

# Ductility and inelastic deformation demands of structures

Cheikh Benazouz<sup>\*1,2</sup>, Leblouba Moussa<sup>2a</sup> and Zerzour Ali<sup>2b</sup>

<sup>1</sup>National Center for Applied Research in Earthquake Engineering, Hussein Dey, Algiers, Algeria

<sup>2</sup>High National School of Public Works, ENSTP, BP. 32, Garidi I, Vieux Kouba 16051, Algiers, Algeria

(Received July 8, 2011, Revised December 15, 2011, Accepted April 24, 2012)

**Abstract.** Current seismic codes require from the seismically designed structures to be capable to withstand inelastic deformation. Many studies dealt with the development of different inelastic spectra with the aim to simplify the evaluation of inelastic deformation and performance of structures. Recently, the concept of inelastic spectra has been adopted in the global scheme of the performance-based seismic design through capacity-spectrum methods. In this paper, the median of the ductility demand ratio for 80 ground motions are presented for different levels of normalized yield strength, defined as the yield strength coefficient divided by the peak ground acceleration (PGA). The influence of the post-to-preyield stiffness ratio on the ductility demand is investigated. For fixed levels of normalized yield strength, the median ductility versus period plots demonstrated that they are independent of the earthquake magnitude and epicentral distance. Determined by regression analysis of the data, two design equations have been developed; one for the ductility demand as function of period, post-to-preyield stiffness ratio, and normalized yield strength, and the other for the inelastic deformation as function of period and peak ground acceleration valid for periods longer than 0.6 seconds. The equations are useful in estimating the ductility and inelastic deformation demands for structures in the preliminary design. It was found that the post-to-preyield stiffness has a negligible effect on the ductility factor if the yield strength coefficient is greater than the PGA of the design ground motion normalized by gravity.

**Keywords:** inelastic spectra; ductility; capacity-spectrum method; ground motion

---

## 1. Introduction

Since the development of the Capacity-Spectrum Method (CSM) (Freeman *et al.* 1975), many response spectra have been proposed to replace the conventional elastic spectra in order to achieve accurate evaluation of the inelastic response of structures.

Newmark and Hall (1973) proposed inelastic response spectra of 5% damped single degree of freedom (SDOF) system based on the elastic spectra.

Sheng and Biggs (1980) constructed inelastic response spectra for different ductility ratios and damping coefficients. Iwan (1980), Kowalsky (1994) developed empirical equations to define the period shift and equivalent viscous damping ratio to estimate the maximum displacement demand of inelastic SDOF system from its linear representation. Ridell *et al.* (1989) introduced reduction

---

\*Corresponding author, Graduate Student, E-mail: [cheikhpbsd@gmail.com](mailto:cheikhpbsd@gmail.com)

<sup>a</sup>Lecturer, E-mail: [mlablouba@enstp.edu.dz](mailto:mlablouba@enstp.edu.dz)

<sup>b</sup>Professor, E-mail: [a.zerzour@enstp.edu.dz](mailto:a.zerzour@enstp.edu.dz)

factors for constructing inelastic design response spectra from elastic spectrum. Similarly, Krawinkler and Nassar (1990) studied strength reductions by using 33 ground motions recorded during the 1989 Whittier Narrow, California earthquake. Later (1991), they evaluated the average inelastic spectra of bilinear and stiffness degrading systems subjected to 15 ground motions and proposed a functional form of the reduction factor with respect to ductility factor, natural period, and post-to-preyield stiffness ratio. Similarly, Miranda (1992) used 124 ground motions recorded during various earthquake events. Emphasis is given to the influence of soil conditions on the inelastic strength and deformation demands of SDOF systems.

The use of inelastic design spectrum in the context of capacity-spectrum methods was suggested by Bertero (1995), and introduced by Reinhorn (1997), Fajfar (1999).

Recently, Miranda (2000) proposed procedures based on displacement modification factors in which the maximum inelastic displacement demand of multi-degree of freedom (MDOF) system is estimated by applying certain displacement modification factors to maximum deformation of equivalent SDOF system having the same lateral stiffness and damping coefficient as that of MDOF system. In another study, Miranda and Ruiz-Garcia (2002) evaluated six possible alternative methods to estimate the maximum inelastic deformations of SDOF systems. The evaluated methods estimate the maximum inelastic deformation using functions of displacement ductility factor to compute equivalent periods and equivalent damping ratios (Rosenblueth and Herrera 1964, Gulkan and Sozen 1974, Iwan 1980, Kowalsky 1994), or to compute displacement modification factors (Newmark and Hall 1982, Miranda 2000). The study aimed at evaluating the accuracy of approximate methods for the preliminary design of structures. Later, Akkar and Miranda (2002) conducted a statistical evaluation of five of the above methods. The study showed that for periods longer than 1 second all methods produce relatively good results. In the short period region, equivalent linear methods proposed by Iwan and Guyader and the one proposed by Kowalsky tend to overestimate deformation demands. The errors produced by any of the evaluated approximate methods can be relatively larger, particularly for lateral strength ratios larger than four.

The use of complicated hysteretic characteristics such as stiffness degradation, strength deterioration, and pinching behavior, enables a wide variety of structural systems to be considered (Ibarra 2003, Vamvatsikos and Cornell 2006, Goda *et al.* 2009). Recently, Goda *et al.* (2009) developed a probabilistic model of the peak ductility demand of inelastic SDOF systems with various hysteretic characteristics using comprehensive sets of ground motions recorded in Japan. In this work, the Bouc-Wen model of hysteresis (Wen 1976, Foliente 1995) including degradation and pinching effects was considered.

Bozorgnia *et al.* (2010) developed deterministic and probabilistic predictions of yield strength and inelastic displacement spectra based on attenuation relationship. The study revealed that over a wide structural period range, the magnitude scaling for an inelastic system is higher than that for an elastic system, especially for ductility levels greater than 2 and magnitude greater than 6.5. In the same study, Bozorgnia *et al.* (2010) showed that the domain of validity of the equal displacement rule is restricted to small to moderate magnitudes and/or for low ductility level, and that this rule underestimates inelastic deformation even for long period structures if the earthquake magnitude is large and the structure needs to sustain a large ductility.

The capacity-spectrum method gives an overall view of the inelastic behavior of structures subjected to ground motions. However, to be as more accurate as possible it requires a realistic capacity curve for the structure that is in accordance with its dynamic behavior when subjected to the design earthquake. Many refinements and corrections have been made to the pushover analysis

to make it in accordance with the nonlinear time history analysis. For instance, the adaptive pushover analysis (Elnashai 2001, among others) actually represents the best choice.

Based on the deficiencies in ATC-40 procedures (Applied Technology Council 1996), Chopra and Goel (1999) proposed an improved Capacity-Demand-Diagram method that uses the constant-ductility design spectrum. Their method has been evaluated using SDOF elastoplastic systems, and to retain the attraction of the graphical implementation of the ATC-40 procedures (A and B) Chopra and Goel developed an alternative graphical implementation of their improved method.

In this paper, we first present the development of the ductility demand response spectrum. This is followed by the development of the response spectra for four selected data-sets of ground motion records, each containing 20 records representing short to long magnitude and short to long distances from fault. The influence of the post-to-preyield stiffness ratio on the ductility demand is investigated. Next, nonlinear regression analysis of the data is conducted, two design equations are developed; one for the ductility demand as function of period, post-to-preyield stiffness ratio, and normalized yield strength, and the other for the inelastic deformation as function of period and peak ground acceleration valid for periods larger than 0.6 seconds. The equations should be useful in estimating the ductility and inelastic displacement demands for structures in the preliminary design.

## 2. Ductility demand response spectrum

### 2.1 Equation of motion in terms of ductility

Considering an inelastic single-degree of freedom system, its motion when subjected to an earthquake ground motion is governed by the following equation

$$m\ddot{x} + c\dot{x} + f(x, \dot{x}) = -m\ddot{u}_g(t) \quad (1)$$

where,  $m$ ,  $c$  and  $f$  represent the mass, damping, and the resisting force of the system, respectively,  $\ddot{u}_g(t)$  denotes the earthquake acceleration. The resisting force  $f$  is defined as the sum of a linear part and a hysteretic part

$$f = k_p x + Qz \quad (2)$$

In the above,  $k_p$  is the postyield stiffness,  $Q$  is the yield strength, and  $z$  represents the dimensionless variable that characterizes the Bouc-Wen model of hysteresis (Wen 1976), it is given by

$$\dot{z} = \frac{\dot{x}}{x_y} [A - |z|^\eta (\gamma \text{sgn}(\dot{x}z) + \beta)] \quad (3)$$

In the above equation,  $x_y$  is the yield displacement,  $A$ ,  $\eta$ ,  $\gamma$ , and  $\beta$  are parameters that control the shape of the hysteresis loop. Material degrading is not considered in this study. However, to accommodate material degradation in the hysteresis model, the Baber-Noori (1985) version of the Bouc-Wen model may be used.

Substituting Eq. (2) into Eq. (1) and dividing by  $m$  yields

$$\ddot{x} + 2\xi\omega\dot{x} + \frac{k_p}{m}x + \frac{Q}{m}z = -\ddot{u}_g(t) \quad (4)$$

or simply

$$\ddot{x} + 2\xi\omega\dot{x} + \alpha\omega^2x + qgz = -\ddot{u}_g(t) \quad (5)$$

In which  $\xi$ ,  $\omega$ ,  $\alpha$ , and  $q$  represent the damping ratio, circular frequency, post-to-preyield stiffness ratio, and the yield strength coefficient (defined as yield strength divided by the system weight  $W$ :  $W = mg$ ,  $g$  stands for the gravity), respectively.

Next, Eq. (5) is rewritten in terms of displacement ductility factor,  $\mu$ . Substituting:  $x = x_y\mu$ ,  $\dot{x} = x_y\dot{\mu}$ , and  $\ddot{x} = x_y\ddot{\mu}$ , in Eq. (5) and dividing by  $x_y$  gives

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \frac{qgz}{x_y} = -\frac{1}{x_y}\ddot{u}_g(t) \quad (6)$$

By doing the same to the dimensionless variable  $z$ , Eq. (3) may be expressed in terms of  $\mu$  as

$$\dot{z} = \dot{\mu}[A - |z|^n(\gamma\text{sgn}(\dot{x}z) + \beta)] \quad (7)$$

The term  $qg/x_y$  in Eq. (6) is rewritten as

$$\frac{qg}{x_y} = \omega^2(1 - \alpha) \quad (8)$$

Solving Eq. (8) for  $x_y$  yields

$$x_y = \frac{qg}{\omega^2(1 - \alpha)} \quad (9)$$

Substituting Eq. (8) and Eq. (9) into Eq. (6) gives

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \omega^2(1 - \alpha)z = -\frac{\omega^2(1 - \alpha)}{qg}\ddot{u}_g(t) \quad (10)$$

We observe from Eq. (10) that for a given ground acceleration,  $\mu(t)$  depends on  $\xi$ ,  $\omega$ ,  $\alpha$ , and  $q$ .

## 2.2 System controlling parameters and normalization

To obtain meaningful system response to an ensemble of ground motions, the system yield strength coefficient has to be defined relative to the intensity of individual ground motions. Using the parameter  $\eta$  introduced by Mahin and Lin (1983) as

$$\eta = \frac{qg}{PGA} \quad (11)$$

where, PGA stands for the Peak Ground Acceleration. Incorporating  $\eta$  into Eq. (10) results

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \omega^2(1 - \alpha)z = -\frac{\omega^2(1 - \alpha)}{\eta}\ddot{u}_g(t) \quad (12)$$

in which,  $\ddot{u}_g(t)$  represents the ground acceleration normalized with respect to the PGA

$$\ddot{u}_g(t) = \frac{\ddot{u}_g(t)}{PGA} \quad (13)$$

The ground acceleration has been normalized such that its value varies from  $-1$  to  $1$ . Eq. (12) implies that for a given inelastic system, if  $\alpha$  and  $\eta$  are fixed, the intensity of the ground motion

has no effect on the peak normalized deformation,  $\mu$ . This permits the construction of the ductility response spectrum for an ensemble of ground motions with common frequency content but variable intensity.

### 2.3 Constant- $\eta$ ductility response spectrum

The procedure to construct the ductility response spectrum for inelastic systems corresponding to specified levels of normalized yield strength  $\eta$ , is summarized in the following steps:

1. Define the ground motion  $\ddot{u}_g(t)$ ;
2. Select and fix the damping ratio  $\xi$  and the post-to-preyield stiffness ratio  $\alpha$  ( $\alpha = 0$  for elastoplastic system) for which the spectrum is to be plotted;
3. Specify a value for  $\eta$ ;
4. Select a value for elastic period  $T$ ;
5. Determine the ductility response  $\mu(t)$  of the system with  $T$ ,  $\xi$ , and  $\alpha$  equal to the values selected by solving Eq. (12) along with Eq. (7). From  $\mu(t)$  determine the peak ductility factor  $\mu$ ;
6. Repeat steps 4 and 5 for a range of  $T$ , resulting in the spectrum values for the  $\eta$  value specified in step 3;
7. Repeat steps 3 to 6 for several values of  $\eta$ .

Given the excitation and the properties  $T$ ,  $\xi$ ,  $\alpha$ , and  $\eta$  of an inelastic SDOF system, it is desired to determine the peak deformation,  $x_m$ . The yield displacement of the system is derived from Eq. (9) and Eq. (11) as follows

$$x_y = \frac{\eta}{\omega^2(1-\alpha)}PGA \quad (14)$$

The value of the ductility factor is read from the spectrum developed by the above procedure and multiplied by  $x_y$  to obtain the peak deformation,  $x_m$ .

## 3. Statistical analysis

Based on statistical analysis of response data, the nonlinear spectra are developed next. Four ensembles of ground motions each with 20 records are included in this study. The ensembles, denoted by LMSR, LMLR, SMSR, and SMLR represent four combinations of large ( $M = 6.6-6.9$ ) or small ( $M = 5.8-6.5$ ) magnitude and short ( $R = 13-30$  km) or long ( $R = 30-60$  km) epicentral distance. These motions were obtained from PEER Strong Motion Database, first used by H. Krawinkler; their parameters are available in (Chopra *et al.* 2003).

### 3.1 Median ductility demand

The median response spectrum for ductility demand is constructed by performing the following sequence of steps:

1. Select ranges for period  $T$  and normalized strength  $\eta$ . We choose  $T$  from 0.02 to 3 seconds and seven values of  $\eta$ : {0.25, 0.5, 0.75, 1, 1.5, 2, 2.5}.
2. Construct the constant- $\eta$  ductility demand response spectrum for each ground motion of the ensembles following the steps in subsection 2.3;

3. Compute the median normalized spectra over all ground motions for each value of  $T$  and  $\eta$ .

Median normalized spectra were computed for each ground motion of the four ensembles. Median ductility-demand spectra for LMSR ensemble are shown in Fig. 1 and Fig. 2 as a function of  $T$  for fixed damping ratio  $\xi = 5\%$ ; all results presented in this paper are for this damping ratio. The spectrum is divided logically into three period regions according to the procedure described in Chopra (2001), where  $T_b$  marks the transition from the acceleration-sensitive region to the velocity-sensitive region which ends at  $T \sim 3$  s. The acceleration-sensitive region is divided judgmentally into two regions at  $T = T_a$ , at this point the spectrum would be easily idealized by a series of straight lines.

The results for fixed post-to-preyield stiffness ratio (Fig. 1) permit the following observations on how the normalized yield strength influences the inelastic action indicated by  $\mu$  in various spectral regions. For systems having normalized yield strength  $\eta < 1$ ,  $\mu$  decreases rapidly with an increase of the normalized yield strength. For systems having  $\eta \geq 1$ , in the acceleration-sensitive region, starting at  $\mu = 1$  at  $T = T_a = 0.1$  s,  $\mu$  increases for shorter periods where  $\mu$  is affected little by  $\eta$ . For periods between  $T_a$  and  $T_b = 0.6$  s, the ductility demand is essentially constant and affected little by  $\eta$ . For periods larger than  $T_b$ , the ductility demand decreases in a manner similar to systems with  $\eta < 1$ .

From Fig. 1 for fixed ductility factor, and consistent with previous studies (e.g., Newmark and Hall 1982, Bozorgnia *et al.* 2010, among others) it is clear that the yield strength can be substantially reduced if a moderate level of ductility (e.g.,  $\mu = 2$ ) is sustained by the structure.

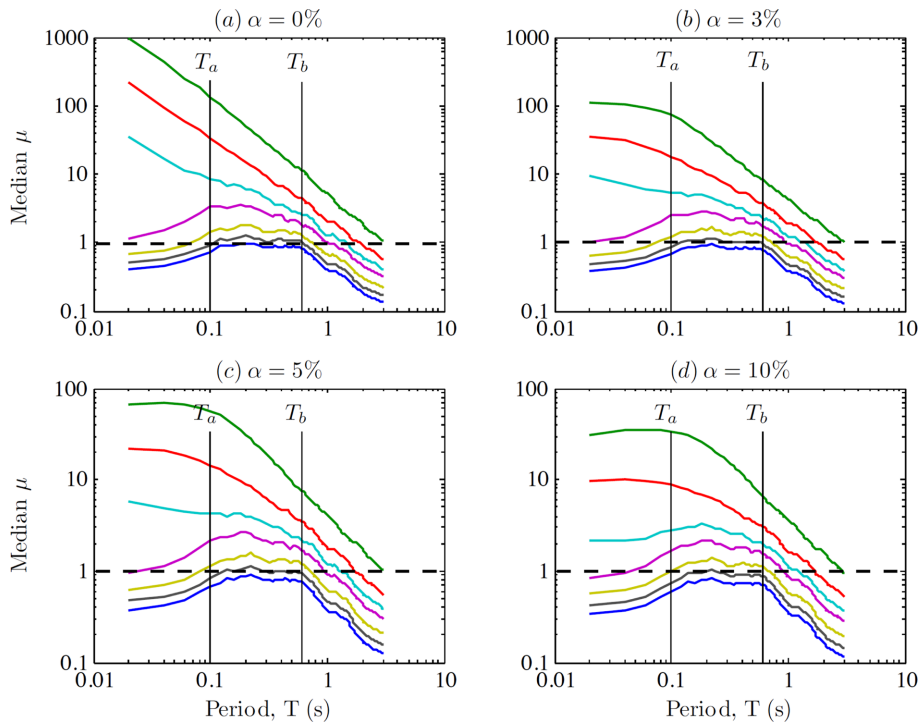


Fig. 1 Median ductility demand for inelastic systems computed for LMSR ensemble of ground motions ( $\eta = 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5$  from top line to bottom line)

Larger available ductility can only result in a moderate reduction in the strength demand, especially for periods larger than 0.5 s.

The results for constant- $\eta$  plots in Fig. 2 indicate that the post-to-preyield stiffness ratio has no effect on the median ductility demand for systems having  $\eta \geq 1$ . For systems with  $\eta < 1$  the Post-to-preyield stiffness ratio reduces the ductility demand only for periods shorter than  $T_b$ , for periods greater than  $T_b$ ,  $\alpha$  has a small effect on the ductility factor.

Computed from the data of Fig. 1 and Fig. 2, the ratio of median  $\mu$  for bilinear and elastoplastic systems is plotted in Fig. 3 against  $T$  for fixed  $\alpha$  and a range of  $\eta$  values. Fig. 3 indicates that the amount of reduction in ductility factor due to post-to-preyield stiffness ratio  $\alpha$  is roughly constant for periods greater than  $T_b$ , increases for shorter periods ( $T < T_b$ ), and disappears for all range of periods for  $\eta \geq 1$ . This observation implied that ignoring post-to-preyield stiffness ratio in estimating the ductility demand is too conservative for seismic evaluation of existing structures with periods in the acceleration-sensitive region; this is similar to what Chopra and Chintanapakdee (2004) concluded. However, the post-to-preyield stiffness ratio may be ignored in estimating the ductility demand for structures with short periods ( $T < T_b$ ) if they have normalized yield strength greater than 1. In other words, for a particular ground motion characterized by PGA, one may ignore the effect of the post-to-preyield stiffness ratio in evaluating structures with periods shorter than 0.6 s (low rise buildings) if their yield strength coefficients are greater than the PGA normalized by  $g$ .

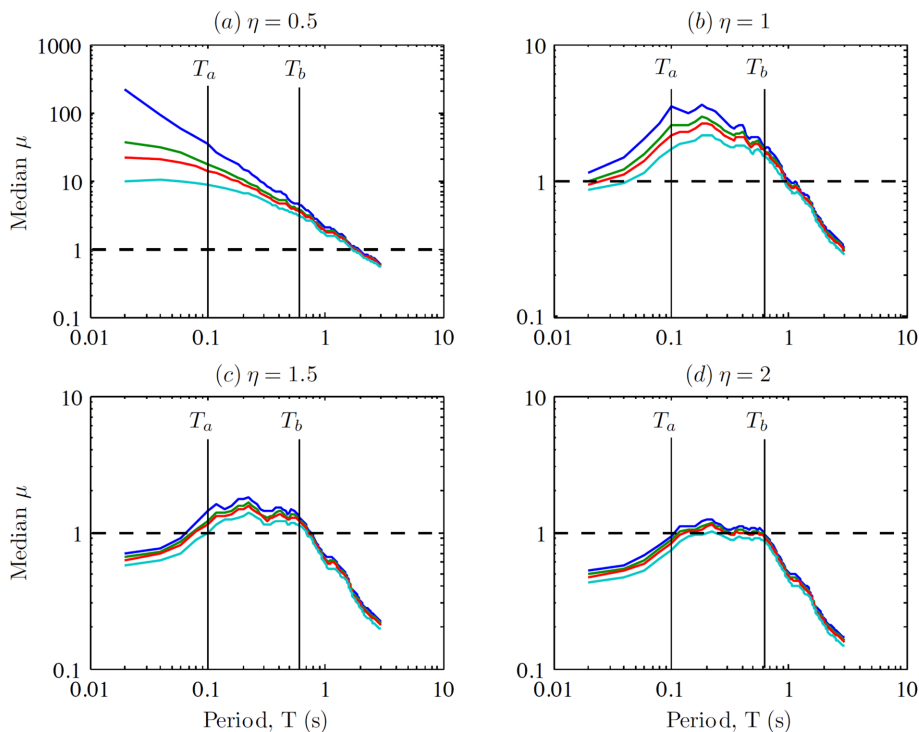


Fig. 2 Influence of post-to-preyield stiffness ratio  $\alpha$  ( $\alpha = 0\%$ ,  $3\%$ ,  $5\%$ ,  $10\%$  from top line to bottom line) on the median of ductility demand  $\mu$  for inelastic systems subjected to LMSR ensemble of ground motions, presented for fixed values of normalized yield strength  $\eta = 0.5, 1, 1.5, 2$

The results for constant- $\eta$  plotted in Fig. 2 indicate that regardless the post-to-preyield stiffness ratio, the response of systems is elastic in the following cases

- $\eta = 0.25$  and  $T \geq 3$  s
- $\eta = 0.5$  and  $T \geq 2$  s
- $\eta = 0.75$  and  $T \geq 1.45$  s
- $\eta = 1$  and  $T \geq 1$  s
- $\eta = 1.5$  and  $T \geq 0.75$  s
- $\eta > 1.5$  at all periods.

Study of the dispersion of the ductility demands was conducted by computing the coefficients of variation (COV) for each ensemble of ground motions. Fig. 4(a) shows the dispersion of  $u$  as a function of natural period for  $\alpha = 3\%$  and different levels of normalized yield strength, while Fig. 4(b) shows the dispersion of  $\mu$  for  $\eta = 1$  and different post-to-preyield stiffness ratios for LMSR ensemble of ground motions. It can be noted that, in general, dispersion of ductility demands is not constant over the whole spectral regions and it depends on the natural period  $T$  and the level of

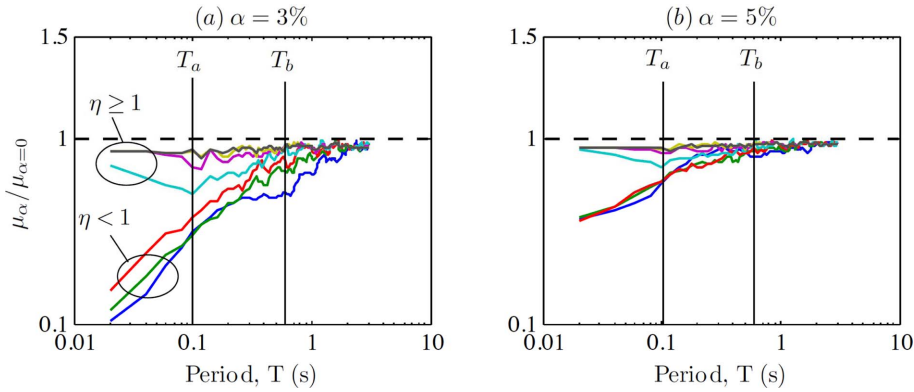


Fig. 3 Ratio of median ductility demand  $\mu$  for bilinear and elastoplastic systems: (a) influence of  $\eta$  for  $\alpha = 3\%$  and (b) influence of  $\eta$  for  $\alpha = 5\%$

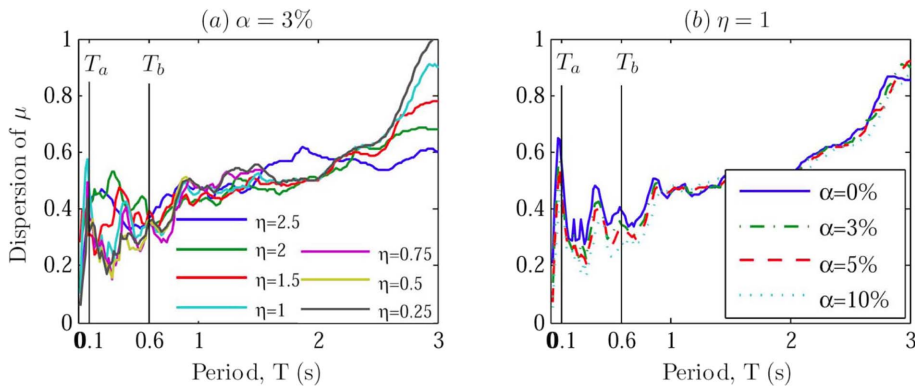


Fig. 4 Dispersion of ductility demand  $\mu$  for inelastic systems subjected to the LMSR ensemble of ground motions: (a) influence of normalized yield strength  $\eta$  at post-to-preyield stiffness ratio  $\alpha = 3\%$  and (b) influence of  $\alpha$  for  $\eta = 1$



normalized yield strength  $\eta$ . In general, dispersion increases as period of vibration increases, this is the case because the PGA has been used in the normalization.

However, this study concerns systems having periods in the acceleration and till velocity-sensitive region. The post-to-preyield stiffness ratio has little effect on the dispersion, especially for periods larger than 1 s.

### 3.2 Effect of earthquake magnitude and distance

The computation that led to the preceding subsection was repeated for the LMSR, LMLR, SMSR, and SMLR of ground motion ensembles. The median  $\mu$  versus  $T$  functions are quite similar, this is demonstrated in the plots for the four ensembles shown in Fig. 5.

This similarity has been observed by Chopra and Chintanapakdee (2004) in the plots of  $C_\mu$  and  $C_R$  coefficients versus  $T$ . Also, Miranda (2000) and Miranda and Ruiz-Garcia (2003) showed that when idealized, peak inelastic displacement demands are not strongly influenced by neither earthquake magnitude nor distance to the fault.

## 4. Estimating of ductility and inelastic deformation demands

Simplified equations for ductility demands  $\mu$  and deformations  $x_m$  would obviously facilitate the estimation of the ductility and deformation of inelastic SDOF system. Such an equation for  $\mu$  could be used to determine the ductility demand for a new or rehabilitated structure with known yield strength coefficient and post-to-preyield stiffness ratio. However, finding a single equation relating all the influencing parameters ( $T$ ,  $\alpha$ ,  $\eta$ ) to ductility factor is not possible. Presented next is an equation that fits the median  $\mu$  data for any of the four ensembles of ground motions, starting from  $T_a$ . Such an equation for  $\mu$  has been derived in terms of the normalized yield strength  $\eta$ , post-to-preyield stiffness ratio  $\alpha$ , and period of vibration  $T$

$$\mu = \frac{a(b)^{1/T}(T)^{\eta-c}}{\eta} \quad (15)$$

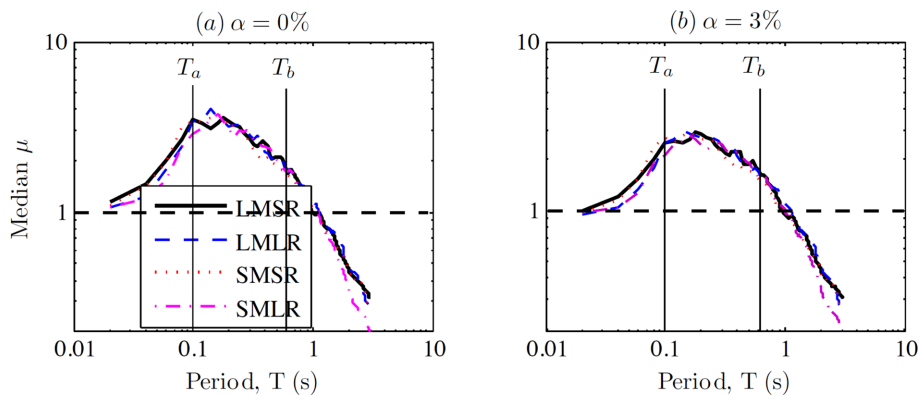


Fig. 5 Comparison of ductility demand  $\mu$  for LMSR, LMLR, SMSR, and SMLR ensembles of ground motions: (a)  $\mu$  for  $\alpha = 0\%$  and  $\eta = 1$ ; (b)  $\alpha = 3\%$  and  $\eta = 1$

The numerical parameters  $a$ ,  $b$ , and  $c$  were determined from the response data by nonlinear regression analysis using Datafit software (Oakdale Engineering), they are tabulated in Table 1 for different ranges of  $\eta$  and  $\alpha$ . However, for values of  $\eta \geq 1$  and periods larger than  $T_b$ , the post-to-preyield stiffness ratio is ignored since it has little influence on the ductility factor. For  $\eta \geq 1$  and for periods between  $T_a$  and  $T_b$ , ductility is observed to be nearly independent of period  $T$ , depending on the value of  $\eta$  and  $\alpha$  (see Table 2).

Fig. 6 shows that Eq. (15) agrees well with median  $\mu$  computed for the LMSR ground motions.

The inelastic deformation  $x_m$  can be determined by multiplying the ductility factor computed from Eq. (15) by the yield displacement  $x_y$  determined from Eq. (14)

$$x_m = \frac{PGA}{4\pi^2} \left[ \frac{a(b)^{1/T} (T)^{\eta+2-c}}{(1-\alpha)} \right] \quad (16)$$

Eq. (15) and Eq. (16) show an advantage over the coefficients like  $C_\mu$  and  $C_R$  that require the determination of the elastic response first.

In addition, for periods longer than  $T_b = 0.6$  s a unified equation is found to fit the median ductility demand

$$\mu = \frac{\omega^2(1-\alpha)}{\eta} 0.027 T^{0.84} \quad (17)$$

The median inelastic deformation  $x_m$  is then determined using Eq. (14) for the yield displacement

$$x_m = 0.027 T^{0.84} PGA \quad (18)$$

This equation provides the deformation for a given median PGA that reflects the intensity of the design ground motions, regardless the yield strength and post-to-preyield stiffness ratio.

Fig. 7 compares Eq. (17) with the median value of  $\mu$  determined earlier for inelastic systems subjected to the LMSR ensemble of ground motions. Eq. (17) provides excellent estimates of the

Table 1 Parameters in Eq. (15) and Eq. (16) for each value of  $\eta$  and  $\alpha$

Parameters	$\eta < 1$				$\eta \geq 1$ (all values of $\alpha$ )
	$\alpha$ (%)				
	0	3	5	10	
$a$	1.24	1.12	1.08	1.04	1.23
$b$	0.98	0.94	0.92	0.88	0.85
$c$	1.69	1.65	1.68	1.68	$1.21+\alpha$

Table 2 Values of ductility  $\mu$  for  $T_a \leq T < T_b$

$\eta$	$\alpha$ (%)			
	0	3	5	10
1	-	2.5	2	1.65
1.5	1.5	1.4	1.3	1.3

$T_a = 0.1$  s,  $T_b = 0.6$  s

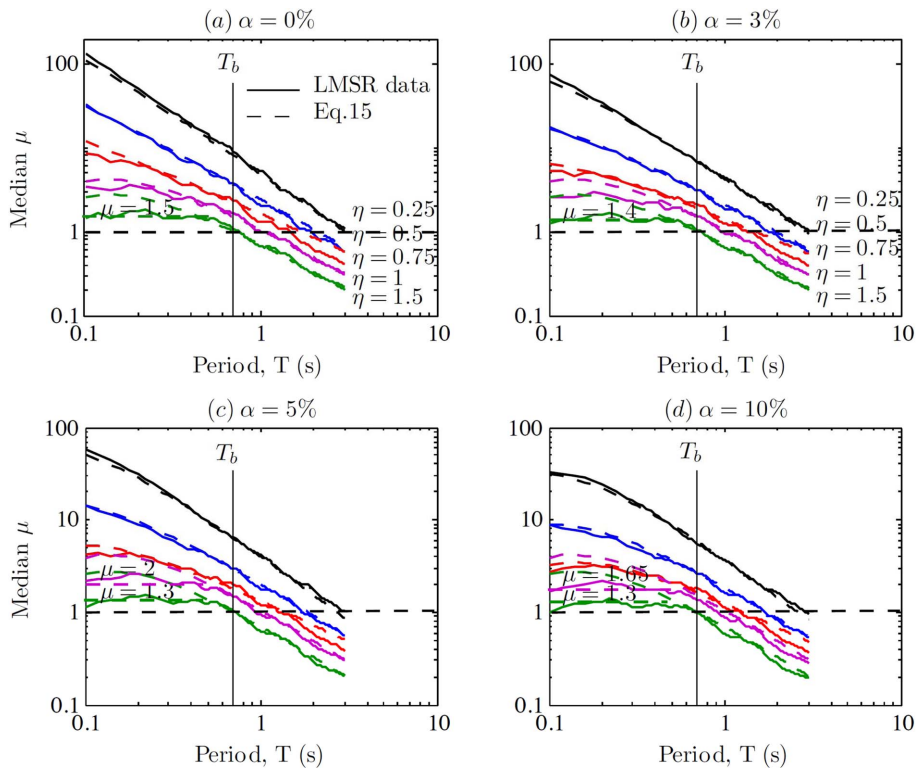


Fig. 6 Comparison of ductility demand  $\mu$  estimated by Eq. (15) with computed data for LMSR ensemble of ground motions for: (a) elastoplastic systems ( $\alpha = 0\%$ ) and (b, c, d) bilinear system ( $\alpha = 3\%, 5\%, 10\%$ )

ductility demand for inelastic systems in the range of periods longer than  $T_b$ .

**Validation of The Proposed Equations:** The Eq. (15) and Eq. (17) are used to compute the displacement ductility of two systems. Considered values of  $T = 0.5$  s in the acceleration-sensitive spectral region and  $T = 0.9$  s in the velocity-sensitive region. The excitation chosen is the North-South component of the El Centro 1940 ground motion (PGA = 0.32 g). Table 3 lists the properties of the two systems and their response in terms of displacement ductility computed using Eqs. (15) and (17) and the nonlinear time history analysis (THA). Results obtained show that the proposed equations give displacement ductilities close to those that would be obtained using THA.

## 5. Conclusions

The main goal of this study was to estimate the displacement ductility and inelastic deformation demands for structures. For this purpose, the ductility demand  $\mu$  was defined. Then, a statistical study of the ductility demands computed for single-degree of freedom systems having different levels of normalized yield strength  $\eta$  and different post-to-preyield stiffness ratios  $\alpha$  was conducted using a total of 80 ground motions divided into four ensembles. The following conclusions can be drawn from the results of this study:

- (1) For systems with normalized yield strength smaller than one, the ductility factor decreases rap-

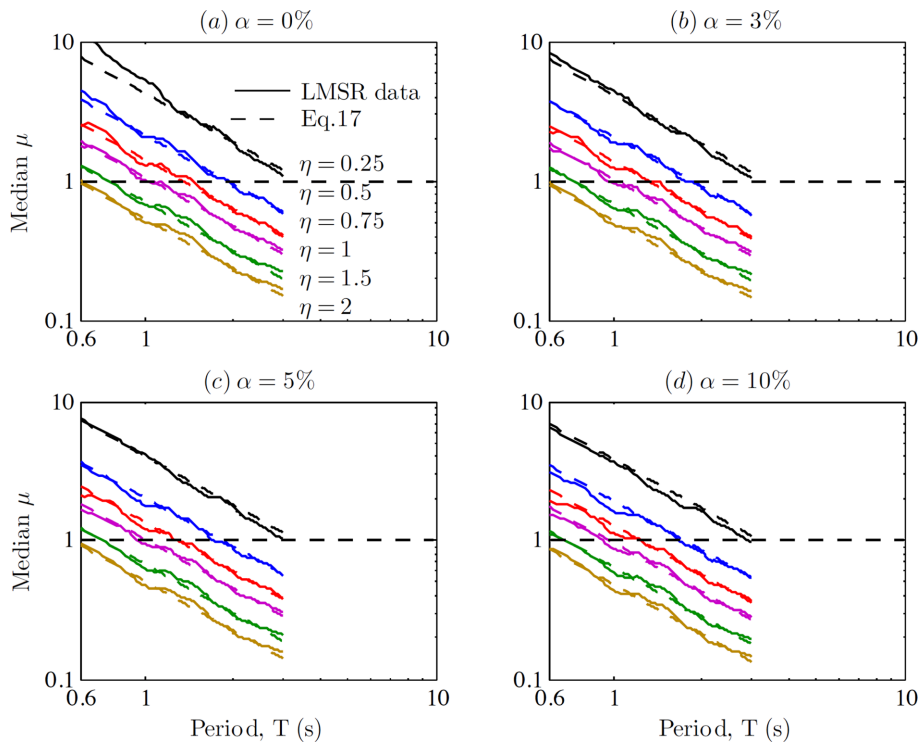


Fig. 7 Comparison of ductility demand  $\mu$  estimated by Eq. (17) with computed data for LMSR ensemble of ground motions for: (a) elastoplastic systems ( $\alpha = 0\%$ ) and (b, c, d) bilinear system ( $\alpha = 3\%$ ,  $5\%$ ,  $10\%$ )

Table 3 Results from Eq. (15) and Eq. (17) of two systems for El Centro (1940) ground motion

System Properties				System Response		
System	$T(s)$	$\alpha$	$\eta$	$\mu$ (THA)	$\mu$ (Eq. (15))	$\mu$ (Eq. (17))
1	0,5	0,05	0,75	2,68	2,32	-
2	0,9	0,05	0,75	1,79	1,45	1,54

idly; this factor is very sensitive to the yield strength.

- (2) For systems with a normalized yield strength greater or equal to one,  $\mu$  decreases in the acceleration sensitive region and it is affected little by the value of  $\eta$ .
- (3) For systems with periods between  $0.1\text{ s}$  and  $0.6\text{ s}$  and normalized yield strength greater or equal to one, the ductility demand is essentially constant.
- (4) For systems with  $\eta > 2$ , the response is elastic regardless the period of the system.
- (5) The post-to-preyield stiffness ratio has no effect on the median ductility demand for systems with a normalized yield strength greater than one. For systems with normalized yield strength smaller than one, the post-to-preyield stiffness ratio reduces the ductility demand only for periods longer than  $0.6\text{ s}$ , and has essentially little effect on the ductility demand for longer periods.
- (6) Consistent with other studies, the yield strength can be substantially reduced if a moderate level of ductility is sustained by the structure (say  $\mu = 2$ ). Larger available ductility can only result in a

- moderate reduction in the strength demand, especially for periods larger than 0.5 s.
- (7) Ignoring post-to-preyield stiffness ratio in estimating the ductility demand is too conservative for seismic evaluation of structures with periods in the acceleration-sensitive region. However, the post-to-preyield stiffness ratio may be ignored when the normalized yield strength is greater than one.
- (8) For the selected accelerograms, it was shown that the ductility demands are not affected by the earthquake magnitude nor by the epicentral distance.
- (9) The dispersion of  $\mu$  is not constant over the whole period range; it depends on  $T$  and  $\eta$ .
- (10) Simplified equations for ductility and inelastic deformation demands for inelastic systems having periods shorter than 3 s have been developed. These equations are simple and provide a good estimation of median ductility demand and inelastic displacement of new or rehabilitated structures when the yield strength is known.
- (11) For systems with periods larger or equal 0.6 s, a simple equation for inelastic deformation of inelastic systems has been developed as function of  $T$  and PGA. This equation provides a good estimation of inelastic deformation of inelastic systems subjected to any earthquake of the considered ground motions, without prior determination of neither yield strength nor the post-to-preyield stiffness ratio.

## References

- Applied Technology Council (1996), "Seismic evaluation and retrofit of concrete buildings", Report ATC 40.
- Baber, T.T. and Noori, M.N. (1984), "Random vibration of pinching hysteretic systems", *J. Eng. Mech.*, **110**(7), 1036-1049.
- Bertero, V.V. (1995), "Tri-service manual methods, in Vision 2000", Part 2, Appendix J, *Structural Engineers Association of California*, Sacramento.
- Bozorgnia, Y., Mahmoud, M.H. and Kenneth, W.C. (2010), "Deterministic and probabilistic predictions of yield strength and inelastic displacement spectra", *Earthq. Spectra*, **26**(1), 25-40.
- Chopra, A.K. and Chatpan, C. (2003), "Inelastic deformation ratios for design and evaluation of structures: single-degree-of-freedom bilinear systems", Report No. 2003/09, Earthquake Engineering Research Center, University of California, Berkeley, California.
- Chopra, A.K. and Chatpan, C. (2004), "Inelastic deformation ratios for design and evaluation of structures: single-degree-of-freedom bilinear systems", *J. Struct. Eng.*, **130**(9), 1309-1319.
- Chopra, A.K. and Goel, R.K. (1999), "Capacity-demand-diagrams based on inelastic design spectrum", *Earthq. Spectra*, **15**(4), 637-656.
- Datafit version 9.0.59, Curve fitting software, Oakdale Engineering, PA, USA.
- Elnashai, A.S. (2001), "Advanced inelastic static (Pushover) analysis for earthquake applications", *J. Struct. Eng. Mech.*, **12**(1), 51-69.
- Fajfar, P. (1999), "Capacity spectrum method based on inelastic demand spectra", *Earthq. Eng. Struct. D.*, **28**(9), 979-993.
- Foliente, G.C. (1995), "Hysteresis modeling of wood joints and structural systems", *J. Struct. Eng.*, **121**(6), 1013-1022.
- Freeman, S.A. Nicoletti, J.P. and Tyrell, J.V. (1975), "Evaluation of existing buildings for seismic risk-a case study of Puget Sound Naval Shipyard, Bremerton, Washington", *Proceedings of 1st U.S. National Conference on Earthquake Engineering*, Earthquake Engineering Research Institute, Berkeley.
- Goda, K. and Atkinson, G.M. (2009), "Seismic demand estimation of inelastic SDOF systems for earthquakes in Japan", *Bull. Soc. Seismol. Am.*, **99**(6), 3284-3299.
- Gülkan, P. and Sözen, M. (1974), "Inelastic response of reinforced concrete structures to earthquake motions", *ACI J.*, **71**(12), 604-610.

- Ibarra, L.F. (2003), "Global collapses of frame structures under seismic excitations", Ph.D. Thesis, Department of Civil and Environmental Engineering, Stanford University, Stanford, California.
- Iwan, W.D. (1980), "Estimating inelastic spectra from elastic spectra", *Earthq. Eng. Struct. D.*, **8**(4), 375-388.
- Kowalsky, M.J. (1994), "Displacement-based design-a methodology for seismic design applied to RC bridge columns", Master's Thesis, University of California at San Diego, La Jolla, California.
- Krawinkler, H. and Nassar, A. (1990), "Strength and ductility demands for SDOF and MDOF systems to Whittier narrows earthquake ground motions", *Proceedings of the Seminar on Seismological and Engineering Implications of Recent Strong-Motion Data*, SMIP, Sacramento.
- Mahin, S.A. and Lin, J. (1983), "Construction of inelastic response spectra for single-degree-of-freedom systems. Computer program and applications", Report No. UCB/EERC-83/17, University of California, Berkeley.
- Miranda, E. (1992), "Nonlinear response spectra for earthquake resistant design", *Proceedings of the Tenth World Conference on Earthquake Engineering*, Balkema, Rotterdam.
- Miranda, E. (1993), "Probabilistic site-dependent non-linear spectra", *Earthq. Eng. Struct. D.*, **22**(12), 1031-1046.
- Miranda, E. (2000), "Inelastic displacement ratios for structures on firm sites", *J. Struct. Eng.*, **126**(10), 1150-1159.
- Miranda, E. (2001), "Estimation of inelastic deformation demands of SDOF systems", *J. Struct. Eng.*, **127**(9), 1005-1012.
- Nassar, A. and Krawinkler, H. (1991), "Seismic demands for SDOF and MDOF systems", *John A. Blume Earthquake Engineering Center*, Report 95, Dept. of Civil Engineering, Stanford University.
- Newmark, N.M. and Hall, W.J. (1973), "Procedures and criteria for earthquake resistant design", *Building Sci. Series No. 46*, National Bureau of Standards, U.S. Dept. of Commerce, Washington, D.C.
- Newmark, N.M. and Hall, W.J. (1982), "Earthquake spectra and design", *Earthquake Engineering Research Institute*, Berkeley, California.
- PEER Strong Motion Database (2011), <http://peer.berkeley.edu/smcat>.
- Reinhorn, A.M. (1997), "Inelastic analysis techniques in seismic evaluations", Eds. P. Fajfar and H. Krawinkler, *Seismic Design Methodologies for the Next Generation of Codes*, Balkema, Rotterdam.
- Riddell, R., Hidalgo, P. and Cruz, E. (1989), "Response modification factors for earthquake resistant design of short period structures", *Earthq. Spectra*, **5**(3), 571-590.
- Rosenblueth, E. and Herrera, I. (1964), "On a kind of hysteretic damping", *J. Eng. Mech.*, **90**(4), 37-48.
- Ruiz-Garcia, J. and Miranda, E. (2003), "Inelastic displacement ratios for evaluation of existing structures", *Earthq. Eng. Struct. D.*, **32**(8), 1237-1258.
- Shih-Sheng, P.L. and John, M.B. (1980), "Inelastic response spectra for aseismic building design", *J. Struct. Div.-ASCE*, **106**(6), 1295-1310.
- Sinan, D.A. and Miranda, E. (2005), "Statistical evaluation of approximate methods for estimating maximum deformation demands on existing structures", *J. Struct. Eng.*, **131**(1), 160-172.
- Vamvatsikos, D. and Cornell, C.A. (2006), "Direct estimation of the seismic demand and capacity of oscillators with multi-linear static pushovers through IDA", *Earthq. Eng. Struct. D.*, **35**(9), 1097-1117.
- Wen, Y.K. (1976), "Method for random vibration of hysteretic systems", *J. Eng. Mech.*, **102**(2), 249-263.