Structural Engineering and Mechanics, Vol. 43, No. 6 (2012) 771-794 771
DOI: http://dx.doi.org/10.12989/sem.2012.43.6.771

Multi-objective optimal design of laminate composite shells and stiffened shells

K. Lakshmi[a] and A. Rama Mohan Rao*

CSIR-Structural Engineering Research Centre, Taramani, Chennai, 600113, Tamil Nadu, India

(Received October 7, 2011, Revised June 22, 2012, Accepted August 9, 2012)

Abstract. This paper presents a multi-objective evolutionary algorithm for combinatorial optimisation and applied for design optimisation of fiber reinforced composite structures. The proposed algorithm closely follows the implementation of Pareto Archive Evolutionary strategy (PAES) proposed in the literature. The modifications suggested include a customized neighbourhood search algorithm in place of mutation operator to improve intensification mechanism and a cross over operator to improve diversification mechanism. Further, an external archive is maintained to collect the historical Pareto optimal solutions. The design constraints are handled in this paper by treating them as additional objectives. Numerical studies have been carried out by solving a hybrid fiber reinforced laminate composite cylindrical shell, stiffened composite cylindrical shell and pressure vessel with varied number of design objectives. The studies presented in this paper clearly indicate that well spread Pareto optimal solutions can be obtained employing the proposed algorithm.

Keywords: laminate composites; cylindrical shell; stiffened shell; multi-objective optimization; Pareto optimal solutions; neighbourhood search; evolutionary algorithm

1. Introduction

Composite materials are being widely used in aero, civil, mechanical, marine and space engineering industries due to their light weight and high specific strength and modulus (Baker *et al.* 2004, Park *et al.* 2009). The greatest advantage of laminated composite materials, in addition to high strength to weight properties, is that they provide designers with the ability to tailor the directional strengths and stiffnesses of the material to the given loading environment of the structure which is certainly not possible in conventional metals. Therefore, laminated composite constructions offer many opportunities for engineers and designers to optimize structures for a particular, or even multiple tasks. A suitable lay-up stacking sequence can improve the resistance of the laminate without increasing the number of plies. Optimization of the stacking sequence is therefore necessary in the design of the composite parts. The angle of composite plies is usually distinct, such as 0, 45, or 90 degrees, for practical manufacturing. This leads to a combinatorial optimization problem with discrete or discontinuous variables and it is difficult to solve.

Several methods for discrete variable optimization have been developed and are being used

*Corresponding author, Ph.D., E-mail: arm2956@yahoo.com
[a]E-mail: lakshmi_44410@yahoo.com

extensively over past few decades. These can be classified into six categories (Arora *et al*. 2004): branch and bound, simulated annealing, sequential linearization, penalty functions, Lagrangian relaxation, and other methods such as rounding-off, heuristic, cutting-plane, pure discrete, and evolutionary algorithms. Each method has advantages and disadvantages depending on the problem type. While conventional methods use a single point at each iteration, evolutionary algorithms on the other hand are population-based approaches. The basic aim of an evolutionary algorithm is to generate a new set of designs (population) from the current set in order to improve the average fitness of the population. Evolutionary algorithms are global-search approaches that can find global optima and are being widely used for various practical optimisation problems.

However, the analysis and design of composite materials is relatively more complex. Composite design optimization typically consists of identifying the optimal configuration that would achieve the required strength with minimum overheads. The possibility to achieve an efficient design that fulfills the global criteria and the difficulty to select the values out of a large set of constrained design variables makes mathematical optimization a natural tool for the design of laminated composite structures. There will be a number of parameters like weight, cost, thickness, etc which have to be taken into consideration for effective design optimization of composites depending on the nature of application for which the component is being designed. Further, due to manufacturing constraints, ply angles and ply thickness are to be selected from a set of discrete values and the design process becomes a combinatorial optimization problem with multiple objectives, which is difficult to solve. Evolutionary algorithms are being popularly used for these class of discrete combinatorial problems with multiple objectives.

Multi-objective optimization problems usually have many optimal solutions, known as Pareto optimal solutions (Miettinen 1999). Each Pareto optimal solution represents a different compromise among design objectives. Hence, the designer is interested in finding many Pareto optimal solutions in order to select a design compromise that suits his preference structure. Number of methods is reported in the literature for solving multi-objective optimization problems. One approach is to optimize one objective while treating other objectives as constraints (Messac 1996). Another popular approach is to condense multiple objectives into a single composite objective function by methods like weighted sum, geometric mean, perturbation, Tchybeshev, min-max, goal programming, and physical programming (Miettinen 1999, Sen *et al*. 1998, Chankong *et al*. 1983, Rama Mohan Rao and Arvind 2005, 2007). Recently, Sepehri *et al*. (2012) have proposed a modified version of particle swarm optimisation algorithm which is stated to be superior in performance when compared to basic particle swarm optimisation (PSO) and the repulsive version of particle swarm optimisation (RPSO) algorithm for multi-objective optimisation of laminate composite cylindrical shells. The modifications suggested by the authors include dynamic acceleration coefficients as well as variable inertia factors apart from variable repulsion coefficient to improve the overall diversification mechanism. The authors have employed weighted sum method to obtain Pareto optimal solutions. However, all these approaches give one Pareto optimal solution in each simulation.

There are number of multi-objective evolutionary algorithms (MOEAs) (Deb 2001) reported in the literature and are currently being used for variety of multi-objective problems to find multiple Pareto optimal solutions in a single simulation run. Some of the popular algorithms include strength Pareto evolutionary algorithm by Zitzler and Thiele (1998), elitist nondominated sorting genetic algorithm (NSGA-II) by Deb *et al.* (2000), micro genetic algorithm by Coello and Pulido (2005). However all these evolutionary algorithms works with population of solutions and through evolutionary process finally converges to Pareto optimal solution. In contrast, the Pareto archived evolutionary strategies (PAES) by Knowles and Corne (2000) works with a single population and the evolutionary process driven by a mutation operator. Some of the recent works report multi-objective optimal design of laminate composite structures employing evolutionary algorithms (Pelletier and Vel 2006, Topal 2009, Irisarri *et al*. 2009, Topal and Uzman 2010).

Eventhough PAES has been successfully employed to solve several problems, this algorithm has not so far been employed to solve the combinatorial problem associated with stacking sequence multi-objective optimisation of laminate composite structures. In this paper we present the implementation of PAES inspired algorithm for optimisation of laminate composite structures.

The proposed evolutionary algorithm draws inspiration from the successful implementation of PAES (Knowles and Corne 2000), Tabu search (Armentano and Claudio 2004), SA (Suppapitnarm *et al.* 2000), and MSGNS algorithm (Rama Mohan Rao and Shyju 2008, 2010) for multi-criteria optimisation techniques in which only single solution is used in the evolutionary process. The proposed algorithm closely follows the implementation of PAES. While PAES uses only mutation to generate new nondominated solutions, the proposed algorithm uses a customized neighbourhood search to generate new neighbourhood solutions. Further in order to improve the diversification mechanism, a crossover operator is also introduced. The proposed elitist multi-objective evolutionary algorithm maintains an archive of non dominated solutions and diversity among the nondominated solutions is achieved using adaptive grid method.

Most applications involve a single material system, with ply orientations being the design variables used to tailor properties. However, in some applications it makes sense to combine more than one material system in order to combine desirable properties. Therefore, hybridization is a simple and effective concept in which a high-stiffness and more expensive material is used in the outer layers and inexpensive low-stiffness material in the inner layers. This method, besides providing suitable structural rigidity, also provides for cost reduction. In this paper, we investigate the combination of two materials for balancing cost and weight considerations. A composite laminate is designed from several materials, and the optimization problem requires choice of material for each ply.

The most widely employed approach in all the meta-heuristic algorithms to deal with constrained search spaces is the penalty method. In the penalty method, the infeasible solutions are penalised by using the amount of constraint violation so that the selection process in the evolutionary algorithms favour feasible solutions. Eventhough the penalty approaches are convenient to employ and hence popular in the meta-heuristic algorithms, the main drawback is with the tuning of the penalty functions. In this paper, the combinatorial constraints are treated using a correction operator (Rama Mohan Rao and Lakshmi 2009, 2011, 2012, Rama Mohan Rao and Arvind 2005, Todorki and Haftka 1998), while the design constraints related to buckling load and natural frequency of the composite laminate cylinder are treated by considering them as additional objectives and solved using the proposed multi-criteria meta-heuristic algorithm.

2. Multi-objective evolutionary algorithm

An elitist evolutionary algorithm has been proposed in this paper to solve the combinatorial optimisation problem associated with stacking sequence optimisation of laminate composite structures for simultaneously optimizing multiple objectives.

The first step in using evolutionary algorithms for laminate composites is to develop a representation of the solution. Every solution is encoded by arranging all ply angles of the given composite laminate in the form of a string s (s_i : i = 1, 2, … N/2), where s_i is an encoded value corresponding to a ply angle and N stands for the number of plies in the laminate composite. Since we use symmetric laminates, N/2 parameters can describe an N-ply laminate. In this work, an array of N/2 variables represents the laminate. Each variable in the string is an integer between 0 and 8, where 0 represents a pair of empty plies, 1, 2, 3 and 4 represents 0^o, $+45^o$, 90^o and -45^o plies of the first material. Similarly 5, 6, 7 and 8 represents 0^o, $+45^o$, 90^o and -45^o plies of the second material. We have to introduce 0 to represent empty plies in the string as the number of individuals in a string are constant where as the number of plies in a laminate is not. In multi-objective optimisation, a Pareto front is the subset of candidate solutions obtained by eliminating any other solution for which an absolute superior one (with respect to the ensemble of objective functions) can be found. According to this definition, the Pareto-front consists of the so called non-dominated (or Pareto-optimal) solutions, in the sense that none of them is absolutely superior to any other constituent of the front. Thus all of them are equally acceptable solutions to the problem and the choice of one of them requires a deep knowledge of the particular problem.

As already mentioned, the proposed multi-objective evolutionary algorithm is PAES influenced one. Unlike the conventional evolutionary algorithms, which operate on a set of population, the multi-objective evolutionary algorithm being discussed in this paper operates only on single population similar to PAES. Intial solution is generated randomly and it is termed as parent solution, P_0. The objective values and the constraints are evaluated for the solution P_0 and then it is mutated to create an offspring, C_0. Here the mutation operator is formulated as a neighbourhood search algorithm. It is well known that the evolutionary algorithms fare better with customized operators. Keeping this in view we have employed a variable depth neighbourhood search algorithm. This algorithm works as follows:

In the variable depth neighbourhood search algorithm, a random cut-off point within the user defined range is used and all the variables to the left of the cut-off point in the candidate solutions are considered for improvement. Series of solutions are then generated from each candidate solution, by using all possible combinations (i.e., 0 to 8) for the chosen variables. Among the several solutions derived from each existing candidate solution, the best non dominated solution is chosen to compete with the parent solution. Since the influence of extreme fibers in the composite laminate is significant in altering the stiffness of the laminate, the variables to the left of cut-off point in the existing candidate solution are considered for swapping to obtain a local optimal value. In the numerical studies carried out in this paper, the cut-off point is chosen randomly between 1 and 4.

The child solution, C_0 thus obtained is compared with the parent solution, p_0 and the winner becomes parent to the next generation. The main crux of the algorithm lies in the way that the winner is chosen in the midst of multiple objectives. The proposed multi-objective evolutionary algorithm also maintains an external archive of non-dominated solutions, in which all the non-dominated solutions obtained during the evolutionary process are stored. The main objective of external archive is to maintain a historical record of nondominated solutions found along the search process. The external archive has infact two main functions: the first one is to control the entry of the nondominated solutions i.e. the decision making process and the second one is density estimation technique to resize and retain the number of nondominated solutions which are uniformly spread across the Pareto front. The nondominated solution (child solution) found in each evolution of our algorithm are compared with respect to the contents of the external archive which, at the beginning of the search will be empty. If the external archive is empty, then the current solution is accepted. If the new solution (i.e., child solution) is dominated by an individual within the external archive, then such a solution is automatically discarded. Otherwise, if none of the solutions contained in the external archive dominates the solution wishing to enter, then such a solution is stored in the external archive. If there are solutions in the archive that are dominated by the new child solution, then such solutions are eliminated from the archive. If child solution finds a place in the external archive, then child solution becomes the parent. Alternatively, if child solution fails to find a place in the external archive, then, we employ a two point crossover operator using the current parent solution and a randomly chosen external archive solution. The two child solutions produced using the crossover operator will be compared with the external archive solutions and if any one among these two child solutions finds a place in the external archive, then that particular child solution will be chosen as a parent for the next generation. If both solutions qualify and finds place in the external archive, then one among these two solutions are randomly chosen as a parent. Alternatively, If both the child solutions fails to qualify to find a place in the external archive, then a new solution is randomly generated and considered as a parent for the next generation.

Finally, if the number of these nondominated solutions in the archive has reached its maximum allowable capacity, the density estimation techniques need to be employed to limit the archive size. Several density estimation techniques (Deb 2001, Zitler and Thiele 1998, Knowles and Corne 2000) are proposed in the literature. In this paper, to produce well-distributed Pareto fronts, we use an adaptive grid procedure, which uses a variation of the grid implementation proposed by Knowles and Corne (2000).

The adaptive grid mechanism proceeds by the division of search space into hypercubes, the coordinates of which are defined by objective space. The particles in the hypercubes are located using their objective function values. If any particle is found eligible to enter the repository and has coordinates lying outside the current bounds of the grid, the grid is recalculated and relocated accordingly. For effective functioning of the grid, the number of divisions of the grid as well as the repository size should be prespecified. The grid separates the objective space into so called hypercubes. The edge length of a cube is calculated as

$$\text{edgelength}_j = \frac{c(\max_j - \min_j)}{\text{ARCH\_SIZE}} \tag{1}$$

where $\max_j$ and $\min_j$ are the maximal and minimal values reached by an archive member, evaluated with the j-th objective function. ARCH_SIZE is the size of the non-dominated set. The real values constant $c \in [0, 1]$ represents a selection pressure. The probability that two solutions share the same hypercube is inversely proportional to the value c.

The fitness of each solution is defined as $f_d = |H|^2$, where $|H|$ is the number of solutions in the same hypercube. This density estimation method is invoked if the external archived population reaches its maximum size. The fitness of all external archived population are calculated and sorted from small to large. The first $N_{\max}$ (maximum size of archive) members are kept whereas the remaining ones are deleted from the archive.

The solution is assumed to have converged if there is no new solutions are added to the external archive for a initially set user defined number of generations or the maximum number of generations.

3. Laminate composite models

The classical laminate theory is used in the present work to describe the behaviour of laminate composite cylindrical shells and stiffened shells with a particular laminate configuration. Since the composite laminate shell is considered as symmetric with respect to the mid-surface and also is balanced, the bending-extensional coupling stiffness matrix [B] can be neglected. The extensional stiffness matrix [A] and the bending stiffness matrix [D] are given as

$$A_{ij} = \sum_{l=1}^{k} \overline{Q}_{ij}(h_l - h_{l-1})$$

$$D_{ij} = \frac{1}{3}\sum_{l=1}^{k} \overline{Q}_{ij}(h_l^3 - h_{l-1}^3) \tag{2}$$

where h_i is the thickness of the ith lamina and $\overline{Q}$ denotes the reduced stiffness of the layers of the laminate. The extensional stiffness matrix and bending stiffness matrix, which are functions of design variables, are the major factors that influence the stability and strength design of the laminated composite structures. These matrices will be used in buckling of laminate composites.

When comparing the stiffnesses of different laminates, especially symmetric laminates that are subjected to in-plane loading, it is often convenient to define the effective extensional modulus $\overline{E}_x$, the effective extensional modulus $\overline{E}_y$, and the effective shear modulus $\overline{G}_{xy}$ of the laminate, as follows

$$\overline{E}_x = \frac{1}{a_{11}Ht}, \quad \overline{E}_y = \frac{1}{a_{22}Ht}, \quad \overline{G}_{xy} = \frac{1}{a_{66}Ht} \tag{3}$$

where Ht is the total thickness of the laminate

3.1 Buckling of composite cylinder

The first case study considered is a composite cylinder subjected to axial compression loading. The stacking sequences are optimised using the proposed multi-objective evolutionary algorithm to maximise the buckling loads of the composite cylinder. The configuration details of the composite cylinder are shown in Fig. 1. The practical significance of the problem is that the fuel tanks of the space shuttles are made of laminated composites and can be modelled as a composite cylindrical shell. The buckling of cylindrical shell under axial compression is of great relevance for the design of fuel tanks. Since the tanks are to be designed leak proof, an additional combinatorial constraint that the difference of ply orientations between contiguous plies should not be greater than 45 degree is imposed. This additional stacking constraint helps in preventing delamination cracks and thereby fuel leakage in the tank. The analytical solutions for buckling of composite cylinders under axial compression are given by Tasi (1966) and are as follows.

(i) Axial symmetric buckling

$$\left(\frac{\overline{N}_x}{t}\right)_s = \frac{2}{Rt}\sqrt{\frac{d_{11}}{a_{22}}}\left(1 + \sqrt{\frac{b_{12}^2}{a_{22}d_{11}}} + \frac{b_{12}}{\sqrt{a_{22}d_{11}}}\right) \tag{4}$$

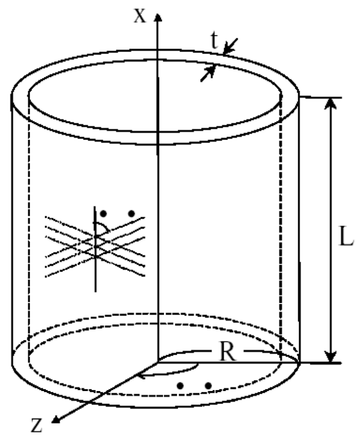

Fig. 1 Composite cylinder

(ii) Non-axial symmetric buckling

$$\left(\frac{\overline{N}_x}{t}\right)_u = \frac{1}{Rt}\sqrt{\frac{d_{22}}{a_{11}}}\left(\Phi_1 + \frac{\left(\Phi_3 + \sqrt{\Phi_1\Phi_2 + \Phi_3^2}\right)^2}{\Phi_2}\right)\left(\Phi_1\Phi_2 + \Phi_3^2\right)^{-1/2} \tag{5}$$

where
$$\Phi_1 = \frac{a_{11}d_{11}}{a_{22}d_{22}}\mu^4 + 2\frac{d_{12}+2d_{66}}{\sqrt{d_{11}d_{22}}}\sqrt{\frac{a_{11}d_{11}}{a_{22}d_{22}}}\mu^2 + 1$$

$$\Phi_2 = \mu^4 + 2\frac{a_{12}+0.50a_{66}}{\sqrt{a_{11}a_{22}}}\mu^2 + 1 \tag{6}$$

$$\Phi_3 = \frac{b_{12}}{a_{22}}\sqrt{\frac{a_{11}}{d_{22}}}\mu^4 + 2\frac{\{0.50(b_{11}+b_{12}) - b_{66}\}}{\sqrt{a_{22}d_{22}}}\mu^2 + \frac{b_{21}}{\sqrt{a_{11}d_{22}}} \tag{7}$$

$$\mu^2 = \frac{\lambda^2}{n^2}\sqrt{\frac{a_{22}}{a_{11}}} \tag{8}$$

$$\lambda = \frac{m\pi R}{L} \tag{9}$$

The elements of matrices [*a*], [*b*], and [*d*] are defined by

$$\begin{aligned}[a] &= [A]^{-1}\\ [b] &= [B].[a]\\ [d] &= [D] - [b].[B]\end{aligned} \tag{10}$$

R is the outer radius, *L* is the length of the cylinder, and *t* is the thickness of the composite laminate. [*A*], [*B*] and [*D*] are the in-plane, coupling and bending stiffness matrices respectively. While m represents the half wave number of buckling mode in the axial direction, *n* represents the circumferential wave number of buckling mode. As mentioned earlier, the elements of B matrix vanish for the symmetric laminates.

The axial symmetric buckling load is calculated using Eq. (4). The non-axial symmetric critical buckling load is taken as smallest buckling load computed for all combinations of m, n (m = 1, 2, …20, n = 1, 2,…20). The objective function for optimal stacking sequence that yields the maximum buckling load of the composite cylinder is formulated as

$$\text{Fitness function} \quad F = \min\left(\frac{\overline{N}_x}{t}\right) \tag{11}$$

In the analysis of the buckling load given by Eqs. (4) to (10), the bending and twisting coupling matrices D_{16} and D_{26} will be neglected. The error induced by this assumption is negligible if the following non-dimensional ratios

$$\xi = \frac{D_{16}}{(D_{11}^3 D_{22})^{1/4}}, \quad \psi = \frac{D_{26}}{(D_{22}^3 D_{11})^{1/4}} \tag{12}$$

satisfy the constraints $\xi \leq 0.20$ and $\psi \leq 0.20$. A detailed discussion of the condition and its implications is given in Nemeth (1995), where it is shown that for buckling problems, the constraints given in Eq. (12) are effective in reducing bending-twisting coupling to a negligible level. In the present implementation, we ignore the solutions failed to satisfy the constraints given in Eq. (12), even if it qualifies as a non-dominated solution.

The natural frequencies of simply supported laminate composite cylindrical shell is given by (Yao and Xiao 1989)

$$\omega^2 = \omega_{mn}^2 = \frac{1}{\rho h}\left\{\begin{array}{c} \left[D_{11}\left(\frac{m\Pi}{l}\right)^4 + 2(D_{12}+D_{33})\left(\frac{n}{R}\right)^2\left(\frac{m\Pi}{l}\right)^2 + D_{22}\left(\frac{n}{R}\right)^4\right] + \\ (A_{11}A_{22}-A_{12}^2)\left(\frac{m\Pi}{l}\right)^4/R^2\left[\begin{array}{c} A_{11}\left(\frac{m\Pi}{l}\right)^4 + A_{22}\left(\frac{n}{R}\right)^4 + \\ (A_{11}A_{22}-A_{12}^2-2A_{12}A_{33})\left(\frac{n}{R}\right)^2\left(\frac{m\Pi}{l}\right)^2/A_{33}\end{array}\right]\end{array}\right\} \tag{13}$$

3.2 Buckling of a fiber-reinforced stiffened laminate composite cylinder

The buckling load computation of stiffened laminate composite circular cylindrical shells are computed using the formulations given by Sun and Mao (1993). The details of these computations are briefly described here for completeness. The laminate composite stiffened cylinder is shown in Fig. 2.

The buckling load is computed for stiffened laminate cylinders using smeared stiffness theory. According to the smeared stiffness theory, the stiffeners are assumed to be closely distributed over the shell surface such that local buckling is not taken into consideration. Due to smeared stiffener assumption, the effect of stiffening can be represented by the equivalent laminate stiffness matrices $[\tilde{A}], [\tilde{B}], [\tilde{D}]$ in which the components $\tilde{A}_{11}, \tilde{A}_{22}, \tilde{B}_{11}, \tilde{B}_{22}, \tilde{D}_{11}, \tilde{D}_{22}$ are obtained by modifying the corresponding shell laminate stiffness coefficients in the following way

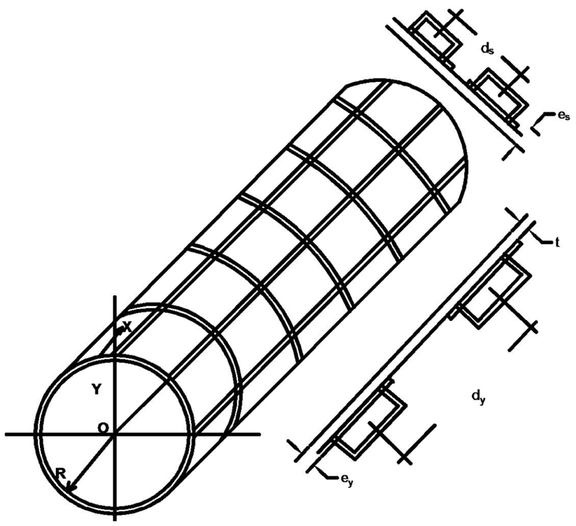

Fig. 2 Stiffened Laminate composite cylinder

$$\tilde{A}_{11} = A_{11} + \frac{A_s}{d_s t}; \quad \tilde{B}_{11} = B_{11} + e_s \frac{A_s}{d_s t^2}; \quad \tilde{D}_{11} = D_{11} + \frac{I_s}{d_s t^3} + e_s^2 \frac{A_s}{d_s t^3}$$

$$\tilde{A}_{22} = A_{22} + \frac{A_r}{d_r t}; \quad \tilde{B}_{22} = B_{22} + e_r \frac{A_r}{d_r t^2}; \quad \tilde{D}_{22} = D_{22} + \frac{I_r}{d_r t^3} + e_s^2 \frac{A_s}{d_r t^3} \tag{14}$$

where A_s and A_r are cross-sectional areas of stringers and rings, d_s and d_r are distances between stringers and rings. I_s and I_r are moments of inertia of stringer and ring about their centroidal axes and e_{ss} and e_{rr} are eccentricities of stringer and ring respectively. It is assumed that the normal strains $\varepsilon_x(z)$ and $\varepsilon_y(z)$ vary linearly in the stringer as well as in the shell and the normal strains are continuous at the interface of the stiffener and shell. The lateral bending stiffness of the stiffener is also neglected and shear membrane force N_{xy} is assumed to be carried entirely by the sheet. The stiffeners are made up of unidirectional fibre-reinforced epoxy. The fibre direction coincides with the stiffener axis and E_{11} is the longitudinal modulus. The shell laminate is composed of the same fibre volume fraction as stiffener. The buckling load factor for stiffened cylindrical shell is given by

$$\lambda(m,n) = \frac{\pi^2\left[\frac{B(m,\bar{n})}{D(m,\bar{n})} - A(m,\bar{n})\right]}{Z(k_1 m^2 + k_2 \bar{n}^2)} \tag{15}$$

where

$$A(m,\bar{n}) = -d_{11}m^4 - 2(d_{12} + 2d_{66})m^2\bar{n}^2 - d_{22}\bar{n}^2$$

$$B(m,\bar{n}) = -d_{21}m^4 - 2(2b_{66} - b_{11} - b_{22})m^2\bar{n}^2 - b_{12}\bar{n}^2 + m^2 z/\pi^2$$

$$D(m,\bar{n}) = a_{22}m^4 - 2(2b_{66} - b_{11} - b_{22})m^2\bar{n}^2 - b_{12}\bar{n}^2 + m^2 z/\pi^2$$

$$\bar{n} = \frac{nL}{\pi R}; \quad z = \frac{L^2}{Rt}$$

Where L is shell length, R is the radius of the middle surface of the shell and t is the total thickness of the shell. a_{ij}, b_{ij} and d_{ij} (i, j = 1, 2, 6) are components of matrices [a], [b], and [d], which are defined as

$$[a] = [\tilde{A}]^{-1}; \quad b = -[\tilde{A}]^{-1}[\tilde{B}]; \; [d] = [\tilde{D}] - [\tilde{B}][\tilde{A}]^{-1}[\tilde{B}] \tag{16}$$

m is the number of half waves in the length of the shell, n is the number of half waves in circumference. The critical buckling load λ_c is obtained by minimising $\lambda(m,n)$ with respect to integer values m and n.

3.3 Laminate composite pressure vessel

A laminated fibre-reinforced composite thin walled pressure vessel of radius R (from the center to the mid-surface of the shell), and total thickness h, subjected to internal pressure p as shown in Fig. 3. The shell has a symmetric layup consisting of K layers of equal thickness t. Let the mid-surface of the shell be the reference surface, and let the origin of the coordinates be located at one end of the pressure vessel. The structure is referenced in an orthogonal coordinate system (x, y, z), where x is the longitudinal, y the circumferential, and z the radial direction. The displacement components u, v and w lie in the x, y and z directions, respectively. The fibre angle is defined as the angle between the fibre direction and the longitudinal (x) axis. The fibre orientations are symmetric with respect to the mid-plane of the shell. The force (N_x, N_y, N_{xy}) and moment resultants (M_x, M_y, M_{xy}) can be written as

$$N_x = \frac{pR}{2}; \quad N_y = pR \quad \text{and}$$
$$N_{xy} = M_x = M_y = M_{xy} = 0 \tag{17}$$

When the applied loads are greater than or equal to the critical values, buckling or overstressing will occur.

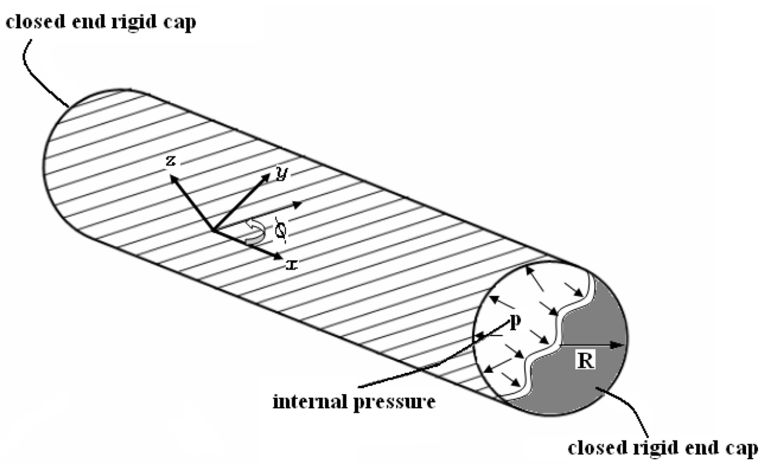

Fig. 3 Laminate composite pressure vessel

The constitutive equation for the fibre-reinforced composite pressure vessel can be rewritten as

$$\{N\}_{xy} = [A]\{\varepsilon_{xy}\}, \quad \{M\}_{xy} = [D]\{K\}_{xy} \tag{18}$$

where A and D are axial and bending stiffness matrices defined in Eq. (2), $\{N\}_{xy}$ is the force resultant tensor, $\{M\}_{xy}$ the moment resultant tensor, $\{\varepsilon\}_{xy}$ the strain tensor, $\{K\}_{xy}$ the curvature tensor. These matrices will be used in failure analysis of the pressure vessel.

3.4 Tsai-wu failure criteria

The Tsai-Wu failure criterion is employed to assess the capability of the composite cylindrical shell to withstand failure due to overstressing. A strength level factor, ζ is used to identify the characteristics of the first-ply failure of the cylindrical shell. Based on the Tsai-Wu failure criterion, the strength level factor ζ is defined as

$$\zeta = \max_k \{F_{11}(\sigma_L^{(k)})^2 + F_{22}(\sigma_T^{(k)})^2 + F_{66}(\sigma_{LT}^{(k)})^2 + 2F_{12}\sigma_T^{(k)}\sigma_L^{(k)} + F_1\sigma_L^{(k)} + F_2\sigma_T^{(k)}\} \tag{19}$$

In Eq. (19), each stress component of the kth layer $\sigma_L^{(k)}, \sigma_T^{(k)}, \sigma_{LT}^{(k)}$ in the material direction can be calculated by

$$\{\sigma\}_{LT}^{(k)} = [T][\bar{Q}]^{(k)}[A]^{-1}\{N\}_{xy} \tag{20}$$

where $\{N\}_{xy}$ is the force resultants $[N_x, N_y, N_{xy}]^T$, defined in Eq. (17); $[A]^{-1}$ the inverse of the extension stiffness matrix $[A]$, given in Eq. (2); $\bar{Q}$ is the reduced stiffness of the kth layer of the laminate; and the strength parameters F_{11}, F_{12}, F_{22}, F_{66}, F_1 and F_2 are given by

$$F_{11} = \frac{1}{\sigma_{LU}^T\sigma_{LU}^C}, \quad F_{22} = \frac{1}{\sigma_{TU}^T\sigma_{TU}^C}, \quad F_{66} = \frac{1}{\sigma_{TLU}^2}$$

$$F_1 = \frac{1}{\sigma_{LU}^T} - \frac{1}{\sigma_{LU}^C}, \quad F_2 = \frac{1}{\sigma_{TU}^T} - \frac{1}{\sigma_{TU}^C}, \quad F_{12} = -\frac{1}{2}\frac{1}{\sqrt{\sigma_{LU}^T\sigma_{LU}^C\sigma_{TU}^T\sigma_{TU}^C}} \tag{21}$$

where σ_{LU}^T, σ_{LU}^C, σ_{TU}^T and σ_{TU}^C are the tensile and compressive strengths of the composite material in the longitudinal and transverse directions, and σ_{TLU} is the in-plane shear strength.

In order to ensure that first-ply failure does not occur in the cylindrical shell, the following condition must be satisfied

$$\zeta \le 1 \tag{22}$$

Eq. (22) is used as a design constraint in the proposed stacking sequence optimisation of cylindrical shell.

4. Numerical studies

Numerical studies are carried out to evaluate the Pareto optimal solutions using the proposed multi objective evolutionary algorithm. The three numerical examples considered are: hybrid laminate composite cylinder, stiffened cylindrical shell and hybrid pressure vessel.

4.1 Laminate composite cylinder

A hybrid laminate composite cylinder of length, L = 0.4 m and radius, R = 0.10 m subjected to in plane compressive load as shown in Fig. 1, is considered as the first numerical example. The material properties of Carbon-epoxy and Glass-epoxy used in this study are given in Table 1.

In this section different sets of numerical studies have been carried out using the laminate composite cylindrical shell problem by varying the number of design objectives. The objective functions and design constraints considered for the studies carried out on the cylindrical shell are

$$\begin{gathered}\text{Minimise, Weight} = 2\pi RL(\rho_c h_c + \rho_g(h-h_c))\\ \text{Minimise, Cost} = 2\pi RL(\rho_c h_c C_c + \rho_g(h-h_c)C_g)\\ \text{Maximise Buckling stress } \overline{N}_x/t\\ \text{Maximise fundamental frequency } f\end{gathered} \tag{23}$$

Where ρ_c material density of the Graphite-epoxy layers and ρ_g is the material density of Glass-epoxy layers and h is the total thickness of the laminate and h_c is the thickness of the Graphite-epoxy plies in the laminate. C_c and C_g are the cost factors for Carbon-epoxy and Glass-epoxy plies respectively. f is the fundamental frequency and $\overline{N}_x/t$ is the buckling stress of the composite cylinder.

In the present study, the following three combinatorial constraints have been considered.

1. Four contiguous ply rule: The same fiber angle cannot be stacked more than four contiguous plies. This is to prevent large matrix cracking
2. Ply angle difference: The fiber angle between two adjacent plies must be less than or equal to 45°. This rule is to prevent matrix cracking due to large thermal stresses.
3. Ply balancing rule: The number of 45° plies is equal to the number of –45° plies. This constraint is to prevent the shear-tension stiffness coupling.

All the combinatorial constraints are treated using a correction operator (Rama Mohan Rao and Lakshmi 2009, Rama Mohan Rao and Arvind 2005, Todorki and Haftka 1998).

Table 1 Material properties of laminate composites

Property	Graphite-epoxy	Glass-epoxy
Long. modulus, E_1	140.68 GPa	44.2 GPa
Trans. modulus, E_2	9.13 Gpa	9.07 GPa
Shear modulus G_{12}	7.24 GPa	4.64 GPa
Poisson Ratio, υ_{12}	0.30	0.27
Density, ρ	1605.434 kg/m^3	1992.95 kg/m^3
Thickness, t	0.000127 m	0.000127 m
Cost factor, C	8	1

Table 2 Pareto optimal solutions of hybrid laminate composite cylinder for optimisation of cost and weight with constraint on buckling stress as 1000 N/m^2

S.No	Cost	Weight (N)	Num. of Layers	Stacking sequence	Buckling Stress (N/m^2)
1	40.52	31.06	52	$[-45^{o}_{(g)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}, -45^{o}_{(gl)}, 0^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}]_s$	1011.59
2	43.2	27.32	52	$[-45^{o}_{(g)}, -45^{o}_{(g)}, 90^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}, 45^{o}_{(gl)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}]_s$	1020.69
3	45.9	23.55	40	$[-45^{o}_{(g)}, 90^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, -45^{o}_{(gl)}, 0^{o}_{(gl)}, 45^{o}_{(gl)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}]_s$	1006.02
4	49.88	20.94	40	$[45^{o}_{(g)}, 0^{o}_{(g)}, -45^{o}_{(g)}, -45^{o}_{(g)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, -45^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl),} 0^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}]_s$	1004.54
5	53.89	18.32	32	$[-45^{o}_{(g)}, -45^{o}_{(g)}, 90^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(g)}, 0^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, 0^{o}_{(gl)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, 90^{o}_{(gl)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}]_s$	1010.55
6	57.91	15.68	28	$[-45^{o}_{(g)}, -45^{o}_{(g)}, 90^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(g)}, 0^{o}_{(g)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}, 0^{o}_{(gl)}]_s$	1007.97
7	63.2	14.24	28	$[-45^{o}_{(g)}, -45^{o}_{(g)}, 0^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(g)}, 90^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}]_s$	1000.25
8	68.51	12.79	28	$[-45^{o}_{(g)}, -45^{o}_{(g)}, 0^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(g)}, 0^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(g)}, 0^{o}_{(gl)}, -45^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}]_s$	1000.48
9	75.07	12.56	28	$[45^{o}_{(g)}, 45^{o}_{(g)}, 90^{o}_{(g)}, -45^{o}_{(g)}, -45^{o}_{(g)}, 0^{o}_{(g)}, -45^{o}_{(g)}, 90^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(gl)}, 90^{o}_{(gl)}, -45^{o}_{(gl)}]_s$	1010.34
10	81.64	12.32	32	$[45^{o}_{(g)}, 90^{o}_{(g)}, -45^{o}_{(g)}, 0^{o}_{(g)}, -45^{o}_{(g)}, 90^{o}_{(g)}, -45^{o}_{(g)}, 0^{o}_{(g)}, 45^{o}_{(g)}, 0^{o}_{(g)}, 45^{o}_{(gl)}, 0^{o}_{(gl)}]_s$	1008.66
11	88.2	12.08	28	$[-45^{o}_{(g)}, -45^{o}_{(g)}, 0^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(g)}, 90^{o}_{(g)}, 45^{o}_{(g)}, 90^{o}_{(g)}, -45^{o}_{(g)}, -45^{o}_{(g)}, 90^{o}_{(g)}, 45^{o}_{(gl)}]_s$	1002.13
12	94.76	11.84	32	$[-45^{o}_{(g)}, 90^{o}_{(g)}, 45^{o}_{(g)}, 45^{o}_{(g)}, 90^{o}_{(g)}, 90^{o}_{(g)}, -45^{o}_{(g)}, 0^{o}_{(g)}, 45^{o}_{(g)}, 0^{o}_{(g)}, -45^{o}_{(g)}, 0^{o}_{(g)}]_s$	1012.76

Table 2 shows the Pareto optimal solutions obtained using the multi-objective evolutionary algorithm for simultaneous optimisation of cost and weight with constraint on buckling stress as 1000 N/m^2. The following observations can be made from the results furnished in Table 2.

i. The optimal stacking sequence with minimum cost is found to be 40.52 and the corresponding weight is found to be 31.06 N. It can also be observed that majority of plies in the stacking sequence corresponding to the minimum cost consists of only Glass-epoxy plies, which are relatively cheaper when compared to graphite-epoxy plies. Infact one can observe that all plies except one extreme ply is made of glass-epoxy.

ii. The minimum weight is found to be 11.84 N and the stacking sequences corresponding to minimum weight consists of only graphite-epoxy plies which are lighter when compared to

Glass-epoxy layers, but expensive. The cost of this stacking sequence is found to be 94.76, which is the highest among all the stacking sequences.

From the stacking sequences given in Table 2, it can be observed that, with the increase in cost (or in other words reduction in weight), the number of graphite-epoxy plies in the outer fibres of the laminate composite cylinder are gradually increasing.

iii. It should be pointed out here that the design constraint is also treated as an additional design objective by recasting the constraint as an objective function and minimized. Hence the problem considered here, in fact attempts to optimise three objectives simultaneously. It can be observed from the constraint value given in Table 2, that the buckling stress values are very close to the specified design constraint value. This study clearly indicates that the proposed constraint handling technique appears to be promising as the difference between constraint value and the buckling load factor is smaller.

Fig. 4 shows the Pareto optimal curve of the hybrid laminate composite cylinder obtained using the multi-objective evolutionary algorithm for simultaneous optimisation of cost as well as weight with constraint on buckling stress. It is clearly evident from Fig. 4, that a Pareto front with well spread Pareto solutions can be obtained using the multi-objective evolutionary algorithm.

The same problem i.e., laminate composite cylinder is solved by considering three objectives. The design objectives considered are minimization of cost, minimization of weight and maximisation of buckling stress. The design constraint considered for this problem is fundamental frequency f and it is set as 5.0 Hz. Table 3 shows the Pareto optimal solutions. The details given in the table include the stacking sequences and the corresponding cost, weight, buckling stress and fundamental frequency. The corresponding Pareto optimal curve with constraint on natural frequency is given in Fig. 5. It can easily verified from the results given in Table 3 and also the Pareto optimal curve given in Fig. 5, that well spread Pareto optimal solutions can be obtained using the proposed multi-objective evolutionary algorithm.

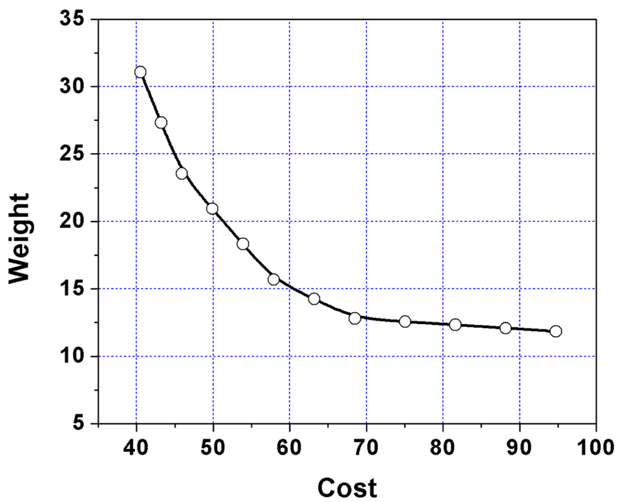

Fig. 4 Pareto optimal curve obtained using multi-objective Evolutionary algorithm for a hybrid laminate composite cylinder with constraint on buckling stress

Table 3 Pareto optimal solutions of hybrid laminate composite cylinder for optimisation of cost, weight and buckling stress with constraint on frequency as 5 Hz

S. NO	Stacking sequence	Number of plies	Cost	Weight (N)	Bucking stress (N/mm^2)	Frequency (Hz)
1	[90$^o_{(g)}$, -45$^o_{(g)}$, 0$^o_{(g)}$, 0$^o_{(g)}$, -45$^o_{(g)}$, 0$^o_{(gl)}$, 45$^o_{(gl)}$, 0$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, 45$^o_{(gl)}$, 90$^o_{(gl)}$, 45$^o_{(gl)}$, 90$^o_{(gl)}$, 45$^o_{(gl)}$, 90$^o_{(gl)}$, -45$^o_{(gl)}$, 90$^o_{(gl)}$]$_s$	38	57.65	21.90	967.4	5.02
2	[45$^o_{(g)}$, 45$^o_{(g)}$, 0$^o_{(g)}$,45$^o_{(g)}$, 90$^o_{(g)}$, -45$^o_{(g)}$, -45$^o_{(g)}$, 90$^o_{(gl)}$, 45$^o_{(gl)}$, 90$^o_{(gl)}$, 90$^o_{(gl)}$, 45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 90$^o_{(gl)}$, -45$^o_{(gl)}$]$_s$	36	69.43	20.23	1215.2	5.07
3	[-45$^o_{(g)}$, -45$^o_{(g)}$, 90$^o_{(g)}$, 45$^o_{(g)}$, 45$^o_{(g)}$, 90$^o_{(g)}$, -45$^o_{(g)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, 45$^o_{(gl)}$, 45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, 45$^o_{(gl)}$, 90$^o_{(gl)}$, 45$^o_{(gl)}$]$_s$	40	71.89	22.61	1330.1	5.39
4	[90$^o_{(g)}$, 90$^o_{(g)}$, 45$^o_{(g)}$, 0$^o_{(g)}$, 45$^o_{(g)}$, 90$^o_{(g)}$, 45$^o_{(g)}$, 0$^o_{(g)}$, 45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 90$^o_{(gl)}$, -45$^o_{(gl)}$, 90$^o_{(gl)}$, -45$^o_{(gl)}$, 90$^o_{(gl)}$, -45$^o_{(gl)}$]$_s$	34	74.71	18.81	1067.6	5.23
5	[-45$^o_{(g)}$, 90$^o_{(g)}$, 45$^o_{(g)}$, 90$^o_{(g)}$, 45$^o_{(g)}$, 0$^o_{(g)}$, 45$^o_{(g)}$, 0$^o_{(g)}$, 45$^o_{(g)}$, 0$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 90$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$]$_s$	34	81.23	18.57	1209.3	5.63
6	[45$^o_{(g)}$, 45$^o_{(g)}$, 90$^o_{(g)}$, -45$^o_{(g)}$, -45$^o_{(g)}$, 0$^o_{(g)}$, 45$^o_{(g)}$, 90$^o_{(g)}$, -45$^o_{(g)}$, 0$^o_{(gl)}$, 45$^o_{(gl)}$, 45$^o_{(gl)}$, 90$^o_{(gl)}$, -45$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 90$^o_{(gl)}$, 45$^o_{(gl)}$]$_s$	38	83.67	20.95	1386.9	5.56
7	[-45$^o_{(g)}$, -45$^o_{(g)}$, 90$^o_{(g)}$, 45$^o_{(g)}$, 45$^o_{(g)}$, 90$^o_{(g)}$, 45$^o_{(g)}$, 90$^o_{(g)}$, 45$^o_{(g)}$, 90$^o_{(g)}$, 90$^o_{(g)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$, 0$^o_{(gl)}$, 45$^o_{(gl)}$, 0$^o_{(gl)}$, -45$^o_{(gl)}$]$_s$	36	95.47	19.29	1322.6	5.32

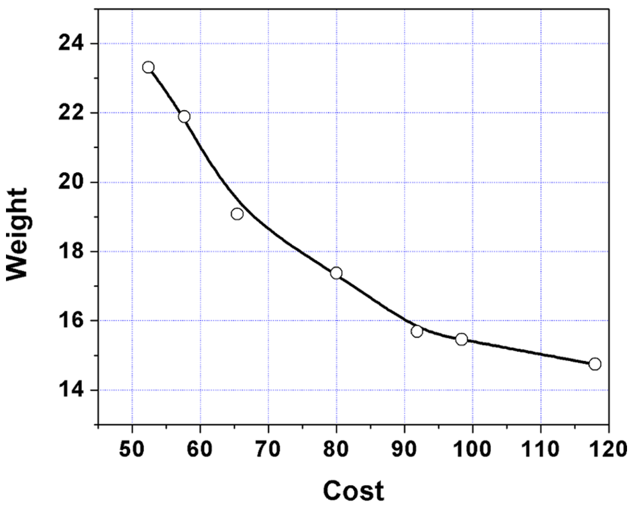

Fig. 5 Pareto optimal curve obtained using multi-objective evolutionary algorithm for a hybrid laminate composite cylinder with constraint on natural frequency

4.2 Optimization of stiffened laminate composite cylinder

This numerical study is concerned with the design of a hybrid stiffened laminate composite cylinder for optimum weight, cost, ply angle sequence and stiffener configuration. The stiffened shell is made up of a graphite-epoxy and/or glass-epoxy material with lamina Properties given in Table 1. In the stiffener, all fibres are laid parallel along the stiffener axis. The shell laminate is composed of N orthotropic layers of equal thickness. The buckling behaviour of the shell may be influenced by a number of factors, such as the shell geometry, the shell laminate stacking sequence, the stiffener eccentricity, the stiffener cross-section parameters, stiffener bending stiffness parameters etc. In the present work, it is proposed to investigate the effect of stacking sequence and also the stiffener configuration on the buckling load of the stiffened cylindrical shell. Accordingly, the shell geometry, the stiffener cross section properties, bending stiffness of stiffeners is kept constant.

The fiber-reinforced stiffened composite cylinder considered for evaluation has a middle radius $R = 0.73$ m and a length $L = 1.25$ m. The cross sectional areas of both the stiffeners i.e., stringer and rings are taken as same, with width, b_s as 0.03 m. and depth, d_s as 0.0182 m. The external stiffener configuration is used for this study. The stiffeners can be of either glass-epoxy or graphite-epoxy.

The ply orientations considered for the optimisation of cylindrical shell are 0^o, ±45^o, 90^o. The laminates are considered to be symmetric and balanced. Only half of the plies of a laminate are considered as design variables because of symmetry. All the three combinatorial constraints considered for earlier problem are considered for this numerical example too and are treated using correction operator.

The optimization problem considered here is to optimize the stacking sequence of the stiffened cylindrical shell with varied number of stingers and rings for simultaneous optimisation of cost and weight with axial buckling load as a design constraint. In the present example, two different materials i.e., graphite-epoxy, which is expensive, but has high stiffness properties and glass-epoxy, which is not as stiff but is relatively cheaper are considered. The design constraint value is taken as 4.0 kN for the problem solved in this paper. The trade off solutions obtained using the proposed evolutionary algorithm are shown in Table 4. The results furnished in the table include the stacking sequences, number of plies, their corresponding weight, cost, buckling load factor and stiffener configuration. A close look at the results indicates that, the minimum weight of the hybrid laminated stiffened cylinder is 385.23 N. It is obtained when majority of plies are made of graphite-epoxy. The corresponding cost is 3081.82. The minimum cost is found to be 504, which is about 16.35% of the cost of stacking sequence corresponding to the optimal weight. The optimum ply sequence corresponding to the minimum cost consists of only glass-epoxy plies. It's weight is found to be 504 N which is approximately 30.83% heavier than the optimum weight laminate sequence. This study clearly confirms the effectiveness of the proposed evolutionary algorithm in obtaining optimal solutions for simultaneous cost/weight minimisation. The trade-off solutions given in Table 4, can be used by the designer to determine the optimal configurations for his problem. The final choice of the best design will depend on additional information that will enable him to evaluate all the points on the Pareto curve and prioritise these values depending on the application on hand. The Pareto optimal curve is given in Fig. 6.

Table 4 Trade-off solutions of hybrid laminate composite stiffened cylinder for multi-objective optimization of both weight and cost using proposed multi-objective evolutionary algorithm with constraint on buckling load (4 kN)

Stacking sequence	Number of plies	Cost	Weight (N)	Buckling Load (kN)	NRG	NST	Stiffener Material type
$[-45^o_{(gl)}, 0^o_{(gl)}, 45^o_{(gl)}, 90^o_{(gl)}, 90^o_{(gl)}, 45^o_{(gl)}, 0^o_{(gl)}, -45^o_{(gl)}, 0^o_{(gl)}]_s$	18	504.00	504.00	4.17	4	4	2
$[90^o_{(g)}, -45^o_{(gl)}, 90^o_{(gl)}, -45^o_{(gl)}, 0^o_{(gl)}, 45^o_{(gl)}, 45^o_{(gl)}, 0^o_{(gl)}]_s$	16	650.87	473.40	5.50	4	4	2
$[-45^o_{(g)}, -45^o_{(g)}, 0^o_{(gl)}, 0^o_{(gl)}, 45^o_{(gl)}, 90^o_{(gl)}, 45^o_{(gl)}, 90^o_{(gl)}]_s$	16	802.70	467.89	5.23	4	4	2
$[-45^o_{(g)}, -45^o_{(g)}, 0^o_{(gl)}, 45^o_{(gl)}, 0^o_{(gl)}, 45^o_{(gl)}, 0^o_{(gl)}, 45^o_{(gl)}]_s$	16	802.70	467.89	5.01	4	4	2
$[45^o_{(g)}, 90^o_{(g)}, 45^o_{(g)}, 90^o_{(g)}, -45^o_{(gl)}, 0^o_{(gl)}, -45^o_{(gl)}, 0^o_{(gl)}]_s$	16	1106.34	456.88	6.24	4	4	2
$[45^o_{(g)}, 90^o_{(g)}, 45^o_{(g)}, 0^o_{(g)}, -45^o_{(gl)}, 0^o_{(gl)}, -45^o_{(gl)}, 0^o_{(gl)}]_s$	16	2170.89	418.27	6.84	4	4	2
$[45^o_{(g)}, 90^o_{(g)}, 45^o_{(g)}, 0^o_{(gl)}, -45^o_{(gl)}, 0^o_{(gl)}, -45^o_{(gl)}, 0^o_{(gl)}]_s$	16	2322.71	412.76	4.02	4	4	1
$[45^o_{(g)}, 90^o_{(g)}, 45^o_{(g)}, 90^o_{(g)}, -45^o_{(gl)}, 0^o_{(gl)}, -45^o_{(gl)}, 0^o_{(gl)}]_s$	16	2474.53	407.26	4.05	4	4	1
$[-45^o_{(g)}, 90^o_{(g)}, -45^o_{(g)}, 0^o_{(g)}, 45^o_{(g)}, 0^o_{(gl)}, 45^o_{(gl)}, 0^o_{(gl)}]_s$	16	2626.36	401.75	4.34	4	4	1
$[-45^o_{(g)}, 90^o_{(g)}, -45^o_{(g)}, 90^o_{(g)}, 45^o_{(g)}, 0^o_{(g)}, 45^o_{(gl)}, 0^o_{(gl)}]_s$	16	3081.82	385.23	4.72	4	4	1

Material Type = 1: Graphite-epoxy; Material Type = 2: Glass-epoxy
NRG: Number of Rings; NST: Number of Stringers

4.3 Hybrid Laminate composite pressure vessel

In the third example, we increase the dimension of the performance space by increasing the number of objectives to four. We have considered the problem of a thin walled pressure vessel. The pressure vessel considered is symmetric and has radius R = 1.2 m, composed of laminate made of Graphite epoxy and Glass epoxy. The pressure vessel is subjected to an internal pressure p, as shown in Fig. 3. The force resultants are calculated using Eqs. (17) and (18). The Tsai-Wu failure criterion yields a quadratic equation in p, which is used to determine the positive and negative pressures p_n^+ and p_n^- respectively, that would cause the nth lamina to fail. We ignore p_n^-, since the pressure vessel is subjected to only positive internal pressures. The first-ply failure pressure, p_f, of the laminated pressure vessel is determined by the smallest positive value of p_n^+.

A multi-objective optimization of hybrid pressure vessel is performed considering the following four objective functions

$$\text{Minimise, Weight} = 2\pi RH(\rho_c h_c + \rho_g(h - h_c))$$

$$\text{Minimise, Cost} = 2\pi RH(\rho_c h_c C_c + \rho_g(h - h_c)C_g)$$

$$\text{Maximise the hoop rigidity} = \bar{E}_y h$$

$$\text{Maximise the axial rigidity} = \bar{E}_x h \tag{24}$$

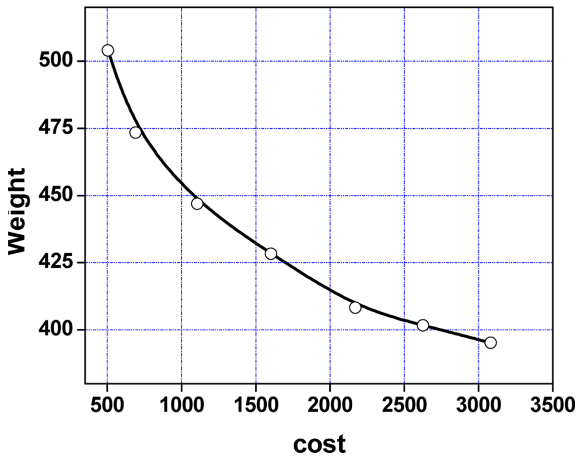

Fig. 6 Pareto optimal curve obtained using multi-objective evolutionary algorithm for a hybrid laminate composite stiffened cylinder with constraint on buckling

Table 5 Multi-objective optimization of pressure vessel with the proposed algorithm with four objectives and constraint on failure pressure as 1.0 MN/mm^2

S. No.	Stacking sequence	Number of plies	Cost	Weight (N)	Axial Rigidity (MN/m)	Hoop Rigidity (MN/m)	Pressure (MN/mm^2)
1	$[45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}]_s$	16	1922.96	240.37	82.18	143.44	1.198
2	$[0^{\circ}_{(g)}, 0^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(gl)}]_s$	18	1963.36	277.63	112.48	144.89	1.150
3	$[90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}, 0^{\circ}_{(gl)}]_s$	20	2203.47	307.64	129.11	152.22	1.260
4	$[90^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(gl)}, -45^{\circ}_{(gl)}, 90^{\circ}_{(gl)}, -45^{\circ}_{(gl)}]_s$	26	2324.53	419.36	157.06	163.51	1.083
5	$[45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(gl)}]_s$	20	2443.53	337.65	130.07	161.20	1.379
6	$[-45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 45^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}, -45^{\circ}_{(gl)}, 90^{\circ}_{(gl)}, -45^{\circ}_{(gl)}, 90^{\circ}_{(gl)}]_s$	28	2564.50	449.35	111.77	186.65	1.010
7	$[0^{\circ}_{(g)}, 45^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(gl)}, 90^{\circ}_{(gl)}, 90^{\circ}_{(gl)}, 45^{\circ}_{(gl)}]_s$	28	2564.50	449.35	167.00	185.78	1.045
8	$[-45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 45^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(gl)}, 45^{\circ}_{(gl)}]_s$	26	2723.85	404.88	113.77	179.04	1.162
9	$[45^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}, 0^{\circ}_{(g)}]_s$	24	2883.22	360.40	200.77	139.01	1.150
10	$[90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 90^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}, 90^{\circ}_{(g)}, -45^{\circ}_{(g)}, 0^{\circ}_{(g)}, 0^{\circ}_{(g)}, 45^{\circ}_{(g)}, 0^{\circ}_{(g)}, -45^{\circ}_{(g)}]_s$	28	3363.04	420.38	241.94	178.59	1.454

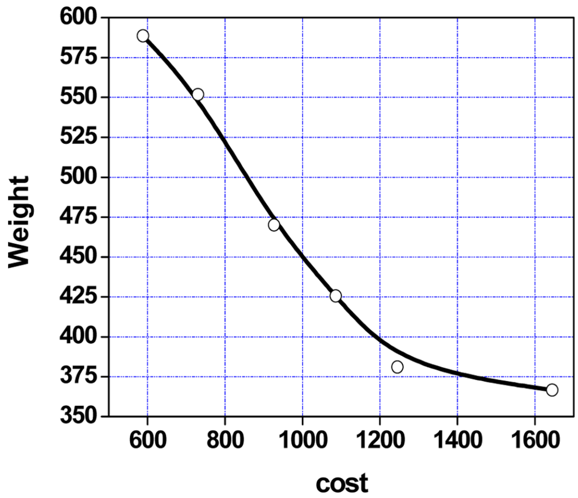

Fig. 7 Pareto optimal curve obtained using multi-objective evolutionary algorithm for a hybrid laminate pressure vessel with constraint on failure pressure

The design constraint is first ply failure pressure $p_f \geq 1.0$ MPa. H is the height of the pressure vessel and we have computed the objective values per meter height. $\overline{E}_x, \overline{E}_y$ are effective modulus as defined in Eq. (3).

All the three combinatorial constraints considered for earlier problem are considered for this numerical example too and are treated using correction operator.

This problem is relevant to the design of stiff, light weight fuel tanks containing compressed gas. Similar to the earlier case studies, the design constraint related to failure pressure is treated as an additional objective by minimising the value $(1 - p_f)$. Table 5 shows the non-dominated solutions obtained using the proposed multi-objective meta-heuristic algorithm. It is evident from this study that the proposed algorithm is scalable and larger number of objectives can be considered. It can also be observed from Table 5 that the failure pressure is very close to the specified design constraint value i.e., 1.0 MPa. This study clearly illustrates that the constraint handling by way of considering it as an additional design objective is found to be effective for laminate stacking sequence optimisation problems. Fig. 7 shows the Pareto optimal curve obtained with two objectives i.e. cost and weight with constraint on failure pressure 1.0 MPa.

5. Performance evaluation of the proposed multi-objective algorithm

The performance of the proposed PAES inspired algorithm is compared with the original version of PAES algorithm. In both the algorithms, the variable depth neighbourhood search has been employed as a mutation operator. Further, the PEAS implementations with other customised neighbourhood search algorithms (mutation operators) are also considered for evaluation to isolate the effect of the adaptive neighbourhood search algorithm. The other mutation operators tested are *invert, permutation, swap* which are earlier used in the GA based stacking sequence algorithms (Le Riche and Haftka 1995) as mutation operators. However, for completeness the customized mutation operators are briefly described here. The neighbourhood search technique called *Invert* assigns each variable a small probability to switch to any other permissible integer value (except the value before alteration). Similarly, the neighbourhood search technique called *Permutation* chooses two locations randomly in the string of design variables and reverses the order of the variables between the two chosen locations. *Swap* is another neighbourhood search technique, which is less disruptive when compared to permutation. The swap technique is implemented by randomly selecting two unique design variables in the string and switching their positions. The *permutation* search technique has been used in combination with *swap* to improve the exploratory characteristics of neighbourhood search.

In order to evaluate the performance of the proposed multi-objective optimisation algorithm, two different criteria are taken into account: closeness of the Pareto front generated by the algorithm to the true Pareto-front and the spread of solutions found. The solution spread should be maximum so that we can have a distribution of vectors as smooth and uniform as possible.

To evaluate the first criterion, it is usually necessary to know the exact location of the true Pareto front. Since we do not have an idea of an exact Pareto front, the usual practice is to combine all the Pareto optimal solutions obtained from all the multi-objective algorithms considered for evaluation. The non-dominated solutions obtained from these combined Pareto front solutions are taken as true Pareto front solutions for evaluating the performance of the proposed algorithm. The number of true Pareto front solutions are however restricted to maximum of ten by using the adaptive grid technique discussed earlier.

Since the evolutionary algorithms are basically stochastic algorithms, there is no assurance that the converged Pareto front is same in each execution. In view of this, the algorithms are executed 100 times (100 independent runs) with the same input data and control parameters. The best Pareto set obtained in majority of executions (say at least 80 times of 100 executions) is considered for evaluation. We have also recorded the practical reliability (Le Riche and Haftka 1993) of the Pareto solutions obtained for each algorithm in order to evaluate the consistency of these algorithms. The practical reliability of an algorithm is given by the percentage of best Pareto set of solutions obtained, after several (100 in this paper) independent runs of the same algorithm.

The performance metrics can be classified into three categories depending on whether they evaluate the closeness to the Pareto front, the diversity in the obtained solutions, or both. We have adopted the following metrics to evaluate the performance of the proposed meta-heuristic algorithm.

i. Metrics to evaluate the closeness to the Pareto-optimal front:
   (a) Error Ratio (b) Set Convergence Metric (SCM) (c) Generational Distance (GD)
ii. Metrics evaluating diversity among Non-dominated solutions
   (a) Spacing (b) Spread (c) Maximum spread (d) Chi-square deviation measure
iii. Metrics evaluating both closeness and diversity to the true Pareto-front
   (a) Hyper volume

The details of these metrics are very well documented in the literature (Deb 2001) and hence not repeated here.

Apart from the above metrics, we have also included a metric called 'dominated solutions', which indicates the number of Pareto solutions of a particular algorithm dominated by the Pareto solutions generated by rest of the algorithms. Hence the performance of an algorithm is considered superior, if the number of dominated solutions is minimum.

For all the numerical examples, we have considered the case with only two objectives i.e., optimisation of cost and weight with buckling load factor as design constraint for cylinder, stiffened cylinder and failure pressure as design constraint for pressure vessel problem. Tables 6, 7 and 8 shows the performance metrics of the algorithms for fibre reinforced cylindrical shell, stiffened shell and pressure vessel respectively. It can be verified from the metrics presented for all the multi-objective algorithms considered for evaluation, that the proposed PAES variant exhibits superior performance in eight out of ten parameters listed in Tables 6 to 8 for cylindrical shell, stiffened shell and pressure vessel respectively. From this results furnished in Tables 6 to 8, it can be easily concluded that the performance of the proposed variant built with variable depth neighbourhood

Table 6 Performance metrics of the proposed multi-objective evolutionary algorithm-Cylindrical shell

Parameter	Proposed algorithm	PAES with variable depth neighbourhood search	PAES with swap	PAES with Permutation	PAES with invert	Preferred value
Error ratio:	0	0	0	0	0	Smaller
Gen dist:	0	0	0	0	0	Smaller
Spacing:	0.143612	0.127138	0.141831	0.084291	0.069921	Smaller
Spread:	0.018712	0.111513	0.20066	0.185615	0.272789	Smaller
Max spread:	57.54464	53.83367	45.18895	50.24364	45.79327	Greater
Chi sqr:	0	1.477098	1.477098	1.477098	1.809068	Smaller
Hyper vol:	36906.83	26855.66	25043.00	23729.04	19229.04	Greater
Set coverage:	0	0	0	0	0	Smaller
Number of solutions dominated	0	2	2	2	3	Smaller
Relaibility	0.98	0.93	0.86	0.82	0.88	Greater

Table 7 Performance metrics of the proposed multi-objective evolutionary algorithm-Stiffened cylindrical shell

Parameter	Proposed algorithm	PAES with variable depth neighbourhood search	PAES with swap	PAES with Permutation	PAES with invert	Preferred value
Error ratio:	0	0	0	0	0	Smaller
Gen dist:	0	0	0	0	0	Smaller
Spacing:	103.9501	109.4923	54.32223	114.6007	54.32223	Smaller
Spread:	0.188198	0.311368	0.50538	0.473407	0.50538	Smaller
Max spread:	2580.554	2432.548	1972.903	2280.618	1972.903	Greater
Chi sqr:	0	1.06066	1.5	1.5	1.5	Smaller
Hyper vol:	17684085	14461191	11202736	12180140	11202736	Greater
Set coverage:	0	0	0	0	0	Smaller
Number of solutions dominated	0	2	2	3	2	Smaller
Relaibility	0.94	0.90	0.86	0.92	0.88	Greater

Table 8 Performance metrics of the proposed multi-objective evolutionary algorithm-pressure vessel

Parameter	Proposed algorithm	PAES with variable depth neighbourhood search	PAES with swap	PAES with Permutation	PAES with invert	Preferred value
Error ratio:	0	0.2	0	0	0.333333	Smaller
Gen dist:	0	0.02381	0	0	0.129289	Smaller
Spacing:	88.36266	84.7658	21.85098	16.73114	199.1523	Smaller
Spread:	0.081911	0.267874	0.403519	0.540211	0.528855	Smaller
Max spread:	1085.765	856.7253	688.7166	469.2056	1424.445	Greater
Chi sqr:	0	1.095445	1.095445	1.549193	1.183216	Smaller
Hyper vol:	6092561	5060194	3706741	2764411	2944451	Greater
Number of solutions dominated	0	2	2	2	2	Smaller
Relaibility	0.96	0.90	0.86	0.83	0.88	Greater

search to maintain intensification and a crossover operator to main diversification help considerably in improving the performance of the algorithm. It is clearly evident from the performance metrics given in Tables 6 to 8, that the variable depth neighbourhood search algorithm acts as an effective mutation operator for the laminate composite problems when compared to other customised mutation operators reported in the literature for the multi-object.

6. Conclusions

This paper presents the details of a multi-objective evolutionary algorithm developed for combinatorial optimisation and applied for optimal stacking sequence design of laminate composite shell structures. The frame work of the proposed algorithm is largely based on the popular multi-objective evolutionary algorithm, PAES. It is well known that the meta-heuristic multi-objective optimisation algorithms perform rather well with customized operators and also multi-objective algorithms work well when a perfect balance of intensification and diversification mechanisms are maintained through out the evolutionary process. Keeping these things in view, we have proposed to employ a customized neighbourhood search algorithm called variable depth neighbourhood search algorithm in the place of mutation operator of PAES. This particular feature built into the algorithm contributes to intensification mechanism. Further a crossover operator is employed to generate new solutions making use of non-dominated (archived) solutions generated already in order to improve the diversification mechanism. If the child solutions produced using either neighbourhood search or crossover operator failed to find non-dominated solution in any particular generation, we generate a random solution and treat it as a parent. This works a multi start mechanism, which is also a very effective search strategy (Rama Mohan Rao and Shyju 2008) for finding Pareto optimal solutions. Similar to PAES, the proposed algorithm uses external archive feature to store the non-dominated individuals found during the search. It also uses the adaptive grid technique for archive management in order to remove individuals from the archive when it becomes full.

Numerical experiments have been conducted by solving a hybrid laminated composite cylindrical shell, stiffened shell and pressure vessel problems. The studies presented in this paper clearly demonstrate that the proposed algorithm yields well spread distinct Pareto optimal solutions.

The studies carried out and presented in this paper clearly indicate that using plies made of different materials and combining them is in fact an effective way to tailor the structural properties according to the design requirements and thereby offer better designs. The adaptive grid feature employed for resizing the archive found to be effective for the problems solved in this paper. This is clearly evident from Pareto optimal curves given in Figs. 4 and 5 for cylindrical shell problems. The Pareto curves are represented by well spread non-dominated solutions.

In the present work, the design constraints are handled by considering the constraint as an additional design objective. From the numerical experiments carried out in this paper, it is clear that the proposed constraint handling technique is appealing for design optimisation of laminate composite structures.

Performance studies carried out and presented in this paper using all the three numerical examples clearly indicate that the proposed algorithm generates superior Pareto optimal fronts for the laminate composite structures.

Acknowledgments

This paper is being published with the permission of the Director, CSIR-SERC, Chennai. This research is supported, in part, by Naval Research Board, New Delhi under the grant DRND/05/4003/NRB/78.

References

Armentano, V.A. and Claudio, J.E. (2004), "An application of a multi-objective tabu search algorithm to a Bicriteria flow shop problem", *J. Heuristics*, **10**(5), 463-481.

Arora, J.S. (2004), *Introduction to Optimum Design,* 2nd Edition, Elsevier/Academic Press, New York.

Baker, A.A., Dutton, S. and Kelly, D. (2004), *Composite Materials for Aircraft Structures*, 2nd Edition, Reston, VA: American Institute of Aeronautics and Astronautics.

Chankong, V. and Haimes, Y.Y. (1983), *Multiobjective Decision Making Theory and Methodology*, Elsevier Science, New York.

Coello, C.A. and Toscano Pulido, G. (2005), "Multiobjective structural optimization using a micro-genetic algorithm", *Struct. Multidiscip. O.*, **30**(5), 388-403.

Deb, K. (2001), *Multi-objective Optimization Using Evolutionary Algorithms*, Wiley, Chichester, UK.

Deb, K., Agrawal, S., Pratap, A. and Meyarivan, T. (2000), "A fast and elitist multiobjective genetic algorithm for multi-objective optimization: NSGAII", *Proceedings of the Parallel Problem Solving from Nature VI Conference*, Paris, 849-858.

Irisarri, F.X., Bassir, D.H., Carrere, N. and Maire, J.F. (2009), "Multiobjective stacking sequence optimization for laminated composite structures", *Compos. Sci. Technol.*, **69**(7-8), 983-990.

Knowles, J. and Corne, D. (2000), "Approximating the non-dominated front using the Pareto archived evolution strategy", *Evol. Comput.*, **8**(2), 149-172.

LeRiche, R. and Haftka, R.T. (1993), "Optimization of laminate stacking sequence for buckling load maximization by genetic algorithm", *AIAA J.*, **31**, 951-956.

Messac, A. (1996), "Physical programming: effective optimization for computational design", *AIAA J.*, **34**(1), 149-158.

Miettinen, K.M. (1999), *Nonlinear Multiobjective Optimization*, Kluwer, Boston.

Nemeth, M.P. (1995), "Importance of Anisotropy on buckling of compression-Loaded symmetric composite plates", *AIAA J.*, **24**, 1831.

Park, I.J., Jung, S.N., Kim, D.H. and Yun, C.Y. (2009), "General purpose cross-section analysis program for composite rotor blades", *Int. J. Aeron. Space Sic.*, **10**, 77-85.

Pelletier, J.L. and Vel, S. (2006), "Multi-objective optimization of fiber reinforced composite laminates for strength, stiffness and minimal mass", *Comput. Struct.*, **84**(29-30), 2065-2080.

Rama Mohan Rao, A. and Arvind, N. (2005), "A scatter search algorithm for stacking sequence optimisation of laminate composites", *Compos. Struct.*, **70**(4), 383-402.

Rama Mohan Rao, A. and Arvind, N. (2007), "Optimal stacking sequence design of laminate composite structures using tabu embedded simulated annealing", *Struct. Eng. Mech.*, **25**(2), 239-268.

Rama Mohan Rao, A. and Shyju, P.P. (2008), "Development of a hybrid meta-heuristic algorithm for combinatorial optimisation and its application for optimal design of laminated composite cylindrical skirt", *Comput. Struct.,* **86**, 796-815.

Rama Mohan Rao, A. and Lakshmi, K. (2009), "Multi-objective optimal design of hybrid laminate composite structures using scatter search", *J. Compos. Mater.*, **43**, 2157-2182.

Rama Mohan Rao, A. and Shyju, P.P. (2010), "A new meta-heuristic algorithm for multi-objective optimal design of Hybrid Laminate Composite structures", *Int. J. Comput. Aided Civil Infra Struct. Eng.*, **25**, 149-170.

Rama Mohan Rao, A. and Lakshmi, K. (2011), "Discrete hybrid PSO algorithm for design of laminate composites with multiple objectives", *J. Reinf. Plast. Compos.*, **30**(20), 1703-1707.

Rama Mohan Rao, A. and Lakshmi, K. (2012), "Optimal design of stiffened laminate composite cylinder using a hybrid SFLA algorithm", *J. Compos. Mater.* doi:10.1177/0021998311435674 (online)

Sen, P. and Yang, J.B. (1998), *Multiple Criteria Decision Support in Engineering Design*, Springer-Verlag, London.

Sepehri, A., Daneshmand, F. and Jafarpur, K. (2012), "A modified particle swarm approach for multi-objective optimization of laminated composite structures", *Struct. Eng. Mech.*, **42**(3), 335-352.

Sun, G. and Renjie Mao. (1993), "Optimization of stiffened laminated-composite circular-cylindrical shells for buckling", *Compos. Struct.*, **23**, 53-60.

Suppapitnarm, A., Seffen, K.A., Parks, G.T. and Clarkson, P.J. (2000), "A simulated annealing algorithm for multiobjective optimization", *Eng. Opt.*, **33**(1), 59-85.

Tasi, J. (1966), "Effect of heterogeneity on the stability of composite cylindrical shells under axial compression", *AIAA J.*, **4**, 1058-1062.

Todoroki, A. and Haftka, R.T. (1998), "Stacking sequence optimization by a genetic algorithm with a new recessive gene like repair strategy", *Compos. Part B*, **29**(3), 277-285.

Topal, U. and Uzman, U. (2010), "Multi-objective optimization of angle-ply laminated plates for maximum buckling load", *Finite Elem. Analy. Des.,* **46**(3), 3273-279.

Topal, U. (2009), "Multi-objective optimization of laminated composite cylindrical shells for maximum frequency and buckling load", *Mater. Des.*, **30**(7), 2584-2594.

Yao, A. and Xiao, F. (1998), "Free vibration analysis of an orthotropic circular cylindrical shell of laminated composite", *J. Southw. Petrol. Univ.*, **11**(2), 65-85.

Zitzler, E. and Thiele, L. (1998), "An evolutionary algorithm for multi-objective optimization: The strength Pareto approach", Technical Report No. 43, Computer Engineering and Networks Laboratory, Zurich Switzerland.