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Lateral capacity of piles in layered soil: a simple approach

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Abstract. Appropriate assessment of lateral capacity of pile foundation is known to be a complex problem involving soil-structure interaction. Having reviewed the available methods in brief, relative paucity of simple and rational technique to evaluate lateral capacity of pile in layered soil is identified. In this context, two efficient approaches for the assessment of lateral capacity of short pile embedded in bi-layer cohesive deposit is developed. It is presumed that the allowable lateral capacity of short pile is generally dictated by the permissible lateral displacement within which pile-soil system may be assumed to be elastic. The applicability of the scheme, depicted through illustration, is believed to be of ample help at least for practical purpose.

Keywords: lateral capacity; piles; soil structure interaction; layered soil; earthquake

1. Introduction

Understanding behavior of pile foundations under lateral load is a challenging and complex problem as the mechanism of transfer of lateral loads to the subsurface strata is essentially dependent on the attributes of sub-soil and pile itself typically known as soil-structure interaction problem. The deformation and flexural stresses in the pile depend on the soil resistance, whilst the soil resistance is a function of the deformations of the pile itself. Furthermore, the ultimate resistance of a vertical pile to a lateral load and the deflection of the pile as the load builds up to its ultimate value is complex and involve the interaction between a semi-rigid structural element and soil which behaves nonlinearly. A number of interrelated factors influence the response of piles under lateral loading. The pile stiffness is a crucial factor as the same controls the deflection and determines the failure mechanism of pile. The nature of loading such as sustained, alternating or

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pulsating also influences the degree of yielding of the soil.

Seismic loading – a major source of lateral loading - is, in practical design to date, typically considered as an additional "pseudo-static" lateral load applied to the pile. Magnitude of such lateral load is considered to be equal to the base shear of the structures resting on the piles. Such base shear is often estimated using the design spectra relevant to the soil under consideration and on the basis of the perceived period of the system. Lateral load may also induce on piles and pile supported systems due to wind and wave action particularly to offshore structures. Piles, almost always, penetrate deep into the underlying soil and encounter soil of various characteristics. It is, though, recognized that often the upper part of a pile meaningfully contributes to carry lateral load (for long pile); it is almost inevitable to experience heterogeneous soil media in such region of interest.

Thus, systematic assessment of lateral capacity of pile incorporating the effect of soil stratification is a problem of overriding importance. In this backdrop, the current investigation attempts to develop some simple guidelines to evaluate the lateral capacity pile embedded in stratified deposit. The outcome of the present work seems to be useful to the practicing engineers pending the emergence of more sophisticated and computationally efficient approach.

2. State-of-the-art and background of the problem

Piles are often subjected to lateral loads in practice and many a time design is dictated by the lateral capacity of pile rather than the vertical capacity. Recognizing the importance of the issue, extensive research works (Matlock and Reese 1960, Broms 1964, Reese *et al.* 1974, Poulos and Davis 1980, Meyerhof *et al.* 1981, Meyerhof and Sastry 1985, Patra and Pise 2001, Shen and Teh 2004, Zhang *et al.* 2005) have been conducted with a view to assessing the resistance of piles under lateral loading over decades. However, behavior of pile-soil system is so complex that the appropriate modeling of such system leading to some acceptable design proposal is shrouded yet.

There exists several approaches for modeling soil-pile system, *viz.*, finite element based approach, pseudo-static approach, *p-y* method, characteristic load approach, strain wedge modeling approach *etc.* In finite element modeling, pile may be modeled as nonlinear beam-column element (Hutchinson *et al.* 2005). On the other hand, simplified approach such as modeling of pile as linear element is also in vogue. This relatively simple scheme seems to capture the buckling failure for piles in liquefied soil (Kerciku 2008) and can represent the effect of vertical load on lateral response of piles (Karthigayan 2007). Concept derived from beams on elastic foundation or *p-y* spring elements is a widely used technique to represent effect of soil on the pile (Hutchinson *et al*. 2005, Kerciku 2008, Rajashree and Sitharam 2001).

The characteristic load method (CLM) for analysis of laterally loaded piles and drilled shafts may be used to accurately estimate ground line deflections and maximum bending moments at free and fixed-head conditions both in clay and sand (Brettmann and Duncan 1996). Strain wedge model has been adopted elsewhere (Ashour and Norris 2003) to examine the response of laterally loaded pile in liquefiable soil. Recognizing 'buckling' as a potential mechanism of failure of pile in liquefiable deposits, such issue is explored through finite difference program Fast Lagrangian Analysis of Continua (FLAC) in a relatively recent work (Halder *et al.* 2008) that proposes certain design guidelines and highlights on the need of identification of proper failure mechanism (besides routine consideration of bending). The basic purpose of the strain wedge model is to study stress-strain-strength (drained or un-drained) relationship of the soil wedge. However, for a more precise evaluation of displacement response of piles under lateral dynamic loading, two-dimensional explicit numerical scheme (Klar and Frydman 2002) has been used. The strain superposition concept, proposed for ballast study, has been adopted (Lin and Liao 1999) to evaluate strain accumulation for laterally loaded piles in sand. Theoretical analysis of the lateral response of vertical piles subjected to lateral soil movements is also attempted via a simplified boundary element analysis (Chen and Poulos 1997). The method tends to yield an upper-bound estimation of the maximum pile bending moment and pile head deflection. However, it leads to satisfactory results for small soil movements. This procedure is limited to the cases of vertical single pile only. Impact of the variability in the soil, structural and other seismic design parameters is examined recently (Halder and Babu 2009) in the probabilistic framework using Monte Carlo simulation technique. Such study proposes a reliability-based design approach for the free-head pile.

However, there exists relative paucity of authoritative experimental works to corroborate the computationally intensive procedures. A few attempts have been made for full scale tests in the field (Burr *et al.* 1997) and also for model tests (Patra and Pise 2001, Hutchinson 2005, Karthigayan 2007). For testing of piles under dynamic or lateral loading, the centrifuge modeling approach is a field of emerging interest (Bruno and Randolph 1999, McVay 1995).

In this context, endeavor has been made in the present study to frame useful guideline to assess lateral load carrying capacity of piles using established finite element method. Recognizing that majority of the existing methods deal with the homogeneous sub-soil characteristics, emphasis is made on stratified soil system – which is often the case in practice. Suitable design recommendation emerged therefrom is also explained through case studies.

3. System idealization and methodology

Single pile along with the surrounding soil mass up to a finite distance of 20D on each side of the pile is considered as per the guideline suggested elsewhere (Karthigayan 2007), where D is the pile diameter. 20-noded brick elements are used to discretize the pile-soil continuum adequately to ensure convergence.

The interface of pile and soil medium has been modeled using 16-noded contact elements of zero thickness. Recognizing that the interface strength may be at variance depending on the type of pile material (wood, steel, or concrete) and method of installation (driven or bored), the same in shear is defined with zero frictional strength and two-third of the cohesive strength of the surrounding soil. However, the shear and normal strength is restricted to a very small value to allow for the relative slip and separation of the pile from the soil under tension.

The nodes on the vertical boundaries on each side are restrained in a direction normal to the surface but free in vertical representing rigid and smooth lateral boundaries. On the other hand, nodes on the bottom surface are restrained in all three directions representing the rough, rigid bottom surface. Number of nodes, brick elements and contact elements in the pile-soil continuum are typically on the order of 61000, 14000 and 4000 respectively. A typical mesh of representative of the pile-soil system with appropriate boundary constraints is presented in Fig. 1.

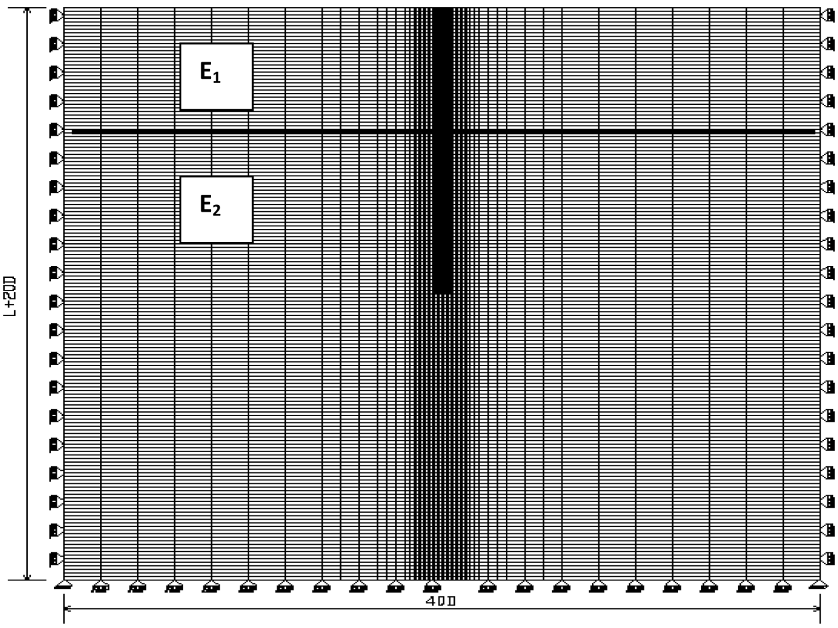


Fig. 1 Schematic representation of soil-pile system discretized in the mathematical model

4. Validation of the model

Success of any numerical formulation is essentially dependent on ensuring convergence criterion. To ensure the same, initially current investigation performs repetitive analysis with gradually refined mesh assuming vertical and horizontal boundaries at 20D apart from the pile surface and base, respectively. In the sample case study presented, a 10 m long reinforced concrete pile (*R/C*, grade of concrete chosen M20) of 500 mm diameter is considered to be embedded in soils with various representative elastic parameters. Response is measured in terms of displacements, maximum bending moment and maximum shear force. Change of such quantities in a particular iteration with respect to the same obtained in the previous iteration is plotted against the current iteration number (Fig. 2). Fig. 2 depicting such variation in percentage shows that the results tend to converge for mesh divisions considered in the fourth iteration. Fig. 2 also suggests that the behavior of pile under lateral load is insensitive to Poisson's ratio of supporting soil.

Further, to decide the cut-off distance of boundary of the infinite media, boundary distance on either side of the pile is increased to 20D, 25D and 30D in successive trials. Response obtained in each case is normalized to the corresponding quantity obtained when the boundary is specified at 20D apart and is graphically presented in Fig. 3. Young's modulus of soil is considered to be equal to 18750 kN/m^2 and Poisson's ratio as 0.3. Fig. 3 shows that the response quantities do not appreciably change if the cut-off distance of such boundary is considered to be 20D apart from the pile (as shown in Fig. 1) and hence adopted in the subsequent analyses.

Analysis is also repeated intensifying the magnitude of lateral load to check the adequacy of the cut-off distance as the sensitivity of numerical scheme may not be properly identified at small load level. Pile response normalized to the same due to the least lateral load applied is presented in Fig. 4. The results described show that displacement, bending moment (B.M.) and shear force (S.F.)

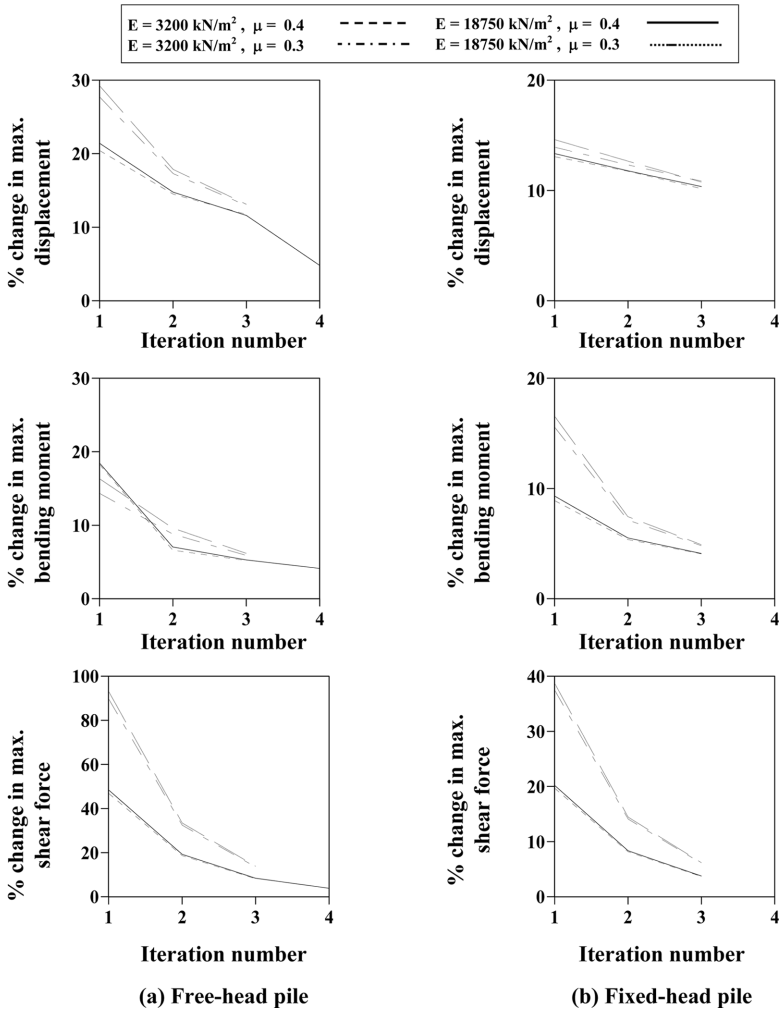


Fig. 2 Variation of change of response with respect to previous iteration

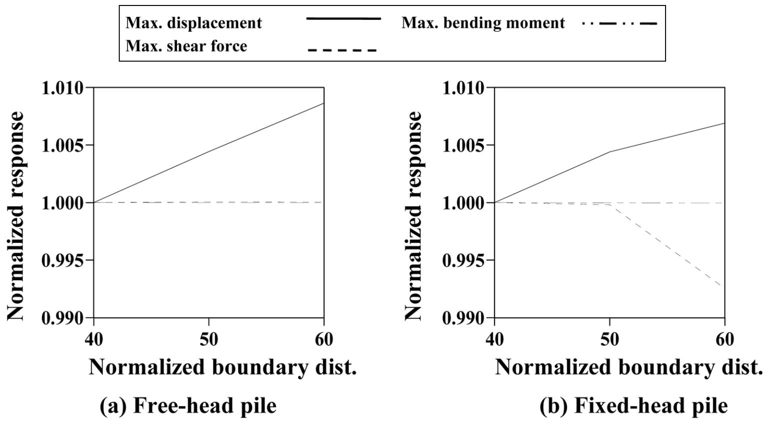
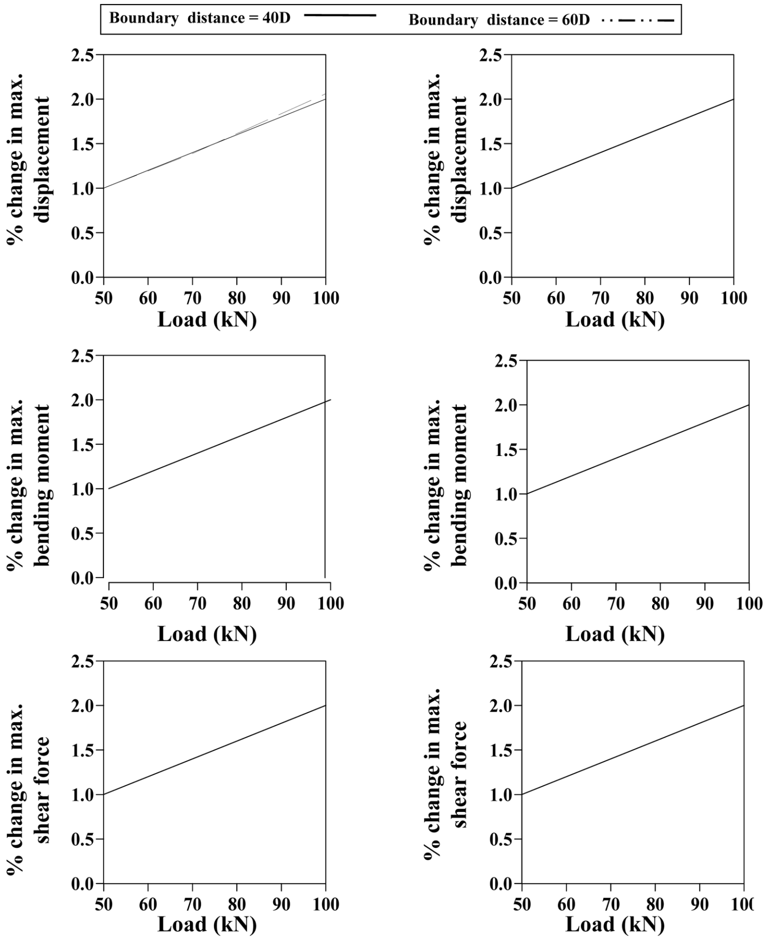


Fig. 3 Variation of normalized maximum response with increase of distance of the boundary



(a) Free-head pile (b) Fixed-head pile

Fig. 4 Variation of normalized maximum response with increase of lateral load

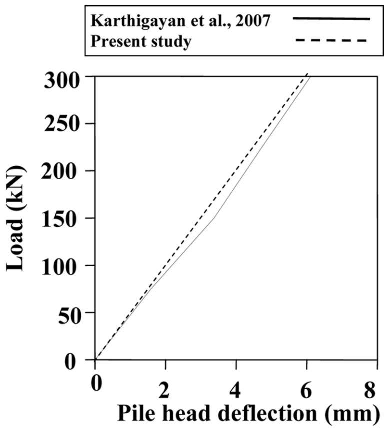


Fig. 5 Comparison of lateral load and pile head deflection with a previous study (Karthigayan *et al.* 2007)

increase proportionately with load implying that change in the magnitude of load does not affect the accuracy of the results at least for the cases considered herein.

The efficacy of the model is further verified through assessing lateral capacity of a 10 m long pile of 1.2 m diameter embedded in homogeneous soil. Elastic properties such as Young's Modulus and Poisson's ratio of pile are taken as 25×10^6 kN/m^2, 0.15 while the same for soil are respectively considered and 40×10^6 kN/m^2, 0.4. Unit weight of pile and soil material is considered as 24 kN/m^2 and 18 kN/m^2, respectively. Un-drained cohesion of soil is assumed as 100 kN/m^2. These set of numerical values of system parameters are considered elsewhere (Karthigayan 2007). Response history, in terms of deflection at pile top, as furnished in Fig. 5 shows a fairly good agreement with the same obtained in a previous work (Karthigayan 2007) at least up to 5 mm deflection – a value often used to evaluate allowable lateral capacity. Thus the present numerical scheme may be used to estimate lateral capacity of pile with reasonable accuracy.

5. Results and discussions

Pile-soil system so modeled is analyzed considering various combinations of sub-soil characteristics representing feasible layered deposits in conjunction with different pile head conditions, *viz.*, free head and fixed head. However, the current investigation exclusively addresses the behavior of short pile embedded in bi-layered cohesive soil. A 5 m long pile of 0.6 m diameter embedded in different combinations of soil is analyzed. Relevant characteristics of soil deposits considered in these analyses are furnished in Table 1. The response of the piles under pure lateral load applied at the pile head is examined and the load applied corresponding to a deflection of 5 mm at pile head is regarded as the allowable lateral capacity.

To examine the effect of the variation of soil characteristics, in the parametric study, properties of the upper layer of a specified thickness is initially set and the attributes of the underlying layer is subsequently changed conforming to the different soil characteristics. Lateral capacity (H) obtained from the same is normalized to what due to in a homogeneous layer with properties of the upper strata. Variation of such normalized parameter is plotted against the un-drained cohesion of the underlying strata. Such analyses are repeated considering three different proportions of the layer-thickness between upper and lower strata over the length of the pile, *viz.*, (i) 1:1; (ii) 3:1 and (iii) 1:3 and the corresponding response curves are presented in the same figure with different symbols. Further, such analyses have been repeated considering various feasible properties of the upper layers. Outcomes of the parametric study delineated herein are presented in Fig. 6 for both free and

Table 1 Modulus of elasticity of various type of soil

Soil type	Undrained cohesion C_u (kN/m^2)	Modulus of Elasticity of soil (kN/m^2)
Soft	20	1350
Medium	50	3200
Stiff	100	5625
Very stiff	200	11250
Hard	250	18750

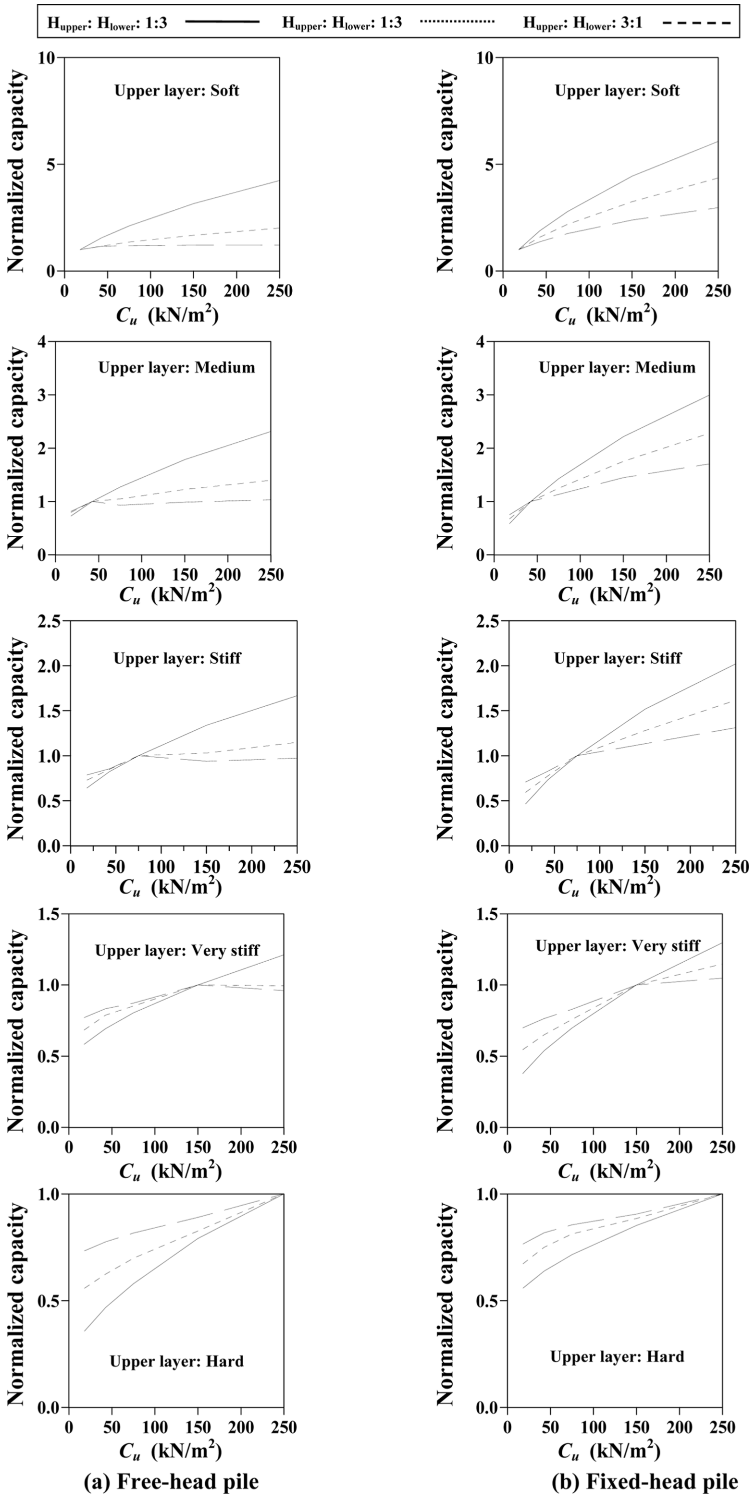


Fig. 6 Variation of lateral capacity in bi-layer media normalized to capacity in homogeneous soil conforming to upper layer

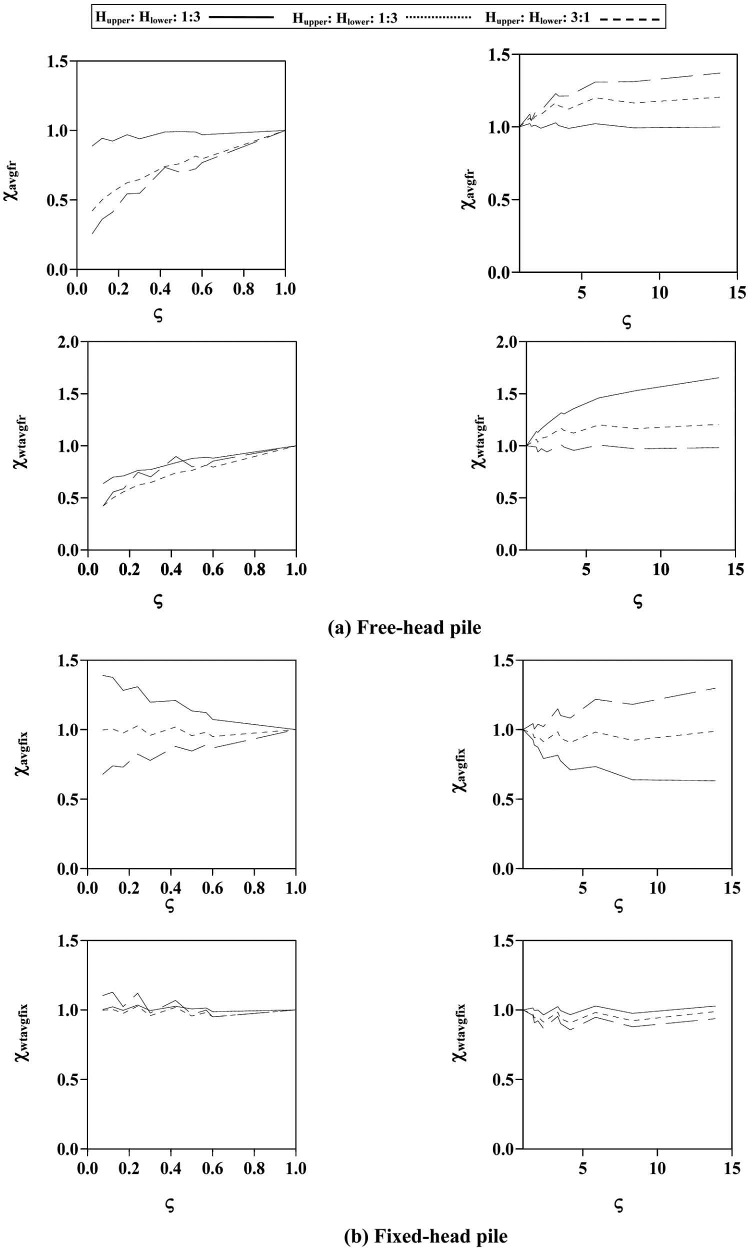


Fig. 7 Correction factors to determine the equivalent lateral capacity of pile in bi-layer soil

fixed head conditions of pile. A careful observation suggests that the change is about 50% to about 400% for free head pile while the same is in the range of about 30% to 600% for fixed head piles due to contrast of properties in bi-layer sub-soil system relative to the homogeneous one.

In this context, H_{avg} and H_{wtavg} are estimated on the basis of the pile capacities in corresponding homogeneous strata by (a) averaging such lateral capacities and (b) averaging such capacities in proportion to the thickness of the relevant strata, respectively. Subsequently, the lateral capacity

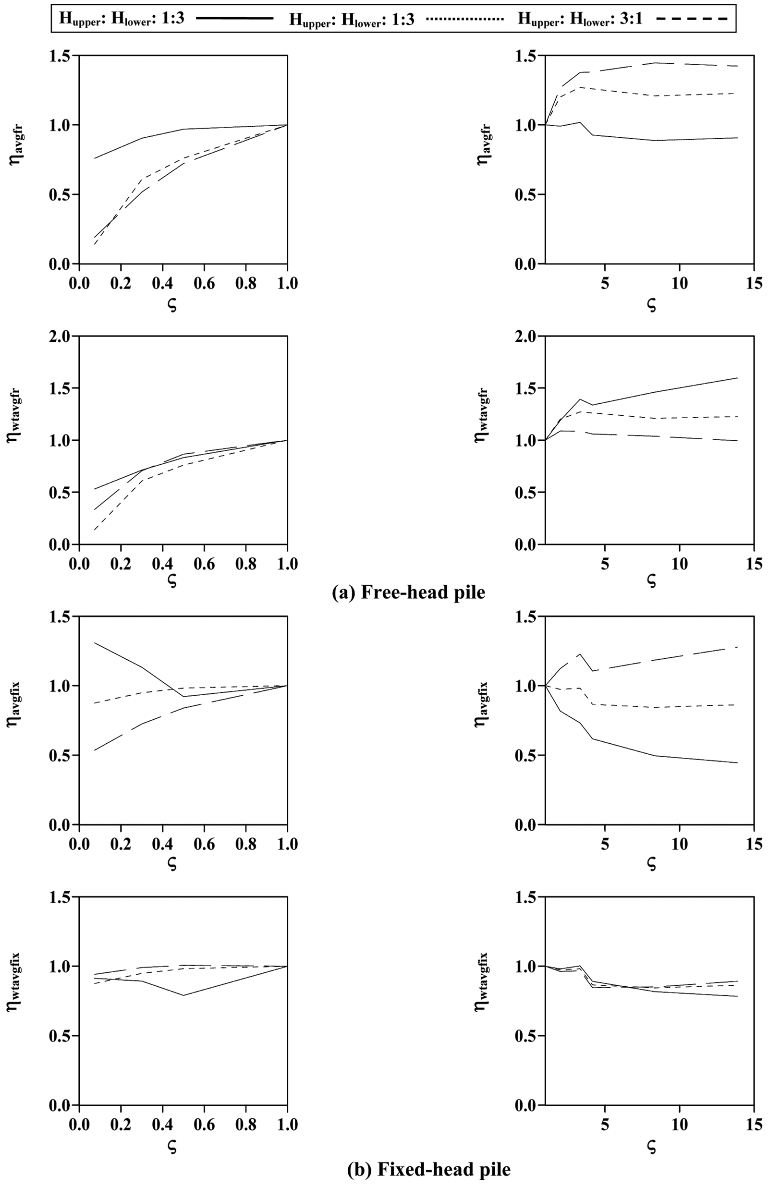


Fig. 8 Correction factors to determine the equivalent Young's Modulus of bi-layer soil for lateral capacity of pile

normalized by the average capacity (H_{avg}) and weighted average capacity (H_{wtavg}) is presented in Fig. 7 for both free and fixed-head piles as a function of the ratio of the modul of the upper and underlying soil layers representing stiffness contrast (ς). Such normalized quantities, also referred to as correction factors, are denoted as χ_{avgfr}, $\chi_{wtavgfr}$ for free-head condition and χ_{avgfix}, $\chi_{wtavgfix}$ for fixed-head condition respectively. Fig. 7 depicts a diverging trend in variation of both χ_{avgfr}, $\chi_{wtavgfr}$ with increase of stiffness contrast. On the other hand, a close scrutiny to the variation curves shows that the lateral capacity of fixed-head pile- relative to H_{avg} - albeit reveals diverging trend with increase of stiffness contrast, varies within a range of around 0.8 to 1.2 times H_{wtavg} over entire range of stiffness contrast.

In the context of routine design, it is perceived that appropriately defined fictitious single layer system in lieu of bi-layer one may be useful. To this end, it is attempted to search for an equivalent Young's modulus (E_{eq}) representative of the layered soil system. This parameter may enable to directly utilize the existing practice of assessing lateral capacity of pile embedded in homogeneous soil. In this process, capacity of the pile for particular combination of layer thickness and properties is evaluated and subsequently the equivalent modulus of elasticity for homogeneous soil layer necessary to predict same lateral capacity is computed through iterations. Subsequently, such equivalent soil modulus is normalized in two simple alternative approaches, *viz.* (a) by average Young's modulus of soil layers (E_{avg}) and (b) by the average Young's modulus weighted in proportion to the thickness of the relevant strata (E_{wtavg}). Such normalized equivalent modulus of elasticity of soil denoted as η_{avgfr}, $\eta_{wtavgfr}$ is presented in Fig. 8 (for free head pile) while the same for fixed head pile, designated as η_{avgfix}, $\eta_{wtavgfix}$ are also furnished in the same figure (Fig. 8).

Fig. 8 demonstrate that, for free-head piles, even if the layer thickness is equal, the equivalent E_{eq} deviates from the average E_{avg} by around 20% for considerable contrast in stiffness of soil layers. On the other hand, it is observed that E_{eq} approaches to the weighted average value E_{wtavg} as the thickness of the upper layer increases and, with decrease of upper layer thickness, tends to be closer to E_{avg}. On the other hand, for fixed-head pile, E_{eq} shows a propensity to be similar to the E_{wtavg}, rather than the average value E_{avg}.

6. Design recommendation

On the basis of the limited study, the present investigation attempts to propose two alternative approaches to predict lateral capacity of pile in bi-layered media. The pile may be assumed to be embedded in homogeneous soil having characteristics similar to each of the individual layer and subsequently H_{avg} and H_{wtavg} may be estimated from such capacities. Appropriate correction factors, *viz.*, χ_{avgfr}, $\chi_{wtavgfr}$, χ_{avgfix}, $\chi_{wtavgfix}$ may then be employed (refer to Fig. 7) to envisage the pile capacity in the layered soil.

Alternatively, observing the properties of the underlying layers (Young's modulus and layer thickness), suitable equivalent modulus E_{eq} may be estimated using relevant variation curves (refer to Fig. 8). This may then be used to assess desired lateral capacity using any suitable method.

It may appear that the relative stiffness of the pile-soil system regulating lateral capacity of pile is also dependent on pile diameter. However, though not included for brevity, such correction factors, *viz.*, χ_{avgfr}, $\chi_{wtavgfr}$, χ_{avgfix}, $\chi_{wtavgfix}$ and η_{avgfr}, $\eta_{wtavgfr}$, η_{avgfix}, $\eta_{wtavgfix}$ as may be obtained from Fig. 7 through Fig. 8, are found to be insensitive to the variation of pile cross-section at least for the cases of short-pile examined herein.

7. Example case studies

The applicability of the design charts and procedure suggested herein is demonstrated through the following sample case studies.

7.1 Fixed-head short pile

A *R/C* pile (concrete grade M20) of 1.0 m diameter and 7.0 m length is embedded in a soil comprising of a 3.5 m thick of hard clay (Layer-I) followed by a 3.5 m thick medium clay (Layer-II). Capacities computed in two alternative approaches as presented in Table 2(a) and Table 2(b) confirm the adequacy of the design strategies proposed herein.

7.2 Free-head short pile

As a second example, a *R/C* pile (concrete grade M20) of 1.5 m diameter and 8.0 m length embedded in a soil comprising of a 2.0 m thick medium clay (Layer-I) followed by stiff clay (Layer-II) is considered. Lateral capacities computed in two alternative approaches are compared to what is obtained from the original system. Results presented in Table 3(a) and Table 3(b) manifest that the scheme emerged from the present investigation may be adopted for practical purpose.

Table 2(a) Example case of a fixed-head pile using equivalent Young's Modulus

Young's Modulus (E) of soil layers (kN/m²)		ς	E_{avg} [*1] or $[E_{wtavg}]$	η_{avgfix} [*2] or $[h_{wtavgfix}]$	E_{eq} (kN/m²)	Capacity (kN)		% deviation
L-I	L-II					Actual	Proposed method	
18750	3200	5.86	10975	0.84	9219	180.25	178.06	1.22

[*1]: E_{avg} and E_{wtavg} are identical as thickness of L-I and L-II is same.
[*2]: Refer to Fig. 8.

Table 2(b) Example case of a fixed-head pile using capacities in individual layers

Young's Modulus (E) of soil layers (kN/m²)		ς	Capacity in individual layer (kN)		H_{avg} or $[H_{wtavg}]$	χ_{avgfix} [*1] or $[\chi_{wtavgfix}]$	Capacity (kN)		% deviation
L-I	L-II		L-I	L-II			Actual	Proposed method	
18750	3200	5.86	298.21	68.90	183.55	0.96	180.25	176.21	2.24

[*1]: Refer to Fig. 7

Table 3(a) Example case of free-head pile using equivalent Young's modulus

Young's Modulus (E) of soil layers (kN/m²)		ς	E_{avg}	E_{wtavg}	η_{avgfix} [*1] (method A)	$\eta_{wtavgfix}$ [*1] (method B)	E_{eq} (kN/m²)		Capacity (kN)			% deviation	
L-I	L-II						A	B	Actual	Proposed method		A	B
										A	B		
3200	5625	0.569	4412.5	5018.7	0.96	0.84	4236.0	4240.8	47.2	45.0	46.5	4.7	1.5

[*1]: Refer to Fig. 8

Table 3(b) Example case of a free-head pile using capacities in individual layers

Young's Modulus (E) of soil layers (kN/m^2)		ς	H_{avg} *1	H_{wtavg} *2	χ_{avgfix} *3 (method A)	$\chi_{wtavgfix}$ *3 (method B)	Capacity (kN)			% deviation	
L-I	L-II						Actual	Proposed method A	Proposed method B	A	B
3200	5625	0.569	47.48	53.76	0.98	0.89	47.2	46.5	47.8	1.5	-1.3

*1: Lateral capacity in Layer-I is 34.91 kN
*2: Lateral capacity in Layer-II is 60.05 kN
*3: Refer to Fig. 7.

8. Conclusions

Evaluation of lateral load carrying capacity of short pile in layered soil is undertaken in the current investigation. Two alternative approaches to this end are illustrated as may be summarized below.

(1) The lateral capacity of pile in bi-layer system may be obtained from two sets of analyses in the first scheme. Capacity of the reference pile embedded in homogeneous soil having characteristics similar to each constituent layer properties is evaluated separately. Such capacities may then be collated employing appropriate correction factors as depicted through Fig. 7.

(2) However, since the procedure depicted above requires two step analyses in case of a bi-layer system, equivalent Young's Modulus may be computed prior to the capacity estimation using the aids furnished in Fig. 8. Such values may subsequently be used to estimate lateral capacity of piles in bi-layer system. Thus, from a single analysis, lateral capacity of short pile may be estimated.

In view of the accuracy of both the proposed approaches (refer to examples shown), it appears that such procedures are rational and useful to exercise in practice. It is believed that for intermediate values of the proportion of layer thickness, appropriate correction factor, based on experience and judgment, may be obtained from the design curves through interpolation. Further, by successive application of this procedure, it is envisaged that the proposed method may be extended to multi-layer deposit – treated beyond the scope of the current investigation. It may be noted that the elastic properties of pile-soil system are conceptually more relevant for analyzing elastic range response and hence used herein as fundamental parameters. However, such parameters are not often estimated with due rigor in routine geotechnical investigation and hence a judicious choice of such parameters based on site-specific correlation or using the strength test data are indispensable.

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