

# Interaction of internal forces of exterior beam-column joints of reinforced concrete frames under seismic action

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**Abstract.** Detailed analysis of internal forces of exterior beam-column joints of RC frames under seismic action is reported in this paper. A formula is derived for calculating the average joint shear from the column shears, and a formula is proposed to estimate torque in eccentric joints induced by seismic action. Average joint shear stress and strain are defined consistently for exterior joints, which can be used to establish joint shear constitutive relationship. Numerical results of shear, bending moment and torque in joints induced by seismic action are presented for a pair of concentric and eccentric exterior connections extracted from a seismically designed RC frame, and two sections located at the levels of beam bottom and top reinforcement, respectively, are identified as the critical joint sections for evaluating seismic joint behavior. A simplified analysis of the effects of joint shear and torque on the flexural strengths of the critical joint sections is made for the two connections extracted from the frame, and the results indicate that joint shear and torque induced by a strong earthquake may lead to “joint-hinging” mechanism of seismically designed RC frames.

**Keywords:** axial strength; beam columns; flexural strength; joints; reinforced concrete; seismic behavior; shear strain; shear stress; torsion

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## 1. Introduction

A beam-column joint in a reinforced concrete (RC) frame refers to a portion of the column within the maximum depth of the framing beams. Beam-column joints of RC frames can be classified into exterior and interior joints (Fig. 1). In the plane of seismic loading under consideration, an exterior joint subassembly (termed as “exterior connection” as per ACI-ASCE 352 2002) includes one framing beam, and an interior connection contains two framing beams. Beam-column joints are critical components of RC frames and are vulnerable to damage during strong earthquakes. Fig. 2 shows a pair of exterior and interior joints damaged during the 2008 China Wenchuan earthquake. Research on seismic joint behavior has been conducted intensively in the past fifty years (Hanson and Connor 1967, Zerbe and Durrani 1989, Jirsa 1991, Shiohara 2001, Kim and LaFave 2007, Mitra and Lowes 2007, Canbolat and Wight 2008, Li and Kulkarni 2010, Alemdar and Sezen 2010,

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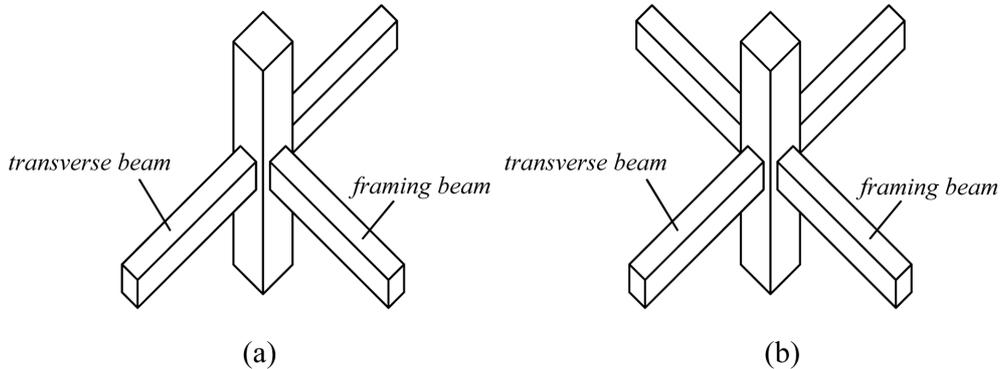


Fig. 1 Reinforced concrete beam-column connections: (a) exterior connection, (b) interior connection



the serious diagonal damage of the joint was mainly due to the lack of joint transverse reinforcement

(a)



concrete cover spalled, joint core concrete seriously damaged, and column longitudinal rebars deformed

(b)

Fig. 2 Beam-column joints damaged during China Wenchuan Earthquake of May 12 2008: (a) exterior joint, (b) interior joint

Fisher and Sezen 2011, Wong and Kuang 2011, and many others). However, conclusive understanding of seismic joint behavior is not yet achieved, and substantial disparity exists in the joint design provisions of the current codes (ACI 318 2005 and its companion document ACI-ASCE 352 2002, MCC 2001a, NZS 1995).

Seismic design of modern building frames requires that inelastic deformation of the frames under a strong earthquake be controlled by the flexural yielding of the framing beams at the column faces. From the free-body force diagrams of exterior and interior joints at the beam yielding at the column faces, it can be seen that joints are subjected to significant shear under seismic action (Hanson and Connor 1967). Zhou (2009) reported that joint shear at the beam-yielding at the column faces is on average 6.3 and 5.9 times the column shear for exterior and interior connections, respectively. Joint shear induced by a strong earthquake causes extensive diagonal cracking in the joints and may result in substantial reduction of the joint concrete strength (Vecchio and Collins 1986). For the joints in the lower stories of a highrise or multi-story RC frame, reduction in the joint concrete strength may lower significantly the flexural strengths of the critical joint sections, since the joints are subjected to large axial compressive force. Thus, the effects of joint shear on the flexural and axial strengths of the critical joint sections should be taken into account in the evaluation of seismic

joint behavior. Furthermore, when eccentric joints are subjected to seismic action, torque is induced in the columns and joints, the effects of the torque should also be considered in the evaluation of seismic behavior of eccentric joints.

One of outstanding issues in the evaluation of seismic joint behavior is with the formulation of joint performance requirements. Zhou (2009) reanalyzed the experimental data of exterior and interior connections reported in the literature, and proposed joint shear deformation, slip of beam reinforcement out of joints and joint axial load capacity as three joint performance indices. On the basis of Zhou's work and the "strong column-weak beam" design requirement for RC frames in the current codes, joint performance requirements during a strong earthquake may be described as follows: (1) column-to-beam flexural strength ratio evaluated at the critical joint sections is kept above unity; (2) joint shear deformation remains elastic during the elastic seismic response of RC frames, and the percentage of joint shear deformation in the lateral story drift ratio of RC frames does not increase during the inelastic seismic response of the frames; (3) joint axial load capacity is maintained above the joint axial force; and (4) percentage of beam rotation at the column faces due to the beam reinforcement slip out of joints in the lateral story drift ratio of RC frames does not exceed a limit value. Joint design requirements can be formulated rationally from the joint performance requirements, if relationships between joint design parameters (such as joint shear stress, amount of joint transverse requirement, and anchorage of beam reinforcement in the joint) and the proposed joint performance indices are established.

This paper presents a detailed analysis of the internal forces of exterior joints under seismic action, proposes consistent definitions of average joint shear stress and strain, identifies critical joint sections for the evaluation of seismic joint behavior, and reports a preliminary analysis of the effects of joint shear and torque on the hinging mechanism of exterior joints. The information presented may provide some new insight into seismic joint behavior and serve as a basis for further analysis of the interaction of shear, bending moment, axial force and torque of exterior joints under seismic action.

## **2. Internal forces of exterior beam-column joints under seismic action**

### *2.1 Designations of five horizontal joint sections*

Five horizontal sections of an exterior joint located at the levels of beam bottom, beam bottom reinforcement, mid-height of joint core, beam top reinforcement, and beam top are designated as S1 through S5, respectively (Fig. 3). In this research, S1 and S2 are taken as sections of the lower column of the connection; S4 and S5 are considered to be sections of the upper column. In the following analysis, the symbols  $V_j$ ,  $M_j$ ,  $T_j$ , and  $N_j$  are used for shear, bending moment, torque, and axial force, respectively, of any horizontal joint section, a superscript is used in these symbols for the five sections S1 through S5 (for instance,  $V_j^1$  represents the shear of S1). At the section S3, there are two values for the bending moment, which are denoted by  $M_j^{31}$  and  $M_j^{32}$ , respectively (to be explained later).

### *2.2 Shear of horizontal joint sections*

Fig. 4(a) shows an exterior connection isolated at the inflexural points from a RC frame under

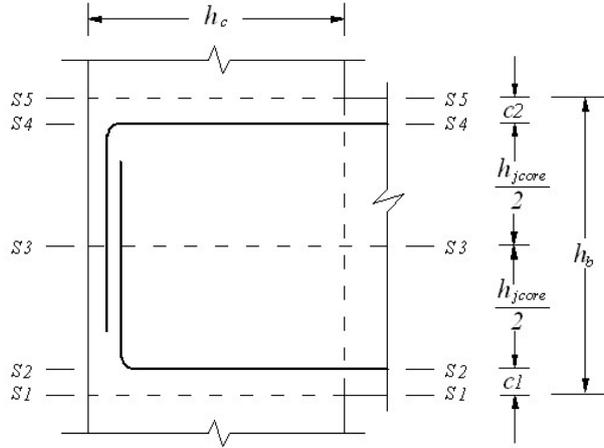


Fig. 3 Designation of five horizontal joint sections

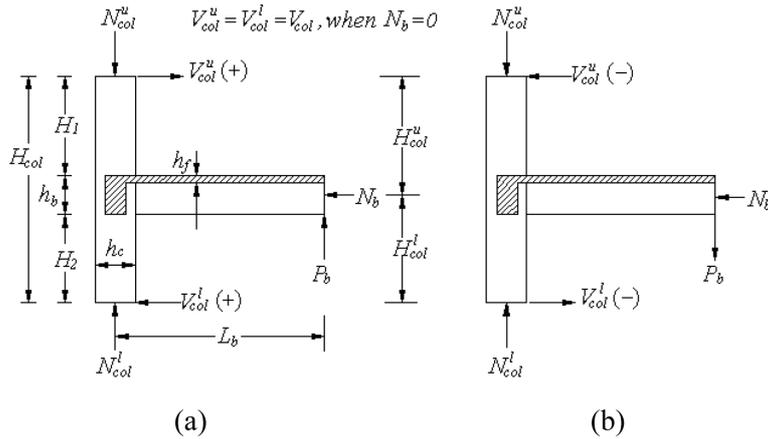


Fig. 4 Force diagrams of exterior connection: (a) under forward loading, (b) under backward loading

forward seismic loading (beam flange in compression). For a given tensile stress of the beam bottom reinforcement at the column face, the joint shear  $V_j$  can be analyzed as follows.

Prior to and at the initial yielding of the beam bottom reinforcement at the column face, beam axial force  $N_b$  is taken as zero, the strain and stress of the beam section at the column face are assumed to be as shown in Fig. 5, and the compressive stress of the top concrete fiber  $\sigma_c$  is related to the corresponding strain  $\varepsilon_c$  by the constitutive relationship (Fig. 6) as per MCC (2002). The concrete strain  $\varepsilon_c$  can be obtained by solving the equilibrium equation of the beam section

$$A_s^b f_s^b = C_c + A_s^t E_s \varepsilon_s^t \tag{1}$$

where  $A_s^b$  = area of beam bottom reinforcement;  $A_s^t$  = area of beam top reinforcement (inclusive of equivalent area of slab reinforcement);  $f_s^b$  = given tensile stress of beam bottom reinforcement at column face;  $\varepsilon_s^t$  = strain of beam top reinforcement determined from the sectional strain;  $C_c$  =

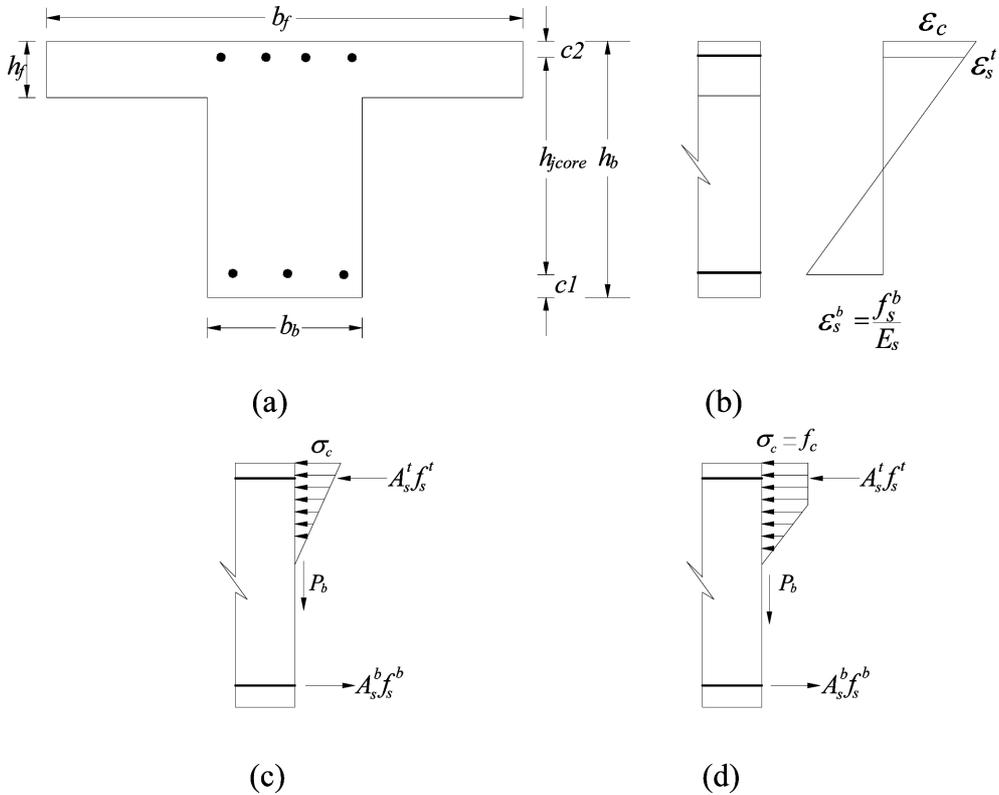


Fig. 5 Beam section at column face prior to and at initial yielding: (a) geometry, (b) strain, (c) stress for  $\epsilon_c \leq 0.002$  ; and (d) stress for  $\epsilon_c > 0.002$

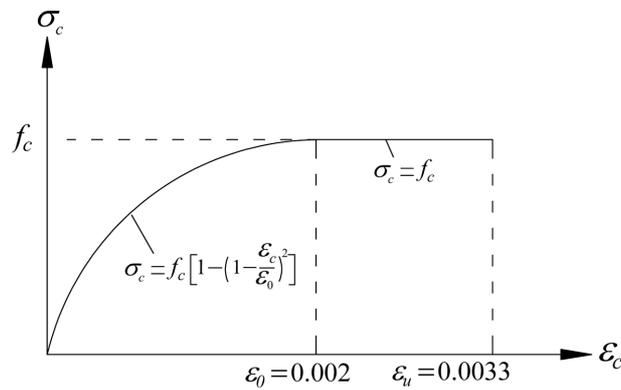


Fig. 6 Concrete stress-strain relationship

resultant of concrete compressive stress;  $E_s$  = elastic modulus of beam reinforcement. After the strain and stress of the beam section at the column face are determined, the column shear  $V_{col}$  ( $=V_{col}^u = V_{col}^l$ ) can be obtained from the moment equilibrium equation of the connection, and the joint shear  $V_j$  can be calculated from the equilibrium equation of the free-body force diagram of the joint.

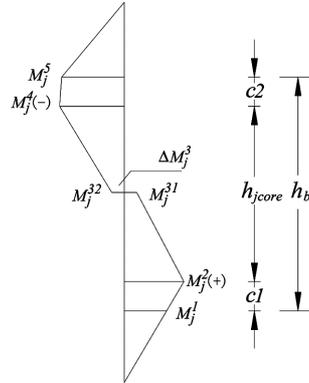


Fig. 7 Illustration of joint bending moment under forward loading

At the ultimate yielding of the beam bottom reinforcement at the column face, the strain and stress of the beam section at the column face can be analyzed in accordance with MCC (2002). Since beam yielding at the column faces leads to beam elongation and induces beam axial compressive force (Zerbe and Durrani 1989), the effect of the beam axial force on the joint shear should be taken into account. For a given beam axial force  $N_b$ , the strain and stress of the beam section at the column face can be determined by solving  $\varepsilon_s^b$  ( $=$  tensile strain of beam bottom reinforcement) from the equilibrium equation of the beam section. Then, the upper and lower column shears  $V_{col}^u$  and  $V_{col}^l$  (Fig. 4(a)) can be obtained from the moment equilibrium equations of the connection, and the joint shear  $V_j$  can be calculated from the force equilibrium equation of the joint.

When the exterior connection is under backward seismic loading (beam flange in tension) (Fig. 4(b)), the joint shear  $V_j$  for a given tensile stress of the beam top reinforcement at the column face can be analyzed in the same way as described above.

### 2.3 Bending moment of horizontal joint sections

Beam-end force  $P_b$  of the exterior connection (Fig. 4) can be determined from the column shears  $V_{col}^u$  and  $V_{col}^l$  and the beam axial force  $N_b$  using the moment equilibrium equation of the connection. For the sake of simplicity in calculating the joint bending moment, the vertical shear imposed by the beam on the column face (equal to  $P_b$  in magnitude) is assumed to act at S3 of the joint. After the forces acting on the column and joint are determined as described above, the joint bending moment  $M_j$  can be calculated from the moment equilibrium equation of the joint. Fig. 7 illustrates the bending moment of the joint under forward loading.

### 2.4 Torque of eccentric exterior joints

In perimeter frames of RC buildings, the perimeter beams are often flushed with the exterior or interior column faces, leading to the formation of one-sided eccentric joints. Eccentric joints also exist in interior frames of RC buildings, where partition walls and the underlying beams are offset from the column centerlines. In order to evaluate the seismic behavior of eccentric joints, torque in the eccentric joints induced by seismic action needs to be estimated.

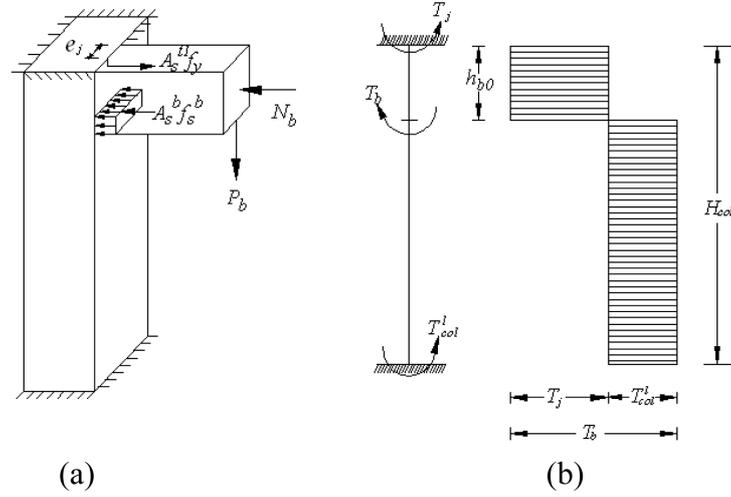


Fig. 8 Illustration of: (a) forces imposed by eccentric beam on column, (b) simplified distribution of torque in column and joint

The floor and roof slabs of RC buildings are normally taken as in-plane rigid diaphragms. When global torsion of RC frame buildings excited by seismic action is negligible, the rotation of the floor and roof slabs in the horizontal plane can be assumed to be zero, and so can the rotation of all the columns in the horizontal plane at the floor and roof levels. This provides a torsional compatibility condition for estimating torque in eccentric joints. Fig. 8(a) shows a column and the forces applied by an eccentric beam on the column under backward loading, and Fig. 8(b) illustrates simplified distribution of torque in the joint and column.

Torque applied by the eccentric beam on the column at the beam yielding  $T^b$  can be estimated by

$$T^b = (A_s^{t1} f_y + N_b) e_j \quad (2)$$

under backward loading, or

$$T^b = (A_s^{b1} f_y) e_j \quad (3)$$

under forward loading, where  $e_j$  = joint eccentricity;  $A_s^{t1}$  = area of beam top reinforcement (exclusive of slab reinforcement);  $f_y$  = yield stress of beam reinforcement. Assuming that  $T^b$  acts at S2 of the eccentric joint, then, the torsional compatibility requirement of the column under seismic action can be expressed as

$$\frac{(T^b - T_{col}^l) h_{b0}}{(GC)_j} - \frac{T_{col}^l (H_{col} - h_{b0})}{(GC)_{col}} = 0 \quad (4)$$

where  $h_{b0}$  = effective beam depth;  $H_{col}$  = column height;  $T_{col}^l$  = torque in lower column;  $(GC)_j$  and  $(GC)_{col}$  = elastic or secant torsional rigidities of joint and column, respectively, depending on whether the joint and the column have cracked or not (Tavio and Teng 2004). The joint torsional rigidity  $(GC)_j$  differs from the column torsional rigidity  $(GC)_{col}$  for two reasons. One reason is the participation of slab and transverse beams in resisting the joint torque, the other is the joint damage

induced by seismic action. It can be assumed that

$$(GC)_j = k(GC)_{col} \quad (5)$$

where  $k$  = joint-to-column torsional rigidity ratio. It can be expected that  $k$  reduces during the seismic response of RC frames. For a given value of  $k$ , the column torque  $T_{col}^l$  can be obtained by

$$T_{col}^l = \frac{T^b}{1 + k(\xi - 1)} \quad (6)$$

where  $\xi = H_{col}/h_{b0}$ , and the joint torque  $T_j (=T^b - T_{col}^l)$  can be determined by

$$T_j = \frac{k(\xi - 1)}{1 + k(\xi - 1)} T^b \quad (7)$$

### 3. Definitions of average joint shear stress and average joint shear strain

#### 3.1 Definitions of average joint shear and average joint shear stress

Once the distribution of joint shear  $V_j$  over the joint height is known, the average shear of joint core  $V_{jave}$  (referred to as ‘‘average joint shear’’ hereafter) can be defined as

$$V_{jave} = \frac{\int_0^{h_{jcore}} V_j dy}{h_{jcore}} \quad (8)$$

where  $h_{jcore}$  = height of joint core, and the average shear stress of joint core  $\tau_{jave}$  (referred to as ‘‘average joint shear stress’’ hereafter) can be obtained by

$$\tau_{jave} = \frac{V_{jave}}{A_j} \quad (9)$$

where  $A_j$  = effective joint area, which may be determined as per ACI 318 (2005).

In addition, the maximum joint shear  $V_{jmax}$  at the beam yielding at the column face can be calculated by

$$V_{jmax} = A_s^b f_y - V_{col}^l = (A_s^b f_y + N_b) - V_{col}^u \quad (10)$$

under forward loading (Fig. 4(a)), or

$$V_{jmax} = A_s^t f_y - V_{col}^u = (A_s^t f_y + N_b) - V_{col}^l \quad (11)$$

under backward loading (Fig. 4(b)), and the corresponding maximum joint shear stress  $\tau_{jmax}$  can be obtained by  $\tau_{jmax} = V_{jmax}/A_j$ .

### 3.2 Formula for calculating average joint shear from column shears

A formula can be derived for calculating the average joint shear  $V_{jave}$  from the column shears  $V_{col}^u$  and  $V_{col}^l$ .

Integrating  $V_j dy = dM_j$  over the upper and lower halves of a joint core (Fig. 7) leads to

$$\int_{h_{jcore}/2}^{h_{jcore}} V_j dy = M_j^A - M_j^{32} \quad (12)$$

and

$$\int_0^{h_{jcore}/2} V_j dy = M_j^{31} - M_j^A \quad (13)$$

respectively. Adding Eqs. (12) and (13) together results in

$$\int_0^{h_{jcore}} V_j dy = (M_j^A - M_j^A) - (M_j^{32} - M_j^{31}) \quad (14)$$

Dividing Eq. (14) with  $h_{jcore}$  yields to

$$V_{jave} = \frac{(M_j^A - M_j^A) - (M_j^{32} - M_j^{31})}{h_{jcore}} \quad (15)$$

It is evident that Eqs. (16) and (17) in the following hold

$$(M_j^{32} - M_j^{31}) = -P_b \left( \frac{h_c}{2} \right) \quad (16)$$

$$P_b L_b = V_{col}^u H_{col}^u + V_{col}^l H_{col}^l \quad (17)$$

where  $h_c$  = column depth;  $L_b$  = distance from column centerline to beam-end;  $H_{col}^u$  and  $H_{col}^l$  = distances from acting line of beam axial force to column top and bottom, respectively (Fig. 4). Solving  $P_b$  from Eq. (17) and substituting it into Eqs. (16) yields to

$$(M_j^{32} - M_j^{31}) = -(V_{col}^u H_{col}^u + V_{col}^l H_{col}^l) \left( \frac{h_c}{2L_b} \right) \quad (18)$$

Substituting Eq. (18),  $M_j^A \approx V_{col}^l (H_{col}^l - h_b/2 + c1)$  and  $M_j^A \approx -V_{col}^u (H_{col}^u - h_b/2 + c2)$  ( $c1$  and  $c2$  = distances from centroids of beam bottom and top reinforcement to beam bottom and top, respectively) into Eq. (15) leads to

$$V_{jave} = -V_{col}^u \left( \frac{H_{col}^u}{h_{jcore}} \right) \beta_1 - V_{col}^l \left( \frac{H_{col}^l}{h_{jcore}} \right) \beta_2 \quad (19)$$

where  $\beta_1 = 1 - h_c/(2L_b) - (h_b/2 - c2)/H_{col}^u$ ;  $\beta_2 = 1 - h_c/(2L_b) - (h_b/2 - c1)/H_{col}^l$ .

For an exterior connection without beam axial force (for instance, an exterior connection tested in laboratory),  $N_b = 0$ , and  $V_{col}^u = V_{col}^l = V_{col}$ . Then, Eq. (19) can be simplified into

$$V_{jave} = -V_{col} \left( \frac{H_{col}}{h_{jcore}} \right) \beta \quad (20)$$

where  $\beta = 1 - h_c/(2L_b) - h_{jcore}/H_{col}$ .

Once the average joint shear  $V_{jave}$  is determined by Eq. (19) or (20), the average joint shear stress  $\tau_{jave}$  can be obtained by Eq. (9).

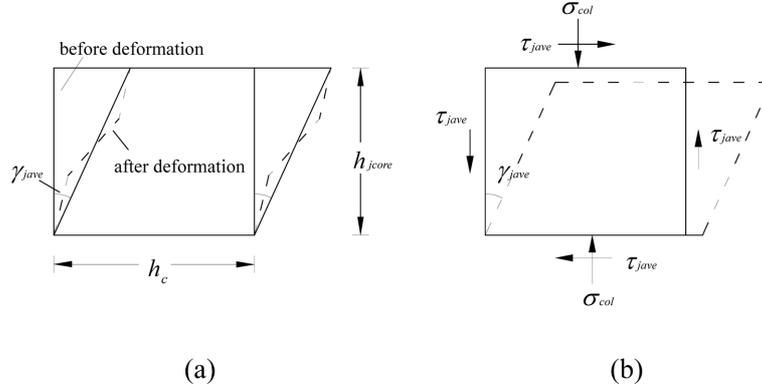


Fig. 9 Illustration of: (a) average joint shear strain, (b) joint model

### 3.3 Definition of average joint shear strain

The average shear deformation of joint core  $\gamma_{jave}$  (referred to as the “average joint shear strain” hereafter) can be defined as (Fig. 9(a))

$$\gamma_{jave} = \frac{\int_0^{h_{jcore}} \gamma_j dy}{h_{jcore}} \quad (21)$$

where  $\gamma_j$  = shear deformation of an infinitesimal portion of joint core.

### 3.4 Significance of definitions of average joint shear stress and strain

Seismic joint behavior should be evaluated at two levels. One level is related to the local behavior of joint panels, the other level is connected with the effect of local joint behavior on the global frame behavior. At the local level of seismic joint behavior, Zhou (2009) reported that for seismically designed joints, the maximum joint shear stress applied during cyclic testing was actually determined by the maximum joint shear stress calculated at the beam yielding, and joint shear failure was characterized by excessive inelastic joint shear deformation. At the global level of seismic frame behavior, joint shear deformation forms an important component of the lateral story drift of RC frames under seismic action (Teng and Zhou 2003). Hence, determination of joint shear deformation plays a critical role in the evaluation of seismic joint behavior.

It can be seen from Eqs. (8), (9) and (21) that the average joint shear stress and strain  $\tau_{jave}$  and  $\gamma_{jave}$  are defined consistently in this research. It can be expected that joint shear constitutive relationship under seismic action exists between  $\tau_{jave}$  and  $\gamma_{jave}$ . When testing an exterior connection in laboratory,  $\tau_{jave}$  can be determined from the applied column shear  $V_{col}$  by Eqs. (20) and (9),  $\gamma_{jave}$  can be measured directly from the joint core, hence, the  $\tau_{jave}$  versus  $\gamma_{jave}$  relationship can be established experimentally. On the other hand, a joint core may be modeled as a panel subjected to the average joint shear stress  $\tau_{jave}$  (Fig. 9(b)), and rational models such as the modified compression-field theory (Vecchio and Collins 1986) and the softened membrane model (Mansour and Hsu 2005) can be used to establish the  $\tau_{jave}$  versus  $\gamma_{jave}$  relationship. The  $\tau_{jave}$  versus  $\gamma_{jave}$

relationship can be used to formulate joint shear strength, evaluate the effect of joint shear on joint concrete strength, and determine the contribution of joint shear deformation to the story drift ratio of RC frames under seismic action.

#### 4. Numerical results of internal forces of two exterior joints under seismic action

##### 4.1 Description of Connections CEC and EEC

A six-story office building in the campus of Hainan University (Fig. 10(a)) is selected as prototype building for this research. An RC frame is designed for this building for a seismic intensity represented by a peak ground acceleration of  $0.2g$  ( $g =$  gravity acceleration) in accordance with the Chinese codes (MCC 2001a, MCC 2001b, MCC 2002) and typical design practice in China, and a commercial software package PKPM (PKPM CAD Division 2008) well established in

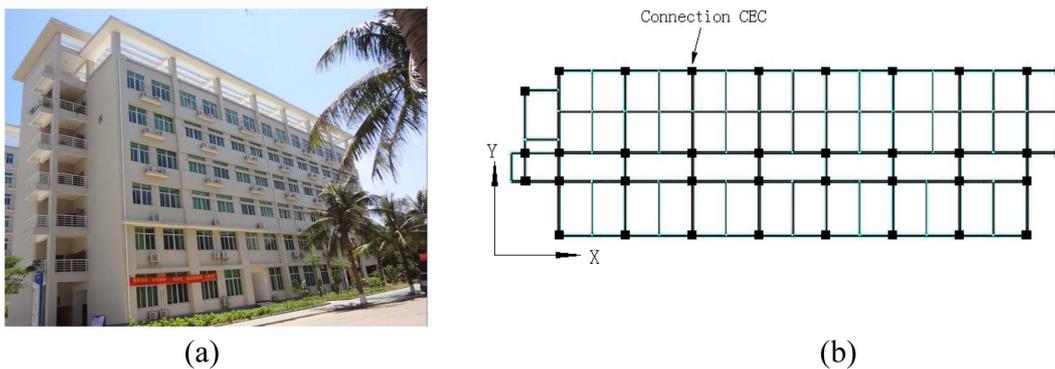


Fig. 10 Prototype building and Connection CEC: (a) photograph, (b) second floor plan

Table 1 Main design information of RC frame for prototype building

Seismic Intensity	Seismic Category	Seismic Class	Column Factor	Fundamental Period (s)		Maximum Story Drift Ratio		Base Shear Ratio	
				X	Y	X	Y	X	Y
0.2g	C	II	1.2	1.05	1.03	1/670	1/588	7.02%	7.04%

- Notes: 1. Seismic Intensity of  $0.2g$  ( $g =$  gravity acceleration) represents the horizontal peak ground acceleration with 10% probability of exceedance in 50 years;  
 2. Seismic Category of C is for ordinary buildings (A for most important buildings, and D for least important buildings);  
 3. Seismic Class of II is jointly determined by the Seismic Intensity of  $0.2g$  and the Seismic Category of C, and is used to determine seismic measures for the frame;  
 4. “Column Factor” = magnification factor for the column design bending moments, the value of 1.2 is determined by the Seismic Class of II;  
 5. “Maximum Story Drift Ratio” = the maximum story drift ratio caused by an earthquake with 63.5% probability of exceedance in 50 years;  
 6. Wind pressure for the design of the frame =  $0.75 \text{ kN/m}^2$ .

Table 2 Geometrical and material data of Connections CEC and EEC

Beam	$L_b$	$b_b$	$b_f$	$h_b$	$h_f$	C1	C2	$A_s^{t1}$	$A_s^{t2}$	$A_s^b$	$f_y$	$f_c$
		4500	300	2060	850	110	37.5	37.5	2700	785	1800	440/360
Column	$H_{col}$	$H_1$	$H_2$	$b_c$	$h_c$	C3	$A_s^u$	$A_s^l$	$f_y$	$f_c$	$N/(Af_c)$	
		4250	1525	1875	600	600	42.5	2600	7000	440/360	23.4/16.7	0.42

- Notes: 1. Units: mm for dimension, mm<sup>2</sup> for area, and N/mm<sup>2</sup> for strength;  
2.  $b_f$  = width of beam flange determined according to Clause 8.10.2 of ACI 318 (2005);  
3.  $A_s^{t2}$  = equivalent area of slab reinforcement within  $b_f$ , including 100% of the top layer and 50% of the bottom layer of slab reinforcement within  $b_f$ ;  
4. C3 = concrete cover + half diameter of column longitudinal rebars;  
5.  $A_s^u$  and  $A_s^l$  = areas of longitudinal rebars of upper and lower columns, respectively;  
6. HRB400 rebars are used for beam and column longitudinal rebars. Both the nominal yield strength (taken as 1.1 times the strength grade) and the design yield strength (determined as per MCC (2002)) are shown for HRB400 rebars;  
7. C30 and C35 concretes are used for beams and columns, respectively. Both the nominal and design compressive strengths determined as per MCC (2002) are shown for C30 and C35, respectively;  
8. In the expression  $N/(Af_c)$ ,  $N$  = column design axial force,  $A$  = column sectional area,  $f$  = design concrete strength of column.

Table 3 Column-to-beam flexural strength ratios of Connections CEC and EEC

Loading direction	Strength type	Beam		Upper column		Lower column		$\frac{\sum M_{col}^R}{M_b^R}$
		$N_b$	$M_b^R$	$N_{col}$	$M_{col}^R$	$N_{col}$	$M_{col}^R$	
Forward loading	Nominal	0	640.4	1513	764.1	1794	1072.5	2.87
	Design	0	521.6	1786	668.2	2114	903.8	3.01
Backward loading	Nominal	0	1170.6	2096	840.9	2553.4	1149	1.7
	Design	0	952.1	2585	715.6	3066	922.6	1.72

- Notes: 1. Units: kN for axial force, and kN.m for flexural strength;  
2. Nominal column axial force is the combined effect of non-factored loads, and design column axial force is the combined effect of factored loads, determined from PKPM results as per MCC (2001a);  
3. Nominal flexural strength is calculated using nominal material strengths and nominal axial force, and design flexural strength is calculated using design material strengths and design axial force.

China is used for the analysis and design of the frame. The frame is modeled as a space frame, the beams and columns of the frame are modeled as three-dimensional bar elements (beam-column joints are not modeled separately), and linear finite element analysis of the frame is carried out under gravity loading, horizontal seismic action, and wind loading, respectively. The main design information of the frame is summarized in Table 1. A concentric exterior connection designated as ‘‘CEC’’ is extracted from the second floor of the frame (Fig. 10(b)) for the analysis in this research. The geometrical and material data of Connection CEC are shown in Table 2. Both the nominal and design strengths of the reinforcement and concrete are shown in Table 2, the nominal strengths are normally used to represent the actual strengths of the materials in the frame, whereas the design strengths are used for the capacity design of the frame. The column-to-beam flexural strength ratio

calculated as per MCC (2002) is shown in Table 3. It can be seen that the “strong column-weak beam” design requirement is satisfied for Connection CEC. Connection EEC is a one-sided eccentric companion connection of CEC with an eccentricity of 150 mm between the column and beam centerlines.

4.2 Internal forces of Connections CEC and EEC under seismic action

Table 4 and Fig. 11 show the numerical results of nominal shears of Connections CEC and EEC. The following points can be observed: (1) joint shear  $V_j$  within the joint core is much larger than outside the joint core, indicating the concentration of seismic joint damage within the joint core; (2) difference between the average joint shear  $V_{jave}$  and the maximum joint shear  $V_{jmax}$  is insignificant under forward loading but significant under backward loading. Hence, in general, the maximum joint shear stress  $\tau_{jmax}$  can not be related to the average joint shear strain  $\gamma_{jave}$  by joint shear constitutive relationship; (3) at the ultimate beam yielding at the column face, the beam axial force  $N_b$  induced by the beam elongation increases  $V_{jave}$  and therefore accelerates seismic damage in the joint core.

Table 5 presents the numerical results of nominal bending moments of Connections CEC and EEC at the beam yielding at the column face. It can be seen that: (1) the maximum bending moment (in terms of absolute value) may not be  $M_j^1$  or  $M_j^5$  but be  $M_j^2$  or  $M_j^4$  instead; (2) an increase in  $N_b$  leads to rapid increase in the maximum bending moment and hence accelerates the yielding of the column/joint; (3) under forward loading,  $M_j^5$  corresponding to  $\alpha = 0.1$  (= 744.1 kN.m) is close to the nominal flexural strength of the upper column (= 764.1 kN.m). According to the research by Zerbe and Durrani (1989), the maximum beam axial force  $N_b$  may be taken as  $N_b = 0.1f_c b_b h_b$ .

Table 6 shows the numerical results of nominal torque in the joint and lower column of Connection EEC at the beam yielding at the column face. It can be seen that: (1) with the reduction of joint-to-column torsional rigidity ratio  $k$  induced by seismic action, the joint torque  $T_j$  reduces,

Table 4 Nominal column and joint shears of Connections CEC and EEC

Loading direction	Item	Prior to initial beam yielding	At initial beam yielding	At ultimate beam yielding		
		$f_s = 0.6f_y$	$f_s = f_y$	$\alpha = 0$	$\alpha = 0.05$	$\alpha = 0.1$
Forward loading	$V_{col}^u$	92.9	154.7	160.3	324.3	488
	$V_{col}^l$	92.9	154.7	160.3	68.1	-24.6
	$V_{jmax}$	-382.3	-637.3	-631.7	-723.9	-816.6
	$V_{jave}$	-377.9	-629.7	-631.7	-723.9	-816.6
Backward loading	$V_{col}^u$	-168	-279	-264.9	-136.6	-5.5
	$V_{col}^l$	-168	-279	-264.9	-392.9	-518
	$V_{jmax}$	752	1254.4	1268.5	1396.8	1527.9
	$V_{jave}$	688.3	1143.6	1085.8	1135.1	1173.5

Notes: 1. Unit: kN for shear;

2. Column and joint shears are calculated using nominal material strengths;

3. Prior to and at initial beam yielding,  $N_b = 0$ ; at ultimate beam yielding,  $N_b = \alpha * (f_c b_b h_b)$ .

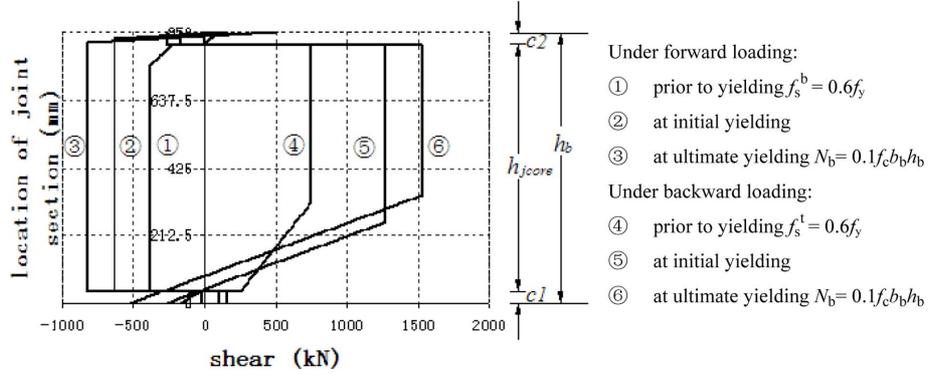


Fig. 11 Nominal joint shear of Connections CEC and EEC

Table 5 Nominal joint bending moment of Connections CEC and EEC at ultimate beam yielding

Loading direction	$\alpha$	$M_j^1$	$M_j^2$	$\Delta M_j^3$	$M_j^4$	$M_j^5$
Forward loading	0	300.6	306.7	-45.4	-228.3	-244.5
	0.05	127.6	130.2	-50	-480.7	-494.6
	0.1	-46.1	-47	-54.3	-734.1	-744.1
Backward loading	0	-496.8	-502.6	75	414	404
	0.05	-736.6	-747.1	80.8	213.4	208.3
	0.1	-971.3	-986.5	85.6	8.6	8.4

Notes: 1. Unit: kN.m for bending moment;

2. Joint bending moment is calculated using nominal material strengths;

3.  $N_b = \alpha * (f_c b_b h_b)$ ;

4.  $\Delta M_j^3 = M_j^{32} - M_j^{31}$ .

Table 6 Nominal joint torque of Connection EEC at ultimate beam yielding

Loading direction	$T_b$	$k = 1.5$		$k = 1.0$		$k = 0.5$		
		$T_{col}$	$T_j$	$T_{col}$	$T_j$	$T_{col}$	$T_j$	
Forward loading	118.8	14.9	103.9	21	97.8	35.8	83.1	
Backward loading	$\alpha = 0$	178.2	22.3	155.9	31.5	146.7	53.5	124.7
	$\alpha = 0.05$	216.6	27.1	189.5	38.3	178.3	65.0	151.6
	$\alpha = 0.1$	255.1	31.9	223.2	45.1	210	76.6	178.5

Notes: 1. Unit: kN.m for torque;

2. Torque is calculated using nominal material strengths.

and the column torque  $T_{col}^l$  increases; (2) the beam axial force  $N_b$  increases  $T_j$  and  $T_{col}^l$  under backward loading, but does not influence  $T_j$  and  $T_{col}^l$  under forward loading.

### 4.3 Identification of critical joint sections

The internal forces of S1 and S2 of Connections CEC and EEC can be compared as follows: under forward loading,  $V_j^2$  is much larger than  $V_j^1$ ,  $M_j^2$  is slightly larger than  $M_j^1$ , and  $N_j^2$  is equal to  $N_j^1$ ; under backward loading,  $V_j^2$  is close to  $V_j^1$ ,  $M_j^2$  is slightly larger than  $M_j^1$ , and  $N_j^2$  is equal to  $N_j^1$ . The comparison clearly indicates that S2 is more critical than S1 in terms of internal forces and capacity requirements. Similar comparison between S4 and S5 indicates that S4 is more critical than S5.

## 5. Effect of joint shear and torque on hinging mechanism of exterior connections under seismic action

### 5.1 General discussion

The compressive strength of joint concrete under seismic action is influenced by two factors, one factor is the diagonal joint cracking caused by joint shear that reduces the concrete strength (Vecchio and Collins 1986), the other is the confining stress provided by the joint transverse reinforcement that increases the concrete strength (Park *et al.* 1982). The combined effect of these two factors may result in substantial reduction of joint concrete strength during a strong earthquake (Hwang and Lee 2002). Reduction of joint concrete strength lowers the axial and flexural strengths of critical joint sections and may lead to yielding and crushing of the joints. For eccentric exterior connections, joint torque induced by seismic action may further reduce the axial and flexural strengths of the critical joint sections.

Complexity of seismic joint behavior results from the interaction of shear, bending moment, axial force and torque. Shear, bending moment and torque in joints are cyclically reversed during an earthquake; however, axial force in joints is mainly due to the gravity loading of frame buildings and may not be significantly influenced by seismic action. Analytical models have been developed by other researchers for analyzing the interaction of internal forces of beams and columns (Rahal and Collins 2003, Mostafaei and Kabeyasawa 2007). The information presented in this paper together with these models may be used for detailed analysis of the interaction of shear, bending moment, axial force and torque in joints. As a starting point, a simplified analysis of the effects of joint shear and torque on the flexural strength and hinging mechanism of Connections CEC and EEC under seismic action is presented in the following.

### 5.2 Effect of joint shear on flexural strength of critical joint sections under seismic action

A critical section of Connection CEC (S2 or S4) is acted by shear, bending moment and axial force under seismic action. The axial force of a critical joint section after the beam yielding at the column faces is assumed to be constant in this research and can be determined from the PKPM results for forward and backward seismic loading, respectively. The joint concrete strength reduced by joint shear can be expressed as  $(1 - \eta)f_c$  ( $\eta$  = reduction factor;  $f_c$  = original joint concrete strength). Then, for a given  $\eta$ , the flexural strengths of S2 and S4 of Connection CEC can be estimated from the axial force and the reduced concrete strength, respectively. Table 7 and Fig. 12 show the results of the flexural strengths for backward loading calculated in accordance with MCC (2002).

Table 7 Flexural strengths of critical joint sections of Connection CEC under backward loading

$\eta$		0%	20%	40%	60%
Nominal strength	$M_j^{1R}$	840.9	785.1	701.7	536.3
	$M_j^{2R}$	1149	1068.7	941.6	723.4
	Total	1989.9	1853.8	1643.3	1259.7
Design strength	$M_j^{1R}$	715.6	619.1	466.1	278.5
	$M_j^{2R}$	922.6	774.9	601.8	402.8
	Total	1638.2	1394	1067.9	681.3

Notes: 1. Unit: kN.m for flexural strength;

2.  $\eta$  = reduction factor of joint concrete strength;

3. Nominal flexural strength is calculated using nominal material strengths and nominal axial force, and design flexural strength is calculated using design material strengths and design axial force.

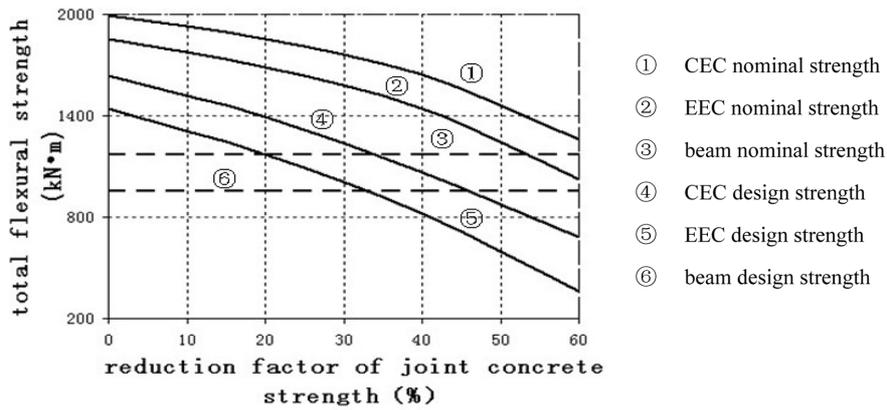


Fig. 12 Total flexural strengths of critical joint sections of Connections CEC and EEC under backward loading

### 5.3 Effect of joint torque on flexural strength of critical joint sections under seismic action

Under seismic action, a critical section of Connection EEC (S2 or S4) is acted by torque in addition to shear, bending moment and axial force. At the beam yielding at the column faces, the joint torque  $T_j$  can be estimated by Eq. (7) for a given joint-to-column torsional rigidity ratio  $k$  and beam axial force  $N_b$ . The effect of  $T_j$  on the flexural strength of a critical joint section is estimated in this research as follows: (1) calculate the area of longitudinal reinforcement required to resist the torque in accordance with MCC (2002) (denote the area as  $A_T$ ); (2) subtract  $A_T$  from the total area of longitudinal reinforcement (denote the result as  $A_F$ ); (3) use  $A_F$  to calculate the flexural strength of the section in the same way as for Connection CEC. Table 8 and Fig. 12 show the results of the flexural strengths of S2 and S4 of Connections EEC under backward loading, calculated for  $T_j$  corresponding to  $k = 1$  and  $N_b = 0$ .

Table 8 Flexural strengths of critical joint sections of Connection EEC under backward loading

$\eta$		0%	20%	40%	60%
Nominal strength	$M_j^{1R}$	768	703.2	604.2	422.5
	$M_j^{2R}$	1086.6	985.8	836.5	604.2
	Total	1854.6	1689	1440.7	1026.7
Design strength	$M_j^{1R}$	618.1	506.8	345.5	113.5
	$M_j^{2R}$	825.2	660.7	474.1	245.6
	Total	1443.3	1167.5	819.6	359.1

- Notes: 1. Unit of flexural strength is kN.m;  
 2. The numerical results in the table are for  $T_j$  corresponding to  $k = 1$  and  $N_b = 0$ ;  
 3. Nominal flexural strength is calculated using nominal material strengths, nominal axial force, and nominal joint torque; design flexural strength is calculated using design material strengths, design axial force, and design joint torque;  
 4. Design joint torque is taken as 1.1 times nominal joint torque.

Table 9 Reduction factor of joint concrete strength at initiation of “joint-hinging” mechanism

Connection	CEC		EEC	
	Nominal	Design	Nominal	Design
Type of flexural strength				
$\eta$	60%	46%	53%	32%

#### 5.4 Effect of joint shear and torque on hinging mechanism of exterior connections under seismic action

Joint shear and torque do not have significant influence on the flexural strength of the framing beams at the column faces but reduce the flexural strength of the critical joint sections. As a result, joint shear and torque reduce the column-to-beam flexural strength ratio evaluated at the critical joint sections. When the column-to-beam flexural strength ratio at a joint reduces to unity, the bending moments of the critical joint sections induced by seismic action reach the flexural strengths. This may be referred to as the “joint-hinging” mechanism of seismically designed connections! Further reduction of joint concrete strength due to the effects of joint shear and torque may lead to axial crushing of the critical joint sections!

Assuming that the flexural strengths of framing beams at the column faces remain constant during a strong earthquake, then, the reduction factor of joint concrete strength  $\eta$  at the initiation of the “joint-hinging” mechanism can be evaluated for Connections CEC and EEC, respectively (see Table 9 and Fig. 12). The values of  $\eta$  for Connections CEC and EEC are about 46% and 32%, respectively, when evaluated using the design material strengths, and are approximately 60% and 53%, respectively, if evaluated using the nominal material strengths.

## 6. Conclusions

Detailed analysis of internal forces of exterior beam-column joints of RC frames under seismic

action is made in this research. A formula is derived for calculating the average joint shear from the column shears, and a formula is proposed to estimate torque in eccentric joints induced by seismic action. Average joint shear stress and strain of exterior joints are defined consistently, which can be used to establish joint shear constitutive relationship.

Numerical results of shear, bending moment and torque in joints induced by seismic action are presented for a pair of concentric and eccentric exterior connections extracted from a seismically designed RC frame. Two joint sections located at the levels of beam bottom and top reinforcement, respectively, are identified as the critical joint sections for evaluating seismic joint behavior.

Joint shear induced by seismic action causes diagonal joint cracking and reduces the joint concrete strength. For the joints in the lower stories of RC frames that are subjected to large axial compressive force, reduction of joint concrete strength may reduce substantially the flexural strengths of the critical joint sections. Joint torque induced by seismic action in eccentric joints further reduces the flexural strengths of the critical joint sections. Hence, joint shear and torque induced by seismic action reduce the column-to-beam flexural strength ratio evaluated at the critical joint sections. When the column-to-beam flexural strength ratio at a joint is reduced to unity during a strong earthquake, the bending moments of the critical joint sections reach the flexural strengths, leading to the initiation of the “joint-hinging” mechanism. A simplified analysis of the effects of joint shear and torque on the “joint-hinging” mechanism is presented for the two exterior connections extracted from the RC frame.

Complexity of seismic joint behavior can be attributed to the interaction of shear, bending moment, torque and axial force in joints. The information presented in this paper serves as a starting point for detailed analysis of the interaction of joint shear, bending moment, torque and axial force.

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## Notations

$A_j$	: effective joint area;
$A_s^b$ and $A_s^{t1}$	: areas of beam bottom and top reinforcement, respectively;
$A_s^{t2}$	: equivalent area of slab reinforcement effective in resisting negative beam moment;
$A_s^t$	: $= A_s^{t1} + A_s^{t2}$ ;
$C_c$	: resultant of concrete compressive stress of beam section;
$C1$ and $C2$	: distances from centroids of beam bottom and top reinforcement to beam bottom and top, respectively;
$E_s$	: elastic modulus of reinforcement;
$e_j$	: joint eccentricity between column and beam centerlines;
$f_c$	: nominal or design concrete compressive strength;
$f_y$	: nominal or design yield strength of reinforcement;
$f_s^b$ and $f_s^t$	: stresses of beam bottom and top reinforcement, respectively;
$H_{col}$	: column height;
$H_{col}^u$ and $H_{col}^l$	: distances from acting line of beam axial force to beam top and bottom, respectively;
$h_b$	: beam depth;
$h_{b0}$	: effective beam depth;
$h_c$	: column depth;
$h_{jcore}$	: height of joint core between centroids of beam top and bottom reinforcement;
$k$	: ratio of joint-to-column torsional rigidity;
$L_b$	: distance between column centerline and beam end;
$M_j$	: joint bending moment;
$M_j^i$ ( $i=1, 2, 4$ or $5$ )	: bending moment of Section Si of joint ;
$M_j^{t1}$ and $M_j^{t2}$	: bending moments of Section S3 of joint;
$N_b$	: beam axial compressive force, acting at the centroid of rectangular part of T-shaped section;
$N_j$	: joint axial compressive force;
$N_j^i$ ( $i=1, 2, 4$ or $5$ )	: axial force of Section Si of joint;
$P_b$	: beam-end force;
$T^b$	: torque imposed by eccentric beam on column;
$T_{col}^l$	: torque of lower column;
$T_j$	: joint torque;
$T_j^i$ ( $i=1, 2, 4$ or $5$ )	: torque of Section Si of joint;
$V_{col}$	: column shear;
$V_{col}^u$ and $V_{col}^l$	: upper and lower column shears, respectively;
$V_j$	: joint shear;
$V_j^i$ ( $i=1, 2, 4$ or $5$ )	: shear of Section Si of joint;
$V_{jave}$	: average shear of joint core, referred to as “average joint shear”;
$V_{jmax}$	: maximum joint shear;
$(GC)_{col}$ and $(GC)_j$	: torsional rigidities of column and joint, respectively;
$\alpha$	: $= N_b/(f_c b_b h_b)$ ;
$\beta$	: $= 1 - h_c/(2L_b) - h_{jcore}/H_{col}$ ;
$\beta_1$	: $= 1 - h_c/(2L_b) - (h_b/2 - c2)/H_{col}^u$ ;
$\beta_2$	: $= 1 - h_c/(2L_b) - (h_b/2 - c1)/H_{col}^l$ ;
$\varepsilon_c$	: compressive strain of extreme concrete fiber;
$\varepsilon_s^b$	: tensile strain of beam bottom reinforcement;
$\varepsilon_s^t$	: compressive strain of beam top reinforcement;

$\gamma_j$	: shear deformation of an infinitesimal height of joint core;
$\gamma_{jave}$	: average shear deformation of joint core, referred to as “average joint shear strain”;
$\eta$	: reduction factor of joint concrete strength;
$\sigma_c$	: compressive stress of extreme concrete fiber;
$\tau_{jave}$	: average shear stress of joint core, referred to as “average joint shear stress”;
$\tau_{jmax}$	: maximum joint shear stress; and
$\xi$	: ratio of column height to effective beam depth.