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Steel fibre and transverse reinforcement effects on the behaviour of high strength concrete beams

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Abstract. An experimental program was carried out to investigate the influence of fibre reinforcement on the mechanical behaviour of high strength reinforced concrete beams. Eighteen beams, loaded in four-point bending tests, were examined by applying monotonically increasing controlled displacements and recording the response in terms of load-deflection curves up to failure. The major test variables were the volume fraction of steel fibres and the transverse steel amount for two different values of shear span. The contribution of the stirrups to the shear strength was derived from the deformations of their vertical legs, measured by means of strain gauges. The structural response of the tested beams was analyzed to evaluate strength, stiffness, energy absorption capacity and failure mode. The experimental results and observed behaviour are in good agreement with those obtained by other authors, confirming that an adequate amount of steel fibres in the concrete can be an alternative solution for minimizing the density of transverse reinforcement. However, the paper shows that the use of different theoretical or semi-empirical models, available in literature, leads to different predictions of the ultimate load in the case of dominant shear failure mode.

Keyword: high strength concrete beam; steel fibres; stirrups; experimental study

1. Introduction

High strength concrete is now readily available for various practical applications as a result of ongoing progress in concrete technology. In civil engineering structures this material can offer many advantages, including excellent mechanical performance and durability, which could enable immediate and long-term reductions in cost. However, there are some concerns about the structural use of high strength concrete in seismic regions, since it exhibits a more brittle behaviour than conventional normal strength concrete.

In recent years experimental and theoretical studies have shown that this disadvantage can be overcome by adding discontinuous steel fibres to a matrix of high strength concrete. Steel fibres mitigate the brittleness because their contribution in tension prevents the formation or limits the growth of cracks (Ashour *et al.* 1992, Imam *et al.* 1995, Kim and White 1998, Khuntia *et al.* 1999,

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Lim and Oh 1999, Noghabai 2000, Cucchiara *et al.* 2004, Parra-Montesinos 2006, Campione and Mangiavillano 2008, Ding *et al.* 2011, Colajanni *et al.* 2012).

In relation to the number of parameters governing the failure mechanisms, the experimental studies carried out on this subject are not enough (Ashour *et al.* 1992, Khuntia *et al.* 1999, Noghabai 2000, Cladera and Marí 2005, Kang *et al.* 2011), therefore, the aim of the present study is to provide further experimental information in order to evaluate the effectiveness of steel fibres on the performance of fibre reinforced high concrete beams, and to verify the reliability of theoretical models that have been proposed to evaluate their bearing capacity.

An experimental investigation was carried out on eighteen beams subjected to bending and shear, generated by means of displacement controlled tests. The research program consisted in two phases: the first concerned with determination of the mechanical characteristics of the materials used to make the beams; the second studying the load displacement-curves of beams subjected to four point bending tests.

For a standard set of beams and unchanged percentage of longitudinal reinforcement, the experimental investigation was carried out by considering different values of the following parameters for two different value of the shear span to depth ratio: the percentage of reinforcing fibres, and the percentage of transverse reinforcement. The present study is related to previous experimental researches (Campione *et al.* 2003, Cucchiara *et al.* 2004, Cucchiara and Priolo 2008) carried out on normal and high strength, plain or fibre reinforced concrete beams.

According to the experimental results found in the literature, this investigation emphasizes the role of the fibres in shear strength mechanism and the advantages of the combined use of fibres and stirrups, which modifies the failure mode from brittle shear to ductile flexural, if an adequate percentage of fibres is utilized.

On the other hand it is shown that a very approximate prediction of the bearing capacity of the examined beams can be made only in the case of dominant bending failure mode, while the expressions of the ultimate load that have been proposed by different authors provide a rather large range of values of the ultimate load in the case of dominant shear failure mode.

2. Materials and mechanical characterization

2.1 Material properties

2.1.1 Plain concrete

The plain concrete utilized consisted of the following composition in kg/m^3: 450 of Cement (Portland 42.5) Tecnocem II-B, 160 of water, 1050 of natural gravel (the maximum diameter of coarse aggregates was 10 mm, chosen in compliance with the RILEM TC 162-TDF recommendations (2000)) and 850 of sand. To improve the workability of the concrete, reduced because of the low ratio water-cement and the high percentage of fibres, 6.75 kg/m^3 of a high-range water-reducing admixture (Rheobuild 1-2) was used.

2.1.2 Fibre reinforced concrete

Fibrous concrete had the same composition as plain concrete, but with the addition of 80 kg/m^3 (V_f = 1%) or 160 kg/m^3 (V_f = 2%) of hooked steel fibres in fresh concrete.

The hooked-end steel fibres used had the following properties: length l_f = 30 mm; equivalent diameter d_f = 0.55 mm (aspect ratio l_f/d_f = 55); nominal tensile strength 1100 MPa; elastic modulus 207000 MPa; specific weight 7869 kg/m^3. The length of the fibres was chosen according to the minimum size of the structural element, as stated in the Italian AICAP Recommendations (1990) and in the ACI rules (1988).

2.1.3 Steel reinforcement

The longitudinal reinforcement used in the specimens consisted of two bars with diameter of 20 mm. For the transverse reinforcement the bars had a diameter of 6 mm. The steel bars in both cases were type $f_{yk} \geq 435$ MPa.

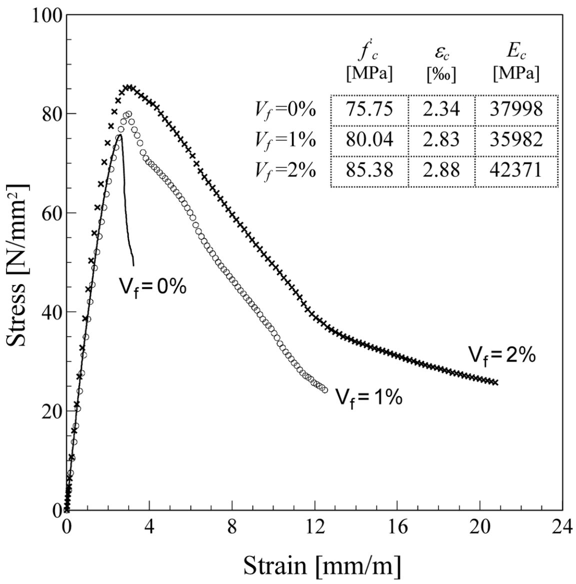
2.2 Mechanical characterization of materials

2.2.1 Plain and fibre reinforced concrete

In order to characterize the mechanical behaviour of concrete, cylindrical specimens 100 × 200 mm were prepared from each type of concrete and tested in compression (four per mix) and in tension (three per mix) after 28 days of curing.

Fig. 1 shows the experimental results of the compression tests in terms of σ–ε mean curves. Each curve was obtained as the average of results recorded in every type of concrete. The comparison between the response of plain and fibrous concrete underlines the advantageous effect offered by the fibres with significant increases in residual strength in the softening branch and of the strain corresponding to the peak stress. Moreover, it was observed that the addition of fibres does not determine significant variations in the peak stress value, as already observed by other authors (Lim and Oh 1999).

The above figure also shows the average values for peak stress f_c' in MPa, the corresponding strain ε_c and the tangent elastic modulus E_c in MPa.



	f_c' [MPa]	ε_c [‰]	E_c [MPa]
V_f=0%	75.75	2.34	37998
V_f=1%	80.04	2.83	35982
V_f=2%	85.38	2.88	42371

Fig. 1 Stress-Strain curves for plain and fibre reinforced concrete

Table 1 Tensile characteristics of the concrete

V_f [%]	F_{max} [kN]	f_{ct} [MPa]
0	126	4.19
1	224	8.34
2	232	9.43

Table 2 Mechanical properties of bars

Diameter 20 mm	$f_{y\ell}$ [MPa]	$E_{y\ell}$ [MPa]
	474	205797
Diameter 6 mm	f_{yw} [MPa]	E_{yw} [MPa]
	559	187700

Table 1 summarizes the mean values for the maximum load applied to the specimens (F_{max}) and the maximum tensile strength ($f_{ct} = 2F_{max} / (\pi DL)$ with D = 100 mm and L = 200 mm) obtained from the split tests. It can be observed that the splitting tensile strength may be increased more than twice at the 2% fibre volume.

It can be seen that the fibre addition greatly enhances the tensile properties of concrete.

2.2.2 Steel reinforcement

Three direct tensile tests were carried out for each type of bar in order to determine the mechanical properties of longitudinal (diameter 20 mm) and transverse (diameter 6 mm) steel reinforcement. The yield stress values $f_{y\ell}$, f_{yw} and the Young's modulus values $E_{y\ell}$, E_{yw} are given in Table 2.

3. Experimental programme

3.1 Test specimens

Eighteen beams were prepared having a rectangular cross-section of dimensions b = 150 mm, h = 250 mm and length L = 2500 mm (Fig. 2). The covering is 15 mm, therefore the distance between the centre of the longitudinal bars and the bottom of the beam is δ = 31 mm, with effective depth of the beams $d = h - \delta$ = 219 mm.

The choice of a high tensile reinforcement ratio $\rho_\ell = A_\ell/(b \cdot d) = 1.91\%$ (A_ℓ = total area of longitudinal bars), was made to obtain the shear failure mode in the absence of specific shear reinforcement.

The main test variables were: percentage of fibres (V_f = 0, 1 and 2% by volume of concrete); transverse reinforcement ratio $\rho_w = A_w/(b \cdot s)$ with A_w = total area of transverse bars and s spacing of stirrups: $\rho_w = 0.188\%$ for s = 200 mm, and $\rho_w = 0.628\%$ for s = 60 mm; two values of the shear span a: a/d = 2.8 and 2.0.

In order to give further data that can turn out useful to estimate the efficiency of constructive solutions in which the use of stirrups or fibrous concrete is preferred, the previous percentages have

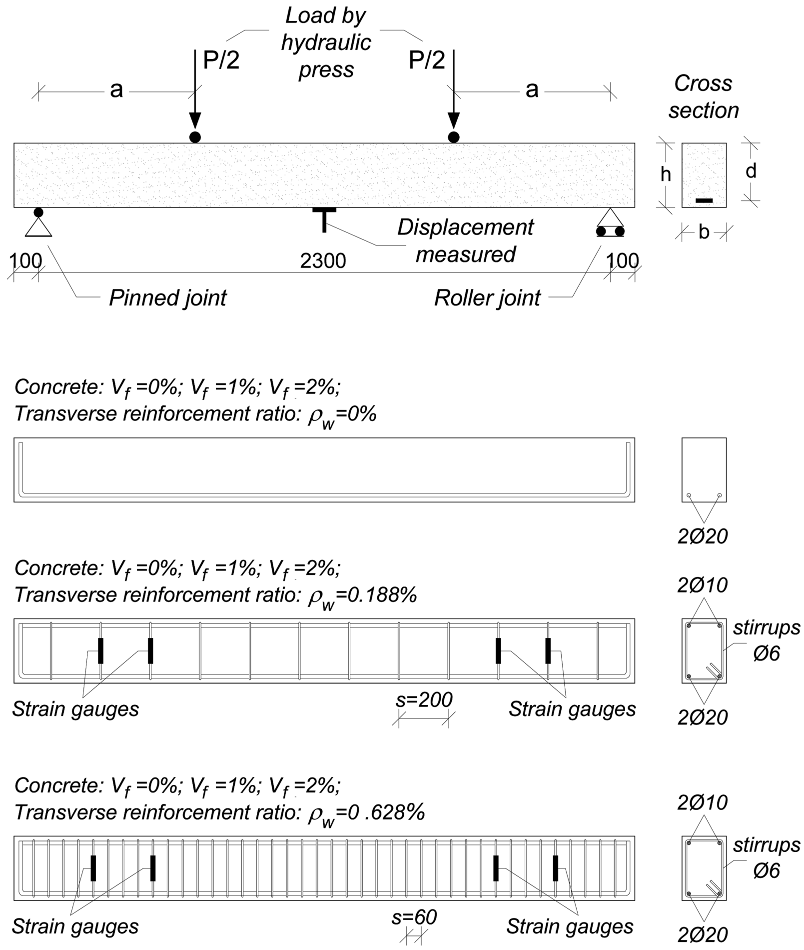


Fig. 2 Test set-up and construction details

to be also expressed in steel weight by volume of concrete. As already said, the fibre percentages 1% and 2% by volume of concrete correspond to 80 and 160 kg/m^3, respectively; the two different values of ρ_w considered (0.188% and 0.628%), taking into account the section geometry and the stirrup spacing, correspond to 23 and 77 kg/m^3.

It should be noted that, as analysed in previous studies (Campione *et al.* 2003, Cucchiara *et al.* 2004), the reason for different value of *a/d*, compatible with the limits of the testing setup arrangement, was to investigate, in the absence of specific shear reinforcement, failure modes in which the "arch effect" (*a/d* = 2.0) or "beam effect" (*a/d* = 2.8) is more significant. This different expected behaviour is also consistent with the recent results shown in Pérez *et al.* (2010), where a large set of experimental data of beams without transverse reinforcement is considered.

The experimental programme is summarized in Table 3. For each specimen the notation *A* and *B* is used with reference to the shear span to depth ratio for testing (*A* for *a/d* = 2.8 and *B* for *a/d* = 2.0), the first digit refers to the percentage of fibres and the last digit refers to the percentage of stirrups.

Table 3 Test beam details

Specimens	Shear span/depth ratio [a/d]	Volume of fibers V_f [%]	Stirrups	
			Pitchs [mm]	ρ_w [%]
A00	2.8	0	/	/
A01			200	0.188
A02			60	0.628
A10		1	/	/
A11			200	0.188
A12			60	0.628
A20		2	/	/
A21			200	0.188
A22			60	0.628
B00	2.0	0	/	/
B01			200	0.188
B02			60	0.628
B10		1	/	/
B11			200	0.188
B12			60	0.628
B20		2	/	/
B21			200	0.188
B22			60	0.628

3.2 Test setup

The flexural tests on the beams were carried out with a hydraulic press (Zwick/Roell & Toni Technik) with maximum load capacity of 4000 kN, which enables tests in displacement controlled mode.

The loading condition was a four point bending test (Fig. 2) and the load was applied to the beams as two equal concentrated loads by means of rigid steel beams. Transmission of concentrated loads (on top of the beams) and the restraints (pinned joint and roller joint under the beams) were effected through the assemblage of steel plates and steel cylinders.

Fig. 2 also shows the location of strain gauges which were attached to the stirrups, inside of the shear span, for monitoring the fibres contribution to the transverse reinforcement response.

All specimens were loaded up to failure with small increments of displacements (0.1 mm/min). During the test, the deflection of the mid span, the strains in the stirrups and the corresponding load were automatically recorded, at each step, by an acquisition system. The test type and the equipment were regulated by an electronic control unit, user interface via personal computer and using the software TestXpert v.7.10. Fig. 3 shows a beam during the test.

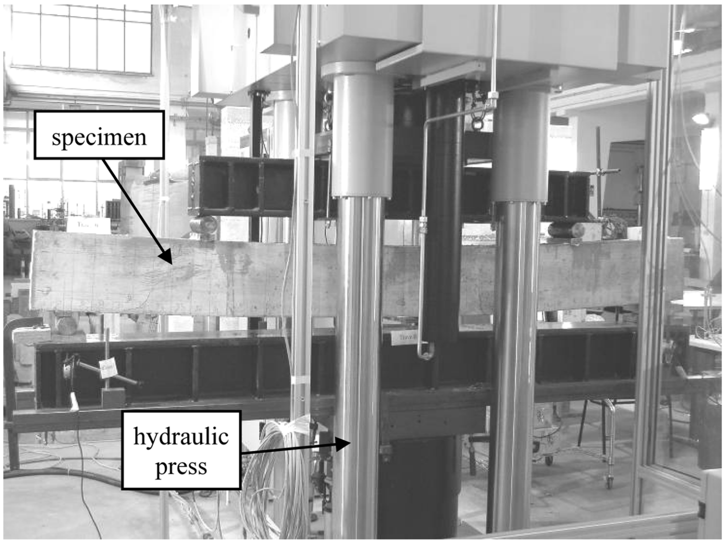


Fig. 3 Beam during the test and details of equipment

4. Reference to some available experimental results

In order to compare the results of the present investigation with other data concerning fibre reinforced concrete beams, this sections includes a summary of some available previous investigations. The details of the test programs considered are given in the papers of the authors below.

Ashour *et al.* (1992) present test results of 18 high-strength fibre reinforced concrete beams subjected to combined flexure and shear. All beams were singly reinforced and without shear reinforcement. The variables were shear-span/depth (a/d) ratio; percentage of longitudinal reinforcement ρ; and steel fibre content V_f. All the beams had a rectangular cross section of dimensions b = 125 mm and h = 250 mm.

The results of this experimental investigation lead to the following conclusions: the presence of steel fibres implies a more ductile collapse mechanism, and this effect is more evident when large values of the a/d ratio are adopted; adding steel fibres increases the beam stiffness, producing a reduced deflection for a given load level; shear strength of beams increases with an increase in the fibre content and a decrease in the a/d ratio.

Noghabai (2000) analyzes high-strength concrete (HSC) beams, tested in shear and bending. The experimental program involved 32 beams with four different depths (A, B, C, and D) and different material properties of the concrete mixtures (HSC^{I}, HSC^{II}, HSC^{III} and HSC^{IV}). Different kinds of fibre were added to the concrete matrix up to 1% by volume of concrete; therefore, for instance, specimen $HSC^{III}_{S60/0.7/0.5}$ was a beam with steel fibre of 60 mm length, 0.7 mm equivalent diameter and 40 kg/m^3 content (V_f = 0.5%). Plain concrete beams without stirrups were marked by REF.

The experimental results confirmed the beneficial effects given by the fibres in terms of stiffness, strength and post-peak behaviour. They also showed that the crack propagation depends on the beam geometry and the amount of reinforcement. Three different failure modes were detected: slender beams of very large depth exhibited loss of stability of the diagonal cracks with reduction in the bearing capacity; for smaller depth compressive failure under concentrated loads or in the web occurred in several cases; a further collapse mechanism was the failure in tension zone as a bond failure, which manifested in longitudinal splitting cracks. The optimal amount and efficiency of fibres are related to the aforementioned failure modes.

A further conclusion given in Noghabai (2000), related to the previous remarks, was that beams of different dimensions behave differently although they are made by using the same fibrous concrete.

Ding *et al.* (2011) consider the influence of steel fibres on the workability of self-consolidating concrete (SCC). The paper presents the experimental results obtained from a series of simply supported SCC rectangular beams, using steel fibre reinforcement with and without stirrups, and subjected to four-point symmetrically placed vertical loads.

The beams had a cross-section of 200×300 mm and a length of 2400 mm. The major test variables were the steel fibre content and the transverse reinforcement percentage. A specimen denoted as SFSCCB is a Steel Fibre Self-Consolidating Concrete Beam; fibre content and spacing of stirrups are indicated after this abbreviation; for instance, specimen SFSCCB25-150 is a beam with 25 kg/m^3 steel fibre content and 150 mm spacing of stirrups.

In relation to the aims of the present paper, the most important conclusions given in Ding *et al.* (2011) concern the amount of stirrups that can be replaced by a fibre content able to maintain a similar behaviour of a beam having the same dimensions and longitudinal reinforcement, and the fibre content able to transform a brittle shear failure mode into a ductile flexural failure mode. With reference to the first subject, Ding *et al.* (2011) show that for the tested beams 25 kg/m^3 steel fibres allow to enlarge the spacing of stirrups from 150 to 250 mm; in relation to the second subject, 50 kg/m^3 steel fibres are needed for a beam with a stirrup spacing of 150 mm.

5. Experimental results and discussion

5.1 Load-deflection characteristics

The typical behaviour of the tested beams is given in Fig. 4 by the curves applied load-measured displacement at the mid span. The curves show quite clearly an initial linear-elastic branch up the formation of first cracks, later following a nonlinear load-deflection pattern. The figure also shows an improvement in the ductility and energy absorption due to the use of fibres; for adequate coupling between fibres and stirrups, the change of failure mode from brittle shear mode into ductile flexural mode can be observed by the post-peak behaviour. However for $\rho_w = 0\%$ it is necessary to use fibre content of 2% to modify the shear failure mechanism in a flexural mechanism.

Furthermore, it can be seen that the beams without stirrups ($\rho_w = 0\%$) and the beams with a low reinforcement ratio ($\rho_w = 0.188\%$) exhibited greater improvement in the mechanical response due to the addition of steel fibres compared to beams with a higher reinforcement ratio ($\rho_w = 0.628\%$).

Avoiding to consider constructive solutions where the presence of steel fibres completely replaces the transverse reinforcement, useful indications are obtained by comparing the responses of test beams with $\rho_w = 0.188\%$ and $\rho_w = 0.628\%$. The curves in Fig. 4 show that beam B11 exhibits a similar behaviour as beams B21 and B12; the same similar behaviour can be observed for beam A11 with respect to beams A12 and A21. Therefore, one can conclude that the percentage of fibre

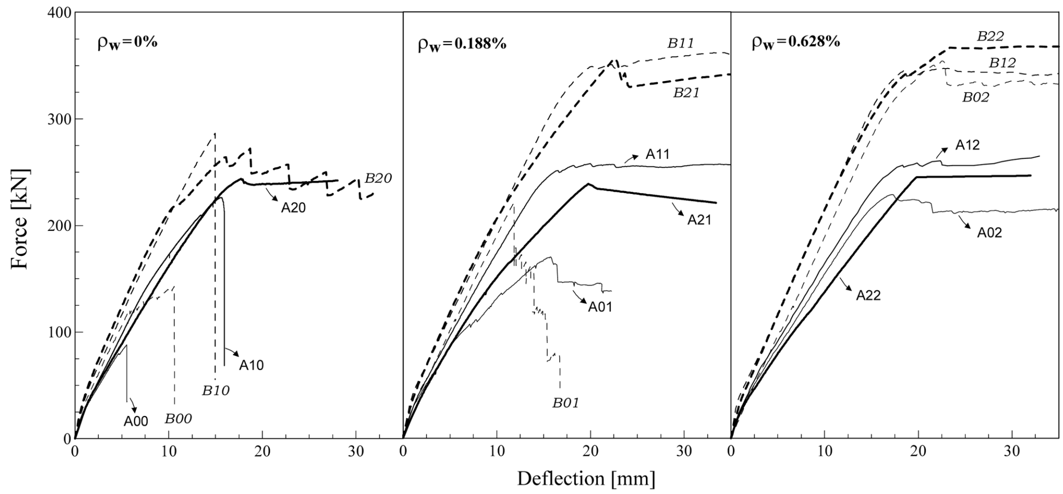


Fig. 4 Load-deflection curves for series *A* and *B* test beams

$V_f = 1\%$ leads to adequate performances also in the presence of a low ratio of transverse reinforcement.

In this connexion it can also be useful to observe that beams A11 and B11 imply a total steel weight of $80 + 23 = 103$ kg/m^3, while for beams A12, B12 and A21, B21 the steel weight is $80 + 77 = 157$ kg/m^3 and $160 + 23 = 183$ kg/m^3, respectively. This occurrence can contribute to choose the optimal solution.

5.2 Strength and stiffness

Fig. 5 summarizes the maximum strength of the curves shown in Fig. 4. In comparison with the plain concrete beams, the strength of the fibre reinforced concrete beams is approximately 90%-150% higher for specimens without stirrups (ρ_w = 0%) and 50%-60% higher for specimens with a low reinforcement ratio (ρ_w = 0.188%). The increase of load carrying capacity due to fibre addition is negligible for beams with a higher reinforcement ratio (ρ_w = 0.628%). It is seen that the use of a greater percentage of fibre (by $V_f = 1$ to $V_f = 2\%$) has no significant effects on the beam strength.

Fig. 6 shows the loss of strength in the softening branch of the load-deflection curves (Fig. 4) compared to the maximum value observed for all beam specimens for the series with ρ_w = 0.188%.

The shifted deflection in the abscissa indicates the difference between the actual mid-span deflection δ and the value δ_0 that was observed in correspondence of the maximum strength.

Note that the addition of fibres in the matrix of concrete greatly improves the post-peak behaviour: the residual strength for fibre reinforced high strength concrete beams is about 97% of the maximum value achieved, while this residual strength quickly decreases for beams without fibres. The figure also shows that a similar conclusion can be derived by showing in suitable form the experimental results of Ashour *et al.* (1992).

Fig. 7 shows the variation in stiffness of some beam specimens of the current study and that of other beam specimens found in literature for series with ρ_w = 0%. The stiffness values are calculated at each load step as the ratio of the load to the corresponding displacement measured at the mid-span of the beams.

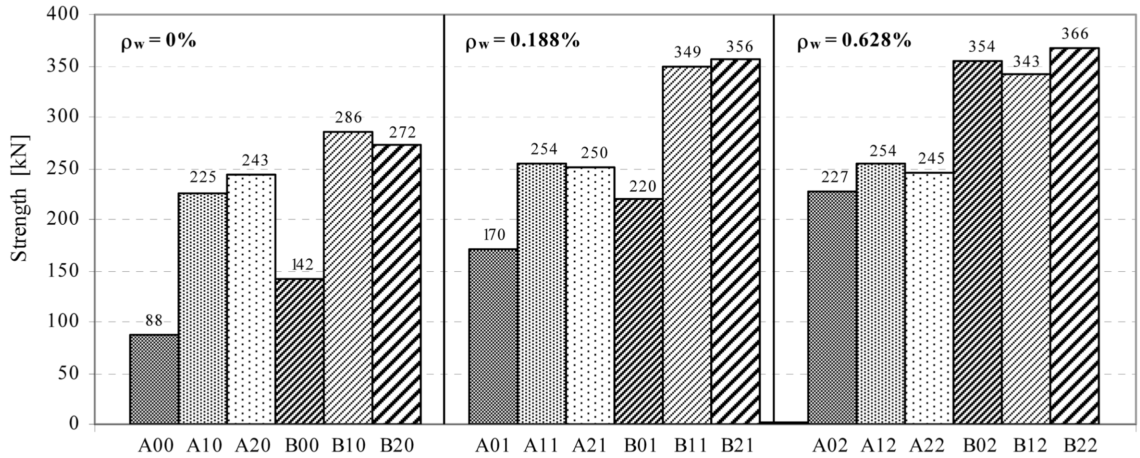


Fig. 5 Effect of steel fibres on beam strength

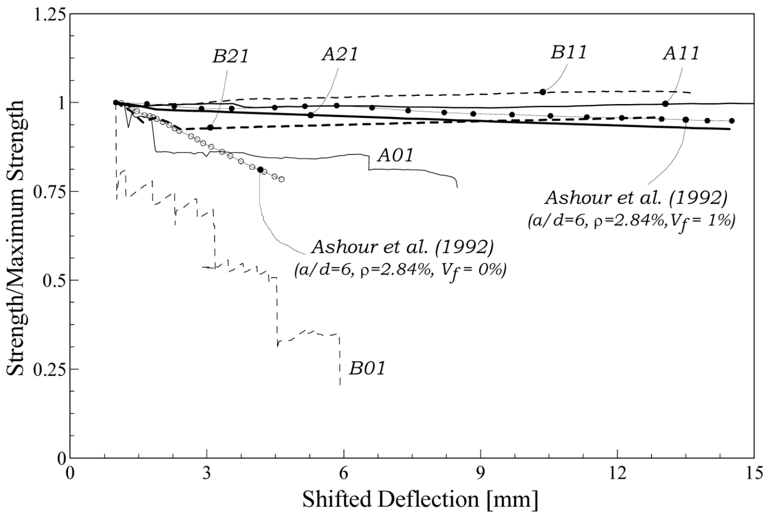


Fig. 6 Loss of strength in the softening branch relative to the maximum value

Both for series A beams and for series B beams the curves in Fig. 7 compare the responses of specimens with maximum fibre content to those of specimens without fibres. The curves relating to test beams A10 and B10, which are omitted to make clear the figure, reveal an intermediate behaviour with respect to those shown.

The figure shows that the initially constant stiffness decreases as the test progresses and the rate of decrease is greater in the initial branch. Also, it shows quite clearly that fibre reinforced concrete beams behaved in a stiffer manner over the entire load range than plain concrete beams. Similar comments apply for the series with $\rho_w \neq 0$.

Fig. 8 shows the loss of stiffness during the test with respect to the initial value for series with a low transverse reinforcement ratio. It can be observed that the normalized stiffness decay can be shown by a single curve. The percentage of fibres does not significantly influence the evolution of loss of stiffness.

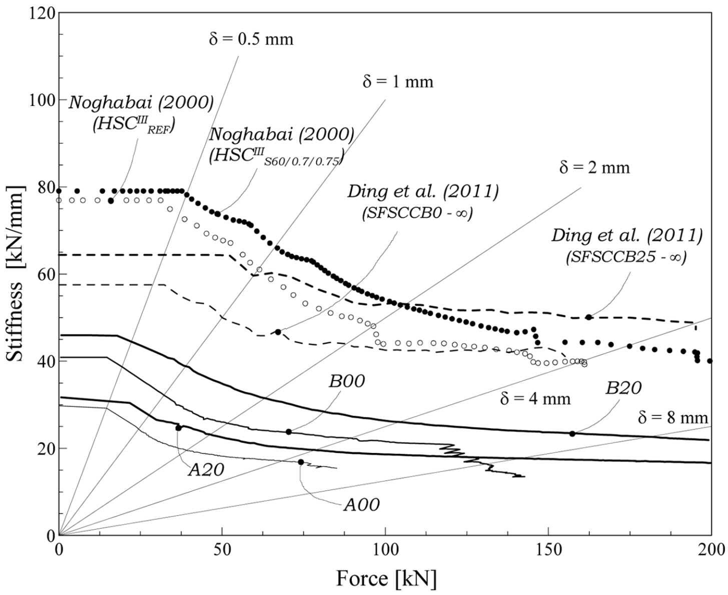


Fig. 7 Stiffness-Load curves and Stiffness-Displacement curves for series test beams with ρ_w = 0%

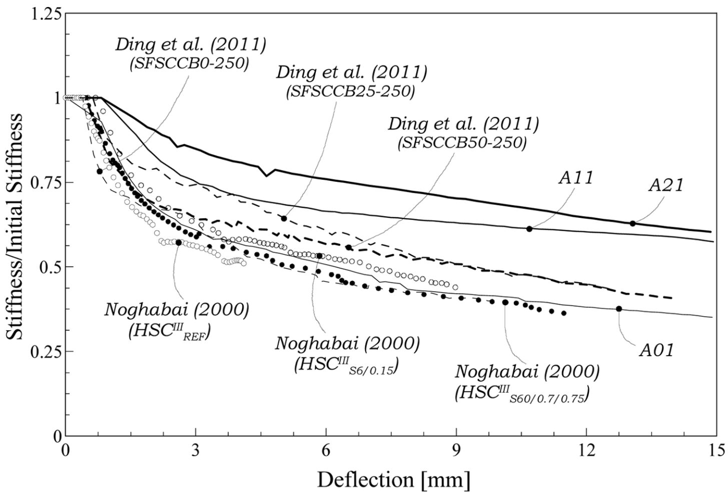


Fig. 8 Loss of stiffness relative to the initial value

5.3 Ductility and energy absorption

Fig. 4 shows that for specimens with ρ_w = 0% the amount of fibres has a significant effect on the ductility of the beam only for V_f = 2%, while for specimens with ρ_w = 0.188%, V_f = 1% is enough to obtain a ductile behaviour of beams. Also, it can be seen from this figure that for specimens with ρ_w = 0.628% the addition of fibres does not influence the ductility of high strength reinforced concrete beams.

The energy absorption, evaluated by calculating the area subtended by the load-deflection curves

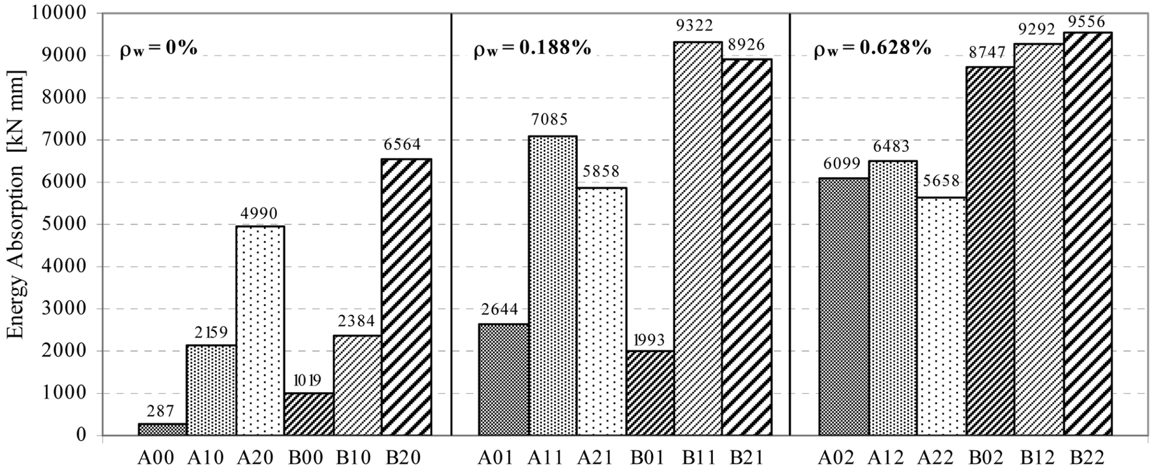


Fig. 9 Comparison of the energy absorption capacity of fibrous and plain concrete beams

(Fig. 4) under monotonic loading, is shown in Fig. 9. The comparison between the energy absorption capacities underlines again the conclusions that only the behaviour of the beams without stirrups or with a low transverse reinforcement ratio is greatly enhanced due to the addition of steel fibres. The fibres produce a considerable improvement in the energy absorption capacity for specimens with ρ_w = 0% and ρ_w = 0.188%, while they do not produce any appraisable effect for specimens with ρ_w = 0.628%.

5.4 Behaviour of steel fibre as transverse reinforcement

Following the discussion on the influence of the fibres addition, Fig. 10 shows the load-steel strain curves for beams fitted with strain gauges at the legs of the stirrups. The figure shows the curves relative to the strain gauges placed on the front right of the beam (see Fig. 2).

For beams without fibres, the presence of a sub-horizontal branch in the curves is observed for a level of load corresponding to the cracking of the concrete. At this stage the stirrups begin to offer a contribution to strength, improving the interlock aggregate and the dowel effect. When the width of the crack is significant, a sudden transmission of load to the stirrups progresses up to reach yielding stress and failure.

For beams with fibres, the more widespread cracks, narrow in width, prevent sudden transmission of load to the stirrups. In this case the curves in Fig. 10 are more regular and are characterized by the absence of a pronounced sub-horizontal branch.

Moreover, it can be seen from this figure that the strain in the transverse reinforcement, for the same load, is reduced as the amount of the fibres in the beam increases, demonstrating once again the fibre contribution to shear reinforcement.

5.5 Failure mode and crack patterns

The beams failed in two different modes, depending on the amounts of fibre and the transverse reinforcement. Types A00, A10, A01 and B00, B10, B01 failed with a large diagonal shear crack,

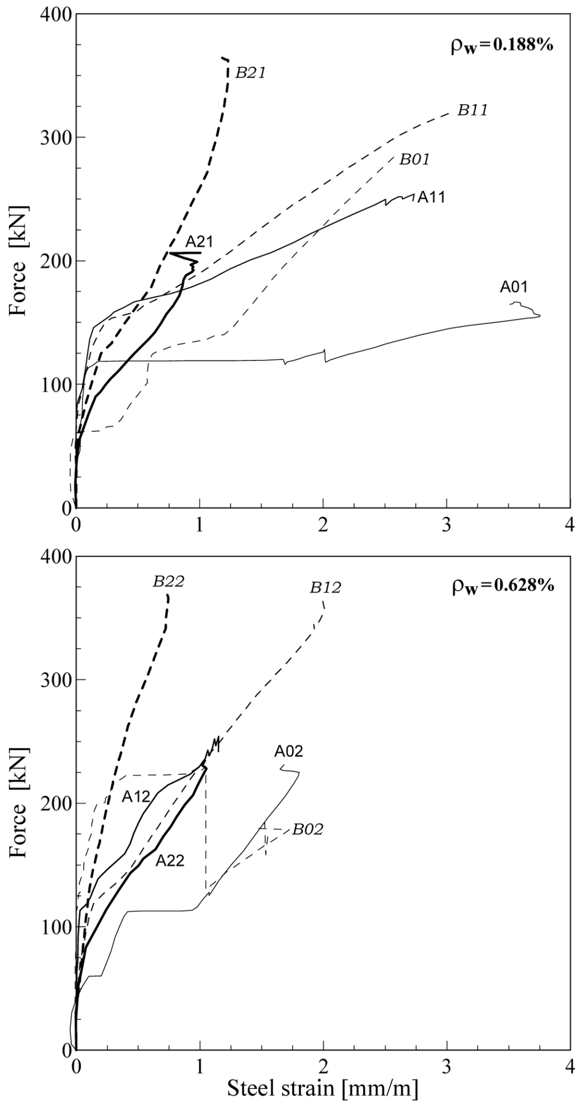


Fig. 10 Load-Steel strain curves for test beams with transverse reinforcement

activated for a low level of load and concentrated in the shear span, going from the loading point up to the support. The collapse occurred after the cover separation and the steel yielding (Fig. 11(a)).

In the remaining types ductile flexural failure was evident. The cracks in the initial phase of loading developed in the central region of the beam (for beams without steel fibres sub-vertical cracks in the central region appeared between the spacing of the stirrups) and after opened in the shear span, while in the compressed region of the beam, some sub-horizontal cracks appeared that quickly propagated leading to a concrete crushing (Fig. 11(b)).

It has been observed that in the beams with stirrups but without steel fibres (A01, A02 and B01, B02), some concrete spalling happened at the ultimate condition, and for specimen A01 rupture of

(a)

(b)

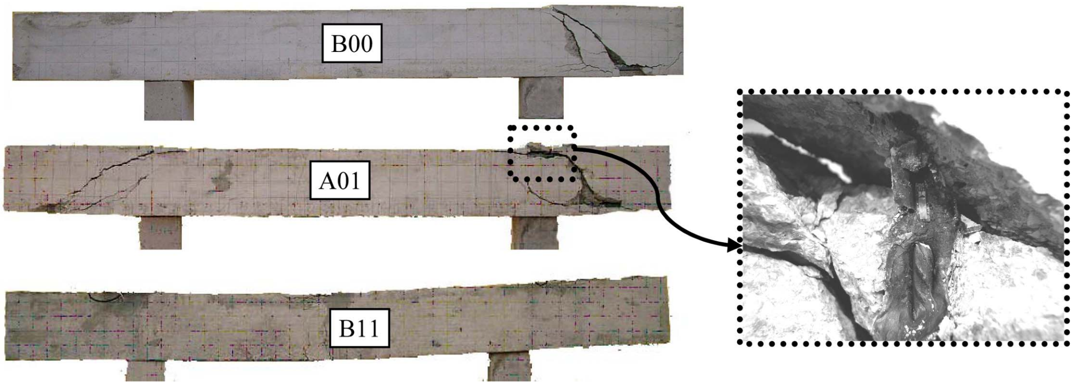
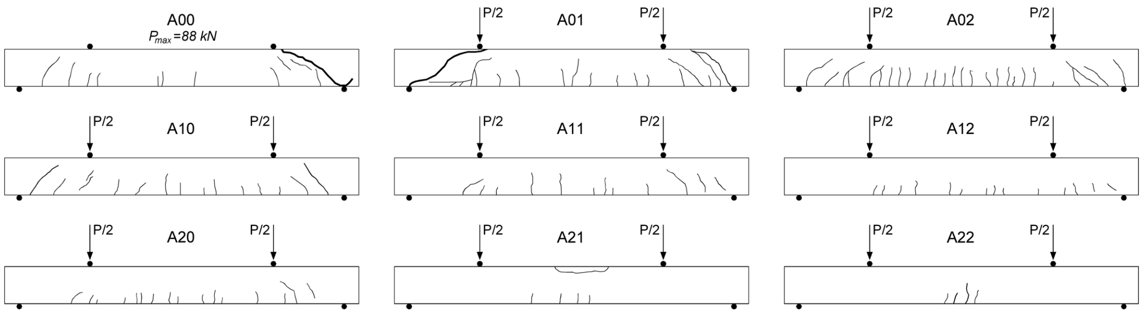


Fig. 11 Typical failure condition: (a) shear failure, (b) flexural failure



(a) (b) (c)

Fig. 12 Cracks patterns at $P = 150$ kN for series A test beams: (a) ρ_w = 0%, (b) ρ_w = 0.188%, (c) ρ_w = 0.628%

the stirrup crossing the cracked section occurred (see Fig. 11(a)). The inclusion of steel fibres eliminated these phenomena: steel fibres improved the capacity of the matrix during the post cracking stage, thus preventing spalling at failure and preventing rupture of the stirrup (beam A01 and A11). Furthermore, since for series *B* test beams the failure was subject to the arch effect, the use of stirrups proved to be less effective than was the case with series *A* test beams.

As further evidence of how the addition of fibre influences the evolution of cracks in high strength reinforced concrete beams, Fig. 12 shows the crack patterns of series *A* test beams at the same applied load $P = 150$ kN. For beam A00, which collapsed for a level of load less than 150 kN, the figure shows the crack pattern at the ultimate state and the value of the maximum load reached.

Fig. 12 indicates that the fibre reinforced concrete has a remarkable resistance to tensile cracking. When the spacing of stirrups is small (ρ_w = 0.628%), the beneficial effect of the fibres only consists of a more controlled crack opening at each load stage. The same comments apply for cracking patterns of series *B* test beams at load $P = 190$ kN.

Similar crack patterns have been observed in other investigations (Ashour *et al.* 1992, Lim and Oh 1999, Parra-Montesinos 2006).

6. Comparison between experimental results and theoretical predictions

In this section a comparison is made between the values of flexural and shear strength obtained by the present experimental investigation and the values deduced from formulas proposed in the literature.

6.1 Beams with dominant flexural failure mode

6.1.1 ACI code 318-02 (2008)

The expression used to calculate the ultimate moment, for beams without steel fibres, is that provided by ACI Building Code Recommendations (2008)

$$M_{f\ell} = bd^2 \rho_\ell f_{y\ell}\left(1 - \frac{\rho_\ell f_{y\ell}}{1.7f_c'}\right) \tag{1}$$

6.1.2 Oh (1992)

To calculate the flexural capacity of the beams with fibrous concrete, the formulation proposed by Oh (1992) was used. The flexural capacity is calculated by assuming the Henager and Doherty strain and stress distributions at cross section of fibre reinforced concrete beam, (quoted in Oh 1992)

$$M_{f\ell} = f_{y\ell}A_\ell(d - 0.4c) + \sigma_{ct}(h - c)(0.5h + 0.1c)b \tag{2}$$

where c is the depth neutral axis and σ_{ct} is the flexural stress of high strength fibre reinforced concrete.

The neutral axis depth is determined from the equilibrium condition of the cross section subjected to a “stress block” distribution, with depth equal to 0.8 c

$$c = \frac{\sigma_{ct}bh + f_{y\ell}A_\ell}{0.85f_c'0.8b + \sigma_{ct}b} \tag{3}$$

The flexural strength σ_{ct} is calculated as the sum of matrix strength σ_{mt} and fibre strength σ_f. Neglecting the strength contribution of the concrete matrix at the ultimate state, it is assumed that the flexural stress corresponds to the fibre strength

$$\sigma_{ct} = \alpha_0\alpha_1\alpha_b\sigma_f\rho_f \tag{4}$$

in which: σ_f is the strength of fibre; ρ_f is the fibre volume ratio; $\alpha_0, \alpha_1, \alpha_b$ are orientation, length efficiency, bond efficiency factor of fibres, respectively.

The strength of fibre σ_f is derived from the bonding characteristic of fibres

$$\sigma_f = 2\tau(l_f/d_f) \tag{5}$$

where τ is the bond strength of fibre, which in this study is expressed according to Khuntia *et al.* (1999)

$$\tau = 0.66\sqrt{f_c'} \text{ (in MPa)} \tag{6}$$

The orientation factor α_0 is about 0.41 for uniformly distributed fibre reinforced concrete and the bond-efficiency factor α_b is about 1 for hooked fibres (Imam *et al.* 1995). The length-efficiency factor α_1 is

$$\alpha_1 = 1 - \tanh(0.5\beta l_f)/0.5\beta l_f \tag{7}$$

$$\beta = \sqrt{(2\pi G_m)/(E_f A_f \ln(q/r_f))} \tag{8}$$

$$q = 25\sqrt{l_f/(\rho_f/d_f)} \tag{9}$$

in which G_m is the shear modulus of concrete matrix; E_f, A_f, r_f are elastic modulus, cross sectional area, and radius of fibre, respectively.

Table 4 shows the comparison of moment deduced from experimental data with that predicted by using Eq. (1) and Eq. (2) for specimens that have exhibited a flexural failure mechanism. The table also includes some results given by the studies commented in Section 4 and in this Section, in the

Table 4 Comparison between analytical and experimental flexural strength

Reference	Specimen	Moment			Observed moment / Predicted moment	
		Predicted		Observed [kN m]	Observed / Eq. (1)	Observed / Eq. (2)
		Eq. (1) [kN m]	Eq. (2) [kN m]			
Test result	A11	60.88	69.94	77.93	1.28	1.11
	A12			77.97	1.28	1.11
	B11			76.37	1.25	1.09
	B12			75.01	1.23	1.07
	A21	61.15	77.66	76.72	1.25	0.99
	A22			75.12	1.23	0.97
	B21			77.96	1.27	1.00
	B22			80.24	1.31	1.03
Ashour *et al.* (1992)	a/d = 6, ρ = 2.84%, V_f = 1%	69.46	79.89	67.95	0.98	0.85
Imam (1995)	Group II B4	177.55	205.56	207.40	1.17	1.01
	Group II B11	177.69	205.99	204.12	1.15	0.99
Noghabai (2000)	$HSC^{II}_{S6/0.15}$	204.39	213.59	235.95	1.15	1.10
	$HSC^{II}_{S30/0.6}$	196.98	206.48	201.50	1.02	0.98
	$HSC^{III}_{S6/0.15}$	523.75	544.69	403.20	0.77	0.74
	$HSC^{III}_{S60/0.7/0.75}$	500.54	529.30	406.80	0.81	0.77
Ding *et al.* (2011)	SFSCCB50-150	152.13	159.36	181.67	1.19	1.14
				Average value	1.15	1.00
				Standard deviation	0.16	0.11

cases in which the data given by the authors make it possible to use Eqs. (1) and (2) and the failure mode is clearly indicated.

As expected, the comparison indicates that the conventional flexural theory, neglecting the fibre contribution, generally underestimates (up to 20%) the flexural capacity of fibre reinforced high strength concrete beams, while the fibre-reinforced concrete theory proposed by Oh (1992) leads to good agreement. The average value and standard deviation of the ratios "Observed value/Predicted value" confirm the importance of using an adequate numerical model, able to include the main phenomena involved in the flexural behaviour of these elements.

6.2 Beams with dominant shear failure mode

6.2.1 Lim and Oh (1999)

The analytical model suggested by Lim and Oh (1999) calculates the shear strength of fibre reinforced concrete as

$$v_u = v_{uc} + v_{uf} \tag{10}$$

where v_{uc} is the shear strength of matrix and v_{uf} is the increase of shear strength due to the presence of steel fibres.

The shear strength of concrete in MPa is given by

$$v_{uc} = \left(10\rho_\ell f_c'\frac{d}{a}\right)^{1/3} + \frac{A_w f_{yw}}{sb} \quad \text{for} \quad \frac{a}{d} \geq 2.5 \tag{11a}$$

$$v_{uc} = (160\rho_\ell f_c')^{1/3}\left(\frac{d}{a}\right)^{4/3} + \frac{A_w f_{yw}}{sb} \quad \text{for} \quad \frac{a}{d} \leq 2.5 \tag{11b}$$

in which f_c' is expressed in MPa.

The shear resistance due to the presence of steel fibre is

$$v_{uf} = 0.5\,\tau\rho_f\,(l_f/d_f) \tag{12}$$

with τ calculated by Eq. (6).

Note that the total shear resistance V_{uf} due to the presence of steel fibres is calculated considering the vertical component of the total force F_{uf} perpendicular to the inclined shear area $b(h-c)/\sin\alpha$

$$V_{uf} = F_{uf}\sin\alpha = v_{uf}b(h-c) \tag{13}$$

In the previous expression the value of c is provided by Eq. (3).

6.2.2 Imam et al. (1995)

These authors evaluate the ultimate shear v_u of high strength concrete beams containing steel fibres as

$$v_u = 0.6\,\psi\sqrt[3]{\omega}\left(f_c'^{0.44} + 275\sqrt{\frac{\omega}{(a/d)^5}}\right) \tag{14}$$

where $\psi = (1+\sqrt{5.08/d_a})/\sqrt{1+d/(25d_a)}$ is the size effect factor, with d_a maximum aggregate size in mm; $\omega = \rho_\ell(1+4F)$ is the reinforcement factor, with $F = (l_f/d_f)\rho_f\alpha_b$ fibre factor in which α_b is the bond efficiency factor previously defined.

6.2.3 Cucchiara and Priolo (2008)

These authors modify the model proposed by Russo *et al.* (2004) to include the contribution of steel fibres to the shear resistance through Eq. (12). Therefore, the ultimate shear stress v_u in MPa is expressed by

$$v_u = \xi[0.97\rho_\ell^{0.46} f_c'^{1/2} + 0.2\rho_\ell^{0.91} f_c'^{0.38} f_{y\ell}^{0.96}(a/d)^{-2.33} + 0.41\,\tau\rho_f\,(l_f/d_f)] + 1.75 I_b \rho_w f_{yw} \quad (15)$$

where $\xi = 1/\sqrt{1 + d/(25d_a)}$ is the aggregate size effect, while I_b is the beam action index, modified here to take the fibre contribution into account, and given by

$$I_b = \frac{0.97\rho_\ell^{0.46} f_c'^{1/2}}{0.97\rho_\ell^{0.46} f_c'^{1/2} + 0.2\rho_\ell^{0.91} f_c'^{0.38} f_{y\ell}'^{0.96}(a/d)^{-2.33} + 0.41\,\tau\rho_f\,(l_f/d_f)} \quad (16)$$

The comparison between experimental shear strength values and corresponding predicted values by using Eqs. (10), (14) and (15) is summarized in Table 5, for specimens that have exhibited a shear failure mechanism. Results given by other authors are also considered.

Table 5 shows that the model proposed by Lim and Oh (1999) proves to be able to give a sufficient approximation level. But it is evident that the aforementioned equations lead to rather different values for a same specimen.

A similar conclusion can be deduced considering also the results shown in Yakoub (2011), where

Table 5 Comparison between analytical and experimental shear strength

Reference	Specimen	Shear				Observed shear / Predicted shear		
		Predicted			Observed [kN]	Observed / Eq. (10)	Observed / Eq. (14)	Observed / Eq. (15)
		Eq. (10) [kN]	Eq. (14) [kN]	Eq. (15) [kN]				
Test result	A00	53.82	63.36	55.62	43.98	0.82	0.69	0.79
	A01	88.44	63.36	75.98	85.16	0.96	1.34	1.12
	A10	100.45	116.81	95.64	112.74	1.12	0.97	1.18
	B00	75.86	88.56	82.78	71.14	0.94	0.80	0.86
	B01	110.47	88.56	103.14	110.23	1.00	1.24	1.07
	B10	122.90	182.95	123.38	142.99	1.16	0.78	1.16
Imam (1995)	Group IV B7	235.25	226.83	195.72	208.80	0.89	0.92	1.07
	Group IV B12	224.17	192.24	174.30	211.86	0.95	1.10	1.22
Noghabai (2000)	$HSC^{I}_{S6/0.15}$	361.14	164.23	152.31	299.00	0.83	1.82	1.96
	$HSC^{I}_{S60/0.7/0.75}$	365.89	189.36	161.95	262.00	0.72	1.38	1.62
Ding *et al.* (2011)	SFSCCB25-250	131.26	134.68	155.11	180.91	1.38	1.34	1.17
	SFSCCB25-150	146.91	134.68	132.83	187.63	1.28	1.39	1.41
				Average value		1.00	1.15	1.22
				Standard deviation		0.19	0.32	0.31

other prediction models are compared, and a further equation to predict the steel fibre contribution is proposed for reinforced concrete beams without stirrups.

The difficulty in defining an expression of general validity to predict the shear resistance of fibre-reinforced concrete beams lies in the fact that there are many parameters that influence the shear strength mechanism, and the separate investigation of each one of them is extremely difficult and costly. In this respect, the different analytical models and empirical expressions available in literature are affected by experimental calibration greatly influenced by the single experimental studies carried out.

7. Conclusions

This study investigated the effect of steel fibres on the mechanical behaviour of high strength reinforced concrete beams. An experimental programme was set up and eighteen beams were tested under combined action of bending and shear. The major test variables were the content of steel fibres and the volume of shear stirrups for two different value of the shear span to depth ratio.

The following considerations may be inferred from the study.

- The residual strength and the tensile splitting strength especially augment with an increase of fibre content; the tensile strength of concrete is greatly improved by steel fibres and the propagation of cracking is more controlled.
- For beam without stirrups the strength, stiffness, ductility and energy absorption capacity are considerably improved with the addition of steel fibres. A fibre content of 1% is sufficient to obtain these improvements in the beam with a low transverse reinforcement ratio. The amount of fibres does not produce significant effects in the beam with a higher transverse reinforcement ratio.
- The addition of steel fibres considerably affects the crack patterns; the fibre reinforced concrete beams have a considerable capacity to tensile cracking and show a significant controlled crack opening at each load stage.
- The amount of transverse reinforcement usually required could be reduced by addition of steel fibres, and the strength and ductility required could be obtained by an optimum combination of steel fibres and shear stirrups. This study seems to confirm that an optimum combination is that obtained in the case of a beam with about 50% of conventional stirrups together with 1% of fibre volume content. For the series of test beams examined this solution leads to optimal performances (strength and ductility) with an acceptable total weight of steel to be utilized (steel fibres and transverse reinforcement); this conclusion can contribute to environmental optimization studies.
- Some analytical methods found in the literature were used to predict the shear and the flexural capacity of fibre reinforced high strength concrete beams. On the one hand they can lead to serious errors in calculations of the flexural strength if the fibre contribution is neglected, on the other they give not univocal values of the shear strength when the beneficial effects of the fibres are considered. This is because each model is affected by calibration heavily influenced by the experimental results utilized for its validation, since the parameters governing the actual behaviour of fibre reinforced concrete beams are very numerous and not always liable to a systematic evaluation.

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