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Buckling of insulated irregular transition flue gas ducts under axial loading

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Abstract. Finite element buckling analysis of insulated transition flue ducts is carried out to determine the critical buckling load multipliers when subjected to axial compression for design process. Through this investigation, the results of numerical computations to examine the buckling strength for different possible duct shapes (cylinder, and circular-to-square) are presented. The load multipliers are determined through detailed buckling analysis taking into account the effects of geometrical construction and duct plate thickness which have great influence on the buckling load. Enhancement in the buckling capacity of such ducts by the addition of horizontal and vertical stiffeners is also investigated. Several models with varying dimensions and plate thicknesses are examined to obtain the linear buckling capacities against duct dimensions. The percentage improvement in the buckling capacity due to the addition of vertical stiffeners and horizontal Stiffeners is shown to be as high as three times for some cases. The study suggests that the best location of the horizontal stiffener is at 0.25 of duct depth from the bottom to achieve the maximum buckling capacity. A design equation estimating the buckling strength of geometrically perfect cylindrical-to-square shell is developed by using regression analysis accurately with approximately 4% errors.

Keywords: ducts buckling; plate buckling; stiffeners; finite element

1. Introduction

Flue gas exits to the atmosphere via a flue, which can be a transition duct, stack or chimney for conveying exhaust gases from a fireplace, boiler or steam generator. Quite often, it refers to the combustion exhaust gas produced at power plants. Therefore, the transition flue gas ducts are considered as a type of chimney through which hot gases are exhausted into the atmosphere. Normally these transition parts are properly insulated internally to prevent steel duct failure induced by high temperature caused by internal gases. Natural draft of passed gases depends on many factors such as: stack and transition ducts dimensions, atmospheric pressure and temperature, frictional resistance to the flow induced by the flue gases through the chimney or stack and finally the heat loss from the flue gases as they flow through the chimney or stack. Additionally, the duct dimensions which are selected by the mechanical designers represent an important factor in reducing the gas distortion internally near inner walls. The transition parts are also indispensable in many engineering flow circuits such as wind-tunnel circuits and fighter gas turbines. Structural

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design of such transition ducts when subjected to different cases of loading is based on buckling criteria of thin shell structures. Additionally, such shells have wide applications in another fields because they have efficient load carrying capacity with weight economy.

Buckling phenomena occurs when most of the strain energy which is stored as membrane energy is converted to bending energy requiring large deformation resulting in catastrophic failure. Two types of buckling analysis can be defined: eigen (or linear) buckling analysis and non-linear buckling analysis. The Eigen buckling analysis or elastic analysis predicts the theoretical buckling strength of an ideal linear elastic structure. This technique is used to determine the critical buckling load multipliers at which a structure becomes unstable, performing unbounded growth of new deformation pattern and finally buckled. Through this analysis, the imperfections and materials nonlinearities are not considered. The non-linear buckling technique provides a non-linear structural analysis with gradually increasing loads to predict the load level at which the structure become unstable. Through this study, only linear buckling analysis is considered.

The stability of different regular steel shells under different loading conditions has been widely investigated. The classical elastic buckling theory predicts the bifurcation buckling pressure of perfect thin cylindrical shell without stiffeners under uniform external pressure and is given in Timoshenko and Gere (1959). Schneider (2005) reported that both analytical and experimental results may be deviating due to inevitable differences called imperfections present in the real structure from the perfect structure. There were many numerical and analytical studies investigating the strength of the cylinders with specific imperfection forms as presented by Teng (2001), Seung (2002). An intensive study considering the material non-linearity and geometric imperfection is conducted by Cai (2003) to investigate the buckling behavior of cylinders when subjected to elevated local axial compressive loads. Buckling behavior of thin-walled square tubes under axial crushing is presented through experimental and analytical investigations done by Aljawi (2004). Finite element analysis of structural elliptical hollow sections when subjected to axial compressing is performed by Zhu (2007). The buckling of conical shells with graded materials is studied by Lavasani (2009) to develop simple and exact producers for conical shell analysis when subjected to axial and external loads. The attention paid to buckling of regular shell shapes such as cylindrical, conical and rectangular shells compared to the irregular ones let the study for the irregular ones insufficient especially with regards to the effect of longitudinal and vertical stiffeners on buckling and stress behavior when subjected to axial loading. Since the buckling is very complex phenomenon and numerical prediction of buckling loads and mode shapes involves very sophisticated analytical techniques, especially when studying irregular shells such as circular-to-square shapes, this necessitates the use of numerical techniques.

The major emphasis of this study is to carry out a buckling analysis of insulated irregular circular-to-square duct shape to determine the critical buckling load multipliers when subjected to axial loading and to ensure that these multipliers should be more than the minimum values required by designers. The load multipliers are determined through detailed buckling analysis taking into account the effects of duct geometrical properties, and plate thickness. In order to enhance the buckling strength, horizontal and vertical stiffeners are introduced at different locations. The analysis is performed by conducting three-dimensional numerical models to the shell structures using ANSYS Version 9.0 (2005) general purpose finite element program. A 4-noded plastic quadrilateral shell element was used to model the steel plates called “Shell 181” from the program library. A design equation estimating the buckling strength of geometrically perfect circular-to-square duct shape is developed by using regression analysis.

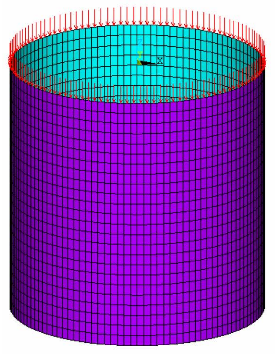

Fig. 1 Finite element modeling of thin shell cylinders

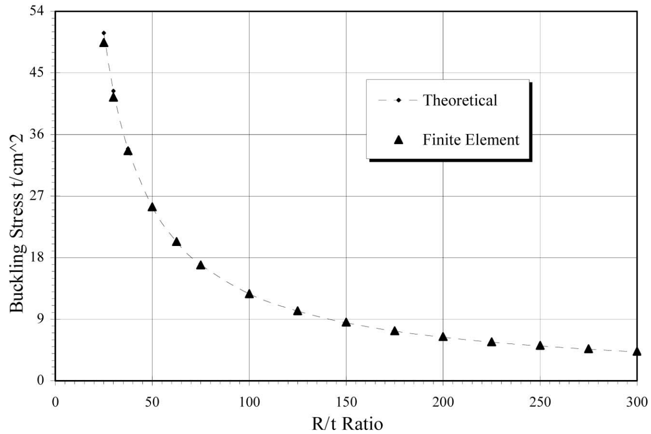

Fig. 2 Buckling stress comparison between finite element and theoretical analysis

2. Finite element modeling for cylinders subjected to axial loading

The general purpose finite element software package ANSYS is employed to simulate cylinders subjected to axial loading. The cylinders were modeled using four nodes shell elements “Shell 181” from ANSYS element library, which is suitable for the studied problem. It is assumed that the steel material is elastic and has initial elasticity modulus value equals to 21E10 N/m^2. The bottom nodes are prevented from movements in all directions meanwhile the top ones are allowed to move only parallel to the load application. As indicated in Fig. 1, model loading is performed by applying uniform axial vertical pressure on the element surfaces at one end of the steel cylinder. Fig. 2 shows a comparison between the finite element results and Timoshenko foot buckling stress obtained from equations for different R/t ratios where R is cylinder radius and t is the cylindrical shell thickness. The finite element procedure yields results in a good agreement with the theoretical values.

3. Finite element modeling for circular-to-square steel ducts under axial loading

Fig. 3 shows the finite element modeling which is employed to analyze the circular-to-square ducts. As shown in Fig. 3, the workshop construction of such ducts is not easy since it is composed of straight and curved steel segments which are considered as parts of inclined cones. The same element type, boundary conditions and material properties previously assumed through cylinder analysis are considered through this analysis. Model loading is performed by applying uniform axial vertical pressure on the element surfaces at one end of the steel ducts as indicated in Fig. 4. Several parameters are considered in this investigation such as: variation of slenderness ratio between upper diameter to shell thickness (D_{top}/t) from 200 to 600, variation of the dimension ratio (δ) between upper diameter to bottom side A from 0.5 to 1.0, variation of side inclination angle α from 0^o to 45^o and duct height H. In all analyses, the wall thickness t is assumed to be 10 mm.

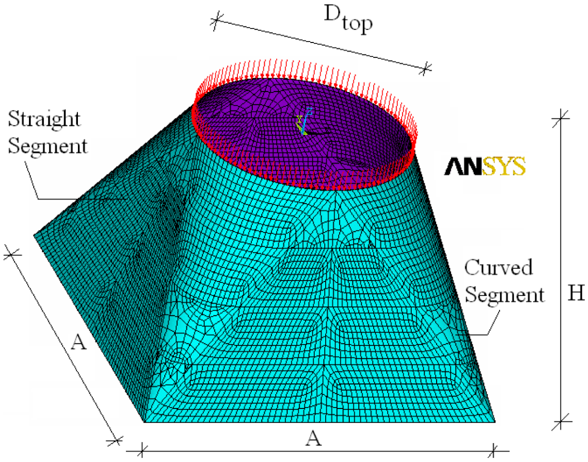

Fig. 3 Finite element modeling of thin shell circular-to-square ducts

4. Analysis of numerical modeling results for circular-to-square steel ducts

In this section, the buckling strength of the geometrically perfect circular-to-square shell when subjected to axial loading is presented. The diameter-to-thickness ratio is chosen to be greater than 200, which is a typical value for flue gases transition parts. Figs. 4 and 5 show the variation of buckling stresses values with respect to duct dimension ratio and its inclination angle cosine for various top diameter-to-thickness ratios and fixed duct height equals 3 m. It is also worth mentioning that all these ducts are subjected to the same stress. It is shown that for the same height, the rate of buckling stress decrease is significant as the inclination angle cosine increases, where as the same rate is slight as the dimension ratio increases. The rate of buckling stress increase seems to be the same for the different duct D_{top}/t ratios. It is also noted that the buckling stress values increase with decrease of the top diameter-to thickness ratio. This trend is different from that in the case of columns, which is attributed to that buckling of a cylindrical shell is governed by not only the axial buckling mode but also the circumferential buckling mode. Figs. 6 and 7 show the parabolic increase of stresses with increase of δ and decrease of α when the height of duct H is taken equal to duct average radius Rav. The duct average radius is defined as radius of a virtual circle whose circumference is calculated by the average circumference of the bottom and top bases for the duct. For a maximum value of δ which corresponds to duct with vertical sides, it is also shown that buckling stresses increased by a ratio of 6.1 to 6.4 for D_{top}/t of 500 to 200, respectively. To study the effect of the height of ducts on its buckling capacity, Figs. 8 and 9 show the variation of buckling stress values with respect to duct dimension ratio and average slenderness ratio defined by average radius Rav to shell thickness for different height to top radius ratios. The top diameter is taken equals to 6 m which means that the applied load for such runs is constant. It is observed that the buckling stress increases with an increase in the value of dimension ratio but is essentially dependent of H/R_{top}. For smaller values of δ and higher values of Rav/t, it is found that buckling stresses are almost the same. However, For higher values of δ and smaller values of Rav/t, it is found that buckling stresses increased by a ratio of 2.9 to 1.7 for H/R_{top} of 1 to 4, respectively.

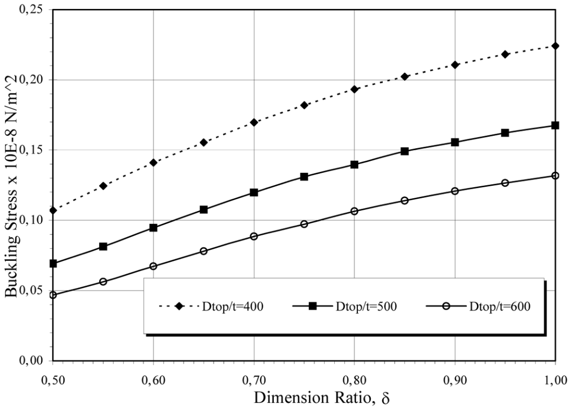

Fig. 4 Buckling stress and duct dimension ratio plot for various values of D_{top}/t, H = 3 m

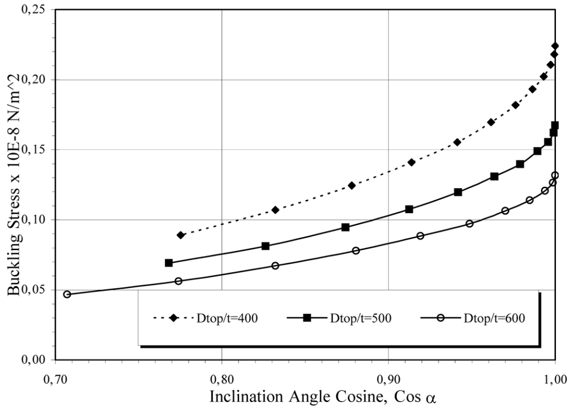

Fig. 5 Buckling stress and duct inclination angle cosine plot for various values of D_{top}/t, H = 3 m

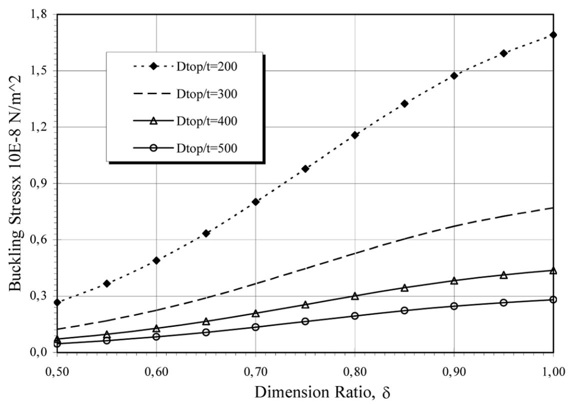

Fig. 6 Buckling stress and duct dimension ratio plot for various values of D_{top}/t, $H = Rav$

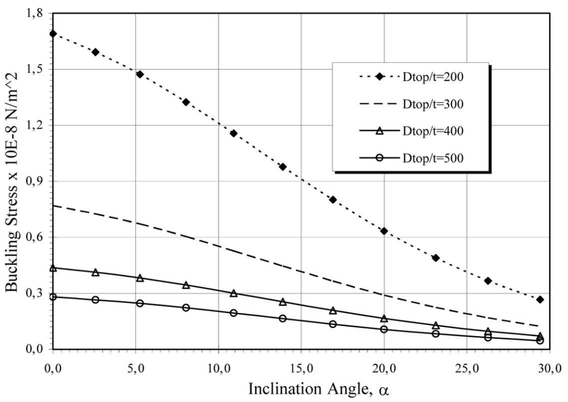

Fig. 7 Buckling stress and duct inclination angle plot for various values of D_{top}/t, $H = Rav$

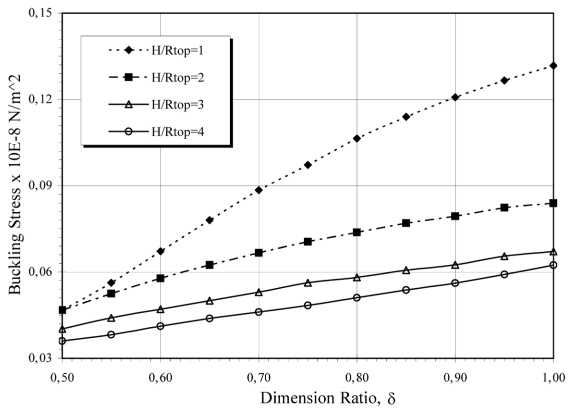

Fig. 8 Buckling stress and duct dimension ratio plot for various values of H/R_{top}, D_{top} = 6 m

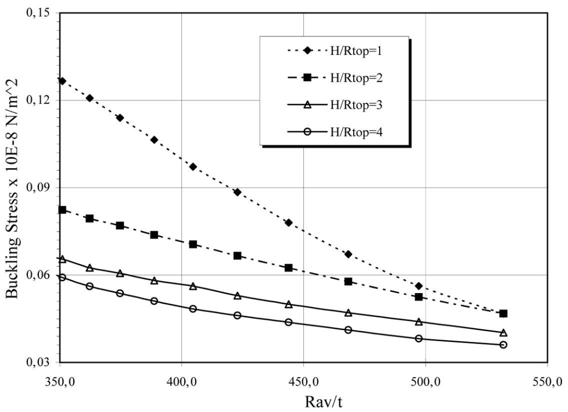

Fig. 9 Buckling stress and duct dimension ratio plot for various values of H/R_{top}, D_{top} = 6 m

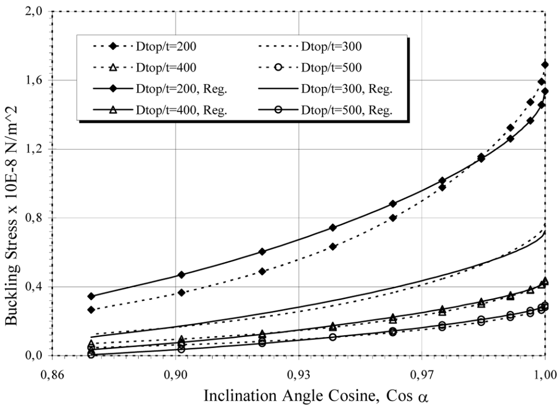

Fig. 10 Comparison between FEM and regression buckling stress with duct inclination angle cosine for various values of D_{top}/t, $H = Rav$

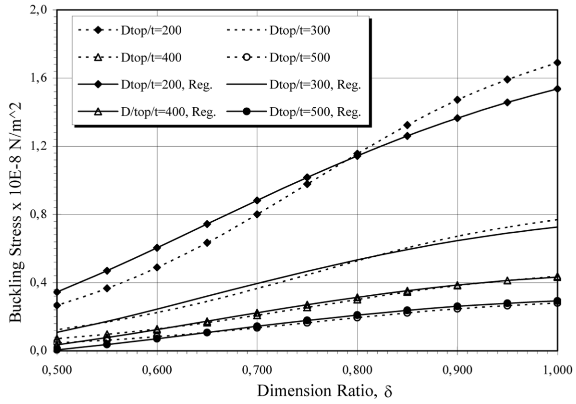

Fig. 11 Comparison between FEM and regression buckling stress with duct dimension ratio for various values of D_{top}/t, $H = Rav$

5. Development of simple formulas

Regression analysis is considered the most common statistical modeling technique, and it is suitable for the majority of problems. Therefore, regression analysis has been performed using finite element modeling results to analyze and investigate the critical buckling stress of circular-to-square steel ducts with constant thickness. It is aimed to develop practical design equations and charts estimating the buckling strength of the duct shell when subjected to axially compressive loads in terms of the cylinder buckling stress. Results presented in Figs. 4 to 9 and conclusions drawn from the study carried out through the previous discussion indicate that the elastic buckling stress depends mainly on the average radius Rav, duct height H, duct slope and the applied pressure which is indicated by D_{top}/t. These parameters were taken as variables to obtain simplified formulas. All regression analysis trials are made using excel and the obtained simplified equation has the following format.

$$\sigma_{cr} = \left[\left(\frac{t}{R_{av}}\right) \times \frac{E}{\sqrt{3(1-\upsilon^2)}}\right] * \left(C1 + C2 \times \left(\frac{R_{av}}{H}\right) + C3 \times \left(\frac{D_{Bott}}{D_{top}}\right)^2 + C4 \times \cos^2\alpha + C5 \times \left(\frac{t}{R_{av}}\right)\right) \quad (1)$$

Where: $C1$, $C2$, $C3$, $C4$ and $C5$ are constants for the evaluation of duct buckling stress obtained from the regression analysis with the following values: –0.324, 0.053, 0.007, 0.28 and 13.64. The stress calculated using Eq. (1) is compared to the obtained ones from the FE modeling and presented in Figs. 10 and 11. It is shown that the design equation predicts buckling strength accurately with approximately 4% errors which is seemed to be practically accepted.

6. Effect of horizontal stiffeners location on buckling strength

To study the effect of horizontal stiffeners location, the duct buckling capacity ratio is plotted

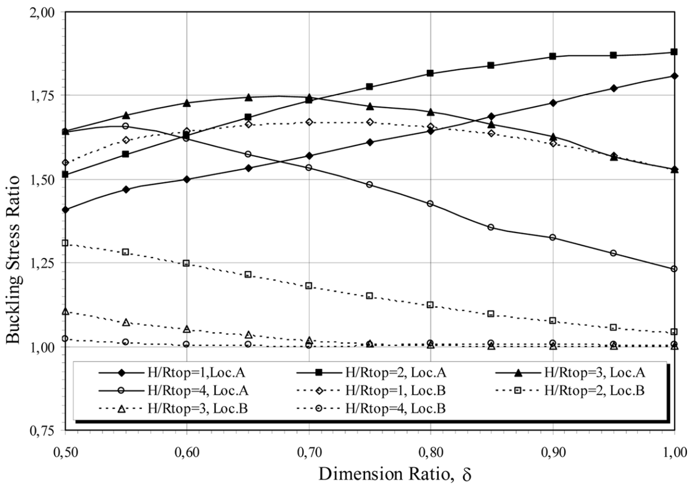

Fig. 12 Effect of horizontal stiffener location on buckling stress ratio for various values of H/R_{top}, D_{top} = 600 cm

against the dimension ratio δ for different ratios of H/R_{top} using horizontal stiffeners located in two positions as shown in Fig. 12. The stiffener locations are indicated by loc.A or loc.B which means 0.25 or 0.5 of duct height from its base, respectively. The values of D_{top}/t are set to 600. The following observations are made:

- The best position of the horizontal stiffener is at 0.25 of duct height from the bottom since this position yields the highest buckling stress ratios for almost all cases. At the same stiffener location, the buckling ratio is found to be increased almost linearly by an average ratio of 1.4 to 1.85 for δ of 0.5 to 1.0 for the case of H/R_{top} ratio lower than two. On the other hand and for higher value of H/R_{top} ratio, the buckling ratio decreased in parabolic trend from 1.6 to 1.4 for δ of 0.5 to 1.0.
- In case of using a horizontal stiffener at the mid height of the ducts, the buckling capacity ratio increases considerably for H/R_{top} ratio equals one compared to the other ratios with an average value equals 1.6. However, the percentage of increase of duct buckling capacity when using horizontal stiffener at is not as high as for the case of positive bending moment.

7. Effect of vertical stiffeners stiffness on buckling strength

To study the effect of vertical stiffener stiffness on the duct buckling capacity when subjected to axial loading, five different properties of vertical stiffeners, as shown in Table 1, located at the mid of each wall side of the duct are used. The obtained results are shown in Fig. 13 through 16 for different stiffeners stiffness. Through this study, the buckling stress ratio which is defined by the ratio of duct buckling stress using vertical stiffener compared to the elastic buckling stress for ducts without provision of any vertical stiffeners is calculated and plotted against duct parameters. In general, it is shown that provision of a vertical stiffener increases the duct bucking capacity by different ratios depending on the stiffener stiffness and duct geometry. For the same duct height, as shown in Fig. 13, the higher stiffness of the vertical stiffness the higher increase of duct buckling

Table 1 List of vertical stiffeners used for the analysis

Stiffener	Description [mm]	Area [cm^2]	Inertia [cm^4]	Section modulus [cm^3]	Radius of gyration [cm]
St1	100 × 100 × 2	7,84	126	25	4
St2	200 × 200 × 2	15,84	1035	104	8
St3	300 × 300 × 2	23,84	3529	235	12
St4	400 × 400 × 2	31,84	8406	420	16
St5	400 × 400 × 3	47,64	12515	626	16

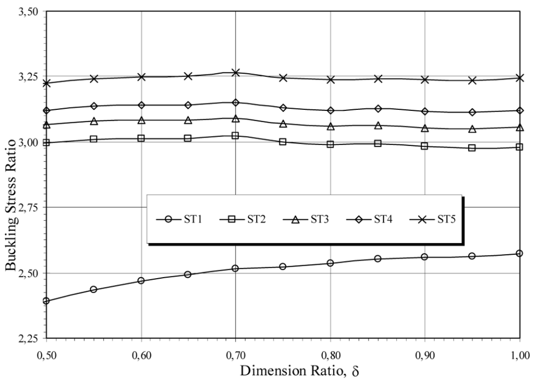

Fig. 13 Effect of vertical stiffener stiffness on buckling stress ratio, H = 3 m

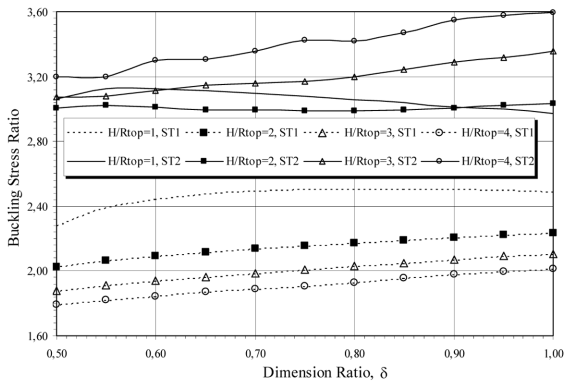

Fig. 14 Buckling stress ratio and duct dimension ratio plot for various values of H/R_{top}, D_{top} = 6 m

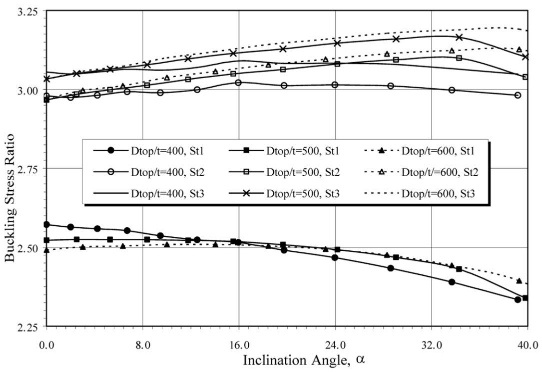

Fig. 15 Effect of vertical stiffeners on buckling stress ratio and duct dimension ratio plot for various values of D_{top}/t, H = 3 m

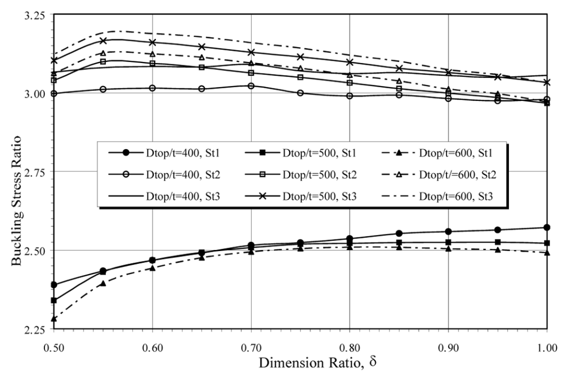

Fig. 16 Effect of vertical stiffeners on buckling stress ratio and duct inclination angle plot for various values of D_{top}/t, H = 3 m

capacity. However, the use of a very high stiffness of vertical stiffener does not affect much the improvement in buckling capacity which is also shown through Figs. 15 and 16. Therefore, from a practical point of view, vertical stiffener with using moderate stiffness is better to be used. It is also found that the minimum value of St2 moment of inertia should be greater than $4{*}H{*}t^3$ where H is the duct height and t is the shell thickness. For the same applied load or the same D_{top}/t using St2 stiffeners, the buckling capacity ratio has an average value equals to 3 for H/R_{top} ratio smaller than 2 as shown in Fig. 14. For values of H/R_{top} ratio greater than 2, the buckling ratio increases with the increase of H/R_{top} and δ ratios. For different D_{top}/t ratios, the buckling capacities increase slightly with the increase of δ. An average value for buckling ratio equals to 3.1 can be used for different D_{top}/t ratios.

6. Conclusions

The aim of this study is to investigate the buckling strength of insulated irregular circular-to-square duct shape when subjected to axial loading. Based on the results and observations of the present study the following conclusions can be drawn:

- A design equation estimating the buckling strength of geometrically perfect cylindrical-to-square shell is developed by using regression analysis accurately with approximately 4% errors.
- The cylinders axially symmetric buckling failure mode occurs when the diameter-to-thickness ratio is greater than 50.
- For the same duct height, the rate of buckling stress decrease is significant as the inclination angle cosine increases where as the same rate is slight as the dimension ratio increases. It is noted that the buckling stress values increase with decrease of the top diameter-to thickness ratio.
- Buckling stress increases in a parabolic trend with increase of δ and decrease of α when the height of duct H is taken equals to duct average radius Rav.
- It is shown that providing a vertical stiffener increases the duct bucking capacity by different ratios depending on the stiffener stiffness and duct geometry.
- From a practical point of view, the vertical stiffener moment of inertia should be greater than $4{*}H{*}t^3$ where H is the duct height and t is the shell thickness since the buckling stress obtained increases slightly for higher values of stiffener inertia.
- The results suggest that the best position of the horizontal stiffener is at 0.25 of duct height from the bottom since this position yields that highest buckling stress ratios for all cases.

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