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Analysis of elastic foundation plates with internal and perimetric stiffening beams on elastic foundations by using Finite Differences Method

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Abstract. The mechanical behavior of rectangular foundation plates with perimetric beams and internal stiffening beams of the plate is herein analyzed, taking the foundation design into account. A series of dimensionless parameters related to the geometry of the studied elements were defined. In order to generalize the problem statement, an initial settlements was considered. A numeric procedure was developed for the resolution by means of the Finite Differences Method that takes into account the stiffness of the plate, the perimetric and internal plate beams and the soil reaction module. Iterative algorithms were employed which, for each of the analyzed cases, made it possible to find displacements and reaction percentages taken by the plate and those that discharge directly into the perimetric beams, practically without affecting the plate. To enhance its mechanical behavior the internal stiffening beams were prestressed and the results obtained with and without prestressing were compared. This analysis was made considering the load conditions and the soil reaction module constant.

Keywords: elastic foundation; plates, prestressed beam; Finite Differences Method; finite elements method

1. Introduction

The evolution of the knowledge associated with the mechanical analysis of foundation plates, developed for the last twenty years, as well as in other fields of Engineering, is related to the development of proper numerical methods to evaluate such items.

To determine the bearing capacity of foundation structures, it is necessary to know the predictable soil settlement according to the soil type. In previous works about elastic foundation plates (Ortega *et al*. 2005, Paloto and Santos 1999, 2000, Paloto *et al*. 2001, Paloto *et al*. 2002), the final settlement was determined and the distribution of the soil reaction was evaluated.

The aim of the work here presented, is the resolution of aspects associated with the Conceptual Design of the foundation, considering some parameters linked to the mechanical behavior.

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Elastic foundation plates with perimetric beams and prestressed and non prestressed internal beams will be herein analyzed and the final settlement and soil reaction distribution between what it is absorbed by the plate and the loads that are directly applied to boundary elements will be determined. The determination of these parameters will be made using the Finite Differences Method. Other authors also used this method for the foundations analysis (Jones *et al.* 2009, Sato *et al.* 2007, Yang *et al.* 2009, Zhang *et al*. 2011). Part of the results will be verified by means of software using the Finite Elements Method (Zienkiewicz and Taylor 1994).

It should be noted, that in the case of soil with a non-linear reaction coefficient (Akgöz and Civalek 2011, Chen *et al.* 2011, Orbanich *et al.* 2003), and in order to solve the problem to solve this problem with a Finite Elements model, the coefficient value should he calculated externally for all the points of the mesh. This does not happen when the Finite Differences Method is used. The advantage of the latter is then evident.

Also, the simulation of the initial settlement of the plate is difficult with this Finite Element software, whereas the modelling with the Finite Differences Method is very simple (Chen *et al.* 2011). On the other hand, results with a good convergence are attained using less dense meshes than with MEF.

In the case of an increase in the load value, and/or if it is necessary to reduce beam height, especially the internal stiffening beams of the plate, it will be necessary to construct prestressing beams (Lee and Kim 2011, Leonhardt 1977, Paloto and Ortega 2000, Tombesi *et al*. 1974). The use of beams with minimum height is beneficial if the ground water is at a high level, so as not to have to reduce this level by pumping for the construction of the foundation, to avoid the contact of water with the structure’s concrete, thus increasing material durability.

Few scientific papers related to the use and analysis of prestressed foundation plates, have been found, although its use is widespread in specific applications. In general, prestressed foundation plates are used for reasons related to important loads or where concrete cracking may be a problem (Ashar 1983, Smitsyn *et al*. 1975, Tovstik 2009). In this paper a calculation method for a foundation plate with prestressed beams, is presented. The developed method allows to evaluate the necessary prestressing so as to improve the mechanical performance, thus allowing an optimum dimensioning of the load bearing elements.

2. Theoretical foundations

2.1 Solution of foundation plates with inner stiffening beams

Elastic Foundation plates with constant soil reaction module (Winkler Foundation Modulus) (Guler and Celep 2005, Chen *et al*. 2011, Ponnusamy and Selvamani 2012, Tanvir 1995, Timoshenko and Woinowsky 1959, Yu and Wang 2010) will be examined so that their settlements exhibit linear variations. The differential equation that explains the mechanical performance is

$$\nabla^2\nabla^2 w = \frac{1}{D}\left[q - k(w + w_0)\right] \tag{1}$$

where:
$w(x, y)$: settlement function
w_0: initial settlement
$q(x, y)$: applied distributed load

k: soil reaction modules
D: plate bending rigidity

$$D = \frac{Ed^3}{12(1-\mu^2)} \tag{2}$$

where:
E: Concrete Elasticity Modulus
d: plate thickness
μ : concrete Poisson´s ratio

With the aim of solving Eq. (1) numerically a Finite Differences approximation is applied, with an "s-side" square mesh (Fig. 1). Settlement function values w_j are thus obtained $w(x, y)$.

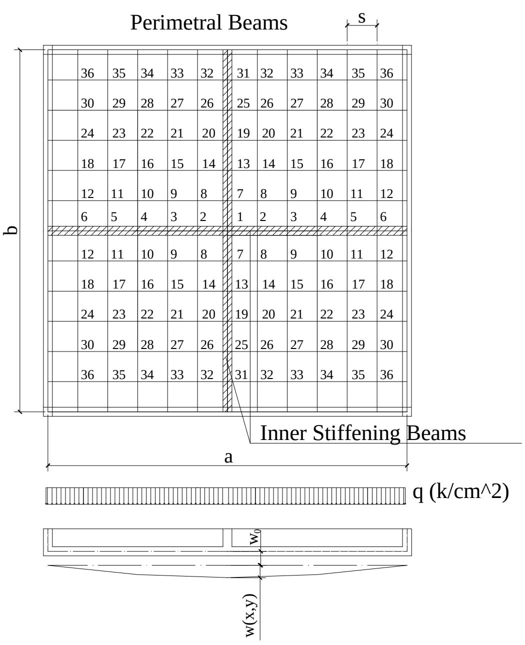

Fig. 1 Plate with internal Stiffening beams

Solving Eq. (1)

$$c_i w_i + \sum_{j=1_{j \neq i}}^{n} c_j w_j = \frac{1}{D} s^4 \left[q - k(w + w_0) \right] \tag{3}$$

where:
s: mesh step

To solve this equation the definition of two factors has been proposed, making it possible to generalize the results.

Applying a uniform initial settlement value to the whole function, so that the limit value is obtained by

$$w_0 = \alpha \frac{q}{k} \tag{4}$$

being α the initial settlement value at the boundary. On the other hand w_o is consider constant over the plate, so the total settlement at each point is: $w + w_o$.

The dimensionless soil relative reaction factor (N'), is expressed by

$$N = ks^4 \tag{5}$$

On the basis of the above definitions, Eq. (3) then becomes

$$c_i w_i + \sum_{j=1_{j \neq i}}^{n} c_j w_j = q \frac{s^4}{D} - \frac{N'w}{D} - k \frac{s^4}{D} \frac{aq}{k} \tag{6}$$

Taking into account beam bending rigidity, we have

$$D_I c_i w_i + D_I \sum_{j=1_{j \neq i}}^{n} c_j w_j = qs^4 - N'w_i - N'a \frac{q}{k} \tag{7}$$

where:
D_I: beam bending rigidity
D: plate bending rigidity (Eq. (2))

By means of successive mathematical operations, we arrive at the following equation

$$d\,[\left(c_i + N\right) w_i + \sum_{j=1_{j \neq i}}^{n} c_j w_j\,] = \frac{q}{k} N(1 - a) \tag{8}$$

where:
$N = N'/D$
$d = D_I / D$

From this Finite Differences system the w_i values which represent displacements at each i-eth plate point are obtained.

Another desired parameter for the analysis of the problem is the percentage of the support reaction, admitted by the plate, and the soil reactions that are discharged directly to the boundary elements.

Once the plate w_i values are obtained, an evaluation of the support reaction discharging upon the plate Rp is evaluated, by means of the following equation

$$Rp = \int_0^a \int_0^b k(w_i)\,dxdy \tag{9}$$

It is worth mentioning that in case that k be constant, the integral for Eq. (9) involves determining the settlement volumes.

Once Rp has been obtained the percentages of the total of the applied load can then be expressed by obtaining the difference of the reaction percentage admitted by the boundary beams. For this system, Eqs. (8) and (9) must be solved. In this case, the solution of these matrix systems was carried out by means of MATLAB software (Matlab 7.0 and Simulink 2004).

2.2 Solution of plates with inner stiffening prestressed beams

An elastic foundation plate of the same geometry as the one presented in Fig. 1, but in this case with prestressed inner stiffening beams, was addressed.

These prestressed beams are analyzed by means of a procedure that we have named Parabola Method. This allows us to find different prestressing forces that originate different plate displacements, including the forces that correspond to null displacements of the plate, a case in which the whole soil reaction is discharged directly upon the boundary elements. This situation is advantageous from the viewpoint of the foundation durability, particularly when it is in contact with the soil or in the presence of, for example, chlorides that attack metallic reinforcement and use this to gain entry into the structure.

When prestressing the inner stiffening beams of the foundation plate, their displacements are opposed to the displacements produced by the load $q(x, y)$ and is partially compensated by the soil reaction.

To determine displacements it is important to remember that only at the central point of the plate it is possible to apply superposition, due to the fact that the resulting displacement, is the sum of all the displacements generated by prestressing forces in both directions, while outside the central zone plate the superposition is not valid due to the effect produced by the perimetric beams. Here the sum of the displacements generated by prestressing forces does not represent the actual plate displacement.

The Parabola Method here applied, consists of representing the displacement function generated by prestressing, by means of parabolic curves, which was determined by knowing the Elastic Modulus value that generates the prestressing at the central point and the actual displacements at the two boundaries. Then, the central parabola, is traced through these points. The corresponding equation is then found by means of a Regression Analysis, in order to calculate displacements $w_1, w_7, w_{13}, w_{19}, w_{25}, w_{31}$.

The foundation plate with inner stiffening beams here analyzed has a side relation equal to 1, and the tracing of the prestressing cable follows a parabolic direction. Under such conditions, for every prestressing force the equivalent load has to be calculated. In this case is considered as uniformly distributed by means of

$$q_{eq} = \frac{8V\,e}{a^2} \tag{10}$$

where:
V: prestressing force.
e: eccentricity of the parabolic wire at the center of the plate.
a: plate side.

Then the desired central displacement is determined as follows

$$\delta_1 = \frac{5q_{eq}\,a^4}{384\,EI} \tag{11}$$

where:

I: inertia of the inner stiffening beam, considering the contribution to the plate, according to the values allowed by CIRSOC 201 (Standard CIRSOC 201 2005).

In the normal direction and in a similar way as in the previous case, we have

$$\delta_2 = \frac{5 q_{eq}\, b^4}{384\, EI} \tag{12}$$

Applying the superposition principle and considering a square plate ($a = b$), the maximum displacement at the center of the plate is obtained as

$$\delta_{\max} = \delta_1 + \delta_2 = \frac{5 q_{eq} a^4}{192\, EI} \tag{13}$$

This maximum displacement is located at the central point of the plate, so that these displacements exhibit a parabolic variation in direction 1 and 2. With these displacements the parabolas whose vertex are points 1, 2, 3, 4, 5 and 6, are traced in normal direction to the central parabola, and this enables us to find the remaining displacements.

3. Analyzed Cases

3.1 Application of the method to a slab

The method presented in this paper was applied to a square slab supported by its four boundaries, 0.20 m high and with an applied load q = 9,8 10^4 N/m^2, mesh density was changed to the effect of examining convergence and analyzing displacements at the central point with 6 × 6 and 12 × 12 and 24 × 24 meshes.

Table 1 Slab displacement at the central point ccording to the type of mesh adopted

Mesh	Central Displacement (*m*)
6 × 6	0.0512
12 × 12	0.0513
24 × 24	0.0513

Checking by means of the Finite Element Method and using a 24 × 24 mesh, the observed displacement at the central point was 0.0511 m.

A verification made with Timoshenko and Woinowsky (1959), for the same conditions, resulted in a displacement of 0.0513 m. Examining these results, it can be said that the accuracy of the results found with the Finite Difference Method attain a high rate of convergence.

3.2 Application of the method to a plate with inner stiffening beams

To study the convergence of the method as applied to elastic foundation plates with inner stiffening beams, a comparison of the results obtained with the Finite Differences Method and with a commercial software that uses the Finite Element Method, Algor software (ALGOR 23 Professional Mech/VE 2008) was carried out. The results obtained are discussed in the following paragraphs.

3.2.1 A foundation plate solved by means of the Finite Differences Method

In this case a foundation plate with two crossing 0.32 × 0.60 m inner stiffening beams was studied. Mesh density was varied in order to establish the minimum mesh density that ensure a good approximation to the results obtained by the Finite Elements Method. The model thus developed has the following characteristics:

$q = 9.8 \quad 10^3$ N/m^2

$k = 9.8 \quad 10^7$ N/m^3

$d = D_1 / D = 24.57$

$\alpha = 0$

By applying the Finite Differences Method for each of the proposed mesh sizes, the displacement values at the central point of the plate, as can be seen in Table 2, are:

Table 2 Plate central point displacement values

Mesh	*N*	Central Displacement (*m*)
6 × 6	0.9750	0.0048
8 × 8	0.3085	0.0059
12 × 12	0.0609	0.0079
24 × 24	0.0038	0.0078

3.2.2 Foundation plate solved by means of the Finite Element Method

The model of a foundation plate was created by means of PLATE elements, with two crossing inner stiffening beams that were simulated by means of BEAM elements, and applying the OFFSET function, to displace plate and beam barycentric axes. Soil was modeled by means of TRUSS elements, located at each node with the aim of simulating soil reaction, and the following analogy was made

$$k\delta_s = \frac{F}{S_n} \tag{14}$$

where:

k: soil reaction modules

δ_s: surface displacement.

F: equivalent force acting on the truss element.

S_n: node influence surface.

$$F = k \quad \delta_s \quad S_n \tag{15}$$

where:

$k_{t:}$ truss element rigidity (EA/L).

The equivalent force acting on the truss element is

$$F = k_t \ \ \delta_n \tag{16}$$

where:
δ_n: node displacement.
A: truss element section.
L: truss element length.

As $\delta s = \delta n$ and considering that node displacement is equal to truss element displacement

$$k \ \ \delta_n \ \ S_n = k_t \ \delta_n = \frac{EA}{L} \delta_n \tag{17}$$

Solving for L

$$L = \frac{EA}{kS_n} \tag{18}$$

For this investigation four models were made, of the same characteristics as those mentioned in 3.2.1., and varying the mesh density, with the aim of ensuring convergence.

The obtained displacements at the plate center point were included in Table 3.

Table 3 Displacements at the plate center point

Mesh	Central Displacement (m)
6 × 6	0.0067
12 × 12	0.0070
24 × 24	0.0078
50 × 50	0.0078

The convergence of the results of the models solved by means of the Finite Difference Method (Table 2) and the Finite Element Method (Table 3) shows that with the Finite Element Method a mesh of at least 24 × 24 should be used. Table 2 shows that with the Finite Differences Method a similar accuracy is obtained by using a 12 × 12 mesh and therefore it may be stated that convergence is faster when using the Finite Differences Method.

3.3 Foundation plate with internal stiffening beams at different heights

One of the models discussed in 3.2.1., whose foundation plate with internal stiffening beams modeled with a 12 × 12 mesh, varying the height of the internal stiffening beams in each case in order to find the soil reaction percentage absorbed by the plate and its boundary elements (Table 4).

3.4 Foundation plates with prestressed internal stiffening beams

In this case prestressed internal stiffening beams were used in a 6 × 6 m foundation plate (Fig. 2). Beam heights were varied and for each different height value the prestressing force was varied until an optimum prestressing stress was obtained. This is the stress that counterbalances deformations produced by the loads.

Table 4 Reaction percentage absorbed by the plate

Beam height (*m*)	Percentage absorbed by the plate with internal stiffening beams
0.00	51.67
0.30	43.66
0.40	41.42
0.60	40.27
0.80	39.96

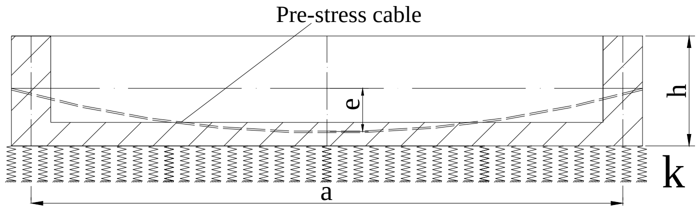

Fig. 2 Prestressed internal stiffening beam

In order to take the prestress force into consideration in the performed analysis, the displacements obtained by means of Eq. (1) that correspond to the load application are superimposed to the ones obtained with the Parabola Method (Subsection 2.2) that account for the displacements due to the prestress of the intermediate beams. The percentage of reaction taken by the plate are determined through the diagram of deformation volumes, built with the superposition of the two above-mentioned effects.

The characteristics of the analyzed model are as follows:

$a = b = 6$ m
$k = 9.8\ \ 10^7$ N/m^3
$q = 9.8\ \ 10^5$ N/m^2
$E = 1.4\ \ 10^{10}$ N/m^2
$\mu = 0.30$
where:
a and b: plate sizes

The first case to be developed was a plate with internal stiffening beams of 0.30 m high and 0.30 m width. It was prestressed by applying force (V) ranging between 15×10^4 N and 56×10^4 N, with a maximum offset of 0.05 m.

Table 5 presents central point displacements due to pre-stressing applied to internal stiffening beams.

Table 5 Central displacements in function of the pre-stressing forces

V (N) ($\times 10^4$)	$a \times b$ (m)	e (m)	q_{eq} (N/m)	E (N/m^2) ($\times 10^{10}$)	I (m^4) ($\times 10^{-3}$)	Maximum Displacement (m)
15	6 × 6	0.05	1633	1.4	1.677	0.00240
30	6 × 6	0.05	3266	1.4	1.677	0.00479
40	6 × 6	0.05	4355	1.4	1.677	0.00639
50	6 × 6	0.05	5444	1.4	1.677	0.00799
56	6 × 6	0.05	6097	1.4	1.677	0.00894

By applying the superposition principle to the displacements produced by prestressing to those of the external load, and within MATLAB software, the reaction percentage applied to plates with internal stiffening beams for each of the prestressing forces, can be observed in Table 6.

Table 6 Load percentages admitted by plate versus the prestressing force

V (N) ($\times 10^4$)	Percentage absorbed by plate
0	43.66
15	31.95
30	21.32
40	14.56
50	7.40
56	3.33

In Fig. 3 and Fig. 4 reaction percentage values absorbed by the plate were plotted versus the prestressing force applied to the internal stiffening beams.

The equation of the straight lines shown in Fig. 4, were determined using a Regression Analysis for different internal stiffening beam heights (Table 7).

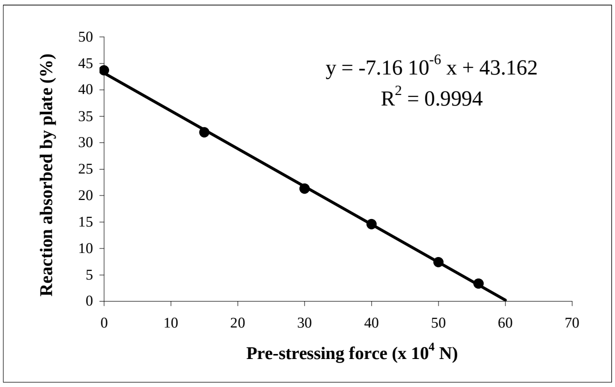

Fig. 3 Reaction absorbed by the plate versus the applied prestressing force for a 0.30 m (h/a=1/20) high internal stiffening beam

Table 7 Equations of the plate absorbed percentage variation versus prestressing for different beam heights

Beam height (m)	Straight line equation
0.30	$P = - 7.16\ 10^{-5}\ V + 43.162$
0.40	$P = - 9.49\ 10^{-5}\ V + 41.113$
0.60	$P = - 4.42\ 10^{-5}\ V + 39.772$
0.84	$P = - 2.46\ 10^{-5}\ V + 39.360$

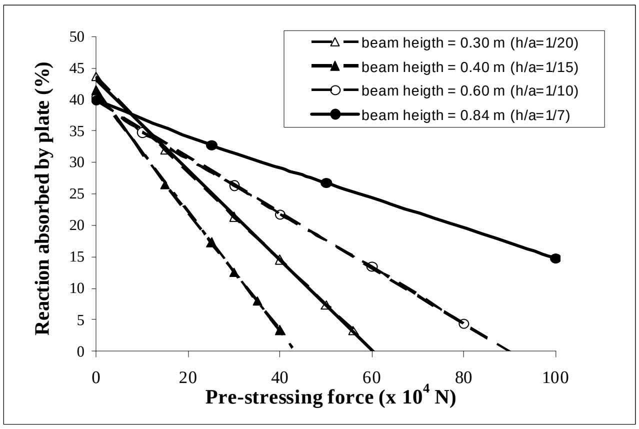

Fig. 4 Reaction absorbed by the plate versus the pre-stressing force for different internal stiffening beam heights

The results presented in Fig. 5 shows the influence of soil type (Table 8) in the distribution of reactions between the plate and the edge beams. To obtain these results was adopted a plate of 6 m × 6m 0.20m thick with internal beams (0.30 m high and 0.30 m wide), to which was applied different pre-stressing force and an external load of $q = 9.8 \times 10^5$ N/m^2.

Table 8 Soil reaction modules (k)

Soil type	k ($\times 10^7$ N/m^3)
Fine sand Filled with silt, sand and gravel	0.98
Wet clay	2.94 to 4.90
Silt compacted with sand and gravel Gravel with very fine sand	9.80
Medium gravel with fine sand	14.7
Coarse gravel with coarse sand	19.6
Quite compacted coarse gravel	24.50

Fig. 5 shows that, when soils are very low bearing capacity ($k = 0.98 \times 10^7$ N/m^3), plates with internal stiffening beams, transferred few load to perimetric beams, regardless the prestressing load applied to the internal stiffening beams.

Moreover, in the case of soil with a high load bearing capacity ($k = 25.00 \times 10^7$ N/m^3), the percentage of load transferred to the perimetric beams are influenced by pre-stressing force, such that as it increases, the reaction taken by the plate decrease. Furthermore it is seen that there is a maximum value of pre-stressing force after which the reaction taken by the plate is null.

For soil bearing capacity between these two values, the behavior is similar varying the maximum pre-stressing load. It should be noted that, the variation is approximately linear and in

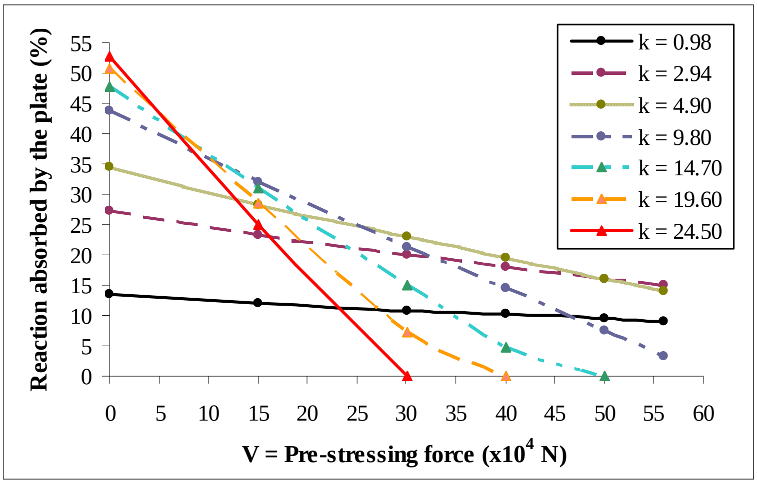

Fig. 5 Reaction absorbed by the plate versus the pre-stressing force for diferents soil reaction modules (*k*)

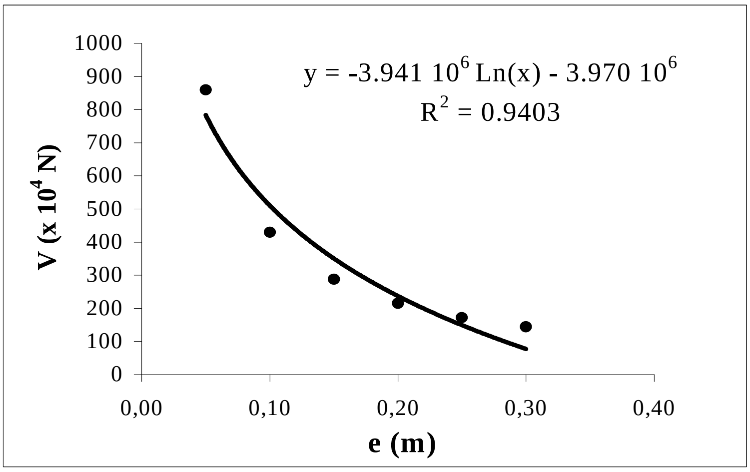

Fig. 6 Pre-stressing force that should be applied to internal stiffening beams for different eccentricity values with beams height of 0.84 m (h/a=1/7)

none of the cases analyzed, the percentage of reaction taken by the plate, exceeds 53 %.

It should be mentioned that, the results obtained analyzing other cases with pre-stressed internal beams with different heights (0.30 to 0.84 m), were similar to those shown in Fig. 5. It is interesting to note that the percentage of load taken by plated do not depend on the applied external load q.

It is interesting to observe that for beams over 0.40 m high, vertical displacement can be varied by changing the eccentricity or prestressing force, such as is shown in Fig. 6, that corresponds to an 0.84 m high beam with null vertical displacement in the midpoint plate.

4. Conclusions

From the above presented results, the following conclusions were drawn:

- A numerical method has been developed to determine the distribution of soil reaction in foundations with inner stiffening beams.
- The accuracy of the developed method was verified by comparing it with other methods, such as the Finite Element Method.
- An improvement in soil reaction percentage was observed as prestressing forces applied to the internal stiffening beams were increased until the optimum prestressing force, namely the one that cancel plate deflection, was reached.
- The herein described method could also be applied to the solution of slabs with internal stiffening beams, with or without pre-stressing, supporting important loads.

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