

Evaluation of structural outrigger belt truss layouts for tall buildings by using topology optimization

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Abstract. The goal of this study is to conceptually orientate optimized layouts of outrigger belt trusses which are in widespread use today in the design of tall buildings by strut-and-tie truss models utilizing a topology optimization method. In this study unknown strut-and-tie models are realized by using a typical SIMP method of topology optimization methods. In tradition strut-and-tie model designs find the appropriate strut-and-tie trusses along force paths with respect to elastic stress distribution, and then engineers or designers determine the most proper truss models by experience and intuition. It is linked to a trial-and-error procedure based on heuristic strategies. The presented strut-and tie model design by using SIMP provides that belt truss models are automatically and robustly produced by optimal layout information of struts-and-ties conforming to force paths without any trial-and-error. Numerical applications are studied to verify that outrigger belt trusses for tall buildings are optimally chosen by the proposed method for both static and dynamic responses.

Keywords: outrigger belt truss; topology optimization; tall buildings; strut-and-ties

1. Introduction

Outrigger structural systems (Stafford *et al.* 1996) are widely used today in the design of tall buildings. Typical outriggers extend from a lateral load-resisting core and columns. The innovative system for tall buildings leads to considerably efficient use of structural materials by activating the axial strength and stiffness of exterior columns to resist part of the overturning moment caused by lateral loadings like wind or earthquake on the tall building. A specific development of outrigger systems is the use of a so-called “belt truss” (Shankar Nair 1998) for tall buildings. Outrigger trusses are connected directly to the core and to outboard columns, and then they transfer overturning moment from the core to elements outboard of the core. While moment transfer of belt trusses is almost equal to that of outrigger trusses, there is no a direct connection between the

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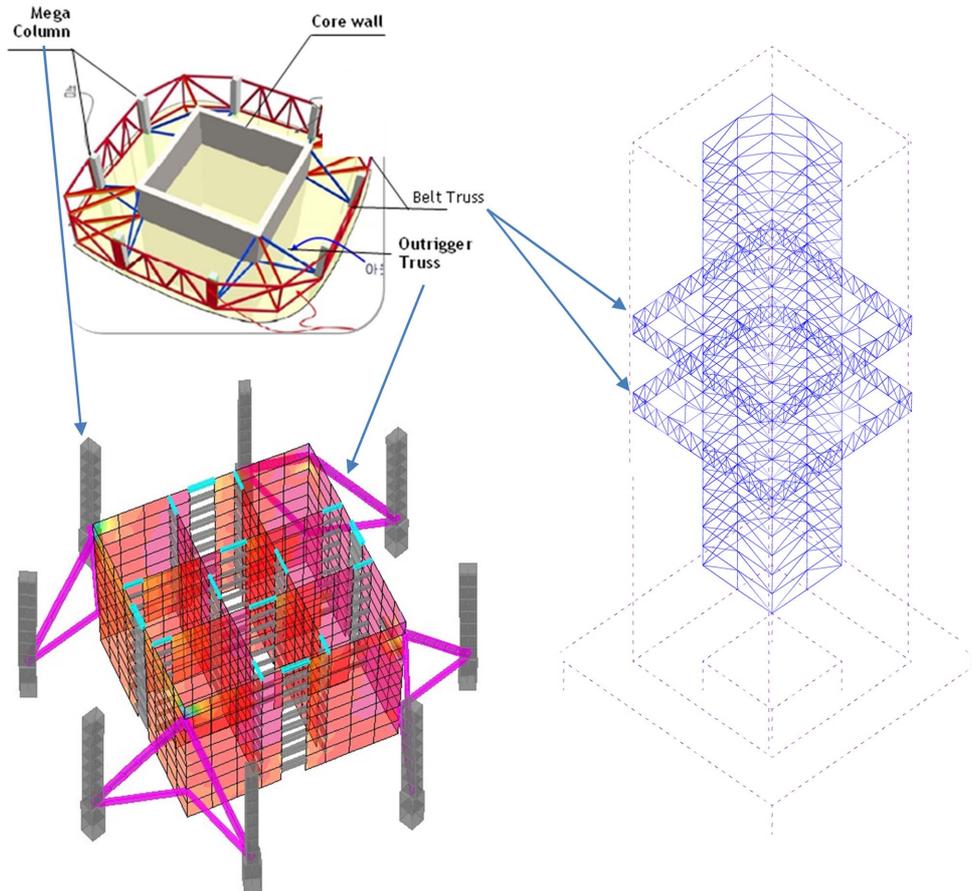


Fig. 1 Outrigger and belt truss systems

outrigger trusses and the core in belt truss systems as shown in Fig. 1. In belt trusses, the removal of a direct connection between the trusses and core keeps away from many of the problems such as architectural and functional constraints on needed floors and complicate connection detail between trusses and core.

As can be seen in Fig. 1 layouts of belt trusses are very similar to those of trusses like building roofs, bridges and cranes which consist of regular triangular modules. Appropriate layouts of these trusses are a significant issue into structural design, since they are directly linked to construction cost, safety and function. Especially the layouts of belt trusses would be determined to synthetically consider given design conditions such as the number of floor occupied by belt trusses, span and material quantity of steel or concrete. In tradition they have a tendency to be decided by trial and error oriented by engineer's experience and intuition.

The issue of layout or connectivity among trusses stems from a strut-and-tie model design whose goal is to reinforce a given structure by using combination of straight bars, i.e., truss like strut or tie. The conventional strut-and-tie model design method (Schlaich *et al.* 1987) requires a trial-and-error procedure oriented by mainly designer's experience and intuitive decision in order to accomplish reinforcement designs. Although the strut-and-tie model design is conceptually simple,

discrete form like a straight formed truss of the strut-and-tie model has some limit to make belt trusses into design space for tall buildings related to complex stress disturbances. In practice it is difficult to realize belt trusses by the simple strut-and tie truss model because of complex stress patterns caused by multi loading and boundary conditions.

To effectively generate the strut-and-tie model in stress disturbance situations without trial and error, discrete truss topology optimization methods by a so-called ground structure approach have been developed to evaluate appropriate reinforced trusses (Ali *et al.* 2001, Biondini *et al.* 2001). Despite of evaluating both optimized shape and topology, the discrete topology optimization method has some shortcoming that in particular potential solution possibilities determining varied optimal shapes and topologies are blocked due to the use of straight truss members with one direction.

In order to resolve the problems, the so-called continuous topology optimization has been introduced (Bendsoe and Kikuchi 1988) as a material approach, i.e., the so-called SIMP (Solid Isotropic Microstructure of Penalization for Intermediate Density). The SIMP presented in this study determines optimal layouts like shapes and topologies of material density of specified volume in a domain that maximizes the stiffness for a given set of loads and boundary conditions. This computer-oriented shape and topology extraction strategy removes trial-and-error procedures for modeling appropriate truss-wise members which often occur in typical strut-and-tie model designs.

The outline of this study is as follows: With respect to SIMP formulation, the static and dynamic material topology optimization problems are described in Section 2. In Section 3, a numerical algorithm for the static and dynamic topology optimization methods is presented. The appropriate belt truss model for tall buildings conformed to load currency and the benefits of automatic optimal shape and topology extraction are studied in several numerical applications of the present method in Section 4. Section 5 presents conclusions of this study.

2. Material topology optimization formulations for static and dynamic problems

In general the field of material topology optimization conveniently deals with voids(0)-solids(1) material distribution and depends on linear elastostatic problems. The schematic of the two-phase material topology optimization of a solid structure with the specified field and boundary conditions is shown in Fig. 2.

2.1 Optimization formulations for static problems

The general problem of the structural topology optimization is specified as the objective function and constraints. Please note that according to the principle of minimum potential energy the objective function can be written as minimum compliance, i.e., minimal strain energy f for static problems as follows. The minimal compliance problem aims to design the stiffest or least compliant structure using a given fixed load, the possible support conditions, and the restrictions on the volume of material used.

$$f = \frac{1}{2} \int_{\Omega_x} \delta \boldsymbol{\varepsilon} \mathbf{C} \boldsymbol{\varepsilon} d\Omega_x \quad (1)$$

where according to discretization, the continuous material tensor \mathbf{C} is dependent on the density-

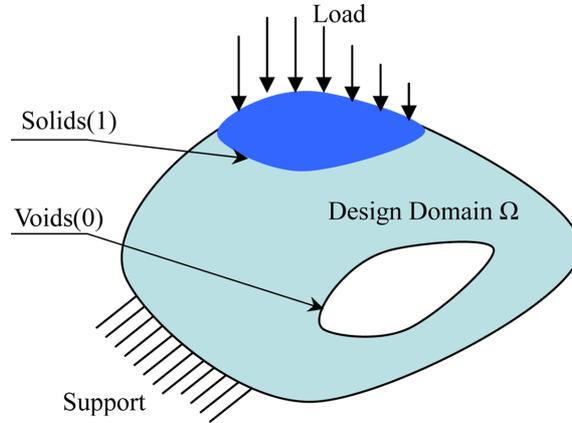


Fig. 2 Design domain for two-phase material topology optimization problems of structures

stiffness relationship of the typical SIMP approach. The discontinuous Heaviside function is regularized for a smoothed and continuous form near the material boundaries. The function can be included in a strain energy formulation since the original Heaviside function determines the solid and void regions in a design domain.

The inequality optimization constraint is $0 \leq \Phi \leq 1$, which ensures that the density stays within reasonable bounds. Equality constraints are a linear elastostatic equilibrium, which clearly presents the state equation, and an equation controlling the volume of the used material under the volume fraction V_{ref} as follows, respectively.

$$\int_{\Omega_x} \delta \boldsymbol{\varepsilon}^T \mathbf{C} \boldsymbol{\varepsilon} d\Omega_x = \int_{\Omega_x} \delta \mathbf{u}^T \bar{\mathbf{b}} d\Omega_x + \int_{\Gamma_f} \delta \mathbf{u}^T \bar{\mathbf{t}} d\Gamma_f \tag{2}$$

$$\int_{\Omega_x} d\Omega_x - V_{ref} = 0 \tag{3}$$

2.2 Optimization formulations for dynamic problems

The governing equation for free vibration systems considered in this study can be written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F} = \underline{\mathbf{0}} \tag{4}$$

By using Laplace transformation Eq. (4) can be rewritten as

$$\mathbf{M}\mathbf{U}(l)^2 + \mathbf{K}\mathbf{U}(l) = \underline{\mathbf{0}} \tag{5}$$

By substituting ω^2 for l into Eq. (5), the final eigenvalue problem is defined as

$$[\mathbf{K} - \omega_i^2 \mathbf{M}]\mathbf{u}_i = \underline{\mathbf{0}} \tag{6}$$

where \mathbf{K} and \mathbf{M} are the global stiffness and mass matrix, respectively. ω_i is the i -th eigenfrequency and \mathbf{u}_i denotes the corresponding eigenvector depending on ω_i . In order to numerically solve Eq. (6), \mathbf{K} and \mathbf{M} have to be symmetrically and positively defined (Lehoucq *et al.* 1998) owing to the finite element-based and generalized structural eigenvalue.

Eigenvalue optimization designs are profitable for mechanical structural systems subjected to dynamic loading conditions like earthquakes and wind loads. The dynamic behaviors of structural systems can be estimated by eigenfrequency which describes structural stiffness. In general maximizing first-order eigenfrequency can be an objective for dynamic topology optimization problems since stiffness of structures also increases when eigenfrequency increases. Problems of topology optimization for maximizing natural eigenfrequencies of vibrating elastostatic structures have been considered in the studies (Diaz *et al.* 1992, Pedersen 2000).

Assuming that damping can be neglected, such a dynamic design problem can be formulated as follows.

$$\max_{\Phi} : \omega_1^2(\Phi) = \frac{\mathbf{u}_1^T \mathbf{K} \mathbf{u}_1}{\mathbf{u}_1^T \mathbf{M} \mathbf{u}_1} \quad (7)$$

$$\text{subject to} : \frac{V(\Phi)}{V_0} \leq g \quad (8)$$

$$: [\mathbf{K} - \omega_i^2 \mathbf{M}] \mathbf{u}_i = \mathbf{0} \quad (9)$$

$$: 0 < \Phi_{\min} \leq \Phi \leq \Phi_{\max} \quad (10)$$

where g is volume fraction which means a specific material constraint. These discrete formulations for the dynamic problem are equal to continuous formulations, i.e., Eqs. (1)-(3) for the static problem except for objective and governing equation.

2.3 Interpolation scheme by Using SIMP material

After discretization of the continuous design domain, the material density Φ_i^h is constantly assigned to each finite element and is defined by applying a penalty contour to the design variable field, i.e. as in the so-called “power law or SIMP approach” (Bendsoe and Kikuchi 1988). According to the SIMP approach, the material density distribution affects element stiffness. This element stiffness-density relationship may be expressed in terms related to Young’s modulus E_i^h . Young’s modulus is associated with the updated element density Φ_i^h . The element stiffness-density relationship is defined as

$$E_i^h(\Phi_i^h) = E_0 \left(\frac{\Phi_i^h}{\Phi_0} \right)^k, \quad k \geq 1, 0 \leq \Phi_i^h \leq 1, i = 1 \dots N_e \quad (11)$$

where E_0 and Φ_0 denote nominal values of Young’s modulus and material density of elements, respectively, and N_e is the number of elements.

According to the penalized Young’s module, element stiffness matrix of four-node square elements with eight-DOF used in this study is written as

$$\mathbf{K}_e = \int_V \mathbf{B}^T \mathbf{C} \mathbf{B} dV$$

$$= \frac{E_i^h(\Phi_i^h)}{1-\nu^2} \begin{bmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\ \cdot & k_1 & k_8 & k_7 & k_6 & k_5 & k_4 & k_3 \\ \cdot & \cdot & k_1 & k_6 & k_5 & k_4 & k_3 & k_2 \\ \cdot & \cdot & \cdot & k_1 & k_8 & k_3 & k_2 & k_5 \\ \cdot & \cdot & \cdot & \cdot & k_1 & k_2 & k_3 & k_4 \\ \cdot & \cdot & \cdot & \cdot & \cdot & k_1 & k_8 & k_7 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & k_1 & k_6 \\ \text{sym.} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & k_1 \end{bmatrix} \quad (12)$$

where $k_1 = \frac{1}{2} - \frac{\nu}{12}$, $k_2 = \frac{1}{8} + \frac{\nu}{8}$, $k_3 = -\frac{1}{4} - \frac{\nu}{12}$, $k_4 = -\frac{1}{8} + \frac{3\nu}{8}$, $k_5 = -\frac{1}{4} + \frac{\nu}{12}$,

$$k_6 = -\frac{1}{8} - \frac{\nu}{8}, k_7 = \frac{\nu}{6}, k_8 = \frac{1}{8} - \frac{3\nu}{8}$$

Please note that the stiffness formulation is used for both static and dynamic problems in this study. For example, an isotropic material model with a plane stress (such as a wall structure) is used here without loss of generality, so that

$$\mathbf{C}_i^h = \frac{E_i^h(\Phi_i^h)}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (13)$$

where \mathbf{C}_i^h is a material tensor of each finite element i and includes the updated term of Young's modulus E_i^h which has been defined by the updated element density average Φ_i^h . ν is Poisson's ratio.

According to dynamic topology optimization problems using SIMP material, mass matrix \mathbf{M}_e of a specific finite element e also includes the same penalty formulation such as the stiffness matrix of Eq. (12) multiplied by original mass matrix \mathbf{M}_0 . Therefore it can be written as

$$\mathbf{M}_e = (\Phi_e)^k \mathbf{M}_0 \quad (14)$$

For the mass matrix, a lumped mass matrix \mathbf{M}_e^L (subscript L =Lumped, superscript e =the number of finite element), a consistent mass matrix \mathbf{M}_C (subscript C =Consistent, superscript e =the number of finite element) or a combination of those two can be used. The lumped and consistent mass matrices are written as respectively in case discretization of eight-node square elements with 8 DOFs.

$$\mathbf{M}_C^e = \int_V \Phi \mathbf{N}^T \mathbf{N} dV$$

$$= \frac{1}{4} \Phi A \begin{bmatrix} 4 & 0 & 2 & 0 & 1 & 0 & 2 & 0 \\ \cdot & 4 & 0 & 2 & 0 & 1 & 0 & 2 \\ \cdot & \cdot & 4 & 0 & 2 & 0 & 1 & 0 \\ \cdot & \cdot & \cdot & 4 & 0 & 2 & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & 4 & 0 & 2 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 4 & 0 & 2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 4 & 0 \\ \text{sym.} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 4 \end{bmatrix} \quad (15)$$

$$\mathbf{M}_L^e = \frac{1}{4} \Phi A \mathbf{I}_{(8 \times 8)} \quad (16)$$

where Φ and A denote the material density and area of elements, respectively and \mathbf{I} is the 8×8 unit matrix.

2.4 Sensitivity analyses for static and dynamic problems

In general, the sensitivity of the optimization problems such as objective functions or constraints is mainly calculated by analytical methods due to small error. The analytical variational approach is used here since it is numerically more efficient than the discrete method for certain optimization problems. With respect design variables s (for instance, material element densities), the total differential form (Haug *et al.* 1986) of the objective function is the combination of parts of an explicit partial derivative and an implicit partial derivative as follows.

$$\nabla_s f = \nabla_s^{ex} f + \bar{\nabla}_s f^T \nabla_s \mathbf{u} \quad (17)$$

According to static topology optimization problem, under the assumptions that external forces $\bar{\mathbf{b}}$, $\bar{\mathbf{t}}$, the differential matrix \mathbf{L} and a Jacobi matrix \mathbf{J} are independent of the design variables, the total partial derivative is written as a simple continuous formulation as

$$\nabla_s f = \frac{1}{2} \int_{\Omega_x} \boldsymbol{\varepsilon}^T \nabla_s \mathbf{C}(\Phi) \boldsymbol{\varepsilon} d\Omega_x \quad (18)$$

According to dynamic topology optimization, the total derivative is written as a simple discrete formulation as follows.

$$\frac{\partial \omega_1^2}{\partial \Phi_e} = \frac{\mathbf{u}_1^{eT} [k(\Phi_e)^{k-1} \mathbf{K}_0 - \omega_1^2 k(\Phi_e)^{k-1} \mathbf{M}_0] \mathbf{u}_1^e}{\mathbf{u}_1^T \mathbf{M} \mathbf{u}_1} \quad (19)$$

3. Strut-and-tie model algorithm generating belt trusses for tall buildings

Since the optimal shape and topology results report the deposition information of stress-resistance

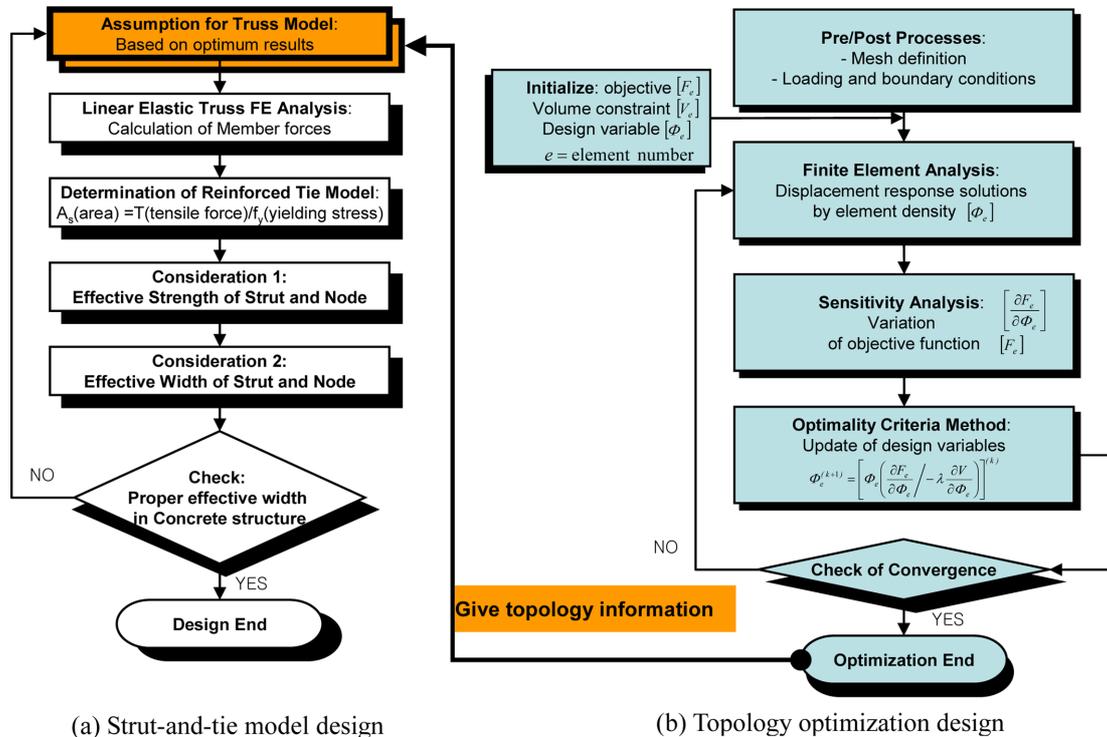


Fig. 3 Synthetic/automatic strut-and-tie model design processes by using topology optimization

truss members in a given design space, it is firstly considered to lay out real strut-and-tie models by using topology optimization.

In general the strut-and-tie model consists of strut, tie and node. The strut is a compression member which denotes a compressible stress region. The tie is a tensile stress member and in general denotes reinforced steels into concrete structures. Areas which the strut, tie and loadings exist together are defined as a node, and then the node takes complex stress states with varied direction.

A classical strut-and-tie model design process is shown in Fig. 3(a). As can be seen this process needs structural knowledge and experience of designers in order to determine initial truss models. As a consequence, the appropriate truss model can be intuitively obtained by trial-and-errors of repetitive analyses and model modification. It serves a large number of designer's efforts and design time.

Fig. 3(b) sketches a typical topology optimization procedure which consists of structural analyses, sensitivity analyses, and optimization methods. Please note that from the solution of the shape and topology design proper truss models for strut-and-tie model design can be automatically produced. This is a key point in this study.

The developed MATLAB code for dynamic topology optimization design is based on MATLAB code (Sigmund 2001) for static designs.

4. Numerical applications and discussion

In general building frame structures are a bending moment-resistance structural system in which beams are connected to columns.

Numerical examples involve generating appropriate layouts of belt trusses on tall buildings in the strut-and-tie model design by using continuous two-phase (0-1) material SIMP topology optimization methods for both static and dynamic problems. The objective function is minimal strain energy (kN·m) for static problem and maximal eigenfrequency (Hz) is defined to objective function for dynamic problem. A plane stress state is assumed to static and dynamic problems.

Design spaces in which belt trusses are located on tall buildings are assumed to be a fixed span length (L) of 32 m and varied floor heights (H) of 1.6 m, 2.0 m, 2.4 m~5.2 m, 5.6 m, 6.0 m as shown in Fig. 4. Thickness of design space is 1.0 m. Ends of both sides of the given design spaces

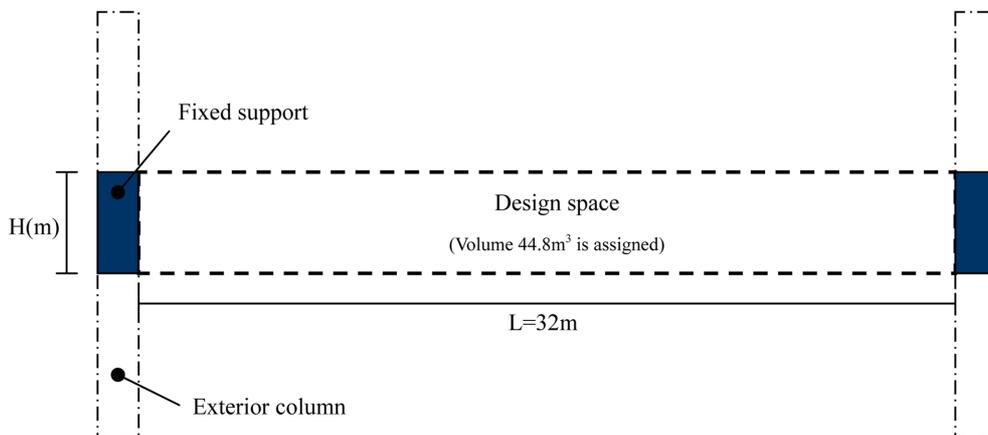


Fig. 4 Design space for belt truss design

Table 1 Design input data

Model number	Design space (L×H×Thickness)	Finite mesh	Relative volume (44.8m ³ is fixed)	Size ratio (L/H)	Material property
Model-1	32 m×1.6 m×1.0 m	80×4	87.60%	20.00	
Model-2	32 m×2.0 m×1.0 m	80×5	70.00%	16.00	
Model-3	32 m×2.4 m×1.0 m	80×6	58.33%	13.33	*Young's modulus: 200GPa
Model-4	32 m×2.8 m×1.0 m	80×7	50.00%	11.43	
Model-5	32 m×3.2 m×1.0 m	80×8	43.75%	10.00	
Model-6	32 m×3.6 m×1.0 m	80×9	38.89%	8.89	*Poisson's Ratio: 0.3
Model-7	32 m×4.0 m×1.0 m	80×10	35.00%	8.00	
Model-8	32 m×4.4 m×1.0 m	80×11	31.82%	7.23	
Model-9	32 m×4.8 m×1.0 m	80×12	29.17%	6.67	
Model-10	32 m×5.2 m×1.0 m	80×13	26.92%	6.15	*Material: steel
Model-11	32 m×5.6 m×1.0 m	80×14	25.00%	5.71	
Model-12	32 m×6.0 m×1.0 m	80×15	23.33%	5.33	

are fixed. In the case of static problem, uniformly distributed load 5,000 kN/m is applied to the upper side of the design space. In dynamic problem any loading condition is not considered due to free vibration response. One square finite element of 0.4 m×0.4 m is assumed for discretizing the given design spaces. 12 design space models (Model 1~12) are shown in Table 1. Material of belt trusses is steel and then Young's modulus of steel is 200 GPa and Poisson's ratio is 0.3. Material quantity which is occupied into each design space to generate belt trusses is fixed to the volume 44.8 m³ during every optimization procedure, and therefore relative volumes of each model differs from each space's size. The relative volumes are used to input data of volume fraction for optimization.

4.1 Belt truss generation for tall-buildings with static and dynamic responses

Figs. 5 and 7 show the final optimal belt truss layouts, for static and dynamic problem, respectively, evaluated by automatic strut-and-tie model results using SIMP with volumes of 23.33~87.60% occupied by steel of 44.8 m³. The layouts are described by collecting 0 (white), 1 (black), and intermediate value (gray) of finite element densities. As can be seen different shapes and topologies of the belt truss layouts depend on design space size such as span and height. The optimal results rely on also static and dynamic problem. The layouts are seemed to combine struts and ties and verify to automatically generate strut-and-tie model.

Figs. 6 and 8 illustrates graphically three dimensionally density contours of Figs. 5 and 7, respectively. Here X and Y grid denote refinement of finite elements of X and Y direction in Fig. 6 and 8.

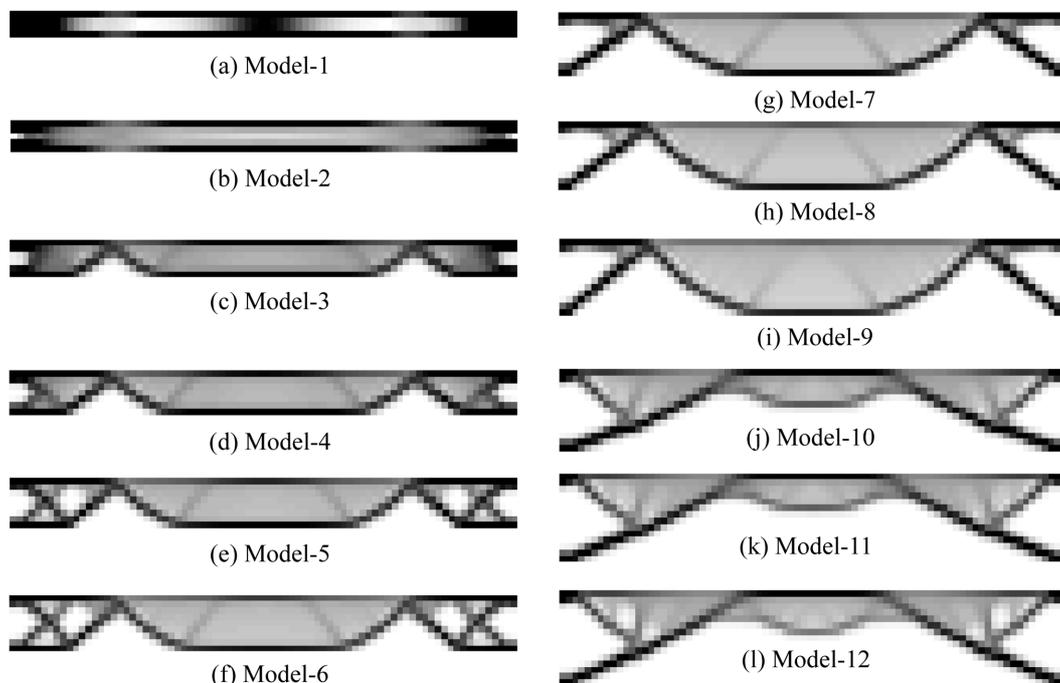


Fig. 5 Belt truss layout results on tall buildings with static response

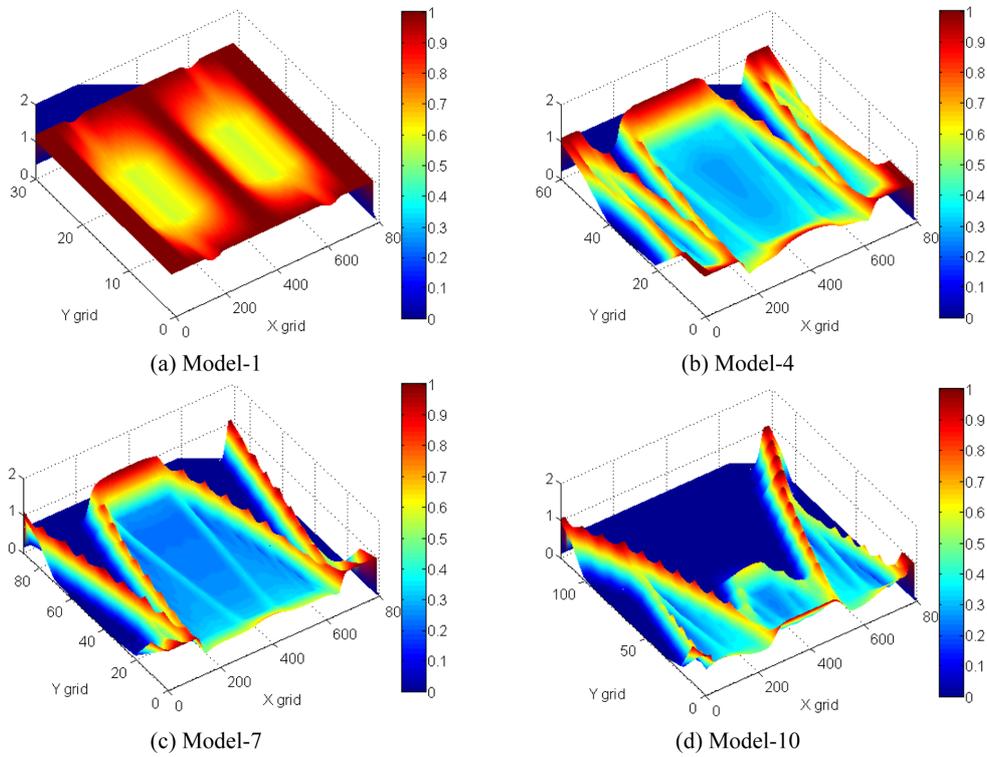


Fig. 6 Graphical density distributions of belt trusses of Model 1, 4, 7, 10 for static response

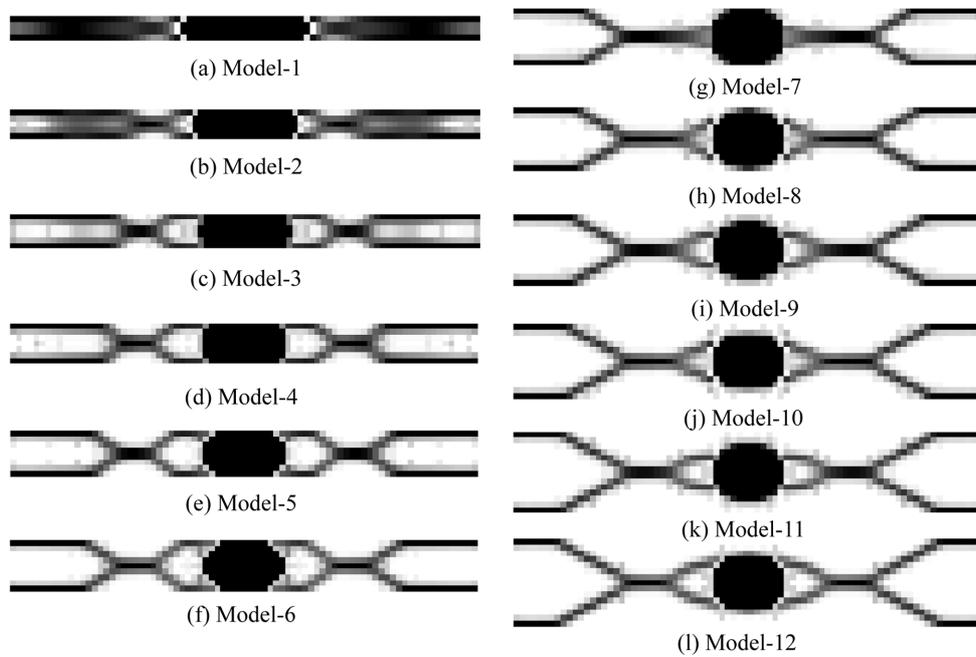


Fig. 7 Belt truss layout results on tall buildings with dynamic response

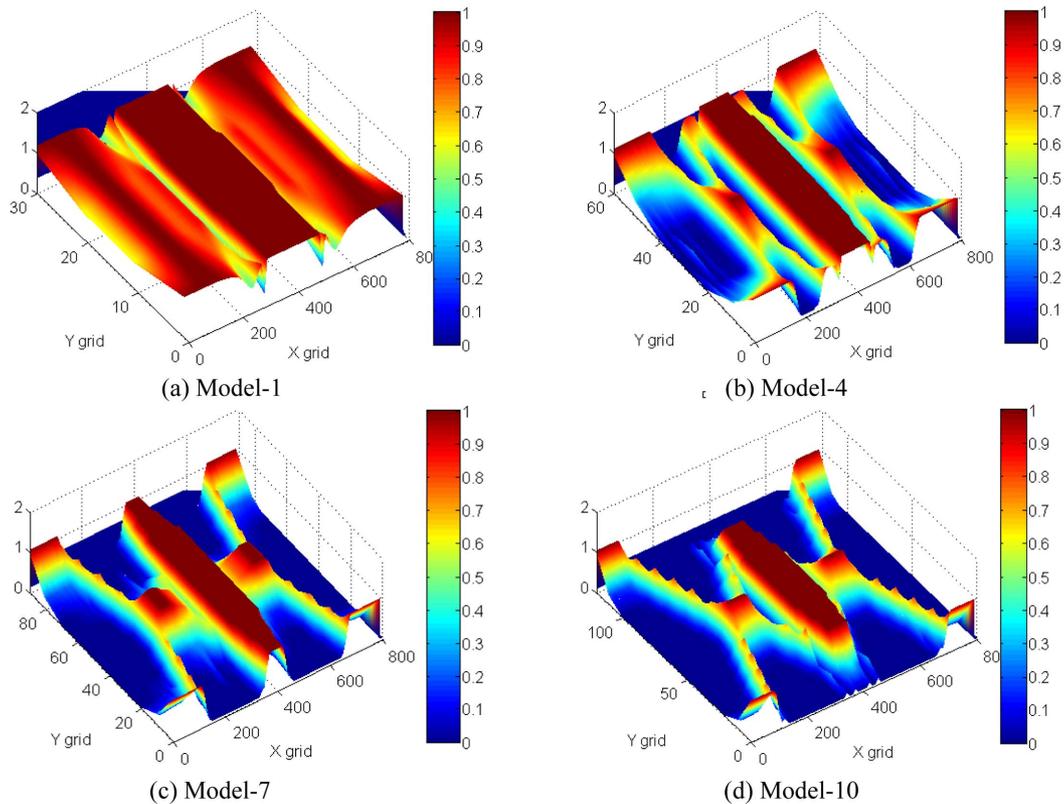


Fig. 8 Graphical density distributions of belt trusses of Model 1, 4, 7, 10 for dynamic response

4.2 Investigation of optimal size ratios (L/H) of belt trusses for static and dynamic problems

Fig. 9 shows C_{opt} and W_{opt} according to size ratios (L/H) on belt truss models 1~12 for static and dynamic problems. Here C_{opt} and W_{opt} denote, respectively, converged minimal strain energy for static problem and converged maximal first order eigenfrequency for dynamic problem. Points in Fig. 9 indicate converged values of model 1, 2, 3 ...12, in turn, from right points. Each model with C_{opt} and W_{opt} leads to each belt truss layout with maximal stiffness.

As can be seen in Figs. 9(a) and (b) stiffness of belt trusses increase, when L/H ratio decreases. It can be found that positions in which L/H ratios converge are almost below $L/H = 10$. According to structural safety and economical aspect, belt trusses at the positions are optimal layouts by using limited steel of 44.8 m^3 . In addition design space size of them is optimized to construct belt trusses. It is very wasteful and inefficient to occupy too much floor space for belt truss construction on tall buildings.

Belt truss layouts of these positions would be chosen to conceptual design practice on tall buildings. Especially model 11 ($32 \text{ m(L)} \times 5.6 \text{ m(H)}$) is the best choice on tall buildings for static problem, and model 6 ($32 \text{ m(L)} \times 3.6 \text{ m(H)}$) for dynamic problem. In order to immediately treat all static and dynamic problems, combining two layouts would be ideal to belt truss design for tall buildings.

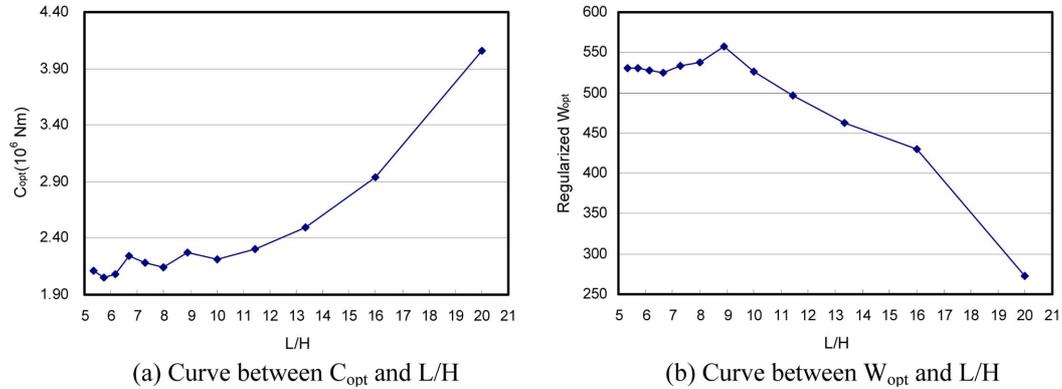


Fig. 9 C_{opt} , W_{opt} according to size ratios (L/H) on belt truss models 1~12 for static and dynamic problems

5. Conclusions

This study targets two facts as follows. One is to evaluate desired outrigger belt trusses of tall buildings with respect to stiffness of optimal shape and topology. The other is to investigate or understand the present strut-and-tie model design by using topology optimization technique. This study provides both a design and analysis tool for outrigger belt truss structural designs and belt truss structural analyses. Appropriate belt truss structures can be achieved or measured as a design model through considering both static and dynamic behaviors.

According to evaluated shape and topology results of numerical examples, it is verified that outrigger belt trusses are redundant and loads follow free formed diagonals like struts and ties structure as it naturally resisting vectors of forces like lateral loadings and uniformly distributed loadings.

The belt truss is used to transfer and resist part of the overturning moment caused by lateral loadings; therefore it may be unreasonable to apply only UDL, i.e., apparently representing gravity to the models of numerical application. If the loading conditions have not been appropriately applied, the consequent results and discussion are not convincing and must be reconsidered. Surely it will be absolutely future's works. This study concentrates on one dimensional design shape results through which engineers or designers firstly make decision to select appropriate belt truss layouts such as member's deposition and topologies.

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