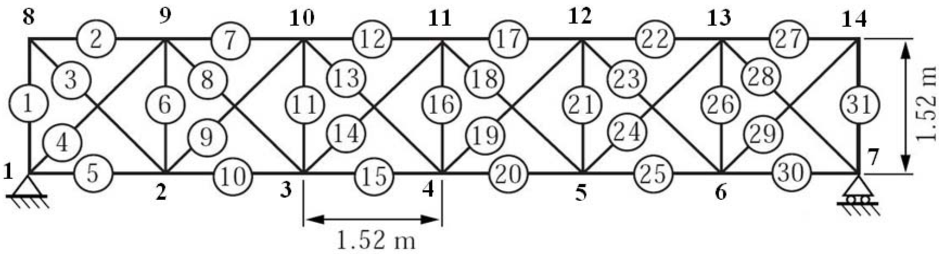


Fig. 5 A planar truss having 31 elements

4.2.1 The efficiency of FSEBI compared to MSEBI

In order to assess the competence of FSEBI for structural damage detection, the efficiency of FSEBI is compared with that of the modal strain energy based index (MSEBI) proposed by Seyedpoor (2012). Figs. 6(a)-(c) show the FSEBI value with respect to element number for damage cases 1 to 3, respectively and compares it with the MSEBI value when the first 5 vibrating modes of the structure are utilized for damage detection. As shown in the figures, the most potentially damaged elements identified by FSEBI for damage scenario 1 are elements 11, 21, 24 and 25; for damage scenario 2 is element 16; and for damage scenario 3 are elements 1 and 2. Here, those elements whose indexes exceed 0.05 are selected as suspected damaged elements. Also, the damaged elements found by MSEBI for damage scenario 1 are elements 11, 21, 25 and 26; for damage scenario 2 is element 16 and for damage scenario 3 are elements 1, 2 and 6. It is revealed that the FSEBI for accurately locating the damaged elements is more efficient than MSEBI while the number of vibrating modes considered for FSEBI and MSEBI, is equal. Also, it can be observed that the magnitudes of the FSEBI are slightly closer to the damage severity when comparing with those of the MSEBI.

Table 2 Three different damage cases induced in 31-bar planar truss

Case 1		Case 2		Case 3	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
11	0.25	16	0.30	1	0.30
25	0.15	-	-	2	0.20

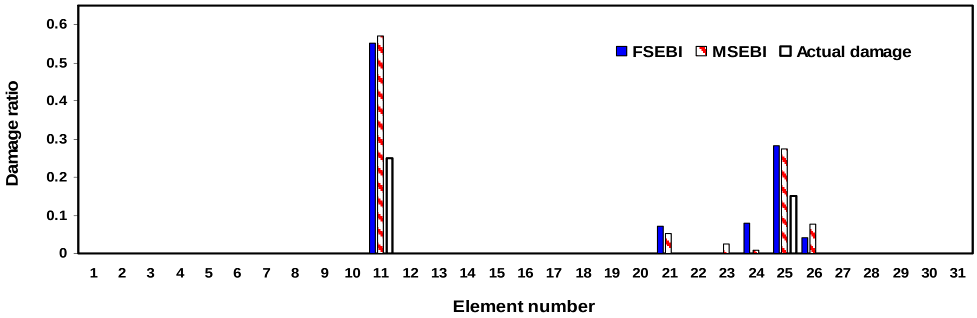


Fig. 6(a) The FSEBI and MSEBI values for damage case 1 of 31-bar planar truss

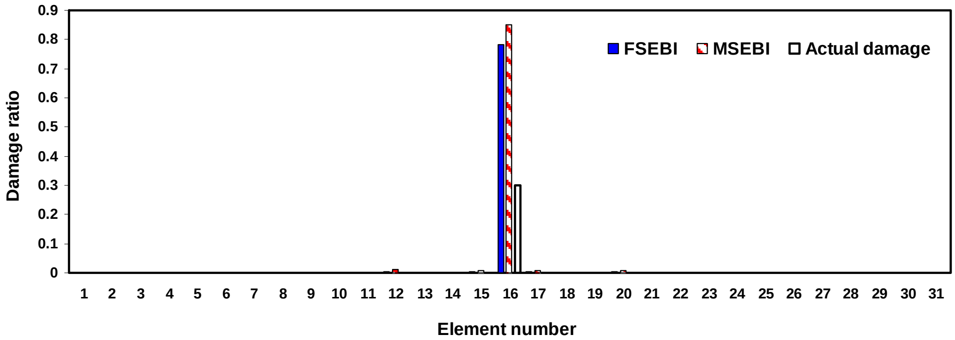


Fig. 6(b) The FSEBI and MSEBI values for damage case 2 of 31-bar planar truss

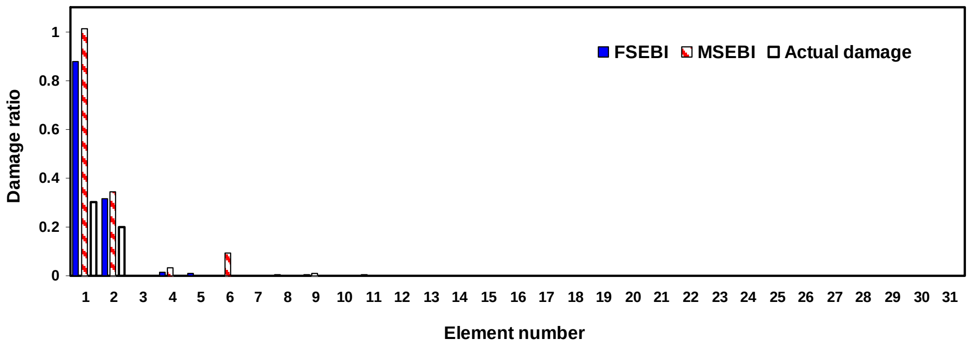


Fig. 6(c) The FSEBI and MSEBI values for damage case 3 of 31-bar planar truss

4.2.2 The effect of measurement noise

In order to evaluate the robustness of the proposed damage detection method, the effect of measurement noise on the performance of the method is investigated. To reduce the noise effect on the accuracy of the method for locating damage, the diagonal matrix $[1/\omega^2]$ in Eq. (2) is assumed to be an identity matrix. Therefore, the measurement noise is applied only to the mode shapes of the structure. To this, the numerical mode shapes are perturbed using a maximum error of ±3% generating by a uniformly distributed random numbers. Figs. 7(a)-(c) show the mean values of

FSEBE for 100 independent runs for damage scenarios 1 to 3, respectively when the first 5 mode shapes considered are randomly polluted using the noise. As can be observed, the high competence of the FSEBI for locating the damaged elements is demonstrated when the measurement noise is considered. As shown in the figures, the potentially damaged elements identified by FSEBI for damage scenario 1 are elements 11, 25 and 26; for damage scenario 2 are element 16; and for damage scenario 3 are elements 1, 2 and 6. In the case of considering noise, those elements whose indices exceed 0.10 are selected as suspected damaged elements.

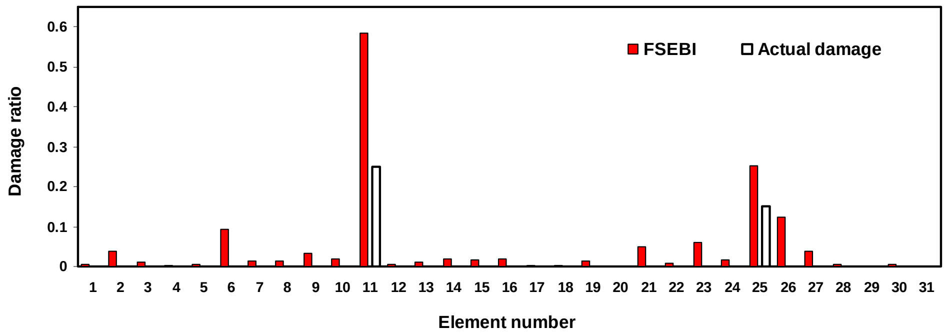


Fig. 7(a) The FSEBI value for damage case 1 of 31-bar planar truss considering noise

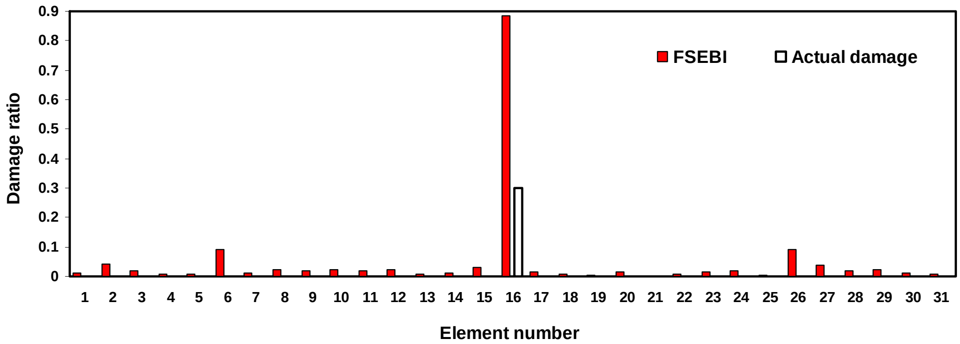


Fig. 7(b) The FSEBI value for damage case 2 of 31-bar planar truss considering noise

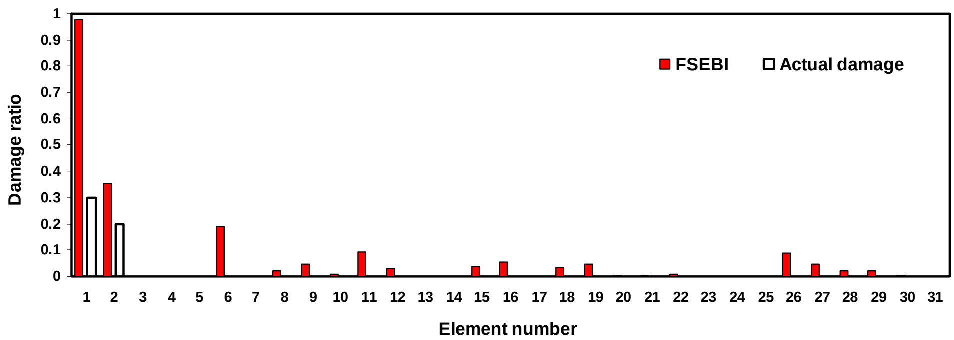


Fig. 7(c) The FSEBI value for damage case 3 of 31-bar planar truss considering noise

5. Conclusions

An efficient method for structural damage identification using a combination of flexibility and strain energy methods has been proposed. The columnar coefficients of the flexibility matrix, approximated through modal analysis information have been used to evaluate strain energy stored in different elements of the structure. A relative change of strain energy of an element before and after damage has been utilized to introduce an indicator named here as FSEBI for structural damage detection. Two illustrative test examples having different damage scenarios have been selected to assess the efficiency of the proposed method. Numerical results have shown that the method can accurately locate the multiple damage cases by considering only the first few modes of the structures. It has also been revealed that the amount of the FSEBI is proportional to the damage severity. The method has shown a superior efficiency when compared with a powerful damage indicator provided in the literature. As a final point, the method has shown a good performance when the measurement noise was considered in damage detection.

References

Abdel-Basset Abdo, M. (2011), “Parametric study of using only static response in structural damage detection”, *Engineering Structures*, **34**, 124-131.

Alvandi, A. and Cremona, C. (2006), “Assessment of vibration-based damage identification techniques”, Journal of Sound and Vibration, **292**(1-2), 179-202.

Au, F.T.K., Cheng, Y.S., Tham, L.G. and Bai, Z.Z. (2003), “Structural damage detection based on a micro-genetic algorithm using incomplete and noisy modal test data”, *Journal of Sound and Vibration*, **259**(3), 1081-1094.

Bakhtiari-Nejad, F., Rahai, A. and Esfandiari, A. (2005), “A structural damage detection method using static noisy data”, *Engineering Structures*, **27**(12), 1784-1793.

Ciambella, J., Vestroni, F. and Vidoli, S. (2011), “Damage observability, localization and assessment based on eigenfrequencies and eigenvectors curvatures”, *Smart Structures and Systems*, **8**(2), 191-204.

Fallahian, S. and Seyedpoor, S.M. (2010) “A two stage method for structural damage identification using an adaptive neuro-fuzzy inference system and particle swarm optimization”, *Asian Journal of Civil Engineering (Building and Housing)*, **11**(6), 797-810.

Gomes, H.M. and Silva, N.R.S. (2008), “Some comparisons for damage detection on structures using genetic algorithms and modal sensitivity method”, *Applied Mathematical Modelling*, **32**(11), 2216-2232.

Guo, H.Y. and Li, Z.L. (2009), “A two-stage method to identify structural damage sites and extents by using evidence theory and micro-search genetic algorithm”, *Mechanical Systems and Signal Processing*, **23**(3), 769-782.

He, R.S. and Hwang, S.F. (2007), “Damage detection by a hybrid real-parameter genetic algorithm under the assistance of grey relation analysis”, *Engineering Application of Artificial Intelligence*, **20**(7), 980-992.

Koh, B.H. and Dyke, S.J. (2007), “Structural health monitoring for flexible bridge structures using correlation sensitivity modal data”, *Computers and Structures*, **85**(3-4), 117-130.

Messina, A., Williams, E.J. and Contursi, T. (1998), “Structural damage detection by a sensitivity and statistical-based method”, *Journal of Sound and Vibration*, **216**(5), 791-808.

Nobahari, M. and Seyedpoor, S.M. (2011), “Structural damage detection using an efficient correlation based index and a modified genetic algorithm”, *Mathematical and Computer Modelling*, **53**(9-10), 1798-1809.

Pandey, A.K. and Biswas, M. (1994), “Damage detection in structures using changes in Flexibility”, *Journal of Sound and Vibration*, **169**(1), 3-17.

Seyedpoor, S.M. (2012), “A two stage method for structural damage detection using a modal strain energy based index and particle swarm optimization”, *International Journal of Nonlinear Mechanics*, **47**(1), 1-8.

Shih, H.W. Thambiratnam, D.P. and Chan, T.H.T. (2009), “Vibration based structural damage detection in flexural members using multi-criteria approach”, *Journal of Sound and Vibration*, **323**(3-5), 645-661.

Wang, X., Hu, N., Fukunaga, H. and Yao, Z.H. (2001), “Structural damage identification using static test data and changes in frequencies”, *Engineering Structures*, **23**(6), 610-621.

Yan, G., Guo, W., Dyke, S.J., Hackmann, G. and Lu, C. (2010), “Experimental validation of a multi-level damage localization technique with distributed computation”, *Smart Structures and Systems*, **6**(5-6), 561-578.

MATLAB (2006), *The language of technical computing (software)*, Math Works Inc.