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Mesh topological form design and geometrical configuration generation for cable-network antenna reflector structures

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Abstract. A well-designed mesh shape of the cable net is of essential significance to achieve high performance of cable-network antenna reflectors. This paper is concerned with the mesh design problem for such antenna reflector structure. Two new methods for creating the topological forms of the cable net are first presented. Among those, the cyclosymmetry method is useful to generate different polygon-faceted meshes, while the topological mapping method is suitable for acquiring triangle-faceted meshes with different mesh grid densities. Then, the desired spatial paraboloidal mesh geometrical configuration in the state of static equilibrium is formed by applying a simple mesh generation approach based on the force density method. The main contribution of this study is that a general technical guide for how to create the connectivities between the nodes and members in the cable net is provided from the topological point of view. With the new idea presented in this paper, multitudes of mesh configurations with different net patterns can be sought by a certain rule rather than by empiricism, which consequently gives a valuable technical reference for the mesh design of this type of cable-network structures in the engineering.

Keywords: cable network; mesh shape; mesh design; topological form; geometrical configuration

1. Introduction

Cable-network antenna reflectors, with superior advantages of lightweight, large aperture and high packaging efficiency, denote the current state-of-the-art large deployable space-borne reflector structure. A representative example is the AstroMesh reflector first developed by TRW Astro Aerospace (Tibert 2003). As shown in Fig. 1, the antenna reflector surface is formed by a metallic reflective mesh attached to a network of thin cables with high axial stiffness, which is known as cable-network. The so-called cable-network is formed by two identical paraboloid-shaped nets (i.e., a front net and a rear net), which are connected by a series of tension ties. An outer unfurlable supporting rim structure, namely ring truss, is coupled to the cable-network and has sufficient stiffness and high stability to support the cable-network (Thomson 2002).

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For concentrating radio frequency (RF) radiation in good condition, the working surface of this kind of antenna reflector needs to maintain an exact paraboloidal shape, which is directly determined by the mesh shape of the cable-network structure. Hence, a well-designed mesh configuration of the cable net is of great importance to achieve high electromagnetic performance of this antenna reflector structure. In recent years, a great deal of interest has been directed toward the problem of mesh shape design. Tibert (2003) presented an approach for the optimal mesh design of tension truss antennas and generated a variety of minimal length mesh configurations. Morterolle *et al.* (2012) proposed a new approach to find mesh configurations of the cable net with a uniform tension and forming exactly a parabolic surface for both prime-focus and offset reflectors. Shi *et al.* (2012) developed a new design method to generate the desired surface mesh geometry automatically with pseudo-geodesic property for spherical and parabolic reflector surfaces. All these studies achieved acceptable mesh configurations for engineering application eventually. However, in most of these available literatures, the topology of the cable net (i.e., the connectivity relationship between the nodes and the members in the cable net) is either formed by an empirical method (which will be described in section 2) or assumed to be known at the start of the form-finding process. Their efforts thus are mainly put to search the optimal mesh configuration of the cable net from a geometrical and mechanical point of view.

Different from these studies as done so far, this paper is focused on the initial guess and design of the mesh shape from the structural topological point of view. Two novel methods named cyclosymmetry method and topological mapping method respectively are developed to create the topology of the cable net in the study. Furthermore, a simple mesh generation approach is proposed to obtain the final spatial mesh configuration. With the idea proposed in this paper, diverse mesh configurations with different net patterns are easy to conceive and design. The main motivation of this study is to provide a general technical guide for how to create the connectivities of the members to the nodes in the cable net. Under the help of this guide, the mesh patterns of the cable net can be no longer designed by empiricism, but by a certain rule. Hence, the present work will be useful for guiding the mesh design of cable-network antenna reflectors in the engineering.

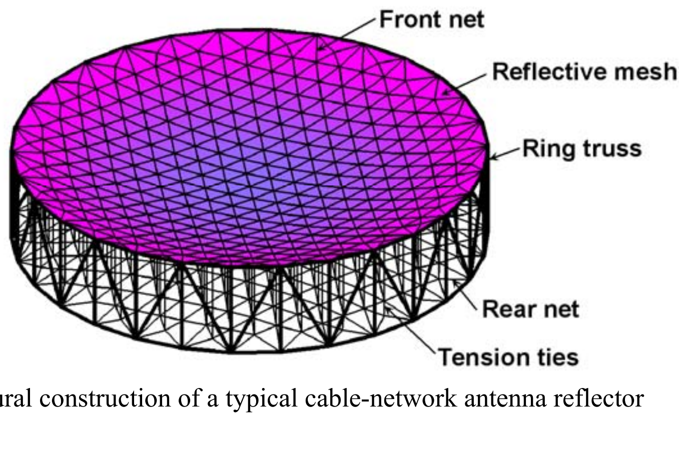
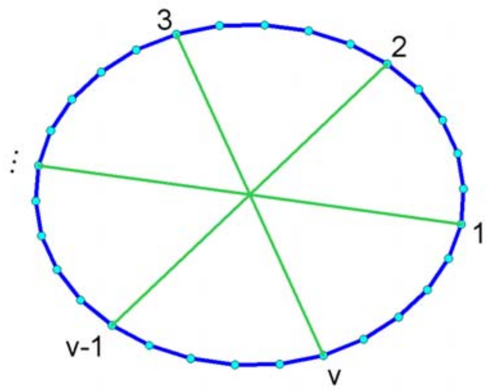
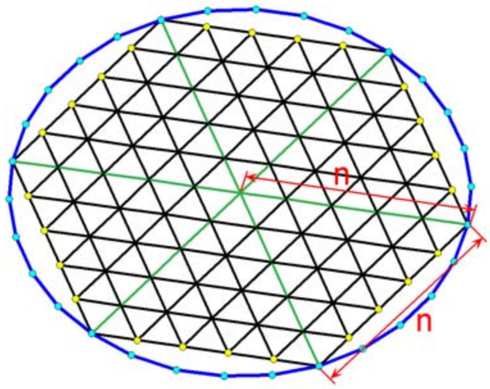


Fig. 1 Structural construction of a typical cable-network antenna reflector

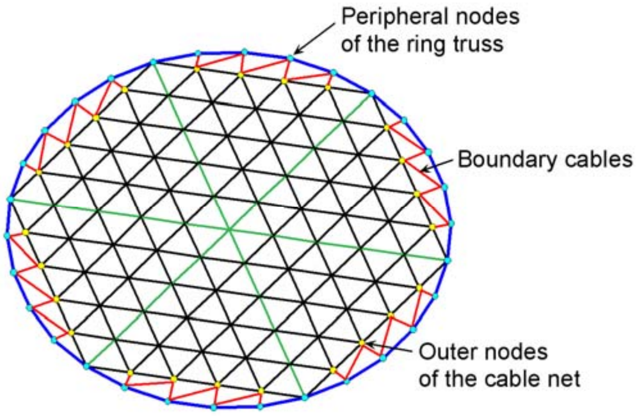
The outline of this paper is as follows. Section 2 introduces the conventional empirical method concerning the generation of the structural topology and presents two new design methods, namely the cyclosymmetry method and the topological mapping method. Section 3 deals with the specific form-finding procedure. The final desired equilibrium mesh configuration is derived by a simple mesh generation approach based on the force density method. Subsequently, numerical examples for generating the spatial paraboloidal mesh shapes with the two topological form design methods are given in Section 4. The results are compared and the advantages of these two presented design methods are clarified. Finally, section 5 concludes the study and discussions some future research opportunities.



(a) Dividing the whole mesh surface into v subregions



(b) Subdividing each subregion into n^2 small triangles



(c) Introducing boundary connection conditions of outer nodes

Fig. 2 Illustration of the traditional subregion dividing method

2. Topological form design

Acquiring the topological relationship of the cable net structure is the first step in the process of mesh generation. Unfortunately, there is only one empirical method, herein we call it as the subregion dividing method, has been employed for the topology form design of cable-network antenna reflectors until now. This method, developed by Tibert and Pellegrino (2002), generates the mesh topological form with the following three steps, as illustrated in Fig. 2:

① The whole mesh surface is divided into v triangular subregions (usually v = 6 is used), see Fig. 2(a);

② By dividing each side of the triangular subregion to n segments, each triangular subregion is subdivided into n^2 small triangles, refer to Fig. 2(b);

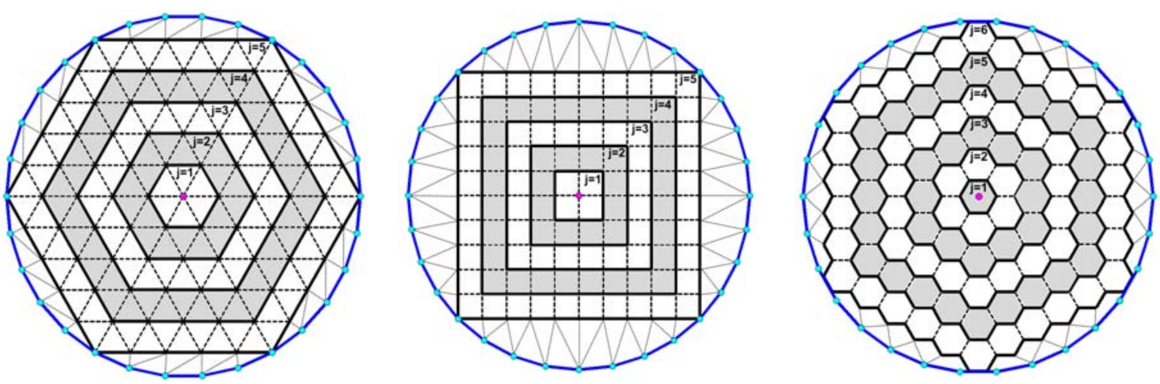
③ As shown in Fig. 2(c), boundary connection conditions between the outer nodes of the cable net and the peripheral nodes of the ring truss are introduced to create the final topology form.

The above subregion dividing method is normally feasible and practical for the engineering application. However, there are some shortages existed, for example, the net pattern is monotonous since only triangle-faceted and partition-symmetric meshes can be obtained by using this method, and again the capacity of varying the mesh grid density can only be achieved by changing the dividing number n.

In this paper, different with this conventional empirical method, two new design methods called cyclosymmetry method and topological mapping method respectively are presented. They will be discussed in detail in the subsequent analysis.

2.1 Cyclosymmetry method

The main idea of the cyclosymmetry method is increasing the mesh grids layer-by-layer along the radial direction of the mesh, while the mesh grids of each layer is generated by way of revolving a basic polygon member in a circle. The whole procedure with this cyclosymmetry rule can be demonstrated clearly in Fig. 3. First, select an appropriate polygon such as a triangle, a quadrangle and a hexagon (shown in Figs. 3(a)-(c) respectively), as a basic member used for revolution and duplication, and choose the center of the mesh as the center of the first layer. Then, revolve and duplicate the basic member in a circle to form the mesh grids of the next layer. This looping process is ended until the generated mesh grids nearly fill with the whole mesh. One can notice that the number of layers j is synthetically depending on the size of the basic member and the aperture of the mesh. Finally, the boundary connection conditions are introduced to create the final topological form of the mesh.



(a) Triangle-faceted mesh (b) Quadrangle-faceted mesh (c) Hexagon-faceted mesh

Fig. 3 Schematic of the cyclosymmetry method

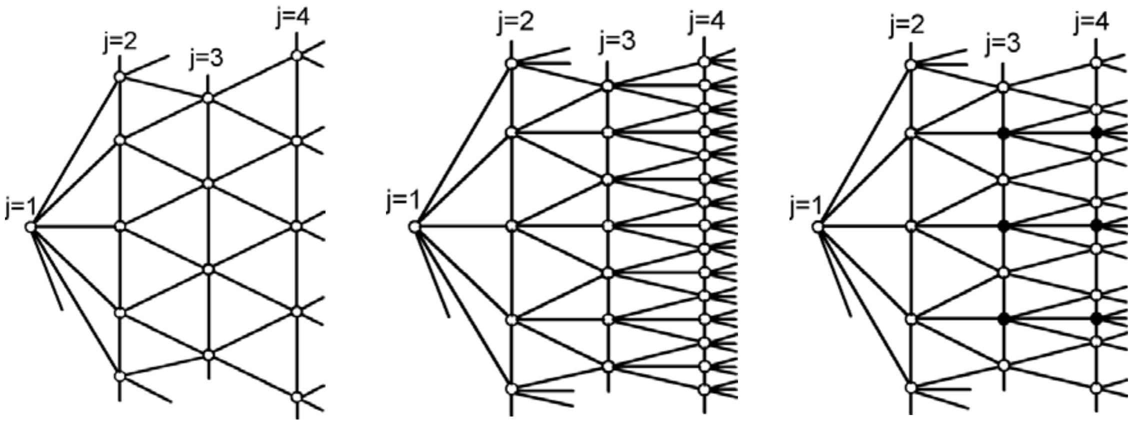
2.2 Topological mapping method

In Hernández-Montes *et al.* (2006) is first introduced a new concept of topological mapping to perform the creation of topological networks for tension structures. The main feature of this topological mapping is independent of the final mesh geometric configuration, and can be applicable to the case that an initial guess of the equilibrium shape is difficult to find. Based on their pioneering research, a mesh topological form design method called topological mapping method is presented in this study for cable-network antenna reflectors. The topological mapping method is to create the connectivities of the cable members to the nodes with a combination of some topological rules. One certain topological rule is corresponding to a specific type of basic mesh. Here three basic mesh types (see Fig. 4) are employed for generating the structural topology relationship, which are explained in detail as below.

① Basic mesh type A. As shown in Fig. 4(a), one node corresponds to the center of the mesh is created in step 1. Then this node is connected to *n* nodes in step 2. Beginning with step 3, each node is connected to two nodes of the next step. At the same time, the nodes in the same step are connected to each other.

② Basic mesh type B. This mesh type is very analogous to type A. The only difference lies in that each node is connected to three nodes of the next step, as illustrated in Fig. 4(b).

③ Basic mesh type C. In this mesh type, if a node at a given step is connected to only one node of the previous step, it will be connected to three nodes of the following step. This manner can be also interpreted as that: for each node at an arbitrary step except step 1, the total number of nodes connected to the previous and following step is four, as shown in Fig. 4(c).



(a) Topology of basic mesh type A (b) Topology of basic mesh type B (c) Topology of basic mesh type C

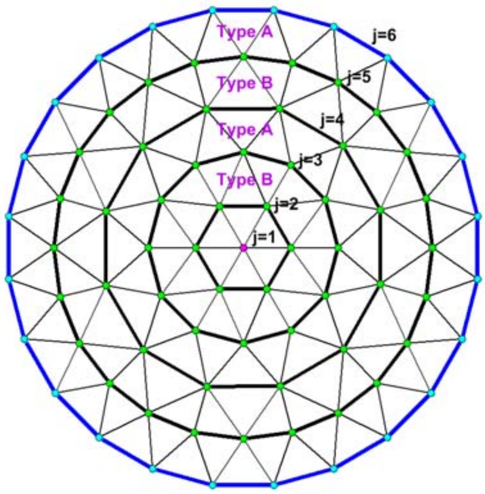
Fig. 4 Three basic mesh types for topology design of the cable net

Table 1 Number of nodes for three basic mesh types

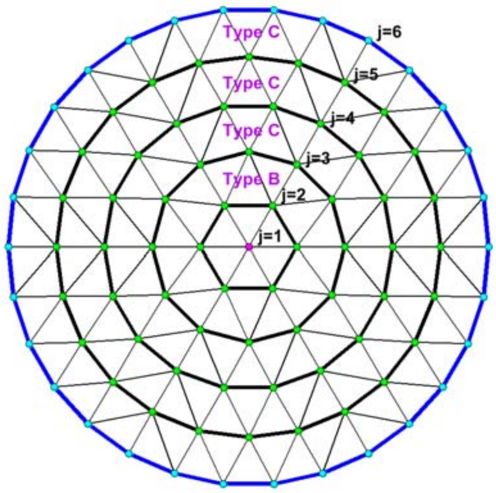
Mesh type	Number of nodes of each step				
	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j \geq 5$
A	1	n	n	n	n
B	1	n	$2n$	$4n$	$2^{(j-2)} \cdot n$
C	1	n	$2n$	$3n$	$(j-1)n$

The number of nodes for three basic mesh types is summarized in Table 1. The characteristic of mesh type A is the number of nodes of each step can be maintained as a constant from the start of step 2, while the number of nodes in mesh type B will double increasing by way of step by step. It is obvious that the mode of mesh type B will lead to a large number of nodes after a few steps. The introduction of mesh type C overcomes this problem well since the number of nodes will increase through a linear way.

To illustrate the above topological mapping method, two examples of mesh topology patterns with a combination of these three basic mesh types are given in Fig. 5. For both of them, the selected number of nodes in step 2 and the total number of steps are equal to 6. Fig. 5(a) illustrates a topology form obtained by a combination of types A and B, while Fig. 5(b) illustrates that using a combination of types B and C.



(a) Mesh pattern obtained by using types A and B



(b) Mesh pattern obtained by using types B and C

Fig. 5 Demonstration of the topological mapping method

3. Geometrical configuration generation

It is noticeable that only mesh patterns of cable nets can be obtained by the work conducted in section 2, yet spatial mesh configurations that satisfy the state of mechanical equilibrium and meanwhile form an exact paraboloidal shape are required to be solved. Hence, the specific form finding procedure for generating the final desired mesh geometric configurations is presented in this section.

Here a simple mesh generation approach is employed. This approach is based on the force density method (FDM) originally proposed by Schek (1974), which is frequently used to analyse tensegrity structures (Zhang and Ohsaki 2006), cable-network structures (Sophianopoulos and Asteris 2006) and membrane structures (Zhang *et al.* 2006). As pointed out by Malerba *et al.* (2012), the FDM is a powerful and extremely versatile tool in form finding of cable nets. Such method can linearize the set of nodal equilibrium equations by introducing the concept of force density, which is defined as a force-length ratio of the cable element. Taking the unique advantage of the FDM that only linear formulation need to be solve in the form finding process, this proposed simple mesh generation approach is considered as the following steps:

Step 1: Form the structural topological matrix $\boldsymbol{C}_{\mathrm{s}}$ that describes the topology of the cable net structure given in section 2.

Number all of the elements and nodes in the cable net in an arbitrary order, and suppose element p matches node i and node j, thus the topological matrix $\boldsymbol{C}_{\mathrm{s}}$ is defined by

$$\mathbf{C}_{\mathrm{s}}(p,q)=\begin{cases}+1 & \text{if } q=\min(i,j)\\ -1 & \text{if } q=\max(i,j).\\ 0 & \text{for other cases}\end{cases} \tag{1}$$

Step 2: Input the position coordinates $\boldsymbol{x}_{\mathrm{f}}$, $\boldsymbol{y}_{\mathrm{f}}$, $\boldsymbol{z}_{\mathrm{f}}$ of the fixed nodes (i.e., the peripheral nodes of the ring truss as mentioned above).

Step 3: Find an appropriate set of force densities q_i (i = 1, 2, …, m; m is the total number of cable elements) with a certain shape design criterion. For the sake of simplicity, the shape design criterion of equal force-density is used in this study, by which we can easily have the force-density vector $\boldsymbol{q}$ as

$$\boldsymbol{q}_{\mathrm{m}\times 1}=\begin{pmatrix}1 & 1 & \cdots & 1\end{pmatrix}^{\mathrm{T}}. \tag{2}$$

Step 4: Solve the position coordinates $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$ of the free nodes in the cable net by applying the FDM. To simply the solution process and simultaneously make all of the free node coordinates be located on an exact paraboloidal surface, an idea of treating the tensions provided by the tension ties as the external loads exerting on the cable net is adopted during the whole procedure of form finding. With this idea, the coordinates $\boldsymbol{x}$, $\boldsymbol{y}$ of the free nodes can be calculated by the following simplified equations

$$\begin{aligned}\boldsymbol{x}&=-\left(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}\right)^{-1}\boldsymbol{C}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}_{\mathrm{f}}\boldsymbol{x}_{\mathrm{f}}\\ \boldsymbol{y}&=-\left(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}\right)^{-1}\boldsymbol{C}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}_{\mathrm{f}}\boldsymbol{y}_{\mathrm{f}}\end{aligned}, \tag{3}$$

while the coordinates $\boldsymbol{z}$ can be computed by applying the paraboloidal equation of the cable net as below

$$\boldsymbol{z}=\frac{1}{4F}\left(\boldsymbol{x}^2+\boldsymbol{y}^2\right)+\frac{1}{2}H_{\mathrm{r}}-H_0, \tag{4}$$

where $\boldsymbol{Q}$ is a diagonal matrix belonging to the force-density vector $\boldsymbol{q}$; matrices $\boldsymbol{C}$ and $\boldsymbol{C}_f$ describe the connectivities of the elements to the free and fixed nodes respectively, and they constitute the topological matrix, i.e., $\boldsymbol{C}_{\mathrm{s}}=[\boldsymbol{C}\quad \boldsymbol{C}_f]$; F is the focal length; H_r is the height of the ring truss; and H_0 is the height of the reflector.

Step 5: Plot the graphics of the spatial mesh geometric configurations of cable-network antenna reflectors by utilizing these obtained coordinates $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$.

4. Numerical examples

In order to verify the feasibility and validity of the proposed mesh design method, several numerical examples with regard to spatial mesh configuration generation of cable-network antenna reflectors are presented in this section.

4.1 Examples with the cyclosymmetry method

A cable-network antenna reflector (see Fig. 1) with an assigned 30-polygonal ring truss structure is considered for all the examples here. The specifications of the antenna reflector used in the numerical simulation are as follows:

- Size of antenna aperture: 10 m
- Focal length: 6.80 m
- Height of the reflector: 0.92 m
- Height of the ring truss: 2.30 m
- Operating frequency: 2.5 GHz (S band)
- Design value of the surface faceting error (RMS): 2.40 mm

Applying the proposed cyclosymmetry method to create the structural topology, and then using the simple mesh generation approach to generate the final configuration, different polygon-faceted mesh configurations such as triangle-faceted, quadrangle-faceted and hexagon-faceted meshes are obtained, as shown in Fig. 6. The relative characteristic parameters of these obtained mesh configurations are listed in Table 2.

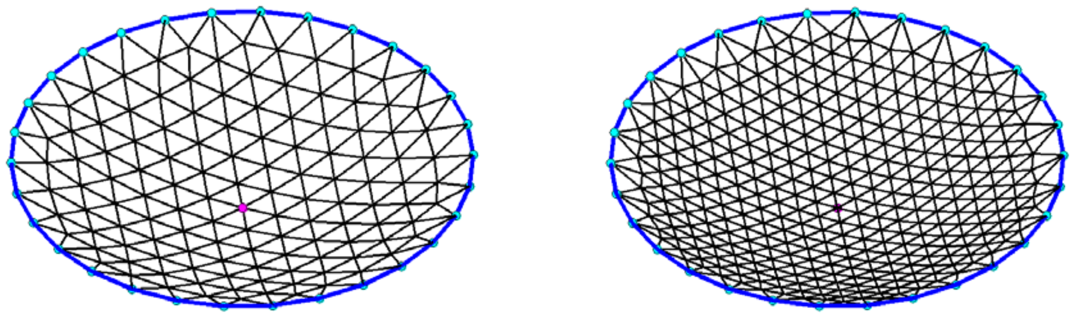
Table 2 Characteristic parameters of mesh configurations using the cyclosymmetry method

Mesh configuration	Triangle-faceted		Quadrangle-faceted		Hexagon-faceted	
	Fig. 6(a)		Fig. 6(b)		Fig. 6(c)	
Number of layers	6	10	6	11	7	11
Number of free nodes	121	325	199	559	294	726
Number of elements	390	1002	390	1128	510	1146
Number of polygons	270	678	222	557	193	421
Surface faceting error (mm)	**3.10**	1.22	**3.50**	**1.17**	**3.76**	1.55

From the diverse mesh configurations illustrated in Fig. 6, we can easily conclude that the cyclosymmetry method is useful to generate different polygon-faceted mesh configurations. Furthermore, to evaluate the performance of these obtained meshes, a root-mean-square (RMS) surface faceting error is employed. The amount of this error is calculated by an estimating method reported by Agrawal *et al.* (1981), as shown in the last row in Table 2. From this table, we can observe that the surface accuracy of the cable net cannot satisfy the design requirement when the used number of layers j is small. Nevertheless, a high surface precision can be achieved by simply increasing j, e.g., the surface faceting error is reduced from 3.50mm to 1.17mm when the used j increases from 6 to 11. In addition, it can be observed that when an identical j is used, the triangle-faceted mesh with the fewest number of cable elements leads to a best surface accuracy, while the hexagon-faceted mesh is worst. For the case of $j = 6$, the surface error of the triangle-faceted mesh is 3.10mm, while that of the quadrangle-faceted mesh is 3.50mm, and even j increases to 7, the hexagon-faceted mesh still has a maximum surface error of 3.76mm.

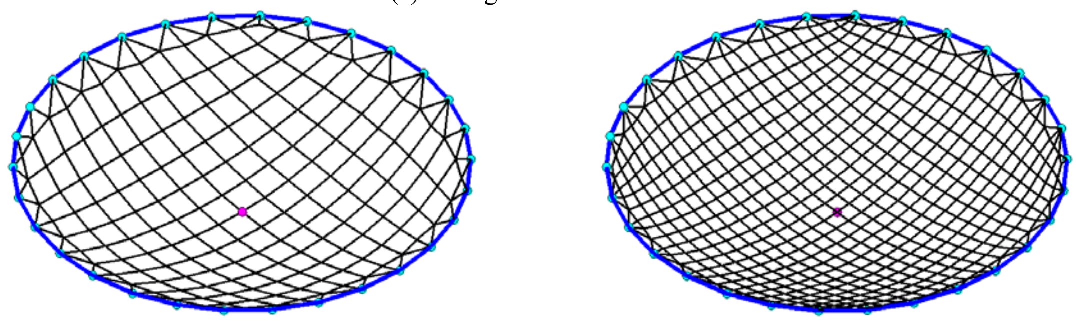
4.2 Examples with the topological mapping method

Now consider the numerical examples concerning spatial mesh configuration generation of the cable net by using the topological mapping method reported in section 2. A closer look at Fig. 5 reveals that the number of nodes in the last step will not equal to that of the ring truss. Hence, a cable-network antenna reflector with a polygonal ring truss structure of which the number of sides is unassigned is taken into consideration here. The specifications of the antenna reflector used here are the same as those in section 4.1. Two types of mesh configurations obtained by using this topological mapping method are given in Fig. 7, and their relative parameters are shown in Table 3.



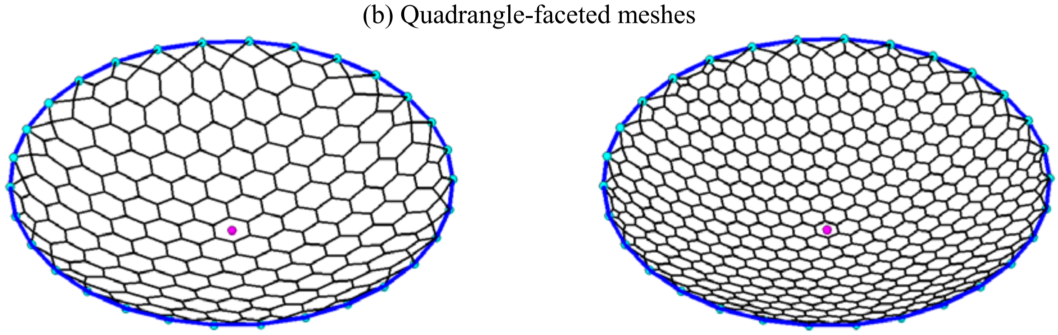
$j = 6$ $j = 10$

(a) Triangle-faceted meshes



$j = 6$ $j = 11$

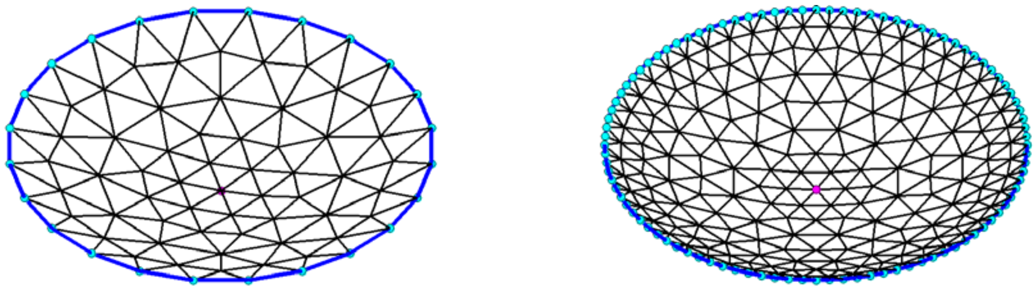
(b) Quadrangle-faceted meshes



$j = 7$ $j = 11$

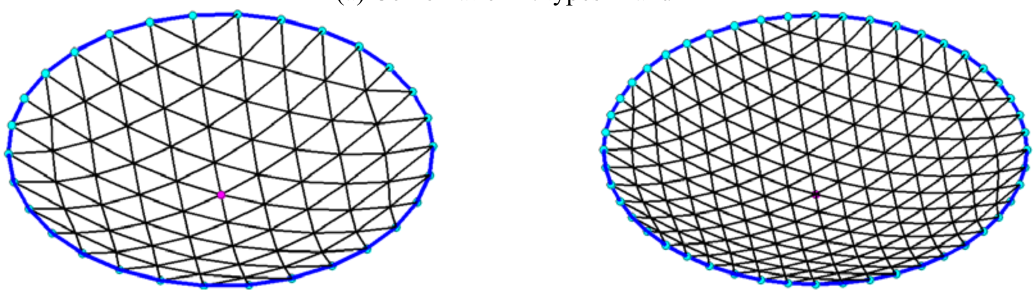
(c) Hexagon-faceted meshes

Fig. 6 Spatial mesh configurations by using the cyclosymmetry method (Perspective view)



j = 6 *j* = 9

(a) Combination 1: types A and B



j = 6 *j* = 9

(b) Combination 2: Types B and C

Fig. 7 Spatial mesh configurations by using the topological mapping method (Perspective view)

Table 3 Characteristic parameters of mesh configurations using the topological mapping method

Mesh configuration	Combination 1		Combination 2		Boundary modified	
	Fig. 7(a)		Fig. 7(b)		Fig. 9(a)	Fig. 9(b)
Number of steps	6	9	6	9	9	9
Number of free nodes	79	271	91	217	205	199
Number of elements	186	618	210	552	552	534
Number of triangles	132	444	180	432	378	402
Surface faceting error (mm)	**6.39**	**2.02**	5.80	**2.27**	2.21	**2.26**

From Fig. 7, we can conclude that this topological mapping method is especially suitable for generating the triangle-faceted meshes. In Fig. 7(a), two mesh configurations with different number of steps *j* are demonstrated, and they are obtained by using a combination of types A and B, which is denoted as Combination 1, while in Fig. 7(b), two mesh configurations by using a combination of types B and C (denoted as Combination 2) are illustrated. Obviously, the aim of acquiring mesh configurations with different mesh grid densities can be easily achieved by changing the number of steps *j*. By referring to Fig. 8 that describes the relationship between the used *j* and the surface faceting error, we can observe that a high surface accuracy (which means a large mesh grid density) can be obtained by increasing *j*. As noted in Table 3, the surface faceting error varies from 6.39mm to 2.02mm when the number of steps varies from 6 to 9.

Additionally, it can be noticed from Fig. 7 that this topological mapping method will lead to a mesh configuration with a large number of ring truss nodes when a large *j* is employed. This problem should be avoided in practical engineering application since a too large number of ring truss nodes will significantly increase the mass of antenna reflector (Tibert 2003). In the study, we settle the problem by the following way: directly apply a proper boundary connection conditions between the nodes of the last but one step and the nodes of the ring truss to substitute for the last step in the process of mesh generation. This way is demonstrated to be eligible by two simulated examples shown in Fig. 9. In both Figs. 9(a) and (b), the number of ring truss nodes is assumed to 30. The relative parameters are listed in the last two columns of Table 3. Compared the results to the mesh configurations with unassigned ring truss sides (see Fig. 7), we find that these new obtained mesh configurations have an approximate surface accuracy for an identical number of steps. For the case of j = 9 and using combination 2, the faceting error of mesh shown in Fig. 7(b) is 2.27mm, while that of mesh shown in Fig. 9(b) is nearly the same (2.26mm), as noted in Table 3.

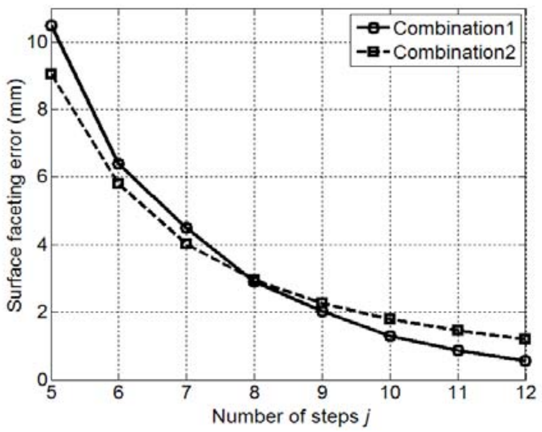
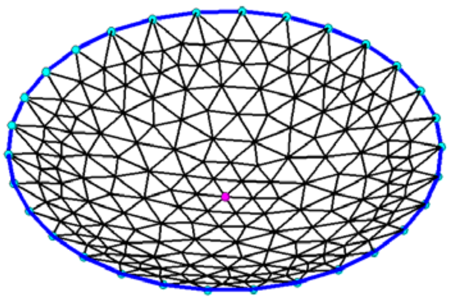
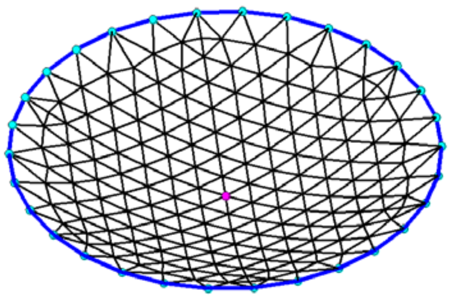


Fig. 8 Relation between the number of steps and the surface faceting error



(a) Combination 1, j = 9



(b) Combination 2, j = 9

Fig. 9 Mesh configurations different from Fig. 7 by modifying boundary connection conditions

5. Conclusions

In the study, mesh topological form design and geometrical configuration generation for cable-network antenna reflectors are addressed. Two new methods, namely the cyclosymmetry method and the topological mapping method, are first developed for creating the topological form of the cable net. Then, a simple mesh generation approach based on the force density method is proposed for acquiring the desired spatial paraboloidal mesh geometrical configuration in the static equilibrium state. Through numerical examples regarding mesh configuration generation of the cable net, the cyclosymmetry method is clarified to be useful to generate different polygon-faceted meshes, while the topological mapping method is demonstrated to be especially suitable for acquiring different density triangle-faceted meshes with modified boundary connection conditions. With the method presented in this study, multitudes of mesh configurations of the cable net with different net patterns can be sought by a certain topological rule, which will provide a valuable guide for the mesh design of this kind of cable-network antenna reflector structures.

However, the present work has not taken into consideration some existing factors, such as the flexibility of the supporting ring truss and the tension distribution of the mesh surface. For obtaining initial mesh shape with high surface precision and uniform tension distribution, more exact and multi-parameter optimal mesh design methods are required further research and development.

References

Agrawal, P.K., Anderson, M.S., and Card, M.F. (1981), "Preliminary design of large reflectors with flat facets", *IEEE Trans. Antenn. Propagat.*, **AP-29**(4), 688-694.

Hernández-Montes, E., Jurado-Piña R. and Bayo E. (2006), "Topological mapping for tension structures", *J. Struct. Eng.-ASCE*, **132**(6), 970-977.

Malerba, P.G., Patelli, M. and Quagliaroli, M. (2012), "An extended force density method for the form finding of cable systems with new forms", *Struct. Eng. Mech.*, **42**(2), 191-210.

Morterolle, S., Maurin, B., Quirant, J. and Dupuy, C. (2012), "Numerical form-finding of geotensoid tension truss for mesh reflector", *Acta Astronaut.*, **76**, 154-163.

Schek, H.J. (1974), "The force density method for form finding and computation of general networks", *Comput. Methods Appl. Mech. Eng.*, **3**(1), 115-134.

Shi, H., Yang, B., Thomson, M. and Fang, H. (2012), "Automatic surface mesh generation for design of space deployable mesh reflectors", *Proceedings of the 53rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, AIAA Paper 2012-1840, Honolulu, Hawaii, April.

Sophianopoulos, D.S. and Asteris, P.G. (2006), "Numerical method for the undamped forced dynamics of steel cable network structures", *Struct. Eng. Mech.*, **23**(4), 449-454.

Thomson, M. (2002), "Astromesh™ deployable reflectors for Ku- and Ka- band commercial satellites", *Proceedings of the 20th AIAA International Communication Satellite Systems Conference and Exhibit*, AIAA Paper 2002–2032, Quebec, Canada, May.

Tibert, A.G. (2003), "Optimal design of tension truss antennas", *Proceedings of the 44th AIAA/ASME/ASCE/AH S Structures, Structural Dynamics, and Materials Conference*, AIAA 2003-1629, Norfolk, Virginia, April.

Tibert, A.G. and Pellegrino, S. (2002), "Deployable tensegrity reflectors for small satellites", *J. Spacecraft Rockets*, **39**(5), 701-709.

Zhang, J.Y. and Ohsaki, M. (2006), "Adaptive force density method for form-finding problem of tensegrity structures", *Int. J. Solids Struct.*, **43**(18-19), 5658-5673.

Zhang, Q.L., Chen, L.X., Luo, X.Q. and Yang, Z.L. (2006), "Equivalent transform from the force-densities of cable nets to the stresses of membrane elements", *Struct. Eng. Mech.*, **26**(4), 479-482.