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Stability and non-stationary vibration analysis of beams subjected to periodic axial forces using discrete singular convolution

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Abstract. Dynamic instability of beams subjected to periodic axial forces is studied using the discrete singular convolution (DSC) method with the regularized Shannon's delta kernel. The principal regions of dynamic instability under different boundary conditions are examined in detail, and the non-stationary vibrations near the stability-instability critical regions have been investigated. It is found that the results obtained by using the DSC method are consistent with the analytical solutions, which shows that the DSC algorithm is suitable for the problems considered in this study. It was found that there is a narrow region of beat vibration existed in the vicinity of one side ($\theta/\Omega > 1$) of the boundaries of the instable region for each condition.

Keywords: dynamic stability; non-stationary vibration; discrete singular convolution (DSC)

1. Introduction

When a transverse vibrating beam is excited under the periodic axial force, parametric resonances related to the dynamic stability is important crucial for engineering design of beam structures. In a parametric resonance problem, the vibration will diverge, and properly devised procedures are needed to capture the important physics. The divergence is not due to the force amplitude reaching to the critical Euler force of buckling. Instead, it is known mainly caused by the relation between the frequency of the axial force and the natural frequency of a beam, and hence care must be taken in the analysis of such beams.

The dynamic stability of beams under periodic loads has been studied by many researches such as Bolotin (1965), Evan-Iwanowski (1965), Iwatsubo *et al.* (1973), Lee (1996), Saffari (2012). The stability-instability critical regions have been investigated by analytical methods (Bolotin 1965, Lee 1996) and numerical methods (Briseghella *et al.* 1998, Iwatsubo *et al.* 1972). Non-stationary vibrations near the stability-instability critical regions for Euler-type and Beck-type problems of a

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cantilevered beam have been investigated by an analogue simulation method (Iwatsubo *et al.* 1972). Recently, a discrete singular convolution (DSC) method proposed by Wei (1999) has emerged as a relatively new approach for numerical solutions to differential equations, which has the global methods' accuracy and the local methods' flexibility for handling complex geometry and boundary conditions. The DSC method has been increasingly applied to many problems in engineering and sciences (Wei 2001, Zhao *et al.* 2002, Lim *et al.* 2005, Civalek 2008a). However, most of works focused on the calculations of natural frequencies of beams (Zhao *et al.* 2005), plates (Civalek and Acar 2007) and shells (Civalek and Gürses 2009), and the discussion of static buckling (Civalek 2008b). To the authors' knowledge, there are no published works which studied the problems of dynamic stability and non-stationary vibrations of beams by using the DSC method.

In this paper, the DSC method is employed to study the stability and non-stationary vibrations of beams subjected to different boundary conditions, including simply supported-simply supported (SS) , clamped-clamped (CC), clamped-simply supported (CS), clamped-free (CF). In the present study, the displacement responses of beams in time domain are calculated by the DSC method. The stability-instability regions are then obtained through stability examination of the responses. Detailed studies on the non-stationary vibrations will be discussed. The calculation results show that the DSC method is effective to study this kind of problems.

2. Discrete singular convolution (DSC)

The DSC method was originally introduced by Wei (1999) as a simple and highly efficient numerical technique, which occurs commonly in mathematical physics and engineering. Details of the DSC method could be referred to the works of Wei and his colleagues (Wei 2001a, b, Wei *et al.* 2002). Here we provide a brief introduction to DSC. Consider a distribution T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by Wei (2001a).

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{1}$$

where $T(t-x)$ is a singular kernel. The DSC algorithm uses many approximation kernels. Recently, the regularized Shannon kernel (RSK) was proposed to solve applied mechanic problems. The RSK is given by Wei (2001b).

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)}\exp\left[\frac{(x-x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0 \tag{2}$$

where Δ is the grid spacing. Eq. (2) can be used to provide discrete approximations to the singular convolution kernels of the delta type (Zhao *et al.* 2005).

$$f^{(n)}(x) = \sum_{k=-M}^{M} \delta_{\Delta}^{(n)}(x-x_k)f(x_k) \tag{3}$$

where $\delta_{\Delta}(x-x_k) = \Delta\delta_{\alpha}(x-x_k)$ and superscript (n) denotes the nth-order derivative, and $2M+1$ is the computational bandwidth which is centred around x and is usually smaller than the whole computational domain. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point xi are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x-x_M,\ x+x_M]$. This can be expressed as (Wei 2001b).

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k)\,, \quad n = 0, 1, 2, \ldots \tag{4}$$

where superscript (n) denotes the nth-order derivative with respect to x.

When the regularized Shannon's delta (RSD) kernel is used, the detailed expressions for $\delta^{(n)}_{\Delta,\sigma}$ can be obtained effectively, readers can refer to some published references (Wei 2001b, Zhao and Wei 2002).

Note the procedure of DSC is quite similar to the so-called smoothed particle hydrodynamics (SPH) (Liu and Liu 2003). The major difference may be in the choice of kernels.

3. Theory and algorithm for vibrating beams

3.1 Equation of motion

The main purpose of this paper is to study the stability and non-stationary vibrations of beams. For simplicity, we assume that the beam has a uniform cross-section. It is supported with different boundary conditions, and is subjected to time-periodic axial forces, as shown in Fig. 1. The equation of motion for governing this type of problems is a 4th order differential equation (Bolotin 1965)

$$EI\frac{\partial^4 u}{\partial x^4} + P_D\cos(\theta t)\frac{\partial^2 u}{\partial x^2} + m\frac{\partial^2 u}{\partial t^2} = 0 \tag{5}$$

where E, I, m are Young's modulus, second moment of area and mass per unit length, respectively. $P_D\cos(\theta t)$ is a time-dependent coefficient in which P_D is the amplitude and θ is the circular frequency of the axial force. $u(x, t)$ is the transverse displacement for any point x on the beam with time t.

For simplicity, one can introduce the dimensionless quantities

$$x = \frac{x}{L}, \quad U = \frac{u}{L}, \quad \beta\frac{P_D}{P_{cr}}, \quad \Theta = \frac{\theta}{\Omega}, \quad \tau = \Omega t, \quad \beta = \frac{\beta}{2}$$

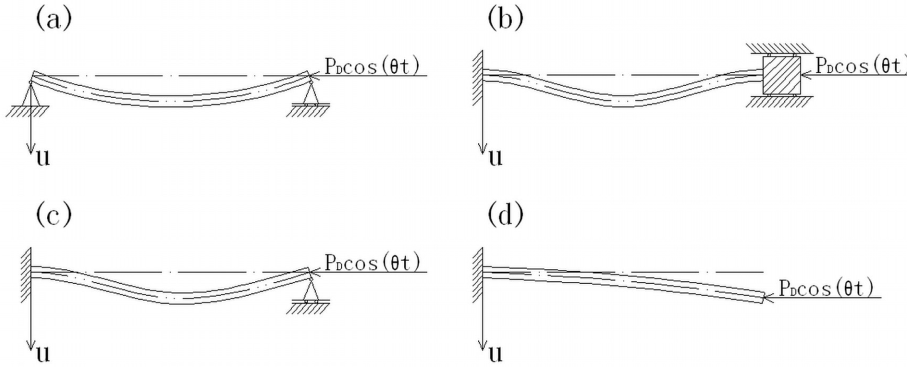

Fig. 1 Analytical models of beams (a) S-S, (b) C-C, (c) C-S, (d) C-F

where the bucking load $P_{cr} = \pi^2 EI/(\eta L)^2$, η is the length coefficient, L is the length of the beam, the natural circular bending frequency $\Omega = \lambda^2\sqrt{EI/m}$, λ is the eigenvalue.

Eq. (5) can then be rewritten as

$$\frac{1}{(\lambda L)^4}\frac{\partial^4 U}{\partial X^4}+\frac{2\mu\cos(\Theta\tau)}{(\lambda L)^4}\frac{\pi^2}{\eta^2}\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial\tau^2} = 0 \tag{6}$$

which also can be changed into

$$\frac{\partial^2 U}{\partial\tau^2} = -\frac{1}{(\lambda L)^4}\frac{\partial^4 U}{\partial X^4}-\frac{2\mu\cos(\Theta\tau)}{(\lambda L)^4}\frac{\pi^2}{\eta^2}\frac{\partial^2 U}{\partial X^2} \tag{7}$$

The boundary conditions can be one of the following:

Simply supported edge (S)

$$U = 0, \quad \frac{\partial^2 U}{\partial X^2} = 0 \tag{8}$$

Clamped edge (C)

$$U = 0, \quad \frac{\partial U}{\partial X} = 0 \tag{9}$$

Free edge with axial compressive force (F)

$$\frac{\partial^2 U}{\partial X^2} = 0, \quad \frac{\partial^3 U}{\partial X^3}+2\mu(\pi/\eta)^2\cos(\Theta\tau)\partial U/\partial X = 0 \tag{10}$$

It is noted this is different from the free edge of cantilever beam without axial compressive force.

3.2 Algorithm

The DSC algorithm is utilized for the spatial discretization and the fourth order Runge-Kutta (RK4) scheme is used for the time discretization. Details of the procedure are as follows:

1) The computational domain of coordinate X is [0, 1];
2) The coordinate X is equally spaced;
3) The grid sizes are denoted by $\Delta X = (1-0)/N$, where N is the total number of partition grid on the computational domain [0, 1];
4) The grid points are denoted by $X_j = j\Delta X\ (j = 1, 2, \ldots, N+1)$, So $X_j - X_{j+k} = -k\Delta X$. The approximate value of U at the grid point X_j is expressed as U_j, Eq. (7) is changed into

$$\begin{cases} \dfrac{\partial U_j}{\partial\tau} = V_j \\ \dfrac{\partial V_j}{\partial\tau} = -\dfrac{1}{(\lambda L)^4}\dfrac{\partial^4 U_j}{\partial X^4}-\dfrac{2\mu\cos(\Theta\tau)}{(\lambda L)^4}\dfrac{\pi^2}{\eta^2}\dfrac{\partial^2 U_j}{\partial X^2} \end{cases} \quad (j = 1, 2, \ldots, N+1) \tag{11}$$

Let

$$\begin{aligned}\{y_j\} &= \{y_1\ \ y_2\ \ \ldots\ \ y_{2N+2}\} \\ &= \{U_1\ \ U_2\ \ \ldots\ \ U_{N+1}\ \ V_1\ \ V_2\ \ \ldots\ \ V_{N+1}\}\end{aligned} \quad (j = 1, 2, \ldots, 2N+2) \tag{12}$$

At $j = 1, 2, \ldots, N+1$, we have

$$f_j = y_{j+N+1} \tag{13}$$

When $j = N+2, N+3, \ldots, 2N+2$, we obtain

$$f_j = -\frac{1}{(\lambda L)^4}\frac{\partial^4 U_{j-N-1}}{\partial X^4} - \frac{2\mu\cos(\Theta\tau)}{(\lambda L)^4}\frac{\pi^2\partial^2 U_{j-N-1}}{\eta^2\partial X^2} \tag{14}$$

Then, we have a unified semi-discretized equation

$$\frac{\partial y_j}{\partial \tau} = f_j \quad (j = 1, 2, \ldots, 2N+2) \tag{15}$$

The temporal discretization expressions for Eq. (15) by using the fourth order Runge-Kutta method are given as

$$y_j^{n+1} = y_j^n + \Delta\tau y_{j+N+1}^n + \frac{\Delta\tau^2}{6}(L_{j+N+1,1} + L_{j+N+1,2} + L_{j+N+1,3}) \quad (j = 1, 2, \ldots, N+1) \tag{16}$$

$$y_j^{n+1} = y_j^n + \frac{\Delta\tau}{6}(L_{j,1} + 2L_{j,2} + 2L_{j,3} + L_{j,4}) \quad (j = N+2, N+3, \ldots, 2N+2) \tag{17}$$

Where

$$L_{j,1} = f_{j,1}^n,\ L_{j,2} = f_{j,2}^n, \quad L_{j,3} = f_{j,3}^n, \quad L_{j,4} = f_{j,4}^n \quad (j = N+2, N+3, \ldots, 2N+2) \tag{18}$$

Here superscript n denotes time level, $\Delta\tau$ is the time step, so $\tau = n\Delta\tau$. Using the DSC discrete scheme in Eq. (3), the discretization expressions for $f_{j,1}^n, f_{j,2}^n, f_{j,3}^n, f_{j,4}^n$ are

$$L_{j,1} = f_{j,1}^n = -\frac{1}{(\lambda L)^4}\sum_{k=-W}^{+W}\delta_{\Delta,\sigma}^{(4)}(-k\Delta X)y_{j+k-N-1}^n - \frac{2\mu\cos(\Theta\tau)}{(\lambda L)^4}\frac{\pi^2}{\eta^2}\sum_{k=-W}^{+W}\delta_{\Delta,\sigma}^{(2)}(-k\Delta X)y_{j+k-N-1}^n$$

$$L_{j,2} = f_{j,2}^n = -\frac{1}{(\lambda L)^4}\sum_{k=-W}^{+W}\delta_{\Delta,\sigma}^{(4)}(-k\Delta X)(y_{j+k-N-1}^n + \frac{\Delta\tau}{2}y_{j+k}^n) -$$
$$\frac{2\mu\cos(\Theta\tau)}{(\lambda L)^4}\frac{\pi^2}{\eta^2}\sum_{k=-W}^{+W}\delta_{\Delta,\sigma}^{(2)}(-k\Delta X)(y_{j+k-N-1}^n + \frac{\Delta\tau}{2}y_{j+k}^n)$$

$$L_{j,3} = f_{j,3}^n = -\frac{1}{(\lambda L)^4}\sum_{k=-W}^{+W}\delta_{\Delta,\sigma}^{(4)}(-k\Delta X)(y_{j+k-N-1}^n + \frac{\Delta\tau}{2}y_{j+k}^n + \frac{\Delta\tau^2}{4}L_{j+k,1}) -$$
$$\frac{2\mu\cos(\Theta\tau)}{(\lambda L)^4}\frac{\pi^2}{\eta^2}\sum_{k=-W}^{+W}\delta_{\Delta,\sigma}^{(2)}(-k\Delta X)(y_{j+k-N-1}^n + \frac{\Delta\tau}{2}y_{j+k}^n + \frac{\Delta\tau^2}{4}L_{j+k,1})$$

$$L_{j,4} = f_{j,4}^n = -\frac{1}{(\lambda L)^4}\sum_{k=-W}^{+W}\delta_{\Delta,\sigma}^{(4)}(-k\Delta X)(y_{j+k-N-1}^n + \Delta\tau y_{j+k}^n + \frac{\Delta\tau^2}{2}L_{j+k,2}) -$$
$$\frac{2\mu\cos(\Theta\tau)}{(\lambda L)^4}\frac{\pi^2}{\eta^2}\sum_{k=-W}^{+W}\delta_{\Delta,\sigma}^{(2)}(-k\Delta X)(y_{j+k-N-1}^n + \Delta\tau y_{j+k}^n + \frac{\Delta\tau^2}{2}L_{j+k,2}) \tag{19}$$
$$(j = N+2, \quad N+3, \quad \cdots, \quad 2N+2)$$

where $[-W, W]$ is the computational bandwidth. Kernels $\delta_{\Delta,\sigma}^{(2)}$ and $\delta_{\Delta,\sigma}^{(4)}$ can be easily obtained (Wei 2001b). All of these coefficients are only dependent on grid size. When the grid point distribution is given, the coefficients can be computed once and stored for use during the computation.

For the simple boundaries such as simply supported edge and clamped edge, antisymmetric extension and symmetric extension are conducted by Zhao and Wei (2002), respectively. For the free edge and other edges, Wei and his group proposed the iteratively matched boundary (IMB) method (Zhao *et al.* 2005) and the matched interface and boundary (MIB) method (Zhao and Wei 2004, Yu *et al.* 2006, Yu *et al.* 2009). In this paper, the IMB method is used to handle the free edge with axial force shown in Eq. (10).

The overall calculation procedure can be summarized as follows:

1) With the initial values for y_j^0 or the values of y_j^n $(j = 1, 2, \ldots, 2N+2)$ at previous time level n;
2) The expressions of Eq. (19) are directly calculated to obtain the values of $f_{j,1}^n, f_{j,2}^n, f_{j,3}^n, f_{j,4}^n$ $(j = N+2, N+3, \ldots, 2N+2)$ at the time level n;
3) Substituting Eq. (18) into Eqs. (16), (17), the values of y_j^n $(j = 1, 2, \ldots, 2N+2)$ at new time level $n+1$ can be calculated;
4) The computational time is advanced (i.e., $\tau = \tau + \Delta\tau$, $n+1$), and the whole procedure above is repeated, until the calculation precision is reached.

4. Instability and non-stationary vibrations of beams

4.1 Instability criteria

Two criteria for unstable vibration have been applied to the dynamic stability analysis by Iwatsubo *et al.* (1965). One of the criteria corresponds to the dynamic stability, and another corresponds to the beating vibration. Criterion means if the amplitude of the vibration of the system increases monotonically during the twenty periods of the periodic load, the system is considered to be dynamically unstable. Criterion means when a vibration is a beat and the maximum amplitude of the beat is larger than double the initial displacement, then the beat is defined as an unstable beat. Otherwise the beat is stable.

4.2 Instability and non-stationary vibrations

For the principal unstable regions of the first mode, the domain of μ is [0, 0.5]. Θ is in the vicinity of two times the natural bending frequency of the first mode. We choose the domain Θ as [1.5, 2.5]. When boundary conditions and initial conditions including initial displacement and velocity are given, the length coefficient η and eigenvalue λ (or λL) will be determined. The displacement responses will be carried out from the governing equation by using the DSC algorithm. Then the stability of the beam will be obtained by judging the stability of the displacement responses. Consequently the unstable region, region of beat and stable region will be obtained by using the two criteria.

For simply supported-simply supported beam, η = 1 for the first bucking, while $\lambda L = \pi$ for the first mode. We set the initial dimensionless displacement as the shape function of the first mode

$U(i\Delta X, 0) = \sin(i\Delta X\pi)$ $(i = 0, 1, \ldots, N)$, the initial dimensionless velocity as $\partial U(X, 0)/\partial\tau = 0$, $N = 18$, $W = 15$, $r = 2.5$, and $\Delta\tau = 2.5\times10^{-6}$.

For clamped-clamped beam, $\eta = 0.5$ for the first bucking, while $\lambda L = 4.7300$ for the first mode. We set the initial dimensionless displacement as the shape function of the first mode $U(X, 0) =$ [0, 0.0325, 0.1191, 0.2435, 0.3900, 0.5435, 0.6901, 0.8178, 0.9164, 0.9787, 1.0000, 0.9787, 0.9164, 0.8178, 0.6901, 0.5435, 0.3900, 0.2435, 0.1191, 0.0325, 0], $\partial U(X,0)/\partial\tau = 0, N = 20$, $W = 15$, $r = 2.5$, and $\Delta\tau = 2.5\times10^{-6}$.

For clamped-simply supported beam, $\eta = 0.6692$ for the first bucking, while $\lambda L = 3.9266$ for the first mode. We set as $U(X, 0) =$ [0, 0.0294, 0.1082, 0.2229, 0.3599, 0.5062, 0.6498, 0.7796, 0.8860, 0.9614, 1.0000, 0.9981, 0.9543, 0.8697, 0.7470, 0.5915, 0.4096, 0.2095, 0], and $\partial U(X, 0)/\partial\tau = 0$, $N = 18$, $W = 15$, $r = 2.5$, and $\Delta\tau = 2.5\times10^{-6}$.

For clamped-free beam, $\eta = 2$ for the first bucking, while $\lambda L = 1.8751$ for the first mode. We set $U(X, 0) =$ [0, 0.0117, 0.0451, 0.0973, 0.1655, 0.2471, 0.3395, 0.4402, 0.5469, 0.6577, 0.7710, 0.8853, 1.0000], $\partial U(X,0)/\partial\tau = 0, N = 12$, $W = 10$, $r = 2.0$, and $\Delta\tau = 2.5\times10^{-6}$

Fig. 2 shows dimensionless displacement responses $U(0.5, \tau)$ of midpoint for simply supported-simply supported beam. It is found that the results in Figs. 2(2), (3), and (8) are unstable, the results

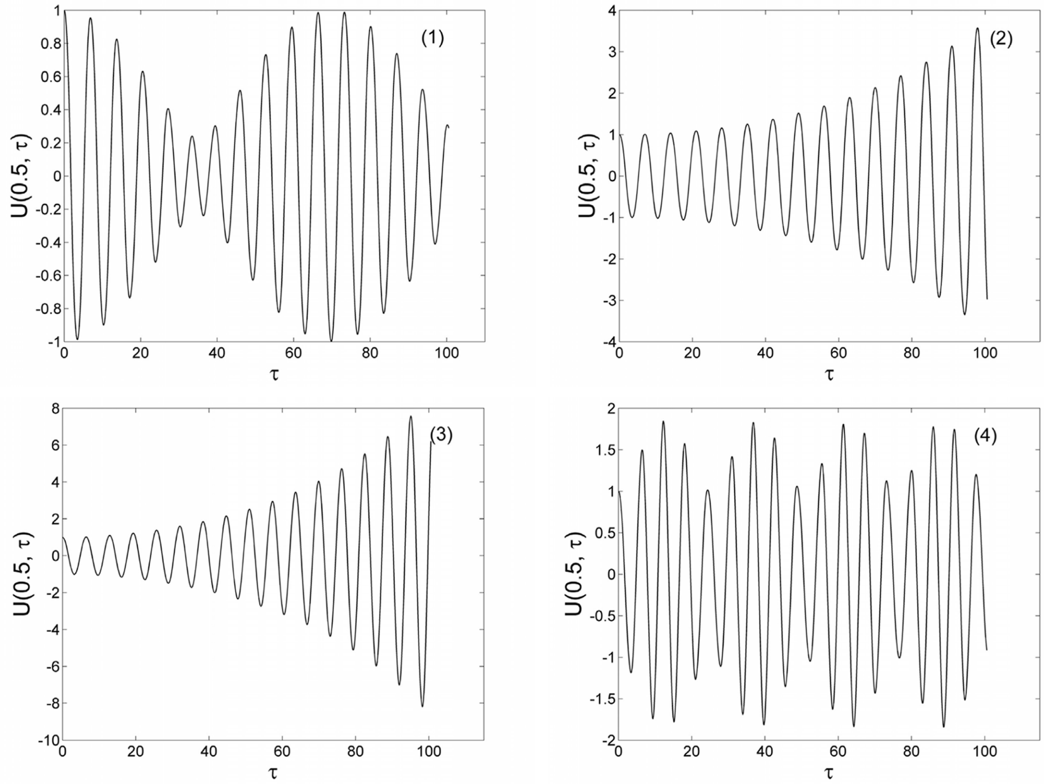

Fig. 2 Dimensionless displacement responses of midpoint for a simply supported-simply supported beam
(1) $\theta/2\Omega = 0.9$, $\mu = 0.175$ (2) $\theta/2\Omega = 0.9$, $\mu = 0.2$ (3) $\theta/2\Omega = 1.0$, $\mu = 0.05$ (4) $\theta/2\Omega = 1.15$, $\mu = 0.175$
(5) $\theta/2\Omega = 1.15$, $\mu = 0.2$ (6) $\theta/2\Omega = 1.15$, $\mu = 0.25$ (7) $\theta/2\Omega = 1.15$, $\mu = 0.3$ (8) $\theta/2\Omega = 1.15$, $\mu = 0.325$

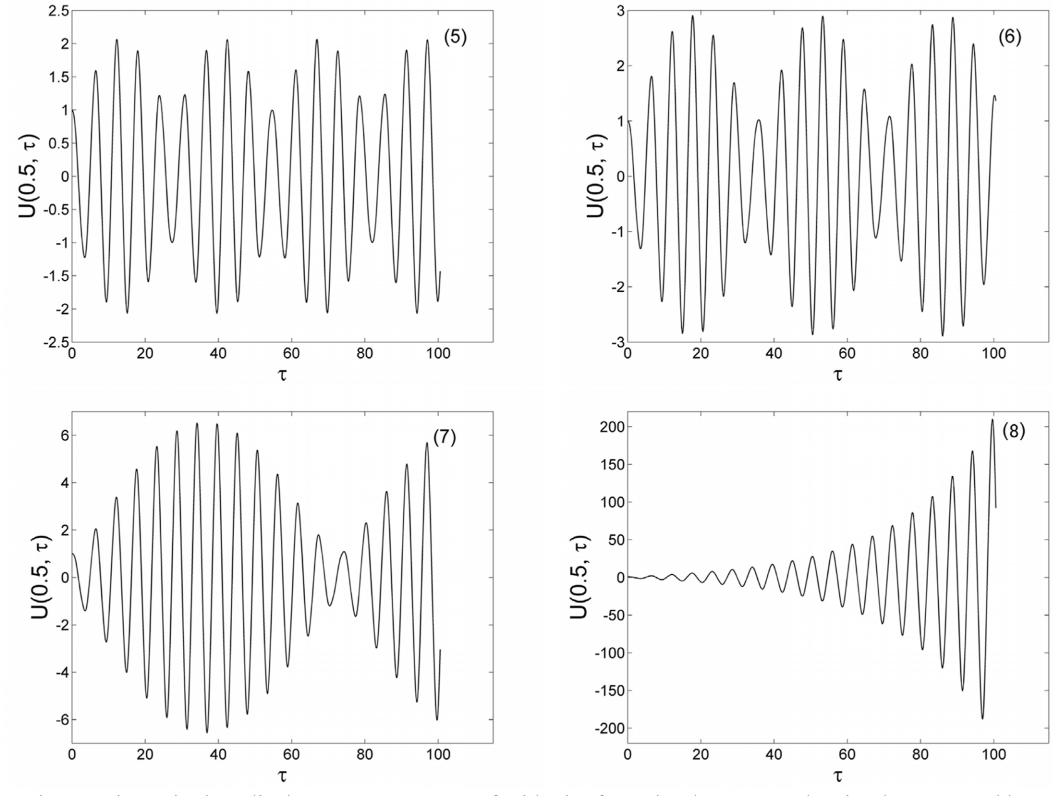

Fig. 2 Continued

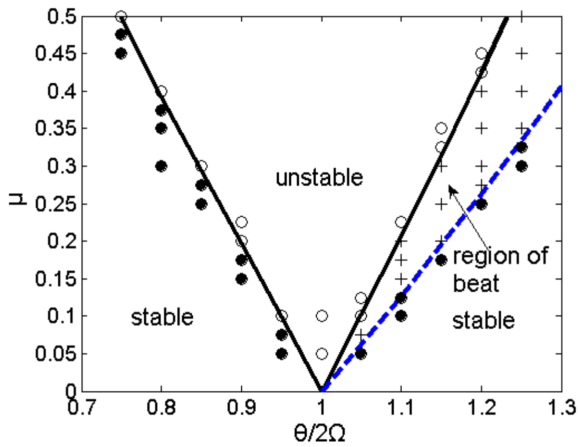

Fig. 3 Principal region of instability and the region of beat for a simply supported-simply supported beam. —, Analytical boundary by criterion I; ---, Analytical boundary by criterion II; '●', Stable results by DSC; '○', Unstable results by DSC; '+', Unstable beat results by DSC

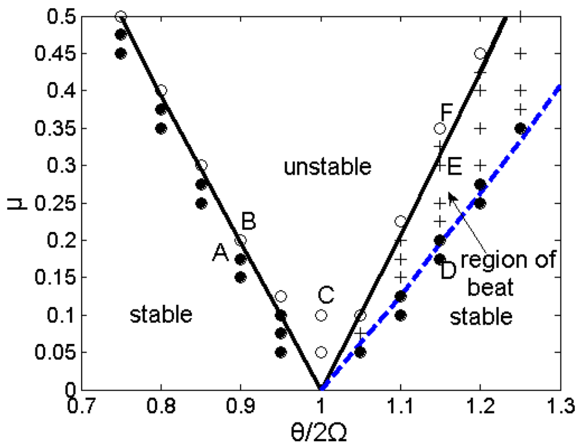

Fig. 4 Principal region of instability and the region of beat for a clamped-clamped beam. The legends are same as those in Fig. 3

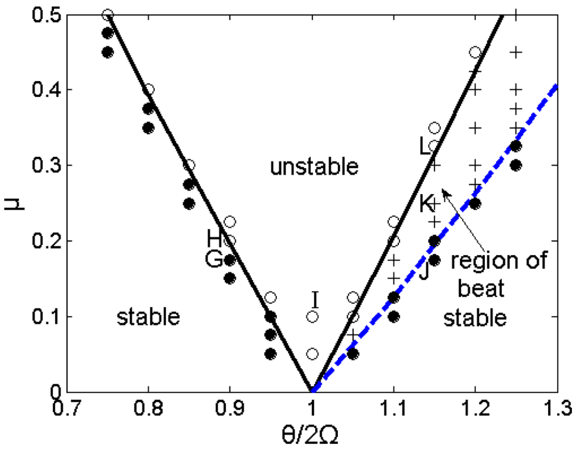

Fig. 5 Principal region of instability and the region of beat for clamped-simply supported beam. The legends are same as those in Fig. 3

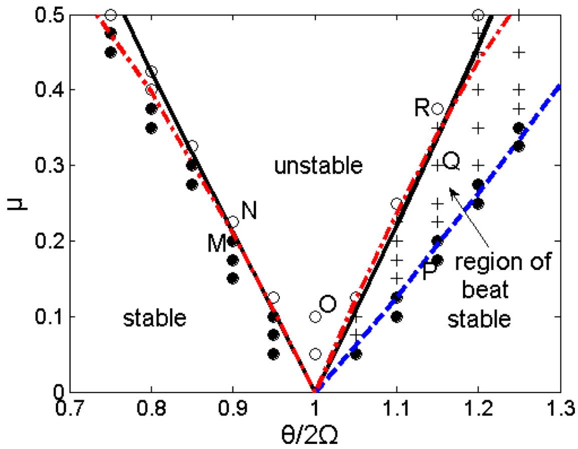

Fig. 6 Principal region of instability and the region of beat for a clamped-free beam, The legends are same as those in Fig. 3, and the dash-dot line is from the works by Iwatsubo *et al.* (1972)

in Figs. 2(1) and (4) are stable, and the maximum amplitudes of the beats are larger than double the initial displacements in Figs. 2(5), (6), and(7), respectively. Meanwhile, it is found that the vibration is from stable and unstable beat with increasing μ from Figs. 2(4)-(8). By repeating the whole calculation procedure with different parameters, the stability for other cases will be obtained. Therefore the chart for principal region of instability and the region of beat shown in Fig. 3 can be obtained.

It could be clearly found that the stability and instability regions by using DSC method agree well with the analytical regions given in Fig. 3, respectively. Meanwhile it is found that there is a narrow region of beat between the stable region and unstable region when $\theta/2\Omega > 1$, where the vibrations are non-stationary vibrations shown in Figs. 2(5)-(7).

Figs. 4-6 show the principal regions of instability and the regions of beat for clamped-clamped, clamped-simply supported and clamped-free beams, respectively. It is obviously that the unstable region, region of beat and stable region obtained by DSC method agree well with the analytical regions shown in each figure. It is also found that there is a narrow region of beat between the stable region and unstable region when $\theta/2\Omega > 1$ in each figure.

In order to demonstrate the vibrational features in different regions under different supporting conditions, some typical points located in different regions are chosen, which are Point A-F in Fig. 4, Point G-L in Fig. 5, Point M-R in Fig. 6. The corresponding displacement responses at each point are shown in Figs. 7-9, where the ordinate is dimensionless displacement, and the abscissa is dimensionless time τ. It is easy to be found that the vibrational curves shown in Figs. 7-9 clearly indicate which points are unstable, which points are stable and which points belong to beat vibration. It means that it is efficient and feasible to judge the vibrational types through the vibrational responses.

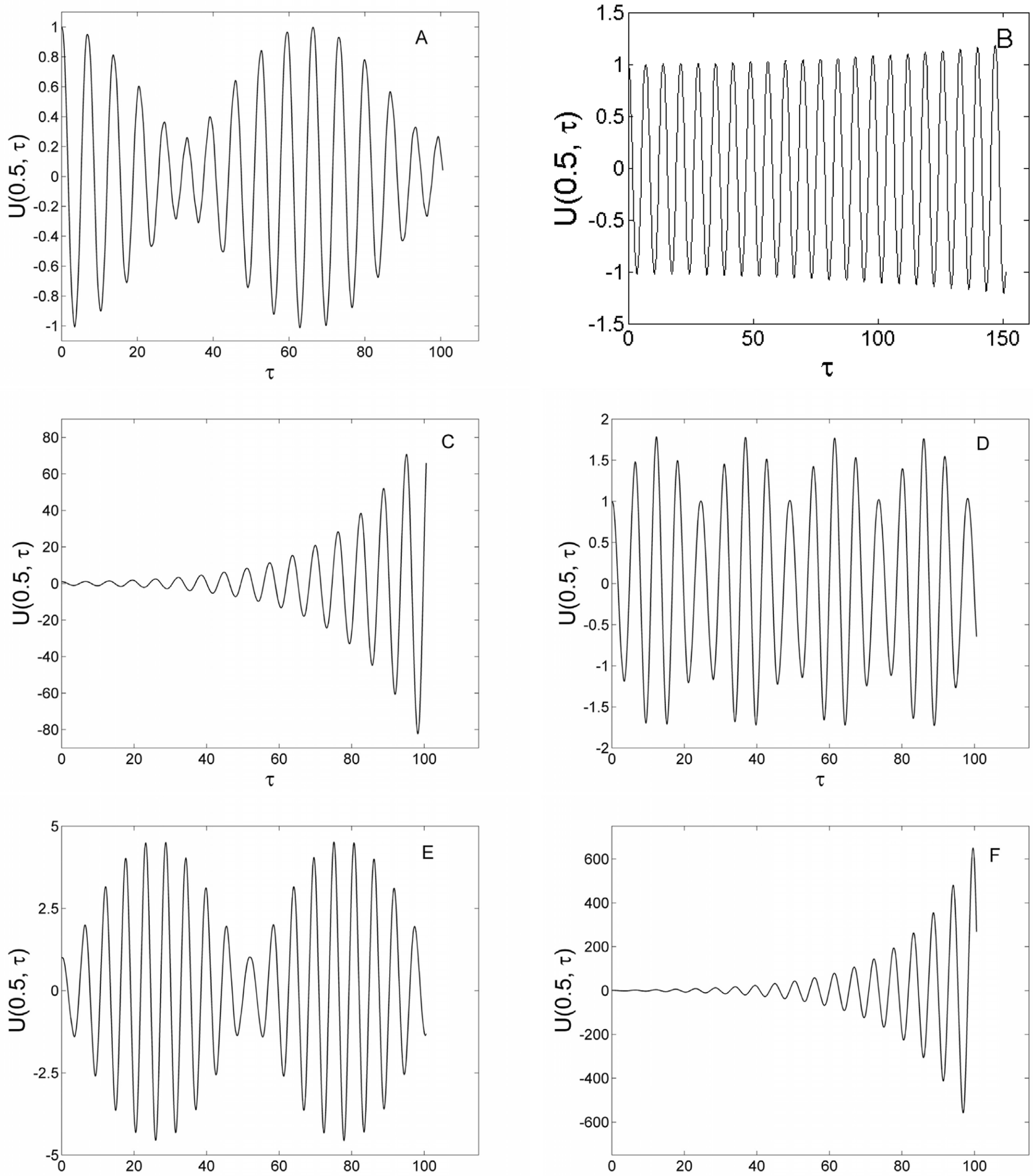

Fig. 7 Dimensionless displacement responses of midpoint for a clamped - clamped beam
(A) $\theta/2\Omega = 0.9$, $\mu = 0.175$ (B) $\theta/2\Omega = 0.9$, $\mu = 0.2$ (C) $\theta/2\Omega = 1.0$, $\mu = 0.1$ (D) $\theta/2\Omega = 1.15$, $\mu = 0.175$ (E) $\theta/2\Omega = 1.15$, $\mu = 0.3$ (F) $\theta/2\Omega = 1.15$, $\mu = 0.35$

5. Conclusions

In this paper, the stability and non-stationary vibrations of beams subjected to four different boundary conditions under periodic axial forces have been studied by the DSC method. Especially, the non-stationary vibrations near the stability-instability critical regions have been discovered by

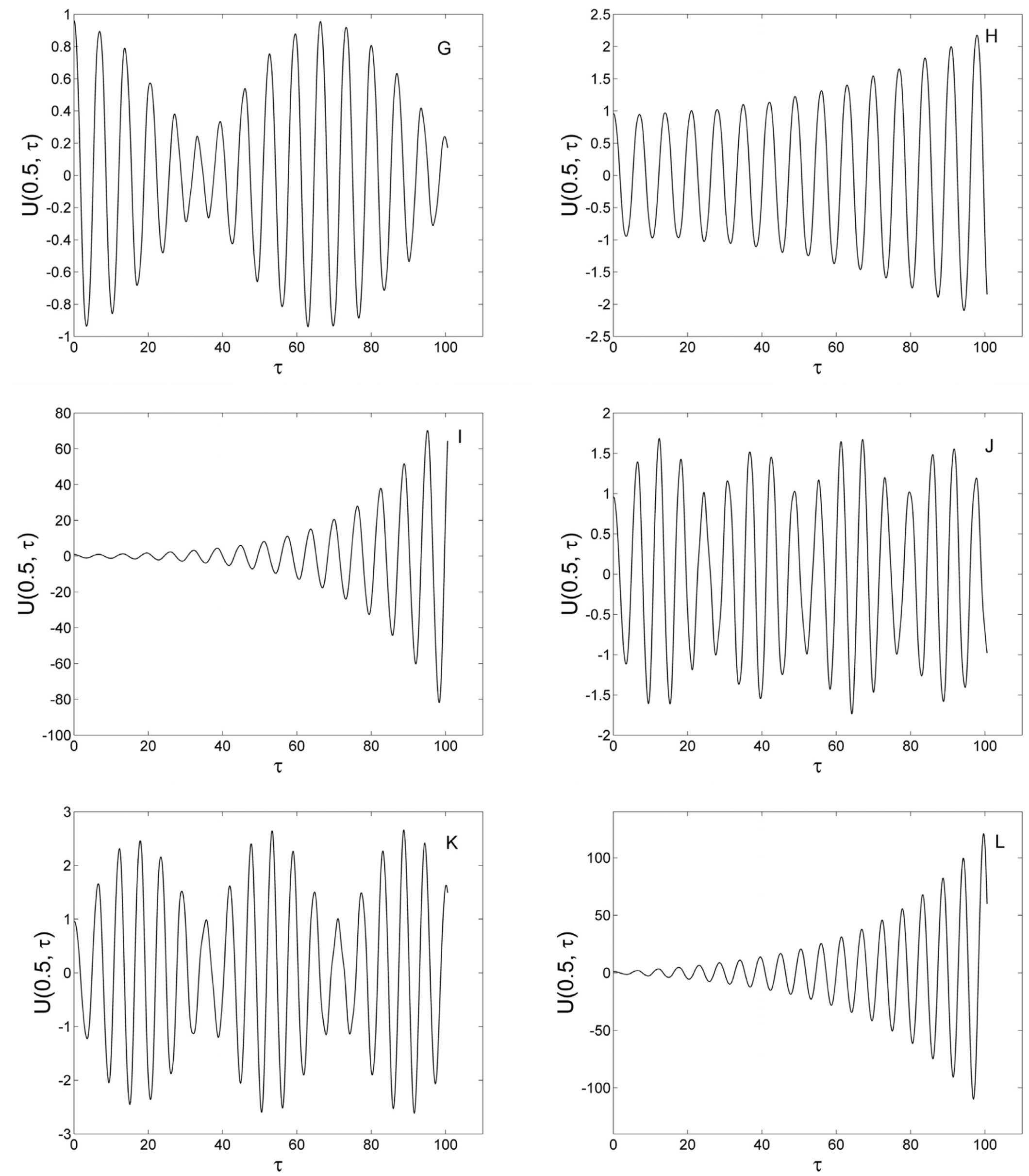

Fig. 8 Dimensionless displacement responses of midpoint for a clamped-simply supported beam
(G) $\theta/2\Omega = 0.9$, $\mu = 0.175$ (H) $\theta/2\Omega = 0.9$, $\mu = 0.2$ (I) $\theta/2\Omega = 1.0$, $\mu = 0.1$ (J) $\theta/2\Omega = 1.15$, $\mu = 0.175$
(K) $\theta/2\Omega = 1.15$, $\mu = 0.25$ (L) $\theta/2\Omega = 1.15$, $\mu = 0.325$

the DSC method. The principal regions of instability obtained by the DSC method are in considerably good agreements with the analytical regions for each condition, which verifies the applicability and effectiveness of the DSC method to the dynamic stability of beams.

In addition, it is found that there is a narrow region of beat vibration existed in the vicinity of one side ($\theta/2\Omega > 1$) of the boundaries of the region of instability for each condition by the DSC method.

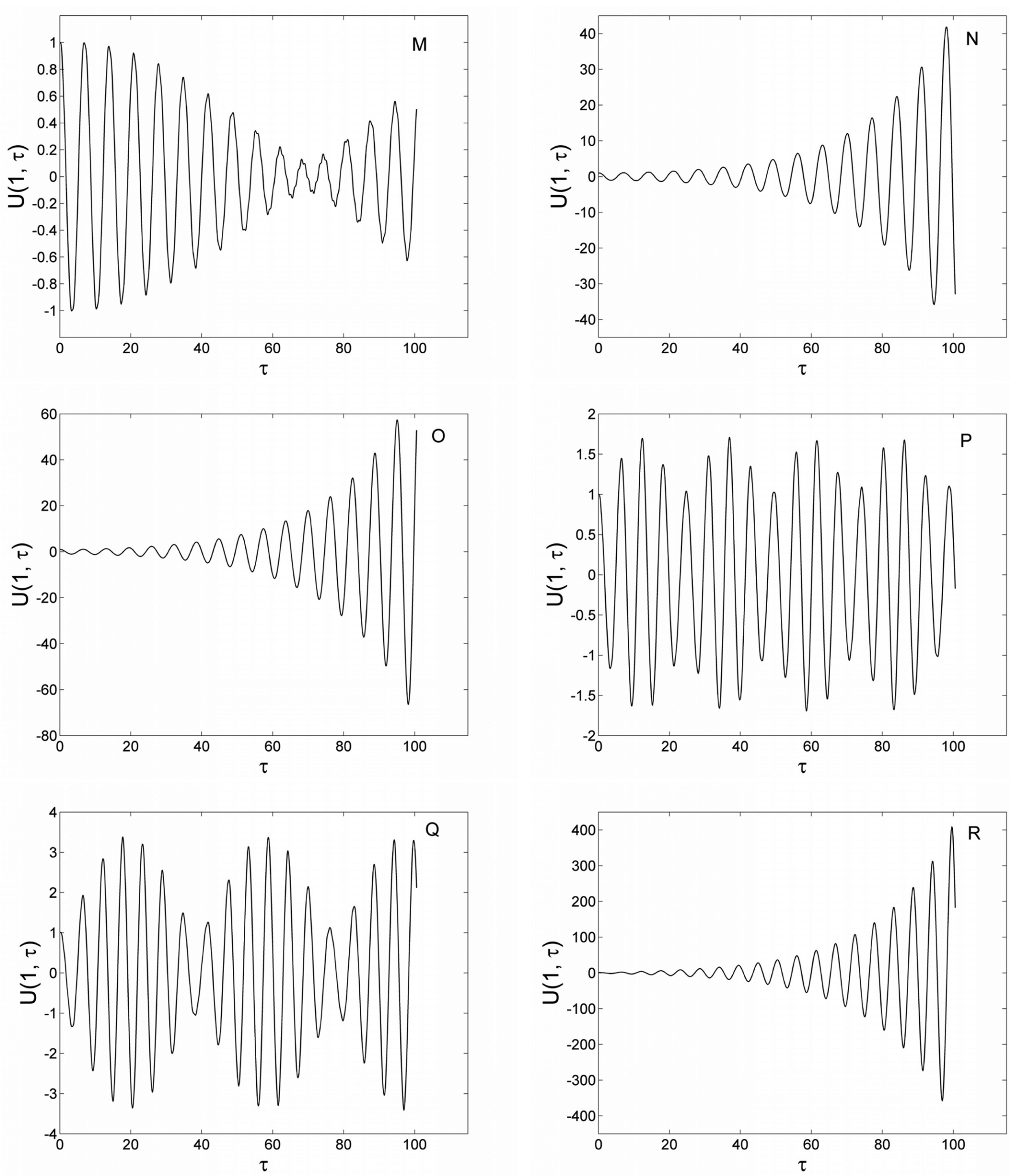

Fig. 9 Dimensionless displacement responses of endpoint at free edge for a clamped-free beam
(M) $\theta/2\Omega = 0.9$, $\mu = 0.2$ (N) $\theta/2\Omega = 0.9$, $\mu = 0.225$ (O) $\theta/2\Omega = 1.0$, $\mu = 0.1$ (P) $\theta/2\Omega = 1.15$, $\mu = 0.175$ (Q) $\theta/2\Omega = 1.15$, $\mu = 0.3$ (R) $\theta/2\Omega = 1.15$, $\mu = 0.375$

Moreover, the vibrational feature varies from stable vibration, stable beat vibration, unstable beat vibration to unstable vibration with the increase of μ when the value of $\theta/2\Omega$ is fixed. The calculated results show that the DSC method is a reliable and effective method for studying the non-stationary vibrations.

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