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Slope variation effect on large deflection of compliant beam using analytical approach

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Abstract. In this study the investigation of large deflections subject in compliant mechanisms is presented using homotopy perturbation method (HPM). The main purpose is to propose a convenient method of solution for the large deflection problem in compliant mechanisms in order to overcome the difficulty and complexity of conventional methods, as well as for the purpose of mathematical modeling and optimization. For simplicity, a cantilever beam of linear elastic material under horizontal, vertical and bending moment end point load is considered. The results show that the applied method is very accurate and capable for cantilever beams and can be used for a large category of practical problems for the aim of optimization. Also the consequence of effective parameters on the large deflection is analyzed and presented.

Keywords: compliant beam; homotopy perturbation method; large deflection; slope variation; non-linear problem

1. Introduction

The study of non-linear problems is important in many areas of science and engineering. One of these problems is the study of the non-linear bending of slender cantilever beams. Conventional mechanisms of engineering applications are strong and rigid. The use of flexibility has primarily been avoided due to the increased difficulty in accounting for flexibility and the large potential for performance degradation. Over the past years compliant mechanisms have become an important area and have vast development. It is usually made of a monolithic piece of material and its members usually undergo large deflection and rotation which leads to nonlinearity in geometry. Compliant mechanisms transmit motion and force by virtue of the elastic deformation of their flexible members. The advantages of these mechanisms over rigid-body mechanisms are due to the absence of rigid-body kinematic joints. Some of the many advantages are less friction and wear (Howell and Midha 1994), design for no assembly (Ananthasuresh and Kota 1995) and provision for non mechanical actuation (Moulton and Ananthasuresh 2001). Compliant mechanisms are used in product design (Howell 2001, Byers and Midha 1991, Crane *et al.* 2000, Ananthasuresh and Saggere 1994), Micro- Electro-Mechanical Systems (Kota *et al.* 1994), nonlinear springs (Jutte and

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Kota 2008), statically balanced mechanisms (Tolou and Herder 2009, Wang *et al.* 1997) and etc.

The analysis of compliant mechanisms, unlike rigid-body mechanisms, relies on a complex model that exactly describes the kinematics of a nonlinearly deflected beam. Therefore, deflections are not easy to determine by any method. Some methods are commonly used to analyze a large deflection of compliant mechanism. The first approach is based on elliptic integral formulation. Wang *et al.* (2008) investigated the large deflection of beams using elliptic integral formulation. They treated a class of large deflection beam problems where one end of the beam is being held while the other end portion is allowed to slide freely over a frictionless support. This approach is suitable for simple loading cases. It cannot solve any prismatic or non prismatic beam with simple uniformly distributed load (Wanga *et al.* 2001, Ohtsuki and Ellyin 2001, Liaoa 2009). The second approach utilizes finite element method (FEM) for solving elastic problems (Bathe 1996, Kačianauskas *et al.* 2005, Shabana 1998). This method requires the generation of a very fine mesh that increases computational time. The determination of the deflected shape of a bar bent through frictionless supports using finite elements was studied by Golley (1997). He concluded that the FEM method is useful in a wide range of applications such as pipeline analysis, curve fitting and etc. The third approach uses the finite differences method. This method requires a large number of nodes for accurate results and it is might to divergence in very large deflection cases. Saje and Srpcic (1985) presented a theory of large deformations of straight slender in-plane beam. In their work, the equilibrium equations are written on a deformed configuration and numerically solved for linear elastic cantilever. The forth method employs numerical integration with iterative shooting techniques. Recently, Lan and Lee (2006) presents a formulation based on shooting method (SM) for analyzing compliant mechanisms consisting of multiple flexible members. They have reported that the generalized SM can achieve higher-order accuracy relatively easily, and hence is more capable. A comprehensive study of this approach was reported by Yin *et al.* (2004). They developed analytical methods for analyzing the design of cantilever-like beams. Their solution was numerically validated by the other three numerical methods (SM, FDM and FEM) for computing the deflected shape of a flexible beam in comparison to the exact closed-form solution (Mattiasson 1981). Their results indicated a good agreement between these methods.

With the rapid development of nonlinear science, many different methods were proposed to solve the wide range of these problems. Addition to mentioned numerical method there has been some analytical method to solving compliant mechanisms such as homotopy analysis method (HAM) (Liao 1992), homotopy perturbation method (HPM) (He 1999, Ganji 2006, Safari *et al.* 2010, Hasanpour *et al.* 2011, Ganji *et al.* 2011) and etc.

Pirbodaghi *et al.* (2009) studied structural non-linearity of beams using the analytical method. They concluded that using the analytical technique leads to highly accurate solutions which are valid for a wide range of vibration amplitudes.

Kimiaeifar *et al.* (2011) used HAM to determine the analytical limit state functions and reliability index for large deflection of a cantilever beam subjected to static co-planar loading.

Wang *et al.* (2012) performed a solution based on HPM for large deflection of a cantilever beam under a terminal follower force. Pasharavesh *et al.* (2011) presented an analytical solution on nonlinear subject of beam. Also the structural modal reanalysis methods using HPM and projection a technique was presented by Massa *et al.* (2011).

The objective of this paper is to apply the HPM as an analytical approach to cantilever beam with constant initial curvature problem initial curvature problem. It is assumed that the beam is loaded by a known force and bending moment. The results have been obtained and the effects of various loading and bending moment have been investigated.

2. Mathematical model

Where the axial dimension is much larger as compared to those in the cross-section a flexible beam is fundamental member of a compliant mechanism. They are common elements of many mechanical, civil and aeronautical engineering systems. In this study the beam is assumed to be initially straight, of constant initial curvature and cross-section and loaded by free end point moment and force. The material has linear elastic stress-strain behavior and assumed that is isotropic. Without loss of generality, the axial and shear deformations are assumed negligible.

2.1 Governing equation

Fig. 1 shows an initially straight beam of length L deflected under a point force F and an external bending moment M at end location.

The bending moment M at a point (x, y) on the beam is obtained as

$$M = EI\frac{d}{dS}(\Psi(S) - \eta(S)) = -F_x(y_C - y) + F_y(x_C - x) + M_0 \tag{1}$$

where E, I, Ψ and η are Young's modulus of the beam material, the moment of area of the beam, the angle of rotation and initial slope of beam, respectively.

In order to express Eq. (1) explicitly in terms of S, it is differentiated with respect to S leading to the following second order differential equation

$$EI\frac{d^2}{dS^2}(\Psi(S) - \eta(S)) = F_x\sin(\Psi(S)) - F_y\cos(\Psi(S)) \tag{2}$$

And the new coordinates of any points are

$$\frac{d}{dS}x = \cos(\Psi(S)) \tag{3}$$

$$\frac{d}{dS}y = \sin(\Psi(S)) \tag{4}$$

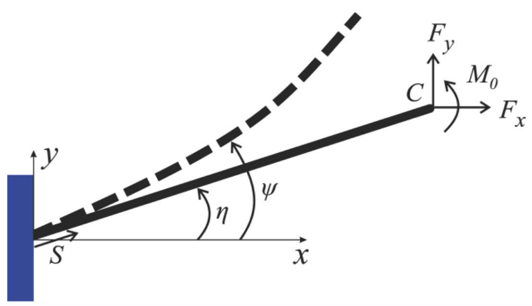


Fig. 1 Initially straight beam

The link is characterized by a dimensionless arc length $u \in [0, 1]$ along its neutral axis that introduced as following

$$u = \frac{S}{L} \tag{5}$$

$$\frac{EI}{L^2}\frac{d^2}{du^2}(\Psi(u) - \eta(u)) = F_x \sin(\Psi(u)) - F_y \cos(\Psi(u)) \tag{6}$$

$$\frac{d}{du}x = L\cos(\Psi(u)) \tag{7}$$

$$\frac{d}{dS}y = L\sin(\Psi(u)) \tag{8}$$

2.2 Basic idea of homotopy perturbation method

To illustrate the basic idea of HPM, the following nonlinear differential equation is constructed. To explain this method the following functions are considered

$$A(u) - f(r) = 0, \quad r \in \Omega \tag{9}$$

with the boundary condition of

$$B(u, \partial u/\partial n) = 0, \quad r \in R \tag{10}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function and Ω is the domain. $A(u)$ is defined as follows

$$A(u) = L(u) + N(u) \tag{11}$$

where $L(u)$ is linear and $N(u)$ is nonlinear part of $A(u)$. Homotopy perturbation structure is shown as (He 1999)

$$H(\upsilon, p) = (1 - p)[L(\upsilon) - L(u_0)] + p[A(\upsilon) - f(r)] = 0 \tag{12}$$

where: $\upsilon(r, p)$: $\Omega \times [0, 1] \rightarrow R$ and $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. The following functions can be obtained by considering Eq. (12)

$$H(\upsilon, 0) = L(\upsilon) - L(u_0) = 0, \quad H(\upsilon, 1) = A(\upsilon) - f(r) = 0 \tag{13}$$

The process of the changes in p from zero to unity is that of $\upsilon(r, p)$ changing from u to u_r.

In HPM the imbedding parameter, p, is used as a small parameter and is assumed that the solution of Eq. (12) can be written as a power series in p

$$\upsilon = \upsilon_0 + p\upsilon^1 + p_2\upsilon^2 + \ldots \tag{14}$$

And the best approximation is

$$u = \lim_{p \rightarrow 1} \upsilon = \upsilon_0 + \upsilon_1 + \upsilon_2 + \ldots \tag{15}$$

2.2 Application of HPM to a cantilever beam

It is described that the main governing equation of present study is in the form of

$$\frac{1}{l^2}\frac{d}{du}\left(EI\left(\frac{d\Psi(u)}{du}-\frac{d\eta(u)}{du}\right)\right)+F_y\cos(\Psi(u))-F_x\sin(\Psi(u)) = 0 \tag{16}$$

By assuming that η has a constant value, this equation is changed to

$$\frac{d^2\Psi(u)}{du^2}+\left(\frac{F_yl^2}{EI}\right)\cos(\Psi(u))-\left(\frac{F_xl^2}{EI}\right)\sin(\Psi(u)) = 0 \tag{17}$$

According to the HPM, the homotopy is constructed.

$$\Psi(u) = \Psi_0(u)+p^1\Psi_1(u)+p^2\Psi_2(u)+p^3\Psi_3(u)+p^4\Psi_4(u)+\ldots$$
$$\Psi(r,p)\colon \Omega\times[0,1]\rightarrow\Re \tag{18}$$

The final homotopy scheme has been constructed in the following form

$$H(\upsilon,p) = (1-p)\left(\frac{d^2}{du^2}\Psi(u)\right)+p\left(\left(\frac{F_yl^2}{EI}\right)\cos(\Psi(u))-\left(\frac{F_xl^2}{EI}\right)\sin(\Psi(u))\right) \tag{19}$$

Taylor expansion is used for sin ($\Psi(u)$) and cos($\Psi(u)$) (Eq. (19)) around the η are expanded by Taylor expansion in form of

$$\sin(\Psi(u)) = \sin(\eta)+(\Psi(u)-\eta)\cos(\eta)-\frac{1}{2!}(\Psi(u)-\eta)^2\sin(\eta)-\frac{1}{3!}(\Psi(u)-\eta)^3\cos(\eta)+O(4) \tag{20}$$

and

$$\cos(\Psi(u)) = \cos(\eta)+(\Psi(u)-\eta)\sin(\eta)-\frac{1}{2!}(\Psi(u)-\eta)^2\cos(\eta)-\frac{1}{3!}(\Psi(u)-\eta)^3\sin(\eta)+O(4) \tag{21}$$

Now by substituting these expansions on the main HPM equation by respect to the terms with identical powers of p, we get

p^0

$$\frac{d^2}{du^2}\Psi_0(u) = 0 \tag{22}$$

p^1 s

$$\frac{d^2}{du^2}\Psi_1(u)-\frac{1}{Ed}\left(F_xl^2\left(\sin(\eta)+\cos(\eta)(\Psi_0(u)-\eta)-\frac{1}{2}\sin(\eta)(\Psi_0(u)-\eta)^2-\frac{1}{6}\cos(\eta)(\Psi_0(u)-\eta)^3\right)\right)+$$
$$\frac{1}{Ed}\left(F_yl^2\left(\cos(\eta)-\sin(\eta)(\Psi_0(u)-\eta)-\frac{1}{2}\cos(\eta)(\Psi_0(u)-\eta)^2+\frac{1}{6}\sin(\eta)(\Psi_0(u)-\eta)^3\right)\right) = 0 \tag{23}$$

p^2

$$\frac{d^2}{du^2}\Psi_2(u)-\frac{1}{Ed}\Big(F_x l^2\Big(\cos(\eta)\Psi_1(u)-\sin(\eta)(\Psi_1(u))(\Psi_0(u)-\eta)-\frac{1}{2}\cos(\eta)(\Psi_1(u))(\Psi_0(u)-\eta)^2\Big)\Big)+$$
$$\frac{1}{Ed}\Big(F_y l^2\Big(-\sin(\eta)\Psi_1(u)-\cos(\eta)\Psi_1(u)(\Psi_0(u)-\eta)+\frac{1}{2}\sin(\eta)\Psi_1(u)(\Psi_0(u)-\eta)^2\Big)\Big)=0 \quad (24)$$

And the boundary conditions for above equations are as follows, respectively

$$\Psi_0(0) = \eta,\ \frac{d}{du}\Psi_0(1) = \frac{Ml}{Ed},\ \ \Psi_1(0) = 0,\ \frac{d}{du}\Psi_1(1) = 0,\ \ \Psi_2(0) = 0,\ \frac{d}{du}\Psi_2(1) = 0$$

and etc. The final solution of $\Psi(u)$ (Forth order) has been obtained by solving Eqs. (22)-(24) and using Eq. (15) in following form

$$Y(u) = -.945u+.5u^2-.361u^4+0.233u^5-.00416u^6-.001u^7+.0008u^8-.00023u^9 \quad (25)$$

3. Result and discussion

In this study, the HPM is applied for solving the problem of cantilever beam with constant initial curvature. It is assumed that the beam is straight with constant initial slope $\eta(u)$ and is loaded by an endpoint vertical and horizontal force and bending moment.

To validate the present simulation, cantilever beam under normalized endpoint load $\bar{F}_y = 0$ to 10 N in $-y$ direction and a cantilever beam under endpoint bending moment $M = \lambda\frac{\pi EI}{L}$ $(0 \le \lambda \le 2)$ were simulated and compared with the previous studies (Mattiasson 1981). The normalized force and moments are

$$\bar{F}_x = \frac{F_x L^2}{EI}$$

$$\bar{F}_y = \frac{F_y L^2}{EI}$$

$$\bar{M} = \frac{ML}{2\pi EI}$$

Fig. 2 displays the normalized endpoint vertical force versus normalized end point displacements and Fig. 3 displays the normalized endpoint bending moment versus normalized endpoint displacements. U and W are endpoint displacement in x and y directions, respectively. It can be seen that a good agreement between the present analytical solutions and the previous solution is obtained.

Beam location for different magnitude of an end point vertical load and moment for $\eta = 0$ is shown in Fig. 4. It is obvious that the displacement is raised by raising the pure force. It can be seen that in Fig. 4(b), the bending is increased by increasing the bending moment. Also it is illustrated that for $\lambda = 2$ the beam is circulated thoroughly.

In this part three load and moment $f_x = -1$n, $f_y = -10$ n and moment with $\lambda = 1$ in r_t incremental

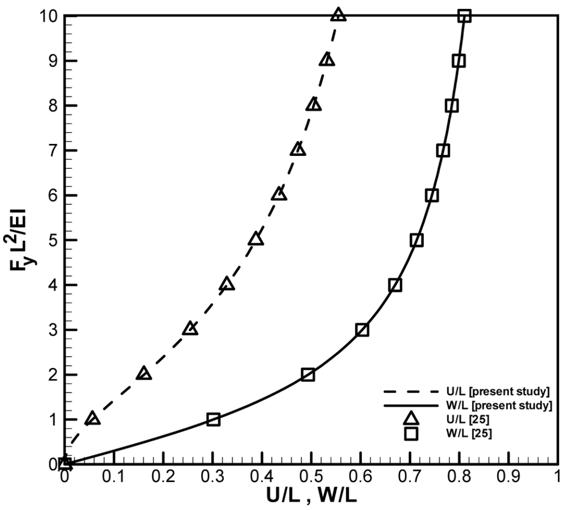


Fig. 2 Comparison of endpoint vertical force versus end point displacement in present study and previous work (Mattiasson 1981)

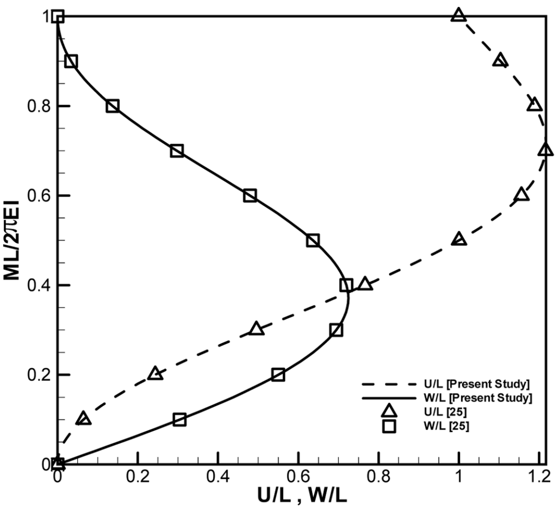


Fig. 3 Comparison of endpoint bending moment versus end point displacement in present study and previous work (Mattiasson 1981)

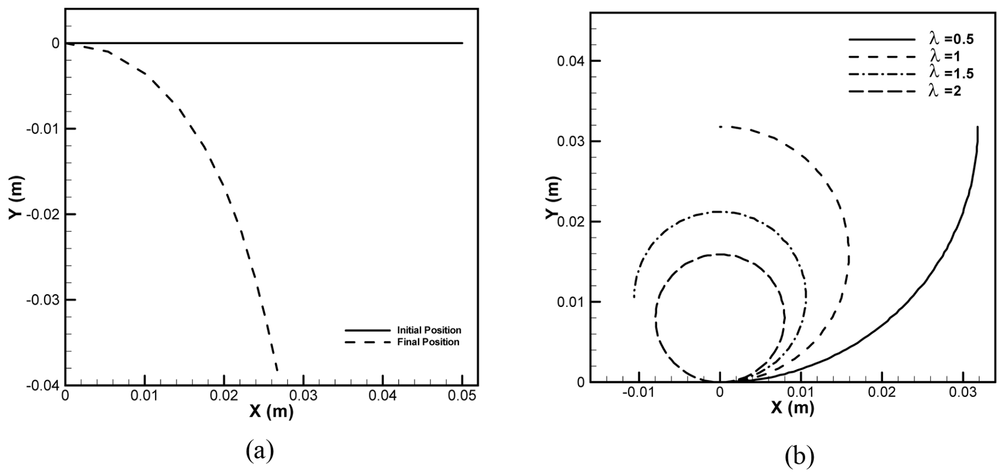
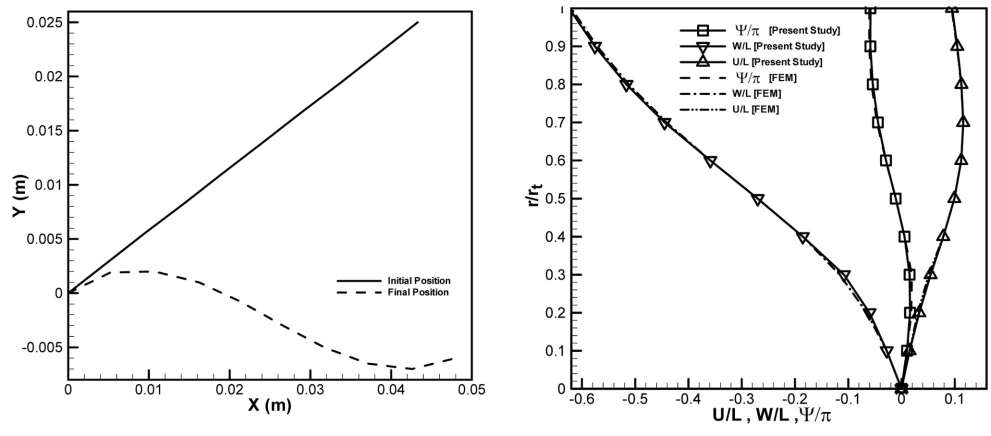


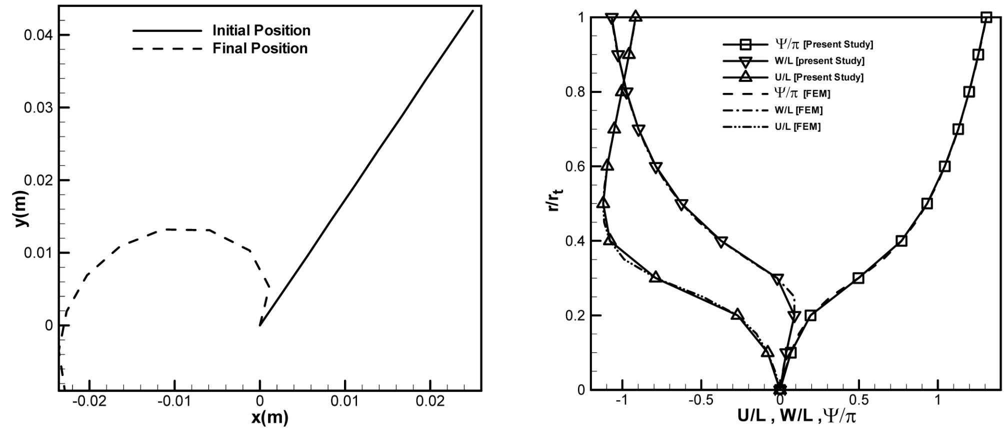
Fig. 4 Beam positions under different values of end point vertical load (a) and moment (b) at $\eta = 0$

steps are exerted at the tip of the beam. The nonlinear FEM result is obtained by the co-rotational procedure implemented in ANSYS®, where beam elements have been used to perform large-displacement static analyses. Other parameters are $l = 0.05$ m and $EI = 0.0037$ nm^2. By implementation the load in r_t incremental steps so that at the r_t h step it is expressed as $[F_x; F_x; M] = r/rt\ [-1; -10; \pi EI/L]$ where $r = 0 \sim r_t$. Figs. 5(a) and 6(a) shows the initial and final location of a beam at angel of $\eta = 30^{\circ}$ and $\eta = 60^{\circ}$, respectively. The normalized force (r/r_t) versus x and y direction displacement and Ψ, is depicted on Fig. 5(b). By attention to Fig. 5(a), it can be seen that in first, the curve slope decreases and in the rest of domain increases. In this part the results for angel of $\eta = 30^{\circ}$ is presented and discussed. The present computation focused on the parameters having the following ranges:



(a) (b)

Fig. 5 (a) Beam positions and (b) rotation angle versus non dimensional arc length at $\eta = 30$



(a) (b)

Fig. 6 (a) Beam positions and (b) rotation angle versus non dimensional arc length at $\eta = 60$

$F_x = 0$ to -10 N, $F_y = 0$ to -10 N, and $M = \lambda \frac{\pi EI}{L}$ where $\lambda = 0$ to 2, $L = 0.05$ m, $EI = 0.0037$ Nm^2.

Beam positions for different value of λ and F_y parameters for $F_x = 0$ is shown in Fig. 7. It can be seen that the displacement is increased by increasing the F_y for $\lambda = 0$ (pure shear). For $F_y = 0$ (pure bending) the displacement is increased by increasing the bending moment. Also the displacement of end point is increased in y direction. It is clear that the beam circulates thoroughly and changes to the full circle bending for $\lambda = 2$. By comparison Figures between $F_y = -5$ N and $F_y = -10$ N at $\lambda = 2$ it is found that the end point dislocation in y direction and $x = 0$ is increased by increasing the F_y.

Fig. 8 show the different beam locations for various value of λ and F_y at $F_x = -5$ N, $\eta = 30^{\circ}$. For $\lambda = 0$ and $F_y = 0$, the effects of bending moment and the moment cause by F_x are in same direction and therefore the end point of beam is shifted to the left. But the displacement of this point in y direction is increased by increasing the F_y. The reason is that with increasing F_y, the emanated moment from F_y is in opposite tendency to the moment that collected the effects of F_x and M.

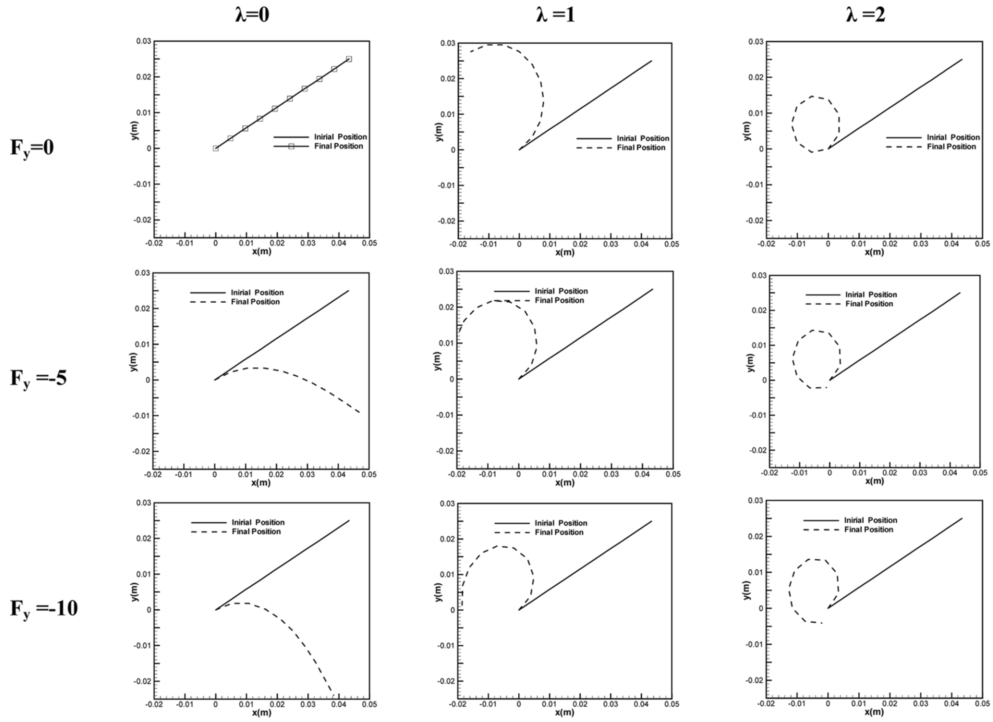


Fig. 7 Beam positions for different value of λ and β numbers for F_x = 0 N and η = 30^o

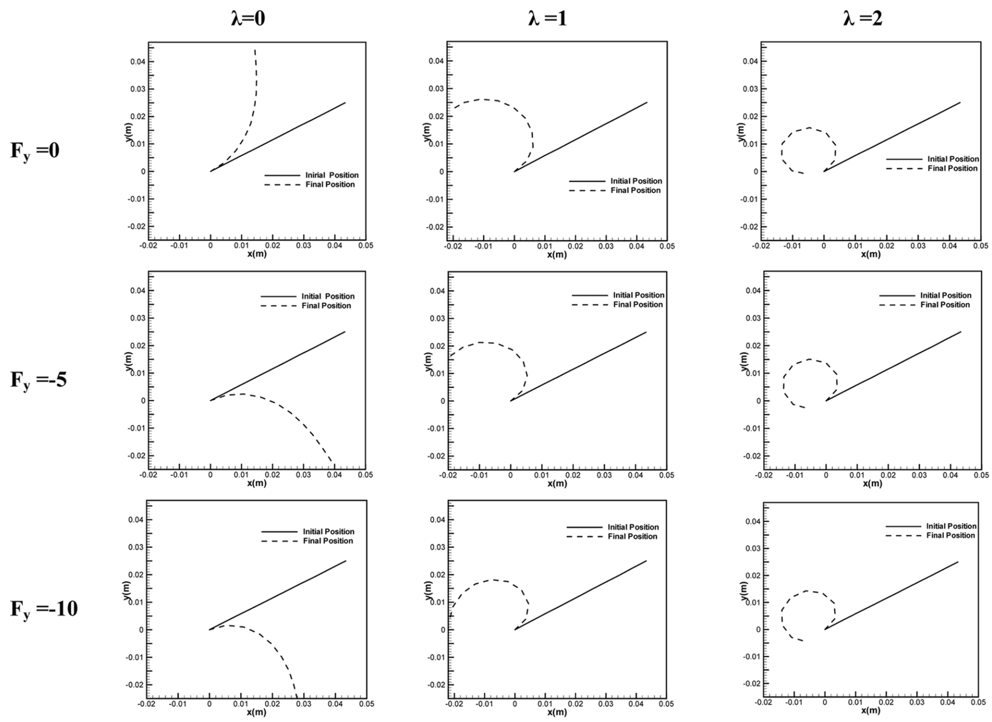


Fig. 8 Beam positions for different value of λ and β numbers for F_x = −5 N and η = 30^o

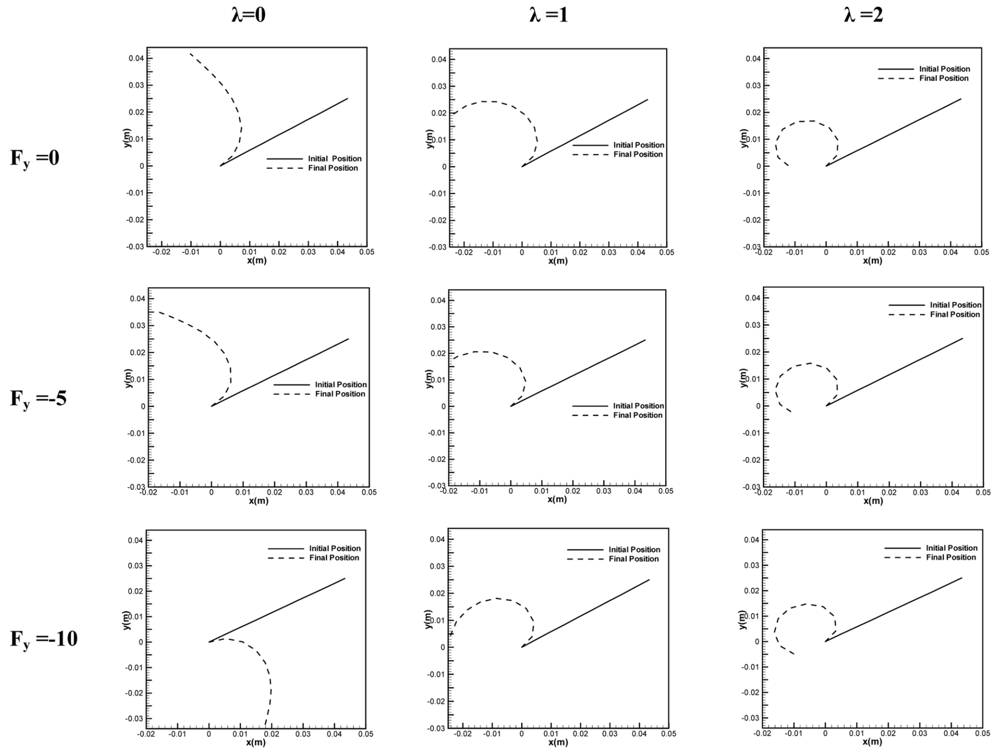


Fig. 9 Beam positions for different value of λ and β numbers for $F_x = -10$ N, $\eta = 30^{\circ}$

By comparison between Figs. 7 and 8 it is found that the end point dislocation in left direction is increased by increasing the F_x.

Finally, the effect of λ and F_y parameters on the beam position at $F_x = -10$ N and $\eta = 30^{\circ}$ is shown in Fig. 9. By comparison of Figs. 8 and 9 It can be seen that for $\lambda = 0$ and $F_y = -5$ N the final location of beam is in the right and left directions of initial location respectively.

The reason of this dissimilarity is that for $F_x = -5$ N the moment that created by F_y can overcome the collection of bending moment (M) and the emanated moment of F_x. But for $F_x = -10$ N this moment has not the enough power to overcome the opposite moments.

4. Conclusions

Homotopy perturbation method (HPM) has been applied to solving the cantilever beam with constant initial curvature problem. It was assumed that the beam is loaded by a known force and bending moment. The HPM results are compared with the pervious numerical techniques and an fascinating agreement is demonstrated. The obtained result is very accurate for cantilever beams and can be used for studying and optimization such the compliant mechanisms. The results of parametric study show that displacement is raised by raising the pure force. Also It is found that the end point dislocation in y direction and $x = 0$ is increased by increasing the F_y.

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Notations

ψ	angle of rotation
A	general differential operator
B	B is a boundary operator
E	Young's modulus
F	known analytic function
H	Homotopy perturbation structure
I	moment of area
L	Length of beam
$L(u)$	linear part of $A(u)$
$N(u)$	nonlinear part of $A(u)$
P	Small parameter
S	curved coordinate along the deflected axis
U	dimensionless arc length
H	initial slope
Ω	Domain

Greek symbol

Y	embedding parameter